Chapter 4 System Modelling

Before modelling, the assumptions of the integrated logistics model (ILM) will be listed in section 4.1. Then, system functions will be enumerated and defined in section 4.2. Finally, objective functions and constraints of the ILM will be described, respectively, in section 4.3 and section 4.4.

4.1 Assumptions

There are some assumptions in the ILM listed as follows.

1. The quantity of demanded products will be given. In other words, the amounts of demanded products, supply nodes (SN), and end-customers are known.
2. The quantity of useless products will be given by the return ratio（γ）. In other words, the amounts of useless products, sold by end-customers and procured by return nodes (RN), are determined by the return ratio.
3. The capacities are known constant.
4. The cost parameters are known.
5. Purchase lead-times ($t^{i}_{products\text{-}members}$) in BLS and transport lead-times in RLS are fixed and known.
6. Up-stream member bears transportation cost (expect end-customers).
7. Shortages are not allowed.
8. Only one item of products could be considered in the proposed model. In

other words, the proposed model could not deal with multi-items simultaneously.

9. The flows of scrap products will not be considered in the proposed model.

10. There is no holding cost in end-customers in the proposed model.

11. The cash flows of RMF will not be considered in the proposed model.

4.2 System Functions and Derivations

All of system functions used in this study are showed in Table 4.2-1, which enumerates the physical definition of system functions as follows.

Table 4.2-1 Definitions of System Functions

System Functions	Definitions
$AMC_{bl}(M,Q)$	Aggregate manufacture cost in the business logistics system
$AAVC_{bl}(AV,Q)$	Aggregate added value cost in the business logistics system
$APC_{bl}(P,Q)$	Aggregate procurement cost in the business logistics system
$AIC_{bl}(I,Q)$	Aggregate inventory cost in the business logistics system
$ATC_{bl}(T,Q)$	Aggregate transportation cost in the business logistics system
$ARFC_{bl}(RF,Q)$	Aggregate recycle fee cost in the business logistics system
$AR_{bl}(R,Q)$	Aggregate revenue in the business logistics system
$APC_{ec}(R,Q)$	Aggregate procurement cost in the end-customers
$AR_{ec}(R,Q)$	Aggregate revenue in the end-customers
$AMC_{rl}(M,Q)$	Aggregate manufacture cost in the reverse logistics system

$APC_{rl}(R,Q)$	Aggregate procurement cost in the reverse logistics system
$AIC_{rl}(I,Q)$	Aggregate inventory cost in the reverse logistics system
$ATC_{rl}(T,Q)$	Aggregate transportation cost in the reverse logistics system
$ATRC_{rl}(TR,Q)$	Aggregate treatment cost in the reverse logistics system
$AR_{rl}(R,Q)$	Aggregate revenue in the reverse logistics system
$AS_{rl}(S,Q)$	Aggregate subsidy in the reverse logistics system
$AC_{bl}(M,AV,P,I,T,RF,Q)$	Aggregate cost in the business logistics system
$APR_{bl}(R,Q)$	Aggregate profit in the business logistics system
$AC_{ec}(R,Q)$	Aggregate cost in the end-customers
$APR_{ec}(R,Q)$	Aggregate profit in the end-customers
$AC_{rl}(M,I,T,TR,Q)$	Aggregate cost in the reverse logistics system
$APR_{rl}(R,S,Q)$	Aggregate profit in the reverse logistics system
$AMC_{bl-rl}(M,Q)$	Aggregate manufacture cost in the integration of the business logistics system and the reverse logistics system
$AAVC_{bl-rl}(AV,Q)$	Aggregate added value cost in the integration of the business logistics system and the reverse logistics system
$APC_{bl-rl}(P,Q)$	Aggregate procurement cost in the integration of the business logistics system and the reverse logistics system
$AIC_{bl-rl}(I,Q)$	Aggregate inventory cost in the integration of the business logistics system and the reverse logistics system
$ATC_{bl-rl}(T,Q)$	Aggregate transportation cost in the integration of the business logistics

	system and the reverse logistics system
$ATRC_{bl-rl}(TR,Q)$	Aggregate treatment cost in the integration of the business logistics system and the reverse logistics system
$ARFC_{bl-rl}(RF,Q)$	Aggregate recycle fee cost in the integration of the business logistics system and the reverse logistics system
$AR_{bl-rl}(R,Q)$	Aggregate revenue in the integration of the business logistics system and the reverse logistics system
$AS_{bl-rl}(S,Q)$	Aggregate subsidy in the integration of the business logistics system and the reverse logistics system
$NPR_{bl}(M,AV,P,I,T,RF,R,Q)$	Net profit in the business logistics system
$NPR_{rl}(M,I,T,TR,R,S,Q)$	Net profit in the reverse logistics system
$AC_{bl-rl}(M,AV,P,I,T,TR,RF,Q)$	Aggregate cost in the integration of the business logistics system and the reverse logistics system
$APR_{bl-rl}(R,S,Q)$	Aggregate profit in the integration of the business logistics system and the reverse logistics system
$NPR_{bl-rl}(M,AV,P,I,T,TR,RF,R,S,Q)$	Net profit in the integration of the business logistics system and the reverse logistics system

The detailed content of system functions used in this study is showed as follows.

1. Aggregate manufacture cost in the BLS ＝

   The unit cost of manufacturing raw materials ╳ The quantity of raw materials ＋ The unit cost of manufacturing demanded products ╳ The quantity of demanded products

$$AMC_{bl}(M,Q)=\sum_{\forall k}\left[\begin{array}{l}\sum_{\forall j_4} M_{rm}^{j_4}(k)\times Q_{rm-M}^{j_4}(k)+\\ \sum_{\forall j_3} M_{p}^{j_3}(k)\times Q_{p-M}^{j_3}(k)\end{array}\right] \tag{4.2-1}$$

2. Aggregate added value cost in the BLS ＝

   The unit cost of adding value in j[th] manufacturer ╳ The quantity of demanded products ＋ The unit cost of adding value in j[th] wholesaler ╳ The quantity of demanded products ＋ The unit cost of adding value in j[th] retailer ╳ The quantity of demanded products

$$AAVC_{bl}\left(AV,Q\right)=\sum_{\forall k}\left[\begin{array}{l}\sum_{\forall j_3} AV_p^{j_3}(k)\times Q_{p-AV}^{j_3}(k)+\\ \sum_{\forall j_2} AV_p^{j_2}(k)\times Q_{p-AV}^{j_2}(k)+\\ \sum_{\forall j_1} AV_p^{j_1}(k)\times Q_{p-AV}^{j_1}(k)\end{array}\right] \quad (4.2\text{-}2)$$

3. Aggregate procurement cost in the BLS ＝

   The unit cost of procuring virgin materials ╳ The quantity of virgin materials ＋ (The unit cost of procuring raw materials ╳ The quantity of raw materials ＋ The unit cost of procuring reused raw materials ╳ The quantity of reused raw materials) ＋ The unit cost of procuring demanded products in j[th] wholesaler ╳ The quantity of demanded products ＋ The unit cost of procuring demanded products in j[th] retailer ╳ The quantity of demanded products

$$APC_{bl}(P,Q)=\sum_{\forall k}\left\{\begin{array}{l}\sum_{\forall j_4}P_{vm}^{j_4}(k)\times Q_{vm-P}^{j_4}(k)+\\ \sum_{\forall j_3}\left[\begin{array}{l}P_{rm}^{j_3}(k)\times Q_{rm-P}^{j_3}(k)+\\ P_{rrm}^{j_3}(k)\times Q_{rrm-P}^{j_3}(k)\end{array}\right]+\\ \sum_{\forall j_2}P_{p}^{j_2}(k)\times Q_{p-P}^{j_2}(k)+\\ \sum_{\forall j_1}P_{p}^{j_1}(k)\times Q_{p-P}^{j_1}(k)\end{array}\right\}$$

$$=\sum_{\forall k}\left[\begin{array}{l}\sum_{\forall j_4}P_{vm}^{j_4}(k)\times Q_{vm-P}^{j_4}(k)+\\ \left[\begin{array}{l}\sum_{\forall j_4}R_{rm-m}^{j_4}(k)\times Q_{rm-m}^{j_4}(k)+\\ \sum_{\forall j_{-4}}R_{rrm-m}^{j_{-4}}(k)\times Q_{rrm-m}^{j_{-4}}(k)\end{array}\right]+\\ \sum_{\forall j_3}R_{p-ws}^{j_3}(k)\times Q_{p-ws}^{j_3}(k)+\\ \left[\begin{array}{l}\sum_{\forall j_2}R_{p-ws}^{j_2}(k)\times Q_{p-ws}^{j_2}(k)+\\ \sum_{\forall j_2}R_{p-r}^{j_2}(k)\times Q_{p-r}^{j_2}(k)\end{array}\right]\end{array}\right] \qquad (4.2\text{-}3)$$

4. Aggregate inventory cost in the BLS ＝

   (The unit cost of stocking virgin materials in j[th] raw material supplier ╳ The quantity of virgin materials ＋ The unit cost of stocking raw materials in j[th] raw material supplier ╳ The quantity of raw materials) ＋ (The unit cost of stocking raw materials in j[th] manufacturer ╳ The quantity of raw materials ＋ The unit cost of stocking demanded products in j[th] manufacturer ╳ The quantity of demanded products) ＋ The unit cost of stocking demanded products in j[th] wholesaler ╳ The quantity of demanded products ＋ The unit cost of stocking demanded products in j[th] retailer ╳ The quantity of demanded products

$$AIC_{bl}(I,Q)=\sum_{\forall k}\left\{\begin{array}{l}\sum\limits_{\forall j_4}\left[\begin{array}{l}I_{vm}^{j_4}(k)\times Q_{vm-I}^{j_4}(k)+\\ I_{rm}^{j_4}(k)\times Q_{rm-I}^{j_4}(k)\end{array}\right]+\\ \sum\limits_{\forall j_3}\left[\begin{array}{l}I_{rm}^{j_3}(k)\times Q_{rm-I}^{j_3}(k)+\\ I_{p}^{j_3}(k)\times Q_{p-I}^{j_3}(k)\end{array}\right]+\\ \sum\limits_{\forall j_2} I_{p}^{j_2}(k)\times Q_{p-I}^{j_2}(k)+\\ \sum\limits_{\forall j_1} I_{p}^{j_1}(k)\times Q_{p-I}^{j_1}(k)\end{array}\right. \tag{4.2-4}$$

5. Aggregate transportation cost in the BLS ＝

   The unit cost of transporting raw materials to manufacturers ╳ The quantity of raw materials ＋ (The unit cost of transporting demanded products to wholesalers ╳ The quantity of demanded products ＋ The unit cost of transporting demanded products to retailers in j^{th} manufacturer ╳ The quantity of demanded products ＋ The unit cost of transporting demanded products to end-customers in j^{th} manufacturer ╳ The quantity of demanded products) ＋ (The unit cost of transporting demanded products to retailers j^{th} wholesaler ╳ The quantity of demanded products ＋ The unit cost of transporting demanded products to end-customers in j^{th} wholesaler ╳ The quantity of demanded products) ＋ The unit cost of transporting demanded products to end-customers in j^{th} retailer ╳ The quantity of demanded products

$$ATC_{bl}(T,Q)=\sum_{\forall k}\left\{\begin{array}{l}\sum\limits_{\forall j_4} T_{rm-m}^{j_4}(k)\times Q_{rm-m}^{j_4}(k)+\\ \sum\limits_{\forall j_3}\left[\begin{array}{l}T_{p-ws}^{j_3}(k)\times Q_{p-ws}^{j_3}(k)+\\ T_{p-r}^{j_3}(k)\times Q_{p-r}^{j_3}(k)+\\ T_{p-ec}^{j_3}(k)\times Q_{p-ec}^{j_3}(k)\end{array}\right]+\\ \sum\limits_{\forall j_2}\left[\begin{array}{l}T_{p-r}^{j_2}(k)\times Q_{p-r}^{j_2}(k)+\\ T_{p-ec}^{j_2}(k)\times Q_{p-ec}^{j_2}(k)\end{array}\right]+\\ \sum\limits_{\forall j_1} T_{p-ec}^{j_1}(k)\times Q_{p-ec}^{j_1}(k)\end{array}\right. \tag{4.2-5}$$

6. Aggregate recycle fee cost in the BLS ＝

   The unit recycle fee ╳ The quantity of demanded products to be manufactured

$$ARFC_{bl}\left(RF,Q\right)=\sum_{\forall k}RF^{j_3}(k)\times Q_{p-M}^{j_3}(k) \tag{4.2-6}$$

7. Aggregate revenue in the BLS ＝

   The unit revenue of selling raw materials ╳ The quantity of raw materials ＋ (The unit revenue of selling demanded products ╳ The quantity of demanded products to be transported to wholesalers ＋ The unit revenue of selling demanded products ╳ The quantity of demanded products to be transported to retailers in j[th] manufacturer ＋The unit revenue of selling demanded products ╳ The quantity of demanded products to be transported to end-customers in j[th] manufacturer) ＋ (The unit revenue of selling demanded products ╳ The quantity of demanded products to be transported to retailers in j[th] wholesaler ＋ The unit revenue of selling demanded products ╳ The quantity of demanded products to be transported to end-customers in j[th] wholesaler) ＋ The unit revenue of selling demanded products ╳ The quantity of demanded products to be transported to end-customers in j[th] retailer

$$AR_{bl}(R,Q)=\sum_{\forall k}\left\{\begin{array}{l}\sum_{\forall j_4}R_{rm-m}^{j_4}(k)\times Q_{rm-m}^{j_4}(k)+\\ \sum_{\forall j_3}\left[\begin{array}{l}R_{p-ws}^{j_3}(k)\times Q_{p-ws}^{j_3}(k)+\\ R_{p-r}^{j_3}(k)\times Q_{p-r}^{j_3}(k)+\\ R_{p-ec}^{j_3}(k)\times Q_{p-ec}^{j_3}(k)\end{array}\right]+\\ \sum_{\forall j_2}\left[\begin{array}{l}R_{p-r}^{j_2}(k)\times Q_{p-r}^{j_2}(k)+\\ R_{p-ec}^{j_2}(k)\times Q_{p-ec}^{j_2}(k)\end{array}\right]+\\ \sum_{\forall j_1}R_{p-ec}^{j_1}(k)\times Q_{p-ec}^{j_1}(k)\end{array}\right\} \tag{4.2-7}$$

8. Aggregate procurement cost in the end-customers ＝

   The unit cost of procurement ╳ The quantity of demanded products

$$APC_{ec}(R,Q) = \sum_{\forall k}\sum_{\forall j_0} P_p^{j_0}(k) \times Q_{p-P}^{j_0}(k)$$

$$= \sum_{\forall k}\begin{bmatrix} \sum_{\forall j_3} R_{p-ec}^{j_3}(k) \times Q_{p-ec}^{j_3}(k) + \\ \sum_{\forall j_2} R_{p-ec}^{j_2}(k) \times Q_{p-ec}^{j_2}(k) + \\ \sum_{\forall j_1} R_{p-ec}^{j_1}(k) \times Q_{p-ec}^{j_1}(k) \end{bmatrix} \quad (4.2\text{-}8)$$

9. Aggregate revenue in the end-customers ＝

   The unit revenue of selling useless products to collecting points ＸThe quantity of useless products ＋ The unit revenue of selling useless products to recycle plants ＸThe quantity of useless products ＋ The unit revenue of selling useless products to disassembly plants ＸThe quantity of useless products

$$AR_{ec}(R,Q) = \sum_{\forall k}\sum_{\forall j_0}\begin{bmatrix} R_{up-cp}^{j_0}(k) \times Q_{up-cp}^{j_0}(k) + \\ R_{up-rp}^{j_0}(k) \times Q_{up-rp}^{j_0}(k) + \\ R_{up-dp}^{j_0}(k) \times Q_{up-dp}^{j_0}(k) \end{bmatrix} \quad (4.2\text{-}9)$$

10. Aggregate manufacture cost in the RLS ＝

    The unit cost of manufacturing reused raw materials ＸThe quantity of reused raw materials

$$AMC_{rl}(M,Q) = \sum_{\forall k}\sum_{\forall j_{-4}} M_{rrm}^{j_{-4}}(k) \times Q_{rrm-M}^{j_{-4}}(k) \quad (4.2\text{-}10)$$

11. Aggregate procurement cost in the RLS ＝

    The unit cost of procuring useless products in j^{th} collecting point ＸThe quantity of useless products ＋ The unit cost of procuring useless products in j^{th} recycle plant ＸThe quantity of useless products ＋ The unit cost of procuring useless products in j^{th} disassembly plant ＸThe quantity of useless products ＋ The unit cost of procuring reusable material ＸThe quantity of reusable materials

$$APC_{rl}(R,Q)=\sum_{\forall k}\begin{bmatrix}P_{up}^{j_{-1}}(k)\times Q_{up-P}^{j_{-1}}(k)+\\P_{up}^{j_{-2}}(k)\times Q_{up-P}^{j_{-2}}(k)+\\P_{up}^{j_{-3}}(k)\times Q_{up-P}^{j_{-3}}(k)+\\P_{rbm}^{j_{-4}}(k)\times Q_{rbm-P}^{j_{-4}}(k)\end{bmatrix}$$

$$=\sum_{\forall k}\begin{bmatrix}\sum_{\forall j_0}\begin{bmatrix}R_{up-cp}^{j_0}(k)\times Q_{up-cp}^{j_0}(k)+\\R_{up-rp}^{j_0}(k)\times Q_{up-rp}^{j_0}(k)+\\R_{up-dp}^{j_0}(k)\times Q_{up-dp}^{j_0}(k)\end{bmatrix}+\\\sum_{\forall j_{-1}}\begin{bmatrix}R_{up-rp}^{j_{-1}}(k)\times Q_{up-rp}^{j_{-1}}(k)+\\R_{up-dp}^{j_{-1}}(k)\times Q_{up-dp}^{j_{-1}}(k)\end{bmatrix}+\\\sum_{\forall j_{-2}}R_{up-dp}^{j_{-2}}(k)\times Q_{up-dp}^{j_{-2}}(k)+\\\sum_{\forall j_{-3}}R_{rbm-smm}^{j_{-3}}(k)\times Q_{rbm-smm}^{j_{-3}}(k)\end{bmatrix} \quad (4.2\text{-}11)$$

12. Aggregate inventory cost in the RLS ＝

The unit cost of stocking useless products in j^{th} collecting point ╳ The quantity of useless products ＋ The unit cost of stocking useless products in j^{th} recycle plant ╳ The quantity of useless products ＋ (The unit cost of stocking useless products in j^{th} disassembly plant ╳ The quantity of useless products ＋ The unit cost of stocking reusable materials in j^{th} disassembly plant ╳ The quantity of reusable materials ＋ The unit cost of stocking derivative waste in j^{th} disassembly plant ╳ The quantity of derivative waste) ＋ (The unit cost of stocking reusable materials in j^{th} secondary material market ╳ The quantity of reusable materials ＋ The unit cost of stocking reused raw materials ╳ The quantity of reused raw materials) ＋ The unit cost of stocking derivative waste in j^{th} landfill/ incinerator ╳ The quantity of derivative waste

$$AIC_{rl}(I,Q)=\sum_{\forall k}\left\{\begin{array}{l}\sum_{\forall j_{-1}} I_{up}^{j_{-1}}(k)\times Q_{up-I}^{j_{-1}}(k)+\\ \sum_{\forall j_{-2}} I_{up}^{j_{-2}}(k)\times Q_{up-I}^{j_{-2}}(k)+\\ \sum_{\forall j_{-3}}\left[\begin{array}{l} I_{up}^{j_{-3}}(k)\times Q_{up-I}^{j_{-3}}(k)+\\ I_{rbm}^{j_{-3}}(k)\times Q_{rbm-I}^{j_{-3}}(k)+\\ I_{dw}^{j_{-3}}(k)\times Q_{dw-I}^{j_{-3}}(k)\end{array}\right]+\\ \sum_{\forall j_{-4}}\left[\begin{array}{l} I_{rbm}^{j_{-4}}(k)\times Q_{rbm-I}^{j_{-4}}(k)+\\ I_{rrm}^{j_{-4}}(k)\times Q_{rrm-I}^{j_{-4}}(k)\end{array}\right]+\\ \sum_{\forall j_{-5}} I_{dw}^{j_{-5}}(k)\times Q_{dw-I}^{j_{-5}}(k)\end{array}\right\} \quad (4.2\text{-}12)$$

13. Aggregate transportation cost in the RLS ＝

(The unit cost of transporting useless products to collecting points in j^{th} end-customer ╳ The quantity of useless products ＋ The unit cost of transporting useless products to recycle plants in j^{th} end-customer ╳ The quantity of useless products ＋ The unit cost of transporting useless products to disassembly plants in j^{th} end-customer ╳ The quantity of useless products) ＋ (The unit cost of transporting useless products to recycle plants in j^{th} collecting point ╳ The quantity of useless products ＋ The unit cost of transporting useless products to disassembly plants in j^{th} collecting point ╳ The quantity of useless products) ＋ The unit cost of transporting useless products to disassembly plants in j^{th} recycle plant ╳ The quantity of useless products ＋ (The unit cost of transporting reusable materials to secondary material market ╳ The quantity of reusable materials ＋ The unit cost of transporting derivative waste to landfills/ incinerators in j^{th} disassembly plant ╳ The quantity of derivative waste) ＋ The unit cost of transporting reused raw materials to manufacturers ╳ The quantity of reused raw materials

$$ATC_{rl}(T,Q)=\sum_{\forall k}\left\{\begin{array}{l}\sum\limits_{\forall j_0}\left[\begin{array}{l}T_{up-cp}^{j_0}(k)\times Q_{up-cp}^{j_0}(k)+\\T_{up-rp}^{j_0}(k)\times Q_{up-rp}^{j_0}(k)+\\T_{up-dp}^{j_0}(k)\times Q_{up-dp}^{j_0}(k)\end{array}\right]+\\\sum\limits_{\forall j_{-1}}\left[\begin{array}{l}T_{up-rp}^{j_{-1}}(k)\times Q_{up-rp}^{j_{-1}}(k)+\\T_{up-dp}^{j_{-1}}(k)\times Q_{up-dp}^{j_{-1}}(k)\end{array}\right]+\\\sum\limits_{\forall j_{-2}}T_{up-dp}^{j_{-2}}(k)\times Q_{up-dp}^{j_{-2}}(k)+\\\sum\limits_{\forall j_{-3}}\left[\begin{array}{l}T_{rbm-smm}^{j_{-3}}(k)\times Q_{rbm-smm}^{j_{-3}}(k)+\\T_{dw-li}^{j_{-3}}(k)\times Q_{dw-li}^{j_{-3}}(k)\end{array}\right]+\\\sum\limits_{\forall j_{-4}}T_{rrm-m}^{j_{-4}}(k)\times Q_{rrm-m}^{j_{-4}}(k)\end{array}\right\} \quad (4.2\text{-}13)$$

14. Aggregate treatment cost in the RLS ＝

The unit cost of treating useless products in j[th] disassembly plant ╳ The quantity of useless products ＋ The unit cost of treating derivative waste ╳ The quantity of derivative waste

$$ATRC_{rl}(TR,Q)=\sum_{\forall k}\left[\begin{array}{l}\sum\limits_{\forall j_{-3}}TR_{up}^{j_{-3}}(k)\times Q_{up-TR}^{j_{-3}}(k)+\\\sum\limits_{\forall j_{-5}}TR_{dw}^{j_{-5}}(k)\times Q_{dw-TR}^{j_{-5}}(k)\end{array}\right] \quad (4.2\text{-}14)$$

15. Aggregate revenue in the RLS ＝

(The unit revenue of selling useless products to recycle plants ╳ The quantity of useless products ＋ The unit revenue of selling useless products to disassembly plants in j[th] collecting point ╳ The quantity of useless products) ＋ The unit revenue of selling useless products to disassembly plants in j[th] recycle point ╳ The quantity of useless products ＋ The unit revenue of selling reusable materials to secondary material market ╳ The quantity of reusable materials ＋ The unit revenue of selling reused raw materials to manufacturers ╳ The quantity of reused raw materials

$$AR_{rl}\left(R,Q\right)=\sum_{\forall k}\left\{\begin{array}{l}\sum\limits_{\forall j_{-1}}\left[\begin{array}{l}R_{up-rp}^{j_{-1}}(k)\times Q_{up-rp}^{j_{-1}}(k)+\\R_{up-dp}^{j_{-1}}(k)\times Q_{up-dp}^{j_{-1}}(k)\end{array}\right]+\\\sum\limits_{\forall j_{-2}}R_{up-dp}^{j_{-2}}(k)\times Q_{up-dp}^{j_{-2}}(k)+\\\sum\limits_{\forall j_{-3}}R_{rbm-smm}^{j_{-3}}(k)\times Q_{rbm-smm}^{j_{-3}}(k)+\\\sum\limits_{\forall j_{-4}}R_{rrm-m}^{j_{-4}}(k)\times Q_{rrm-m}^{j_{-4}}(k)\end{array}\right\} \tag{4.2-15}$$

16. Aggregate subsidy in the RLS ＝

The unit subsidy of treating useless products ㄨ The quantity of useless products

$$AS_{rl}\left(S,Q\right)=\sum_{\forall k}\sum_{\forall j_{-3}}S^{j_{-3}}(k)\times Q_{up-TR}^{j_{-3}}(k) \tag{4.2-16}$$

All derivations of system functions used in this study are showed as follows.

17. Aggregate cost in the BLS ＝

Aggregate manufacture cost in the BLS ＋ Aggregate added value cost in the BLS ＋ Aggregate procurement cost in the BLS ＋ Aggregate inventory cost in the BLS ＋ Aggregate transportation cost in the BLS ＋ Aggregate recycle fee cost in the business logistics system in the BLS

$$AC_{bl}\left(M,AV,P,I,T,RF,Q\right)=AMC_{bl}+AAVC_{bl}+APC_{bl}+AIC_{bl}+ATC_{bl}+ARFC_{bl} \tag{4.2-17}$$

18. Aggregate profit in the BLS ＝ Aggregate revenue in the BLS

$$APR_{bl}(R,Q)=AR_{bl} \tag{4.2-18}$$

19. Aggregate cost in the RLS ＝

Aggregate manufacture cost in the RLS ＋ Aggregate procurement cost in the RLS ＋ Aggregate inventory cost in the RLS ＋ Aggregate transportation cost in the RLS ＋ Aggregate treatment cost in the RLS

$$AC_{rl}\left(M,I,T,TR,R,Q\right)=AMC_{rl}+APC_{rl}+AIC_{rl}+ATC_{rl}+ATRC_{rl} \quad (4.2\text{-}19)$$

20. Aggregate profit in the RLS ＝

Aggregate revenue in the RLS ＋ Aggregate subsidy in the RLS

$$APR_{rl}(R,S,Q)=AR_{rl}+AS_{rl} \quad (4.2\text{-}20)$$

21. Aggregate cost in the end-customers ＝

Aggregate procurement cost in the end-customers

$$AC_{ec}\left(R,Q\right)=APC_{ec} \quad (4.2\text{-}21)$$

22. Aggregate profit in the end-customers ＝

Aggregate revenue in the end-customers

$$APR_{ec}\left(R,Q\right)=AR_{ec} \quad (4.2\text{-}22)$$

23. Aggregate manufacture cost in the integration of the BLS and the RLS ＝

Aggregate manufacture cost in the BLS ＋ Aggregate manufacture cost in the RLS

$$AMC_{bl-rl}(M,Q)=AMC_{bl}+AMC_{rl} \quad (4.2\text{-}23)$$

24. Aggregate added value cost in the integration of the BLS and the RLS ＝

Aggregate added value cost in the BLS

$$AAVC_{bl-rl}(AV,Q)=AAVC_{bl} \quad (4.2\text{-}24)$$

25. Aggregate procurement cost in the integration of the BLS and the RLS ＝

Aggregate procurement cost in the BLS ＋ Aggregate procurement cost in the RLS

$$APC_{bl-rl}(P,Q)=APC_{bl}+APC_{rl} \quad (4.2\text{-}25)$$

26. Aggregate inventory cost in the integration of the BLS and the RLS ＝

Aggregate inventory cost in the BLS ＋ Aggregate inventory cost in the RLS

$$AIC_{bl-rl}(I,Q) = AIC_{bl} + AIC_{rl} \quad (4.2\text{-}26)$$

27. Aggregate transportation cost in the integration of the BLS and the RLS ＝

Aggregate transportation cost in BLS ＋ Aggregate transportation cost in RLS

$$ATC_{bl-rl}(T,Q) = ATC_{bl} + ATC_{rl} \quad (4.2\text{-}27)$$

28. Aggregate treatment cost in the integration of the BLS and the RLS ＝

Aggregate treatment cost in BLS ＋ Aggregate treatment cost in RLS

$$ATRC_{bl-rl}(TR,Q) = ATRC_{rl} \quad (4.2\text{-}28)$$

29. Aggregate recycle fee cost in the integration of the BLS and the RLS ＝

Aggregate recycle fee cost in the BLS

$$ARFC_{bl-rl}(RF,Q) = ARFC_{bl} \quad (4.2\text{-}29)$$

30. Aggregate revenue in the integration of the BLS and the RLS ＝

Aggregate revenue in BLS ＋ Aggregate revenue in RLS

$$AR_{bl-rl}(R,Q) = AR_{bl} + AR_{rl} \quad (4.2\text{-}30)$$

31. Aggregate subsidy in the integration of the BLS and the RLS ＝

Aggregate subsidy in RLS

$$AS_{bl-rl}(S,Q) = AS_{rl} \quad (4.2\text{-}31)$$

32. Net profit in the BLS ＝

Aggregate profit in the BLS － Aggregate cost in the BLS

$$NPR_{bl}(M, AV, P, I, T, RF, R, Q) = APR_{bl} - AC_{bl}$$
$$= AR_{bl} - \begin{pmatrix} AMC_{bl} + AAVC_{bl} + APC_{bl} + \\ AIC_{bl} + ATC_{bl} + ARFC_{bl} \end{pmatrix} \quad (4.2\text{-}32)$$

33. Net profit in the RLS ＝

Aggregate profit in the RLS － Aggregate cost in the RLS

$$NPR_{rl}(M, I, T, TR, R, S, Q) = APR_{rl} - AC_{rl}$$
$$= (AR_{rl} + AS_{rl}) - \begin{pmatrix} AMC_{rl} + APC_{rl} + AIC_{rl} \\ + ATC_{rl} + ATRC_{rl} \end{pmatrix} \quad (4.2\text{-}33)$$

34. Aggregate cost in the integration of the BLS and the RLS ＝

Aggregate cost in the BLS ＋ Aggregate cost in the RLS

$$AC_{bl-rl}(M, AV, P, I, T, TR, RF, Q) = AC_{bl} + AC_{rl}$$
$$= AMC_{bl-rl} + AAVC_{bl-rl} + APC_{bl-rl} + AIC_{bl-rl} + ATC_{bl-rl} + ATRC_{bl-rl} + ARFC_{bl-rl} \quad (4.2\text{-}34)$$
$$= \begin{pmatrix} AMC_{bl} + AAVC_{bl} + APC_{bl} + \\ AIC_{bl} + ATC_{bl} + ARFC_{bl} \end{pmatrix} + (AMC_{rl} + APC_{rl} + AIC_{rl} + ATC_{rl} + ATRC_{rl})$$

35. Aggregate profit in the integration of the BLS and the RLS ＝

Aggregate profit in the BLS ＋ Aggregate profit in the RLS

$$APR_{bl-rl}(R, S, Q) = APR_{bl} + APR_{rl}$$
$$= AR_{bl-rl} + AS_{bl-rl} \quad (4.2\text{-}35)$$
$$= AR_{bl} + (AR_{rl} + AS_{rl})$$

36. Net profit in the integration of the BLS and the RLS ＝

Net profit in the BLS ＋ Net profit in the RLS

$$
\begin{aligned}
NPR_{bl-rl}\begin{pmatrix} M, AV, P, I, T, \\ TR, RF, R, S, Q \end{pmatrix} &= NPR_{bl} \quad + NPR_{rl} \\
&= \left(APR_{bl} - AC_{bl}\right) + \left(APR_{rl} - AC_{rl}\right) \\
&= \left[AR_{bl} - \begin{pmatrix} AMC_{bl} + AAVC_{bl} + APC_{bl} + \\ AIC_{bl} + ATC_{bl} + ARFC_{bl} \end{pmatrix} \right] + \\
&\quad \left[\left(AR_{rl} + AS_{rl}\right) - \begin{pmatrix} AMC_{rl} + APC_{rl} + AIC_{rl} + \\ ATC_{rl} + ATRC_{rl} \end{pmatrix} \right] \\
&= APR_{bl-rl} - AC_{bl-rl} \\
&= \left(AR_{bl-rl} + AS_{bl-rl}\right) - \\
&\quad \begin{pmatrix} AMC_{bl-rl} + AAVC_{bl-rl} + APC_{bl-rl} + AIC_{bl-rl} + \\ ATC_{bl-rl} + ATRC_{bl-rl} + ARFC_{bl-rl} \end{pmatrix} \\
&= \left(APR_{bl} + APR_{rl}\right) - \left(AC_{bl} + AC_{rl}\right) \\
&= \left[AR_{bl} + \left(AR_{rl} + AS_{rl}\right) \right] - \\
&\quad \left[\begin{pmatrix} AMC_{bl} + AAVC_{bl} + APC_{bl} + \\ AIC_{bl} + ATC_{bl} + ARFC_{bl} \end{pmatrix} + \right. \\
&\quad \left. \begin{pmatrix} AMC_{rl} + APC_{rl} + AIC_{rl} + \\ ATC_{rl} + ATRC_{rl} \end{pmatrix} \right]
\end{aligned}
\qquad (4.2\text{-}36)
$$

4.3 Objective Function

In this study, the most interesting issue will be discussed in this section. Since there is the trade-off between the business logistics system (BLS) and the reverse logistics system (RLS), the ILM will be proposed to optimize the trade-off in the ILS. To maximize the net profit in the ILS, the objective function is formulated as follows.

Objective funtion: $Max\ Z = w_1 NPR_{bl} + w_2 NPR_{rl}$ (4.3-1)

where, Z : Objective value for net profit in the ILS.
w_1: The weight for net profit in the BLS.
w_2: The weight for net profit in the RLS.

4.4 Constraints

This section will represent the constraints about capacity, inventory, manufacture, treatment, procurement, and the weight separately in followed subsection.

4.4.1 Capacity Constraints

In this study, there are lower limit quantity and upper limit quantity with state variables described in Table 4.4-1. First of all, general forms aimed at the lower limit quantity and the upper limit quantity will be represented as follows.

1. The general form of the lower limit quantity is like $\underline{Q}^{j_i}_{products-Activity}(k)$ or $\underline{Q}^{j_i}_{products-members}(k)$. The physical meaning of $\underline{Q}^{j_i}_{products-Activity}(k)$ is the lower limit quantity of products done by some activity in time interval k in j^{th} member in layer i. The physical meaning of $\underline{Q}^{j_i}_{products-members}(k)$ is the lower limit quantity of products transported to some members in time interval k in j^{th} member in layer i.

2. The general form of the upper limit quantity is like $\overline{Q}^{j_i}_{products-Activity}(k)$ or $\overline{Q}^{j_i}_{products-members}(k)$. The physical meaning of $\overline{Q}^{j_i}_{products-Activity}(k)$ is the upper limit quantity of products done by some activity in time interval k in j^{th} member in layer i. The physical meaning of $\overline{Q}^{j_i}_{products-members}(k)$ is the upper limit quantity of products transported to some members in time interval k in j^{th} member in layer i.

Table 4.4-1 The Limit of Quantity

Constraints		
$\underline{Q}_{rm-M}^{j_4}(k) \le Q_{rm-M}^{j_4}(k) \le \overline{Q}_{rm-M}^{j_4}(k)$	$\forall j,k$	$(4.4-1)$
$\underline{Q}_{p-M}^{j_3}(k) \le Q_{p-M}^{j_3}(k) \le \overline{Q}_{p-M}^{j_3}(k)$	$\forall j,k$	$(4.4-2)$
$\underline{Q}_{rrm-M}^{j_4}(k) \le Q_{rrm-M}^{j_4}(k) \le \overline{Q}_{rrm-M}^{j_4}(k)$	$\forall j,k$	$(4.4-3)$
$\underline{Q}_{rbm-M}^{j_3}(k) \le Q_{rbm-M}^{j_3}(k) \le \overline{Q}_{rbm-M}^{j_3}(k)$	$\forall j,k$	$(4.4-4)$
$\underline{Q}_{p-AV}^{j_3}(k) \le Q_{p-AV}^{j_3}(k) \le \overline{Q}_{p-AV}^{j_3}(k)$	$\forall j,k$	$(4.4-5)$
$\underline{Q}_{p-AV}^{j_2}(k) \le Q_{p-AV}^{j_2}(k) \le \overline{Q}_{p-AV}^{j_2}(k)$	$\forall j,k$	$(4.4-6)$
$\underline{Q}_{p-AV}^{j_1}(k) \le Q_{p-AV}^{j_1}(k) \le \overline{Q}_{p-AV}^{j_1}(k)$	$\forall j,k$	$(4.4-7)$
$\underline{Q}_{vm-P}^{j_4}(k) \le Q_{vm-P}^{j_4}(k) \le \overline{Q}_{vm-P}^{j_4}(k)$	$\forall j,k$	$(4.4-8)$
$\underline{Q}_{rm-P}^{j_3}(k) \le Q_{rm-P}^{j_3}(k) \le \overline{Q}_{rm-P}^{j_3}(k)$	$\forall j,k$	$(4.4-9)$
$\underline{Q}_{rrm-P}^{j_3}(k) \le Q_{rrm-P}^{j_3}(k) \le \overline{Q}_{rrm-P}^{j_3}(k)$	$\forall j,k$	$(4.4-10)$
$\underline{Q}_{p-P}^{j_2}(k) \le Q_{p-P}^{j_2}(k) \le \overline{Q}_{p-P}^{j_2}(k)$	$\forall j,k$	$(4.4-11)$
$\underline{Q}_{p-P}^{j_1}(k) \le Q_{p-P}^{j_1}(k) \le \overline{Q}_{p-P}^{j_1}(k)$	$\forall j,k$	$(4.4-12)$
$\underline{Q}_{p-P}^{j_0}(k) \le Q_{p-P}^{j_0}(k) \le \overline{Q}_{p-P}^{j_0}(k)$	$\forall j,k$	$(4.4-13)$
$\underline{Q}_{up-P}^{j_{-1}}(k) \le Q_{up-P}^{j_{-1}}(k) \le \overline{Q}_{up-P}^{j_{-1}}(k)$	$\forall j,k$	$(4.4-14)$
$\underline{Q}_{up-P}^{j_{-2}}(k) \le Q_{up-P}^{j_{-2}}(k) \le \overline{Q}_{up-P}^{j_{-2}}(k)$	$\forall j,k$	$(4.4-15)$
$\underline{Q}_{up-P}^{j_{-3}}(k) \le Q_{up-P}^{j_{-3}}(k) \le \overline{Q}_{up-P}^{j_{-3}}(k)$	$\forall j,k$	$(4.4-16)$
$\underline{Q}_{rbm-P}^{j_{-4}}(k) \le Q_{rbm-P}^{j_{-4}}(k) \le \overline{Q}_{rbm-P}^{j_{-4}}(k)$	$\forall j,k$	$(4.4-17)$
$\underline{Q}_{vm-I}^{j_4}(k) \le Q_{vm-I}^{j_4}(k) \le \overline{Q}_{vm-I}^{j_4}(k)$	$\forall j,k$	$(4.4-18)$

$$\underline{Q}_{rm-I}^{j_4}(k) \le Q_{rm-I}^{j_4}(k) \le \overline{Q}_{rm-I}^{j_4}(k) \qquad \forall j,k \qquad (4.4-19)$$

$$\underline{Q}_{rm-I}^{j_3}(k) \le Q_{rm-I}^{j_3}(k) \le \overline{Q}_{rm-I}^{j_3}(k) \qquad \forall j,k \qquad (4.4-20)$$

$$\underline{Q}_{p-I}^{j_3}(k) \le Q_{p-I}^{j_3}(k) \le \overline{Q}_{p-I}^{j_3}(k) \qquad \forall j,k \qquad (4.4-21)$$

$$\underline{Q}_{p-I}^{j_2}(k) \le Q_{p-I}^{j_2}(k) \le \overline{Q}_{p-I}^{j_2}(k) \qquad \forall j,k \qquad (4.4-22)$$

$$\underline{Q}_{p-I}^{j_1}(k) \le Q_{p-I}^{j_1}(k) \le \overline{Q}_{p-I}^{j_1}(k) \qquad \forall j,k \qquad (4.4-23)$$

$$\underline{Q}_{up-I}^{j_{-1}}(k) \le Q_{up-I}^{j_{-1}}(k) \le \overline{Q}_{up-I}^{j_{-1}}(k) \qquad \forall j,k \qquad (4.4-24)$$

$$\underline{Q}_{up-I}^{j_{-2}}(k) \le Q_{up-I}^{j_{-2}}(k) \le \overline{Q}_{up-I}^{j_{-2}}(k) \qquad \forall j,k \qquad (4.4-25)$$

$$\underline{Q}_{up-I}^{j_{-3}}(k) \le Q_{up-I}^{j_{-3}}(k) \le \overline{Q}_{up-I}^{j_{-3}}(k) \qquad \forall j,k \qquad (4.4-26)$$

$$\underline{Q}_{rbm-I}^{j_{-3}}(k) \le Q_{rbm-I}^{j_{-3}}(k) \le \overline{Q}_{rbm-I}^{j_{-3}}(k) \qquad \forall j,k \qquad (4.4-27)$$

$$\underline{Q}_{dw-I}^{j_{-3}}(k) \le Q_{dw-I}^{j_{-3}}(k) \le \overline{Q}_{dw-I}^{j_{-3}}(k) \qquad \forall j,k \qquad (4.4-28)$$

$$\underline{Q}_{rbm-I}^{j_{-4}}(k) \le Q_{rbm-I}^{j_{-4}}(k) \le \overline{Q}_{rbm-I}^{j_{-4}}(k) \qquad \forall j,k \qquad (4.4-29)$$

$$\underline{Q}_{rrm-I}^{j_{-4}}(k) \le Q_{rrm-I}^{j_{-4}}(k) \le \overline{Q}_{rrm-I}^{j_{-4}}(k) \qquad \forall j,k \qquad (4.4-30)$$

$$\underline{Q}_{dw-I}^{j_{-5}}(k) \le Q_{dw-I}^{j_{-5}}(k) \le \overline{Q}_{dw-I}^{j_{-5}}(k) \qquad \forall j,k \qquad (4.4-31)$$

$$\underline{Q}_{up-TR}^{j_{-3}}(k) \le Q_{up-TR}^{j_{-3}}(k) \le \overline{Q}_{up-TR}^{j_{-3}}(k) \qquad \forall j,k \qquad (4.4-32)$$

$$\underline{Q}_{dw-TR}^{j_{-5}}(k) \le Q_{dw-TR}^{j_{-5}}(k) \le \overline{Q}_{dw-TR}^{j_{-5}}(k) \qquad \forall j,k \qquad (4.4-33)$$

$$\underline{Q}_{rm-E}^{j_3}(k) \le Q_{rm-E}^{j_3}(k) \le \overline{Q}_{rm-E}^{j_3}(k) \qquad \forall j,k \qquad (4.4-34)$$

$$\underline{Q}_{rbm-E}^{j_{-4}}(k) \le Q_{rbm-E}^{j_{-4}}(k) \le \overline{Q}_{rbm-E}^{j_{-4}}(k) \qquad \forall j,k \qquad (4.4-35)$$

$$\underline{Q}_{dw-D}^{j_{-2}}(k) \le Q_{dw-D}^{j_{-2}}(k) \le \overline{Q}_{dw-D}^{j_{-2}}(k) \qquad \forall j,k \qquad (4.4-36)$$

$$\underline{Q}_{dw-D}^{j_{-3}}(k) \le Q_{dw-D}^{j_{-3}}(k) \le \overline{Q}_{dw-D}^{j_{-3}}(k) \qquad \forall j,k \qquad (4.4-37)$$

$$\underline{Q}_{dw-D}^{j_{-4}}(k) \le Q_{dw-D}^{j_{-4}}(k) \le \overline{Q}_{dw-D}^{j_{-4}}(k) \qquad \forall j,k \qquad (4.4-38)$$

$$\underline{Q}_{rm-m}^{j_4}(k) \le Q_{rm-m}^{j_4}(k) \le \overline{Q}_{rm-m}^{j_4}(k) \qquad \forall j,k \qquad (4.4-39)$$

$$\underline{Q}_{p-ws}^{j_3}(k) \le Q_{p-ws}^{j_3}(k) \le \overline{Q}_{p-ws}^{j_3}(k) \qquad \forall j,k \qquad (4.4-40)$$

$$\underline{Q}_{p-r}^{j_3}(k) \le Q_{p-r}^{j_3}(k) \le \overline{Q}_{p-r}^{j_3}(k) \qquad \forall j,k \qquad (4.4-41)$$

$$\underline{Q}_{p-ec}^{j_3}(k) \le Q_{p-ec}^{j_3}(k) \le \overline{Q}_{p-ec}^{j_3}(k) \qquad \forall j,k \qquad (4.4-42)$$

$$\underline{Q}_{p-r}^{j_2}(k) \le Q_{p-r}^{j_2}(k) \le \overline{Q}_{p-r}^{j_2}(k) \qquad \forall j,k \qquad (4.4-43)$$

$$\underline{Q}_{p-ec}^{j_2}(k) \le Q_{p-ec}^{j_2}(k) \le \overline{Q}_{p-ec}^{j_2}(k) \qquad \forall j,k \qquad (4.4-44)$$

$$\underline{Q}_{p-ec}^{j_1}(k) \le Q_{p-ec}^{j_1}(k) \le \overline{Q}_{p-ec}^{j_1}(k) \qquad \forall j,k \qquad (4.4-45)$$

$$\underline{Q}_{up-cp}^{j_0}(k) \le Q_{up-cp}^{j_0}(k) \le \overline{Q}_{up-cp}^{j_0}(k) \qquad \forall j,k \qquad (4.4-46)$$

$$\underline{Q}_{up-rp}^{j_0}(k) \le Q_{up-rp}^{j_0}(k) \le \overline{Q}_{up-rp}^{j_0}(k) \qquad \forall j,k \qquad (4.4-47)$$

$$\underline{Q}_{up-dp}^{j_0}(k) \le Q_{up-dp}^{j_0}(k) \le \overline{Q}_{up-dp}^{j_0}(k) \qquad \forall j,k \qquad (4.4-48)$$

$$\underline{Q}_{up-rp}^{j_{-1}}(k) \le Q_{up-rp}^{j_{-1}}(k) \le \overline{Q}_{up-rp}^{j_{-1}}(k) \qquad \forall j,k \qquad (4.4-49)$$

$$\underline{Q}_{up-dp}^{j_{-1}}(k) \le Q_{up-dp}^{j_{-1}}(k) \le \overline{Q}_{up-dp}^{j_{-1}}(k) \qquad \forall j,k \qquad (4.4-50)$$

$$\underline{Q}_{up-dp}^{j_{-2}}(k) \le Q_{up-dp}^{j_{-2}}(k) \le \overline{Q}_{up-dp}^{j_{-2}}(k) \qquad \forall j,k \qquad (4.4-51)$$

$$\underline{Q}_{rbm-smm}^{j_{-3}}(k) \le Q_{rbm-smm}^{j_{-3}}(k) \le \overline{Q}_{rbm-smm}^{j_{-3}}(k) \qquad \forall j,k \qquad (4.4-52)$$

$$\underline{Q}_{dw-li}^{j_{-3}}(k) \le Q_{dw-li}^{j_{-3}}(k) \le \overline{Q}_{dw-li}^{j_{-3}}(k) \qquad \forall j,k \qquad (4.4-53)$$

$$\underline{Q}_{rrm-m}^{j_{-4}}(k) \le Q_{rrm-m}^{j_{-4}}(k) \le \overline{Q}_{rrm-m}^{j_{-4}}(k) \qquad \forall j,k \qquad (4.4-54)$$

4.4.2 Inventory Constraints

After introducing the limit of quantity, then we will represent the inventory constraints in various nodes with different products as follows.

1. In j^{th} raw material supplier

(1) For virgin materials

The quantity of virgin materials stocked in k^{th} time interval ＝ The quantity of virgin materials stocked in $(k-1)^{th}$ time interval ＋ The quantity of virgin materials procured in k^{th} time interval － The quantity of virgin materials exhausted in k^{th} time interval

$$Q_{vm-I}^{j_4}(k) = Q_{vm-I}^{j_4}(k-1) + Q_{vm-P}^{j_4}(k) - Q_{vm-E}^{j_4}(k) \tag{4.4-58}$$

in which, $Q_{vm-E}^{j_3}(k) = \tau_{vm/rm} \times Q_{rm-M}^{j_3}(k)$,

$\tau_{vm/rm}$: The transition ratio between virgin materials and raw materials.

(2) For raw materials

The quantity of raw materials stocked in k^{th} time interval ＝ The quantity of raw materials stocked in $(k-1)^{th}$ time interval ＋ The quantity of raw materials manufactured in k^{th} time interval － The quantity of raw materials transported to manufactures in k^{th} time interval

$$Q_{rm-I}^{j_4}(k) = Q_{rm-I}^{j_4}(k-1) + Q_{rm-M}^{j_4}(k) - Q_{rm-m}^{j_4}(k) \tag{4.4-59}$$

2. In j^{th} manufacturer

(1) For raw materials

The quantity of raw materials stocked in k^{th} time interval ＝ The quantity of raw materials stocked in $(k-1)^{th}$ time interval ＋ The quantity of raw materials procured in k^{th} time interval ＋ The quantity of reused raw materials procured in k^{th} time interval － The quantity of raw materials exhausted in k^{th} time interval

$$Q_{rm-I}^{j_3}(k) = Q_{rm-I}^{j_3}(k-1) + Q_{rm-P}^{j_3}(k) + Q_{rrm-P}^{j_3}(k) - Q_{rm-E}^{j_3}(k) \quad (4.4\text{-}60)$$

in which, $Q_{rm-E}^{j_3}(k) = \tau_{rm/p} \times Q_{p-M}^{j_3}(k)$,

$\tau_{rm/p}$: The transition ratio between raw materials and demanded products.

(2) For demanded products

The quantity of demanded products stocked in k^{th} time interval ＝ The quantity of demanded products stocked in $(k-1)^{th}$ time interval ＋ The quantity of demanded products manufactured in k^{th} time interval － The quantity of demanded products transported to wholesalers in k^{th} time interval － The quantity of demanded products transported to retailers in k^{th} time interval － The quantity of demanded products transported to end-customers in k^{th} time interval

$$Q_{p-I}^{j_3}(k) = Q_{p-I}^{j_3}(k-1) + Q_{p-M}^{j_3}(k) - Q_{p-ws}^{j_3}(k) - Q_{p-r}^{j_3}(k) - Q_{p-ec}^{j_3}(k) \quad (4.4\text{-}61)$$

3. In j^{th} wholesaler

The quantity of demanded products stocked in k^{th} time interval ＝ The quantity of demanded products stocked in $(k-1)^{th}$ time interval ＋ The quantity of demanded products procured in k^{th} time interval － The quantity of demanded products transported to retailers in k^{th} time interval － The quantity of demanded products transported to end-customers in k^{th} time interval

$$Q_{p-I}^{j_2}(k) = Q_{p-I}^{j_2}(k-1) + Q_{p-P}^{j_2}(k) - Q_{p-r}^{j_2}(k) - Q_{p-ec}^{j_2}(k) \tag{4.4-62}$$

4. In j^{th} retailer

The quantity of demanded products stocked in k^{th} time interval ＝ The quantity of demanded products stocked in $(k-1)^{th}$ time interval ＋ The quantity of demanded products procured in k^{th} time interval － The quantity of demanded products transported to end-customers in k^{th} time interval

$$Q_{p-I}^{j_1}(k) = Q_{p-I}^{j_1}(k-1) + Q_{p-P}^{j_1}(k) - Q_{p-ec}^{j_1}(k) \tag{4.4-63}$$

5. In j^{th} collecting point

The quantity of useless products stocked in k^{th} time interval ＝ The quantity of useless products stocked in $(k-1)^{th}$ time interval ＋ The quantity of useless products procured in k^{th} time interval － The quantity of useless products transported to recycle plants in k^{th} time interval － The quantity of useless products transported to disassembly plants in k^{th} time interval

$$Q_{up-I}^{j_{-1}}(k) = Q_{up-I}^{j_{-1}}(k-1) + Q_{up-P}^{j_{-1}}(k) - Q_{up-rp}^{j_{-1}}(k) - Q_{up-dp}^{j_{-1}}(k) \tag{4.4-64}$$

6. In j^{th} recycle plant

The quantity of useless products stocked in k^{th} time interval ＝ The quantity of useless products stocked in $(k-1)^{th}$ time interval ＋ The quantity of useless products procured in k^{th} time interval － The quantity of useless products transported to disassembly plants in k^{th} time interval

$$Q_{up-I}^{j-2}(k) = Q_{up-I}^{j-2}(k-1) + Q_{up-P}^{j-2}(k) - Q_{up-dp}^{j-2}(k) \tag{4.4-65}$$

7. In j^{th} disassembly plant

(1) For useless products

The quantity of useless products stocked in k^{th} time interval ＝ The quantity of useless products stocked in $(k-1)^{th}$ time interval ＋ The quantity of useless products procured in k^{th} time interval － The quantity of useless products treated in k^{th} time interval

$$Q_{up-I}^{j-3}(k) = Q_{up-I}^{j-3}(k-1) + Q_{up-P}^{j-3}(k) - Q_{up-TR}^{j-3}(k) \tag{4.4-66}$$

(2) For reusable materials

The quantity of reusable materials stocked in k^{th} time interval ＝ The quantity of reusable materials stocked in $(k-1)^{th}$ time interval ＋ The quantity of reusable materials manufactured in k^{th} time interval － The quantity of reusable materials transported to secondary material market in k^{th} time interval

$$Q_{rbm-I}^{j-3}(k) = Q_{rbm-I}^{j-3}(k-1) + Q_{rbm-M}^{j-3}(k) - Q_{rbm-smm}^{j-3}(k) \tag{4.4-67}$$

in which, $Q_{rbm-M}^{j-3}(k) = \tau_{rbm/up} \times Q_{up-TR}^{j-3}(k)$,

$\tau_{rbm/up}$: The transition ratio between reusable materials and useless products.

(3) For derivative waste

The quantity of derivative waste stocked in k^{th} time interval ＝ The quantity of derivative waste stocked in $(k-1)^{th}$ time interval ＋ The quantity of derivative waste derived in k^{th} time interval － The quantity of derivative waste transported to landfills/ incinerators in k^{th} time interval

$$Q_{dw-I}^{j-3}(k) = Q_{dw-I}^{j-3}(k-1) + Q_{dw-D}^{j-3}(k) - Q_{dw-li}^{j-3}(k) \tag{4.4-68}$$

in which, $Q_{dw-D}^{j-3}(k) = \tau_{dw/up} \times Q_{up-TR}^{j-3}(k)$,

$\tau_{dw/up}$: The transition ratio between derivative waste and useless products.

8. In j^{th} secondary material market

(1) For reusable materials

The quantity of reusable materials stocked in k^{th} time interval ＝ The quantity of reusable materials stocked in $(k-1)^{th}$ time interval ＋ The quantity of reusable materials procured in k^{th} time interval － The quantity of reusable materials exhausted in k^{th} time interval

$$Q_{rbm-I}^{j-4}(k) = Q_{rbm-I}^{j-4}(k-1) + Q_{rbm-P}^{j-4}(k) - Q_{rbm-E}^{j-4}(k) \tag{4.4-69}$$

in which, $Q_{rbm-E}^{j-4}(k) = \tau_{rbm/rrm} \times Q_{rrm-M}^{j-4}(k)$,

$\tau_{rbm/rrm}$: The transition ratio between reusable materials and reused raw materials.

(2) For reused raw materials

The quantity of reused raw materials stocked in k^{th} time interval ＝ The quantity of reused raw materials stocked in $(k-1)^{th}$ time interval＋ The quantity of reused raw materials manufactured in k^{th} time interval － The quantity of reused raw materials transported to manufacturers in k^{th} time interval

$$Q_{rrm-I}^{j_{-4}}(k) = Q_{rrm-I}^{j_{-4}}(k-1) + Q_{rrm-M}^{j_{-4}}(k) - Q_{rrm-m}^{j_{-4}}(k) \quad (4.4\text{-}70)$$

9. In jth landfill/ incinerator

The quantity of derivative waste stocked in k^{th} time interval ＝ The quantity of derivative waste stocked in $(k-1)^{th}$ time interval ＋ The quantity of derivative waste transported from disassembly plants in k^{th} time interval － The quantity of derivative waste to be treated in k^{th} time interval

$$Q_{dw-I}^{j_{-5}}(k) = Q_{dw-I}^{j_{-5}}(k-1) + Q_{dw-li}^{j_{-3}}(k) - Q_{dw-TR}^{j_{-5}}(k) \quad (4.4\text{-}71)$$

4.4.3 Manufacture Constraints

It will represent the manufacture constraints of different products as follows.

1. Raw materials

The transition ratio between virgin materials and raw materials ╳ The quantity of raw materials manufactured in k^{th} time interval ≦ The quantity of

virgin materials procured in k[th] time interval ＋ The quantity of virgin materials stocked in (k-1)[th] time interval

$$\tau_{vm/rm} \times Q^{j_4}_{rm-M}(k) \le Q^{j_4}_{vm-P}(k) + Q^{j_4}_{vm-I}(k-1) \quad (4.4\text{-}72)$$

2. Reused raw materials

The transition ratio between reusable materials and reused raw materials ╳ The quantity of reused raw materials manufactured in k[th] time interval ≦ The quantity of reusable materials procured in k[th] time interval ＋ The quantity of reusable materials stocked in (k-1)[th] time interval

$$\tau_{rbm/rrm} \times Q^{j_{-4}}_{rrm-M}(k) \le Q^{j_{-4}}_{rbm-P}(k) + Q^{j_{-4}}_{rbm-I}(k-1) \quad (4.4\text{-}73)$$

3. Demanded products

The transition ratio between raw materials and demanded products ╳ The quantity of demanded products manufactured in k[th] time interval ≦ The quantity of raw materials procured in k[th] time interval ＋ The quantity of reused raw materials procured in k[th] time interval ＋ The quantity of raw materials stocked in (k-1)[th] time interval

$$\tau_{rm/p} \times Q^{j_3}_{p-M}(k) \le Q^{j_3}_{rm-P}(k) + Q^{j_3}_{rrm-P}(k) + Q^{j_3}_{rm-I}(k-1) \quad (4.4\text{-}74)$$

4.4.4 Treatment Constraints

It will represent the treatment constraints of different products as follows.

1. Useless products

The quantity of useless products treated in k[th] time interval ≦ The quantity

of useless products procured in k[th] time interval ＋ The quantity of useless products stocked in (k-1)[th] time interval

$$Q_{up-TR}^{j_{-3}}(k) \le Q_{up-P}^{j_{-3}}(k) + Q_{up-I}^{j_{-3}}(k-1) \tag{4.4-75}$$

2. Derivative waste

The quantity of derivative waste treated in k[th] time interval ≦ The quantity of derivative waste transported in k[th] time interval ＋ The quantity of derivative waste stocked in (k-1)[th] time interval

$$Q_{dw-TR}^{j_{-5}}(k) \le Q_{dw-li}^{j_{-3}}(k) + Q_{dw-I}^{j_{-5}}(k-1) \tag{4.4-76}$$

4.4.5 Procurement Constraints

It will represent the inventory constraints in various nodes with different products as follows.

1. In j[th] raw material supplier

The quantity of raw materials transported in k[th] time interval ≦ The quantity of raw materials manufactured in (k-t)[th] time interval ＋ The quantity of raw materials stocked in (k-1)[th] time interval

$$Q_{rm-m}^{j_4}(k) \le Q_{rm-M}^{j_4}(k - t_{rm-m}^4) + Q_{rm-I}^{j_4}(k-1) \tag{4.4-77}$$

in which, $t^4_{rm\text{-}m}$: The lead-times of procuring raw materials from layer 4 in manufacturers.

2. In j^{th} manufacturer

The quantity of demanded products transported in k^{th} time interval ≦ The quantity of demanded products manufactured in $(k-t)^{th}$ time interval ＋ The quantity of demanded products stocked in $(k-1)^{th}$ time interval

$$Q_{p-ws}^{j_3}(k)+Q_{p-r}^{j_3}(k)+Q_{p-ec}^{j_3}(k)\le Q_{p-M}^{j_3}(k-t_{p-members}^{3})+Q_{p-I}^{j_3}(k-1) \qquad (4.4\text{-}78)$$

in which, $t^3_{p\text{-members}}=t^3_{p\text{-ws}}$ or $t^3_{p\text{-r}}$ or $t^3_{p\text{-ec}}$

$t^3_{p\text{-ws}}$: The lead-times of procuring demanded products from layer 3 in wholesalers.

$t^3_{p\text{-r}}$: The lead-times of procuring demanded products from layer 3 in retailers.

$t^3_{p\text{-ec}}$: The lead-times of procuring demanded products from layer 3 in end-customers.

3. In j^{th} wholesaler

The quantity of demanded products transported in k^{th} time interval ≦ The quantity of demanded products procured in $(k-t)^{th}$ time interval ＋ The quantity of demanded products stocked in $(k-1)^{th}$ time interval

$$Q_{p-r}^{j_2}(k)+Q_{p-ec}^{j_2}(k)\le Q_{p-P}^{j_2}(k-t_{p-members}^{2})+Q_{p-I}^{j_2}(k-1) \qquad (4.4\text{-}79)$$

in which, $t^2_{p\text{-members}}=t^2_{p\text{-r}}$ or $t^2_{p\text{-ec}}$

$t^2_{p\text{-ec}}$: The lead-times of procuring demanded products from layer 2 in end-customers.

$t^1_{p\text{-}ec}$: The lead-times of procuring demanded products from layer 1 in end-customers.

4. In j^{th} retailer

The quantity of demanded products transported in k^{th} time interval $\leqq$ The quantity of demanded products procured in k^{th} time interval $+$ The quantity of demanded products stocked in $(k\text{-}1)^{th}$ time interval

$$Q^{j_1}_{p-ec}(k) \leq Q^{j_1}_{p-P}(k) + Q^{j_1}_{p-I}(k-1) \tag{4.4-80}$$

5. In j^{th} end-customer

(1) For demands

The quantity of demanded products transported in k^{th} time interval $=$ The quantity of demanded products procured in k^{th} time interval $=$ The quantity of demands in k^{th} time interval

$$Q^{j_3}_{p-ec}(k) + Q^{j_2}_{p-ec}(k) + Q^{j_1}_{p-ec}(k) = Q^{j_0}_{p-P}(k) = D^j(k) \tag{4.4-81}$$

in which, $D^j(k)$: The quantity of demands in k^{th} time interval.

(2) For returns

The quantity of useless products transported in k^{th} time interval $=$ The quantity of returns in k^{th} time interval

$$Q^{j_0}_{up-cp}(k) + Q^{j_0}_{up-rp}(k) + Q^{j_0}_{up-dp}(k) = R^j(k) \tag{4.4-82}$$

in which, $R^j(k)$: The quantity of returns in k^{th} time interval.

6. In j^{th} collecting point

The quantity of useless products transported in k^{th} time interval ≦ The quantity of useless products procured in $(k-t)^{th}$ time interval ＋ The quantity of useless products stocked in $(k-1)^{th}$ time interval

$$Q_{up-rp}^{j_{-1}}(k) + Q_{up-dp}^{j_{-1}}(k) \le Q_{up-P}^{j_{-1}}(k - t_{up-members}^{-1}) + Q_{up-I}^{j_{-1}}(k-1) \quad (4.4\text{-}83)$$

in which, $t^{-1}_{up\text{-}members} = t^{-1}_{up\text{-}rp}$ or $t^{-1}_{up\text{-}dp}$

$t^{-1}_{up\text{-}rp}$: The lead-times of transporting useless products to recycle plants in layer -1.

$t^{-1}_{up\text{-}dp}$: The lead-times of transporting useless products to disassembly plants in layer -1.

7. In j^{th} recycle plant

The quantity of useless products transported in k^{th} time interval ≦ The quantity of useless products procured in $(k-t)^{th}$ time interval ＋ The quantity of useless products stocked in $(k-1)^{th}$ time interval

$$Q_{up-dp}^{j_{-2}}(k) \le Q_{up-P}^{j_{-2}}(k - t_{up-dp}^{-2}) + Q_{up-I}^{j_{-2}}(k-1) \quad (4.4\text{-}84)$$

in which, $t^{-2}_{up\text{-}dp}$: The lead-times of transporting useless products to disassembly plants in layer -2.

8. In j^{th} disassembly plant

(1) For reusable materials

The quantity of reusable materials transported in k^{th} time interval ≦ The

quantity of useless products treated in (k-t)th time interval ＋ The quantity of reusable materials stocked in (k-1)th time interval

$$Q_{rbm-smm}^{j_{-3}}(k) \le Q_{up-TR}^{j_{-3}}(k - t_{rbm-smm}^{-3}) + Q_{rbm-I}^{j_{-3}}(k-1) \quad (4.4\text{-}85)$$

in which, $t^{-3}_{rbm\text{-}smm}$: The lead-times of transporting reusable materials to secondary material market in layer -3.

(2) For derivative waste

The quantity of derivative waste transported in kth time interval ≦ The quantity of useless products treated in (k-t)th time interval ＋ The quantity of derivative waste stocked in (k-1)th time interval

$$Q_{dw-li}^{j_{-3}}(k) \le Q_{up-TR}^{j_{-3}}(k - t_{dw-li}^{-3}) + Q_{dw-I}^{j_{-3}}(k-1) \quad (4.4\text{-}86)$$

in which, $t^{-3}_{rbm\text{-}smm}$: The lead-times of transporting derivative waste to landfills/ incinerators in layer -3.

9. In jth secondary material market

The quantity of reused raw materials transported in kth time interval ≦ The quantity of reused raw materials manufactured in (k-t)th time interval ＋ The quantity of reused raw materials stocked in (k-1)th time interval

$$Q_{rrm-m}^{j_{-4}}(k) \le Q_{rrm-M}^{j_{-4}}(k - t_{rrm-m}^{-4}) + Q_{rrm-I}^{j_{-4}}(k-1) \quad (4.4\text{-}87)$$

in which, $t^{-4}_{rrm\text{-}m}$: The lead-times of transporting reusable materials to manufacturers in layer -4.

4.4.6 Weight Constraint

Finally, the constraint of weights is represented as follows.

$$w_1 + w_2 = 1 \tag{4.4-88}$$