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平行模擬車流波茲曼方程式之研究

A Parallel Monte Carlo Computing Technique for the Numerical Simulation of Traffic Boltzmann Transport Equation

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ABSTRACT

Lighthill and Whitham proposed their kinematic traffic flow model five decades ago, then the mathematical description of traffic flow operations has been a lively subject of research and debate for traffic scientists. There were a wide range of traffic flow theories and models, which were developed to describe traffic flow operations.

In this study, we focus on mesoscopic traffic flow model. The gas-kinetic traffic flow model of Prigogine and Andrews, one of mesoscopic traffic flow model, was developed during 1960's. They modified some of the key concepts in the kinetic theory of gases and wrote down an equation alike to the Boltzmann transport equation. We present a new traffic Boltzmann transport equation describes the dynamics of the velocity distribution functions of vehicles in the traffic flow. From the traffic Boltzmann transport equation, we consider the vehicles governed by drift, traffic field, deceleration, and lane-changing. The Monte Carlo simulation technique plays an important role in solving the complex equation.

The name of Monte Carlo simulation technique is usually given to stochastic methods that employ a stochastic process to simulate a system. In this study, Monte Carlo simulation technique is introduced to directly solve the traffic Boltzmann transport equation by direct physical simulation. Monte Carlo simulation technique offers an accurate description of transport, but it requires intensive computation and hence has not found wide use for traffic flow applications. Then we introduce the parallel Monte Carlo simulation technique used for improving the drawback of Monte Carlo simulation. Finally, comparison the simulation results with real VD data is discussed.

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Chapter 1 Introduction

Intelligent transportation systems (ITS) are the systems that employ advanced information and communication technologies to the operations of existing transportation systems in order to ensure traffic safety and transportation mobility, and improve traffic congestion and environmental impacts. For the purpose of ITS applications, real-time prediction is needed. Managing traffic real-time in congested road or networks requires a clear understanding of traffic flow operations. For this purpose, during the past fifty years, there were a wide range of traffic flow theories and models, which were developed to answer these research questions.

1.1 Motivation and Objective

Modeling and computer simulation play an increasing role in the optimization of traffic flow. Traditionally there have been two types of approach to the problem. Macroscopic models based on fluid dynamic equations have been proposed by a large number of authors, see, e.g., Lighthill and Whitham [1], May [2] and Payne [3]. However, some of these models have been subject to considerable controversy, concerning their validity and applicability to traffic flow. Microscopic or car-following model are the most basic models, modeling the actual response of individual vehicles to their predecessor by ordinary differential equations based on Newton’s law. They have been investigated with many authors, e.g. Prigogine [4,5], Reuschel [6], Chandler [7], Gazis [8] and Herman [9]. From the viewpoint of applicability to model-based estimation, prediction, and control, the absence of a closed analytic solution presents a problem that is not easily solved. That is the reason that microscopic simulation models are ideally suited for off-line simulations, for instance to test roadway geometry.

Another interesting approach to the study of traffic flow is the kinetic models. Kinetic models in traffic flow started originally with the work of Prigogine et al. [4,5], who introduced a kinetic term to account for the slowing down interactions. Kinetic models may present an intermediate step between the above two types of model, even though they are included of microscopic models. They are based on Boltzmann type kinetic equation. On the one hand they can be more fundamentally justified than the standard macroscopic models, leading to a better justification of the macroscopic models and potentially to more accurate results. On the other side, compared to microscopic models, computation time is strongly decreased. This may make the kinetic models applicable to the description of real life situations and traffic control problems. For this reason, some scholars think this kind of model as a mesoscopic model.

Cho and Lo [10] improved Prigogine model by considering acceleration as influence of traffic field, namely traffic Boltzmann transport equation or Boltzmann-like model. Boltzmann-like model is a mesoscopic model, which can infer to macroscopic models and can be developed with behavioral analysis, so as to improve the lack of behavior of macroscopic model. Nevertheless, the resulting equations have been criticized for having too many parameters and high dimensionality, hampering calibration and their real-time applicability. This is an important cause that the thesis will be focus on Boltzmann-like model. We will simulate the Boltzmann-like model in parallel to give the usefully real-time data for ITS, and to make the ITS approach effective (e.g. traffic signal control, incident management, integrated traffic responsive metering).

1.2 Study Procedure

As show in Figure 1.1, we first survey and confirm the study issue for the thesis. The second frame reviews some dynamic traffic flow models, some computational methods for the Boltzmann transport equation, and the development of Message Passing Interface (MPI). The third frame shows our main research methodologies that include of Boltzmann transport equation for microscopic traffic flow, Monte Carlo simulation method, and parallelism architecture. In the forth step, we will use Monte Carlo simulation method and parallel method to solve the traffic Boltzmann transport equation. Furthermore, we analyze the simulation results and discuss the performance of the parallel system in this study. After that we will compare simulation results with real VD data. Finally, some conclusions and further works are given.

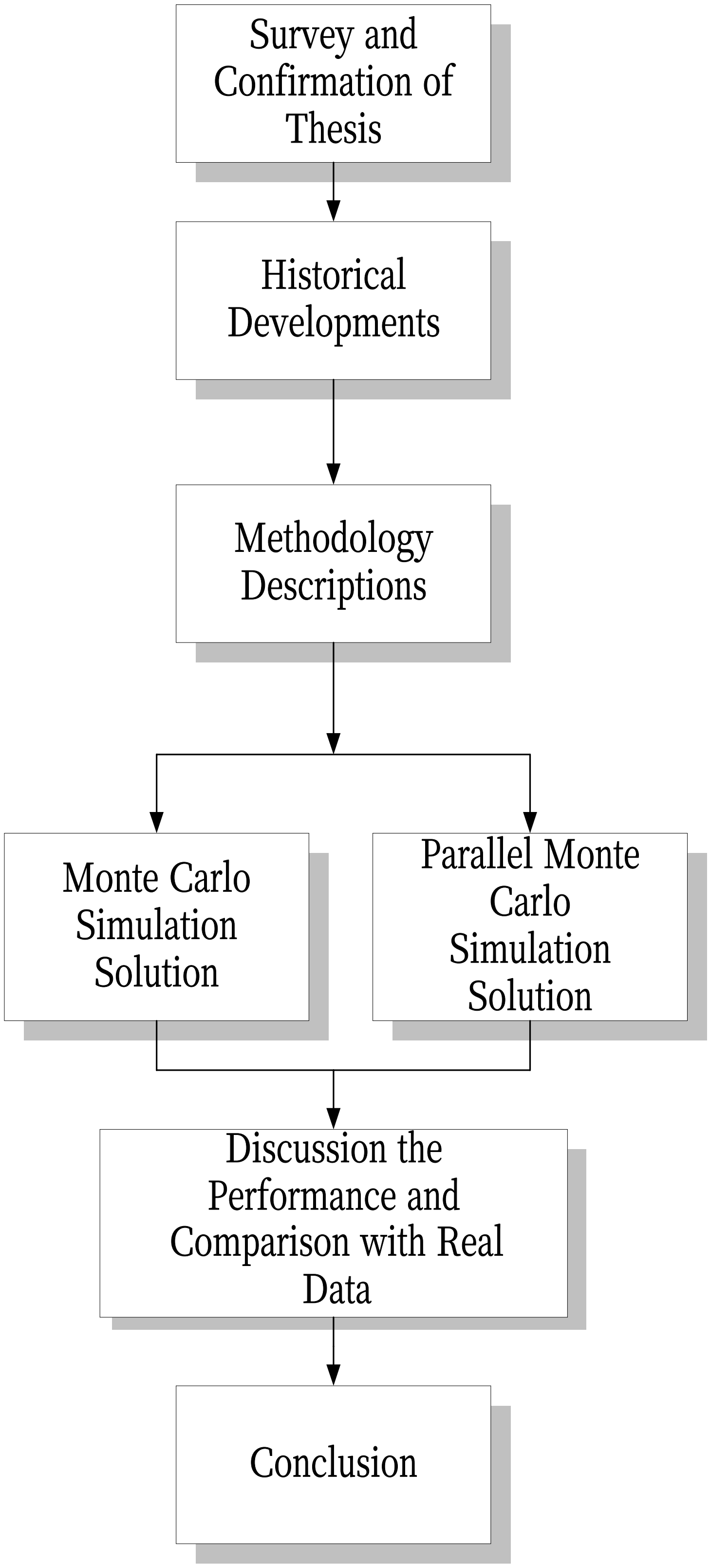


Fig.1.1 A flowchart of this study

1.3 Outline of Thesis

The rest of the thesis is organized on the following way: In chapter 2 we will review some relative literatures. In chapter 3, we formulate the traffic flow model. Our main methodologies of the thesis are stated in chapter 4. Numerical simulation of freeway traffic flow is discussed in the chapter 5. Finally, expected results are given in chapter 6.

In chapter 2, we will briefly review the macroscopic traffic flow models and microscopic traffic flow models. Then we will focus on the discussion about mesoscopic traffic flow models, some computational methods for the Boltzmann transport equation, and the development of Message Passing Interface (MPI).

In chapter 3, we particularly discuss our modeling, included of traffic Boltzmann transport equation and scattering mechanisms.

In chapter 4, we will discuss the purpose of this thesis and our methodologies. Parallel Monte Carlo computing technique for the numerical simulation of traffic Boltzmann transport equation will be used in this study.

In chapter 5, some example is experimental to obtain a first impression of the validity of the traffic flow model. Furthermore, performance results for parallel Monte Carlo simulation technique are discussed. Eventually, comparison with VD real data is required to obtain accuracy of the model and numerical scheme.

In chapter 6, the conclusion and results are stated.

Chapter 2 Literatures Review

In this chapter, we first review the dynamic traffic flow model, including the macroscopic traffic flow models and microscopic traffic flow models. Then we focus on the discussion about the mesoscopic traffic flow models and some computational methods for the Boltzmann transport equation, and the development of Message Passing Interface (MPI).

2.1 A Review on Dynamic Traffic Flow Model

Traffic operations on roadways can be improved by field research and field experiments of real-life traffic flow. However, apart from the scientific problem of reproducing such experiments, costs and safety play a role of dominant importance as well. Due to the complexity of the traffic flow system, analytical approaches may not provide the desired results. Therefore, for almost half a century physicists have been trying to understand the fundamental principles governing the flow of vehicular traffic using theoretical approaches based on statistical physics. A physicist would like to develop a model of traffic by incorporating only the most essential ingredients which absolutely necessary to describe the general features of typical real traffic.

Consequently, traffic flow or simulation models designed to characterize the behavior of the complex traffic flow system have become an essential tool in traffic flow analysis and experimentation. Usually, we can develop traffic analytical techniques by two different points of view, microscopic and macroscopic analysis.

2.1.1 Macroscopic Traffic Flow Models

Macroscopic traffic flow models assume that the aggregate behavior of drivers depends on the traffic conditions in the drivers' direct environments. In the "coarse-grained" fluid –dynamical description, the traffic is viewed as a compressible fluid formed by the vehicles but these individual vehicles do not appear explicitly in the theory. Macroscopic system may be selected for higher-density, large-scale systems in which a study of the behavior of groups of units is sufficient. Generally, calibration of macroscopic models is relatively simple (compared to microscopic models). Most macroscopic traffic flow models describe the dynamics of the density *k*, the velocity *u*, and the flow *q* and the relationship between them is $q = k \times u$. Some researches investigated speeds, flows and densities from low quality time lapse film and got the information of vehicle tracking which proved vehicular platoon can be treated as stream of fluid. The general macroscopic traffic flow models are always noted, include Lighthill-Whitham-Richards models [1], Payne-type models [3] and Helbing-type models [11].

However, macroscopic models are generally too coarse to correctly describe microscopic details and impacts, for instance caused by changes in roadway geometry. Due to the availability of closed analytical solutions, there are however very suitable for application in model-based estimation, prediction, and control of traffic flow.

2.1.2 Microscopic Traffic Flow Models

In contrast, in the so-called "microscopic" models of vehicular traffic attention is explicitly focused on individual vehicles each of which is represented by a particle; the nature of the interactions among these particles is determined by the way the vehicles influence each other's movement. In other words, in the "microscopic" theories vehicular traffic is treated as a system of interacting particles driven far from equilibrium.

In this section, we discuss microscopic traffic flow models, the development of which started during the sixties with the so-called car-following models. We will concisely review some different types of microscopic models.

2.1.2.1 Car-Following Models

During the 1960's, research efforts focused on the so-called follower-the-leader models [6,12]. The basic idea of the models is that the following drivers will correspond to the action of the preceding vehicle. This car-following process is base on the following principle:

$$[response]_n \propto [stimulus]_n \tag{2.1}$$

for the *n*-th vehicle (n = 1, 2, …). In general, the *response* is the braking or the acceleration of the following vehicle, delayed by an overall reaction time T. Each driver can respond to the surrounding traffic conditions only by accelerating or decelerating the vehicle. Different forms of the equations of motion of the vehicles in the different versions of the car-following model arise from the differences in their assumption regarding the nature of the stimulus. The stimulus may be compose of the speed of the vehicle, the difference in the speeds of the vehicle under consideration and its lead vehicle, the distance-headway, etc., and, therefore, in general,

$$a_n(t) = f_{sti}(v_n, \Delta x_n, \Delta v_n), \tag{2.2}$$

where the function f_{sti} represents the stimulus received by the *n*-th vehicle. If the function f_{sti} is assumed or modeled, a different car-following model is derived. For example, a well-known model specification is [6]:

$$a_n(t+T) = \gamma(\dot{x}_{n+1}(t) - \dot{x}_n(t)), \tag{2.3}$$

where the sensitivity coefficient γ is a constant, which is independent of *n*, *T* is a response time lag. A survey can be found in Gazis, Herman and Potts [8]. The model is a second order ODE

$$a_n(t+T) = \frac{c(\dot{x}_n(t))^m}{(x_{n-1}(t) - x_n(t))^l}(\dot{x}_{n-1}(t) - \dot{x}_n(t)), \tag{2.4}$$

with parameters *T*, *c*, *m*, *l*. the basic idea is that the acceleration at time *t*+*T* depends on the speed of the vehicle at time *t*, the relative speeds of the vehicle and its leading vehicle at time *t* and the distance between the vehicles. *T* is a typical reaction time of the driver. *c*, *m*, *l* are fitted to special situations. Since lane-changing processes cannot be easily described, car-following models have been mainly applied to single lane traffic [13] and traffic stability analysis [2,9]. Todosiev [14] and Wiedemann [15] introduced psycho-physiological considerations into the car-following models. Wiedemann considers so-called reaction thresholds to distinguish different regions of driver behavior.

2.1.2.2 Cellular Automaton Models

A more recent addition to the development of microscopic traffic flow theories are the so-called Cellular Automaton (CA) or Particle Hopping models. CA-models describe the traffic system as a lattice of cells of equal size (typically 7.5m). A CA-model describes in a discrete way the movements of vehicles from cell to cell [16,17]. CA-models aim to combine advantages of complex micro-simulation models, while remaining computationally efficient. However, the car-following rules of both the space-oriented and time-oriented CA-models lack intuitive appeal and their exact mechanisms are not easily interpretable from the driving-task perspective. Moreover, they are too crude to describe and study microscopic details of traffic flow sufficiently accurate from a single driver's perspective.

2.1.3 Mesoscopic Traffic Flow Models

Instead of describing the traffic dynamics of individual vehicles, mesoscopic traffic flow model describe the dynamics of the velocity distribution functions of vehicles in the traffic flow. In this section, we first review the seminal models, gas-kinetic traffic flow models of Prigogine and coworkers [4,5], after which a few extensions to this model type are dealt with.

In the kinetic theory of gases, the Boltzmann transport equation, which describes the time-evolution of distribution $f(x, v, t)$, is given by

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla_r + \frac{\vec{F}}{m} \cdot \nabla_v \right) f(\vec{r}, \vec{v}, t) = \left(\frac{\partial f}{\partial t} \right)_{collision}, \tag{2.5}$$

where the symbols ∇_r and ∇_v denote gradient operations with respect to $\vec{r}$ and $\vec{v}$, respectively, while $\vec{F}$ is the external force. The term $\left(\frac{\partial f}{\partial t} \right)_{collision}$ represents the rate of change of *f*, with time, which is caused by the mutual collisions of the molecules.

Prigogine and coworkers modified some of the key concepts in the kinetic theory of gases and wrote down an equation alike to the Boltzmann transport equation. Suppose the velocity distribution function, $f(x, v, t)dxdv$, denotes the number of vehicles, at time *t*, between *x* and *x*+*dx*, having actual velocity between *v* and *v*+*dv*. In addition, Prigogine and coworkers introduced a desired distribution $f_0(v)$ which is a mathematical idealization of the goals that the population of the drivers collectively strives to achieve. Prigogine and coworkers suggested that the analogue of the

Boltzmann transport equation for the traffic should have the form

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = \left(\frac{\partial f}{\partial t}\right)_{relaxation} + \left(\frac{\partial f}{\partial t}\right)_{interaction}, \tag{2.6}$$

The first term on the right-hand side accounts for the relaxation of f towards f_0 in the absence of mutual interactions of the vehicles, and may be interpreted as the counterpart of the term $\vec{F} \cdot \nabla_v f(\vec{r}, \vec{v}, t)$ in the above equation (2.5). The second term takes in to account traffic interaction. For the relaxation and interaction terms can take an expression of the form:

$$\left(\frac{\partial f}{\partial t}\right)_{relaxation} = -\frac{f - f_0}{T}, \tag{2.7}$$

where T is relaxation time.

$$\left(\frac{\partial f}{\partial t}\right)_{interaction} = (1-P)k(\bar{v} - v)f\ , \tag{2.8}$$

where P is the probability of a car passing another one, $\bar{v}$ is the average speed of traffic, and k is the concentration of the traffic flow.

Then Anderson, Herman, and Prigogine [18] discussed the homogeneous time-independent solution for a one-car speed distribution function, that is derived for a Boltzmann-like approach to the statistical theory of traffic flow. Numerical results are discussed for a number of different desired speed-distribution functions. Herman and Lam [19] extended the model as a kinetic equation consisting of three additive terms, each approximating one of the following traffic processes: relaxation, interaction, and adjustment, as

$$\left(\frac{\partial f}{\partial t}\right)=\left(\frac{\partial f}{\partial t}\right)_{relaxation}+\left(\frac{\partial f}{\partial t}\right)_{interaction}+\left(\frac{\partial f}{\partial t}\right)_{adjustment} \tag{2.9}$$

Paveri-Fontana [20] argued that each vehicle, in contrast to the molecules in a gas, has a desired velocity towards which its actual velocity tend to “relax” in the absence of “interaction” with other vehicles. Thus, Paveri-Fontana’s model is based on a scenario of relaxation of the velocities of the individual vehicles rather than a collective relaxation of the distribution of the velocities. Paveri-Fontana introduced an additional phase-space coordinate, namely, the desired velocity. Suppose, $g(x,v,v_0,t)dxdvdv_0$ denotes the number of vehicles at time t between x and $x+dx$, having actual velocity between v and $v+dv$ and desired velocity between v_0 and v_0+dv_0. The one-vehicle actual velocity distribution function is described as

$$f(x,v,t)=\int g(x,v,v_0,t)dv_0 \tag{2.10}$$

Similarly, the one-vehicle desired velocity distribution function is described as

$$f_0(x,v_0,t)=\int g(x,v,v_0,t)dv \tag{2.11}$$

However, Paveri-Fontana’s model has one more speed dimension, increasing the complexity of the problem considerably. Edie et al. [21] determined the reasonable accuracy what the speed distributions are on a multiple-lane roadway under steady state (spatially homogeneous and time independent) conditions and to compare the characteristics of the observed distributions with those derived from the theory.

Lampis [22] modified the Prigogine kinetic equation. There, queueing vehicles have been included in the Prigogine equation by introducing a speed distribution for queues. Nelson [23] and Nelson et al. [24] derived a model for the usual distribution function $f(x,v,t)$ strictly from microscopic considerations. He treated the acceleration term in a way similar to the one Prigogine used for the braking term. However, as he himself states, his model is a caricature of traffic flow and should be seen only as a first step in obtaining a kinetic equation that is also suitable for real application. Helbing [25] resents a gas-kinetic model for multilane traffic flow operations. The approach is similar to the approach of Paveri-Fontana, although lane changing is explicitly considered. Another multilane gas-kinetic model was proposed by Klar and Wegener in 1998 [26].

Recently, Hoogendoorn et al. [27,28] developed a platoon-based multilane multi-class model that is describing the dynamics of general traffic flow systems. They consider that each class has different behavior and describe by different conservation law, and space of a road section is limit and each class of user shares the space. Nevertheless, these models have been criticized for having too many parameters and high dimensionality, increasing the difficulty to find the numerical solutions.

Cho and Lo [10] improved Boltzmann transport equation by considering acceleration as influence of traffic field. It may assume that there exists a velocity distribution function $f(\mathbf{x},\mathbf{v},t)$, where $\mathbf{x}=(x,y)$, $\mathbf{v}=(v_x,v_y)$. The Boltzmann transport equation is shown by

$$\frac{df(\mathbf{x},\mathbf{v},t)}{dt}=\frac{\partial f(\mathbf{x},\mathbf{v},t)}{\partial t}+\mathbf{v}\cdot\nabla_{\mathbf{x}}f(\mathbf{x},\mathbf{v},t)+e\mathbf{E}\cdot\nabla_{\mathbf{v}}f(\mathbf{x},\mathbf{v},t)=\left(\frac{\partial f(\mathbf{x},\mathbf{v},t)}{\partial t}\right)_{collision}$$

$$f(\mathbf{x},\mathbf{v},t)\big|_{\partial\Omega_{\mathbf{v}}}=0\,, \tag{2.12}$$

where f is defined on Ω and $\partial\Omega_v$ is the boundary of individual velocity, $\mathbf{E}$ is defined traffic field. Traffic field is employed to describe the dependency among vehicles traveling in a platoon. The concept of traffic field is extended from car-following theory. The relation between traffic field and density results in a Poisson equation [29,30]. Nevertheless, the resulting equations have been criticized for having too many parameters and high dimensionality, hampering calibration. We will simulate the traffic Boltzmann transport equation by Monte Carlo simulation technique with a simpler traffic field function that is considered in the study.

2.2 A Review on the Computational Methods for the Boltzmann Transport Equation

Boltzmann transport equation describes physical phenomena, which are often of great engineering and technological importance (in aerospace industry, semiconductor design, or currently in traffic flow research). For this reason, analytical and computational methods of solving have Boltzmann transport equation has been studied extensively since the first computer hardware making these calculations feasible. In general, the computational techniques for the Boltzmann transport equation can be divided into two categories: deterministic methods and stochastic methods.

The deterministic methods combine finite difference [31], finite element or finite volume approximations of the free flow equation with an appropriate evaluation method for the collision operator. In these schemes the substantive difficulty is the evaluation of the collision operator. These evaluation methods can be divided into two groups: statistical quadratures and regular quadratures. In the first group the Monte Carlo quadratures are applied to evaluate the collision operator. This approach was initiated from Nordsieck in 1955 [32]. In the second group the collision operator is evaluated analytically or numerically (using regular quadratures) for particular discretizations of the distribution function, as was done e.g. by Aristov [33] and Tan et al. [34].

The stochastic methods constitute the other important area within computational techniques for solving the Boltzmann transport equation. Flow simulation methods, were known as Direct Simulation Monte Carlo methods, were initiated by Bird [35]. Since then his method was undergoing subsequent improvements and modifications. The simulation method, was derived from the Boltzmann transport equation, was presented by Nanbu [36,37,38,39] in the series of his papers. Babovsky [40] modified the Nanbu method essentially. The new model reduced its high computational complexity and that made the algorithm applicable.

Monte Carlo methods are in fact computationally effective, compared with deterministic methods when treating many dimensional problems. That is partly why their use is so widespread in operations research, in radiation transport or the particles' transport in semiconductor devices (where problems in up to seven dimensions must be dealt with), and especially in statistical physics and chemistry (where systems of hundreds or thousands of particles can now be treated quite routinely). Monte Carlo simulation offers an accurate description of transport, but it requires intensive computation and hence has not found wide use for traffic Boltzmann transport equation.

2.3 Introduction to the Development of Message Passing Interface

We will simulate the traffic Boltzmann transport equation in parallel with Message

Passing Interface (MPI). In this section, we will review the development of MPI.

MPI is a library of routines that can be used to create parallel programs in C, C++ or Fortran. Standard C, C++ and Fortran include no constructs supporting parallelism so vendors have developed a variety of extensions to allow users of those languages to build parallel applications. The result has been a spate of non-portable applications, and a need to retrain programmers for each platform upon which they work.

The MPI standard was developed to improve these problems. It is a library that runs with standard C, C++ or Fortran programs, using commonly available operating system services to create parallel processes and exchange information among these processes.

MPI is designed to allow users to create programs that can run efficiently on most parallel architectures. The design process included vendors (such as IBM, Intel, TMC, Cray, Convex, etc.), parallel library authors (involved in the development of PVM, Linda, etc.), and applications specialists. The final version for the draft standard (MPI-1.0) became available in May of 1994 [41]. Beginning in March 1995, the Message Passing Interface Forum reconvened to correct errors and make clarifications in the MPI document of May 5, 1994, referred to below as Version 1.0. These discussions resulted in Version 1.1. The changes from Version 1.0 are minor. It extended to the enhanced standard (MPI-2.0) in 1998. From MPI specifications, we know MPI is a message-passing model, not a compiler specification and a specific product. The more explicit discussion of parallelism architecture is stated in section 4.4.

Chapter 3 Traffic Boltzmann Transport Equation

The purpose of this thesis is to use parallel Monte Carlo computing technique for the numerical simulation of Traffic Boltzmann transport equation. Firstly, how to describe the motion of the vehicle through its distribution function in the Traffic Boltzmann transport equation is what we want to know. We will discuss particularly in this chapter. Then, we discuss Monte Carlo computing technique and construct a programming environment of parallel computers included of hardware and software in the next chapter.

3.1 Boltzmann Transport Equation for Mesoscopic Traffic Flow

Boltzmann transport equation, which is a mesoscopic kinetic equation, is widely applied in applied science, such as gas dynamics, population analysis, semiconductor, traffic flow and so on [42]. The Boltzmann transport equation is a continuity equation for the single particle distribution function $f(x,v,t)$ of a molecular substance. How to describe the motion of the vehicle through its distribution function in the Boltzmann transport equation for mesoscopic traffic flow is the point we want to know.

Consider a multilane freeway on which passing is allowed to occur. We consider a multilane road as a two-dimensional space herein as been presented in Figure 3.1. We may assume that there exists a distribution function $f(\mathbf{r},\mathbf{v},t)$, where $f(\mathbf{r},\mathbf{v},t)d\mathbf{r}d\mathbf{v}$ represents the number of vehicles dN with position $\mathbf{r}(x,y)$ and velocity $\mathbf{v}(v_x,v_y)$ inside the volume element $d\mathbf{r}d\mathbf{v}$ at time t.

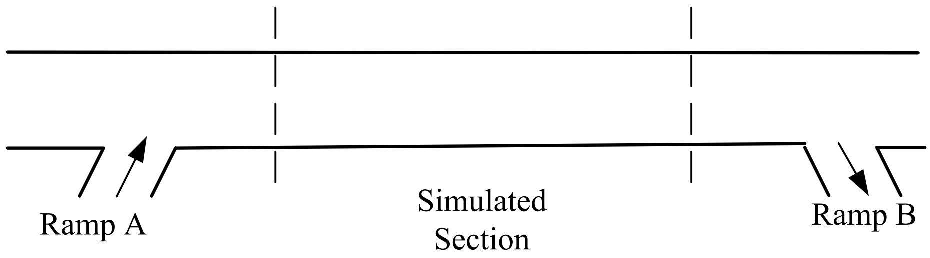


Fig. 3.1 The simulated section of the highway.

The time evolution of this distribution function is influenced by several factors, including the drift of vehicles into or out of the region $\mathbf{r}+d\mathbf{r}$, the presence of an externally applied field and the scattering of vehicles with other vehicles in the system. Each of these events brings about a different change to the distribution as time changes and all need to be handled separately.

If we consider vehicles moving at a velocity of **v** at time *t*, we may assume that the number of vehicles that will drift into the region **r** will be the same that exists in the region **r**-**v***t* at *t*=0. What this means, is that in some time interval *t*, particles move a distance of **v***t* which brings them into a region **r**. These vehicles, which now reside in **r** at time *t*, must have resided in **r**-**v***t*, the state before the moved at *t*=0. Therefore, from this it can be determined that the distribution function at **r** at time *t* is the same as that at region **r**-**v***t* at time *t*=0. This can be written as

$$f(\mathbf{r},\mathbf{v},t)=f(\mathbf{r}-\mathbf{v}t,\mathbf{v},0)\,. \tag{3.1}$$

By Taylor's expansion and dropping the high-order terms, the time change of the distribution function due to drift can be shown to be

$$f(\mathbf{r}-\mathbf{v}t,\mathbf{v},0)=f(\mathbf{r},\mathbf{v},t)+f_{\mathbf{r}}(\mathbf{r},\mathbf{v},t)\cdot(\mathbf{r}-\mathbf{v}t-\mathbf{r})+f_{t}(\mathbf{r},\mathbf{v},t)\cdot(0-t)+\cdots$$

$$\left(\frac{\partial f(\mathbf{r},\mathbf{v},t)}{\partial t}\right)_{drift} = -\mathbf{v}\cdot\nabla_{\mathbf{r}} f(\mathbf{r},\mathbf{v},t)\,. \qquad (3.2)$$

Next we consider an externally applied field, traffic field, on the system. When an external traffic field is applied, the momentum (velocity) of the vehicles is changed by the virtual force field that excerpts on them. We assume that the acceleration characteristics only vary between vehicle-type and roadway geometry. This rate of change is given within Newton's equation of motion as

$$\frac{d\mathbf{v}}{dt} = e\mathbf{E}\,, \qquad (3.3)$$

where e is a scalar which is changed with the vehicle-type, and **E** denotes the traffic field, which is the vector component directed in either x and y direction. By the virtual force field, the vehicles are accelerated towards their desired velocity or the maximum velocity of lane. Therefore, similarly to the drift term, vehicles are accelerated out of certain regions and into new regions by this force. Vehicles that resided in region **r** at time t were moved to this region by a force from their original location, $\mathbf{v}-\dot{\mathbf{v}}t$, at time t=0. This gives us a similar expression to (3.1) and we can write

$$f(\mathbf{r},\mathbf{v},t) = f(\mathbf{r},\mathbf{v}-\dot{\mathbf{v}}t,0)\,. \qquad (3.4)$$

Therefore, similar to drift term, the time rate of change of the distribution function due to this external virtual force may be written as

$$\left(\frac{\partial f(\mathbf{r},\mathbf{v},t)}{\partial t}\right)_{field} = -\dot{\mathbf{v}}\cdot\nabla_{\mathrm{v}} f(\mathbf{r},\mathbf{v},t)\,. \tag{3.5}$$

In order to derive the changes in the distribution function due to scattering mechanisms, certain assumptions are made. Firstly, these scattering mechanisms are independent of any spatial or time dependence. For this to occur, the scattering mechanisms are assumed to be instantaneous and that the vehicles distributions are homogeneous in space. This allows the time and spatial terms to be dropped from the equation leaving an equation in terms of only the phase space. We can now write

$$\left(\frac{\partial f(\mathbf{r},\mathbf{v},t)}{\partial t}\right)_{scattering} = -\int\left[S(\mathbf{v},\mathbf{v}')f(\mathbf{v}) - S(\mathbf{v}',\mathbf{v})f(\mathbf{v}')\right]d\mathbf{v}'\,, \tag{3.6}$$

where $S(\mathbf{v},\mathbf{v}')$ is the transition rate from **v** to $\mathbf{v}'$. This equation therefore describes the change, e.g. due to interactions with ahead slower vehicle or accident etc, in the distribution function in terms of vehicles scattering into a state and vehicles scattering out of that same state. We will discuss the scattering mechanisms in this thesis particularly in next section.

The final equation which describes the total change in the distribution function as a function of time, which is the known as the time dependant Boltzmann transport equation for mesoscopic traffic flow, is given by

$$\frac{\partial f(\mathbf{r},\mathbf{v},t)}{\partial t} = \left(\frac{\partial f(\mathbf{r},\mathbf{v},t)}{\partial t}\right)_{drift} + \left(\frac{\partial f(\mathbf{r},\mathbf{v},t)}{\partial t}\right)_{field} + \left(\frac{\partial f(\mathbf{r},\mathbf{v},t)}{\partial t}\right)_{scattering}\,. \tag{3.7}$$

In general, we can write the traffic Boltzmann transport equation as

$$\frac{\partial f(\mathbf{r},\mathbf{v},t)}{\partial t}+\mathbf{v}\cdot\nabla_{\mathbf{r}} f(\mathbf{r},\mathbf{v},t)+\dot{\mathbf{v}}\cdot\nabla_{\mathbf{v}} f(\mathbf{r},\mathbf{v},t)=\left(\frac{\partial f(\mathbf{r},\mathbf{v},t)}{\partial t}\right)_{scattering}$$

$$\left(\frac{\partial f(\mathbf{r},\mathbf{v},t)}{\partial t}\right)_{scattering}=-\int\left[S(\mathbf{v},\mathbf{v}')f(\mathbf{v})-S(\mathbf{v}',\mathbf{v})f(\mathbf{v}')\right]d\mathbf{v}'. \tag{3.8}$$

Next chapter, we will simulate the traffic Boltzmann transport equation (3.8) by using Monte Carlo simulation technique.

Except for describing the behavior of vehicles in a multilane freeway, this distribution function also can be used to obtain various macroscopic quantities interested and some of the important macroscopic variables are defined below. For example, density is given by

$$k(\mathbf{r},t)=\int_{\mathbf{v}} f(\mathbf{r},\mathbf{v},t)d\mathbf{v}, \tag{3.9}$$

and flow density, is defined by Cho and Lo [10], is given by

$$q(\mathbf{r},t)=\int_{\mathbf{v}} \mathbf{v}f(\mathbf{r},\mathbf{v},t)d\mathbf{v}=k(\mathbf{r},t)\mathbf{u}(\mathbf{r},t), \tag{3.10}$$

where $\mathbf{u}(\mathbf{r},t)$ denote group velocity, which is defined as

$$\mathbf{u}(\mathbf{r},t)=\frac{\int_{\mathbf{v}}\mathbf{v}f(\mathbf{r},\mathbf{v},t)d\mathbf{v}}{\int_{\mathbf{v}}f(\mathbf{r},\mathbf{v},t)d\mathbf{v}}. \tag{3.11}$$

3.2 Scattering Mechanisms

The knowledge of scattering mechanisms is essential for the Monte Carlo simulation, since they control the nature of the vehicle transport. Scattering mechanisms are very “flexible”. With more knowledge of scattering mechanisms, more the accuracy of simulation result will be obtained. Scattering is the process whereby vehicles undergo a transition from one state, $\mathbf{r}$ and $\mathbf{v}$, to a new state $\mathbf{r}'$ and $\mathbf{v}'$. In this study, we consider that this may occur due to vehicle’s lane-changing and deceleration caused by interactions with ahead slower vehicle. When a vehicle driving with velocity $\mathbf{v}$ catches up with a slower vehicle, it means that the deceleration from filed term is not enough. It either needs to more reduce its velocity, or perform an immediate lane change. Scattering in the traffic system is responsible for limiting and controlling the vehicle dynamics. Without the scattering events, vehicles would constantly increase their velocity to their excepted one by a factor proportional to any applied bias field that may exist. And the event that one vehicle covering the other one may happen.

In this study, the vehicle’s scattering is not real collision as one between particles in gas or electrons in device and the length of vehicles can’t be neglected. Therefore, we assumed that the virtual length of vehicle (safe distance plus assumptive length of vehicle) depends on velocity. The safe distance data of vehicles extracted from video file taken of the National Freeway No.1 (Shijr-Wuku Overpass Southbound 14.733km) are used. This gives a 200m field of view, and data in Figure 3.2 are obtained on the passage of a total of 200 vehicles that have the follow behavior obviously. From empirical value and simplifying the complication of the problem, the virtual length of vehicle is stated in Table 3.1, and the ellipse in the Figure 3.3.

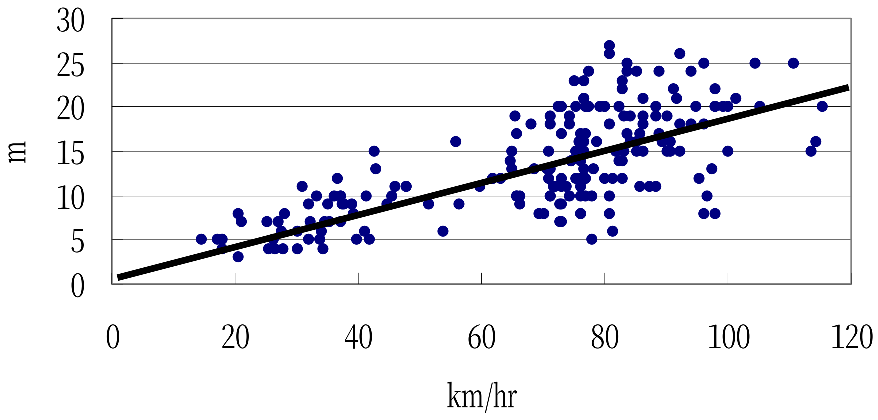


Fig. 3.2 The safe distance data of 200 vehicles extracted from video file.

Table 3.1 The virtual length of vehicle.

Velocity	The virtual length of vehicle h: $v*\alpha$ +7.50 (m)	
27.78m/s (100km/hr)	when $\alpha = 0.7$	19.45+7.50=26.95m
22.22m/s (80km/hr)		15.55+7.50=23.05m
16.67m/s (60km/hr)		11.67+7.50=19.17m
11.11m/s (40km/hr)		7.78+7.50=15.28m
5.56m/s (20km/hr)		3.89+7.50=11.39m

Firstly, we assume that when an incoming fast moving vehicle with velocity $\mathbf{v}_1$ reaches a slow moving vehicle with velocity, $\mathbf{v}_1 > \mathbf{v}_2$, as shown in Figure 3.3 (a), it either passes directly as shown in (b), or it slows down to $\mathbf{v}_1'$ in (c)

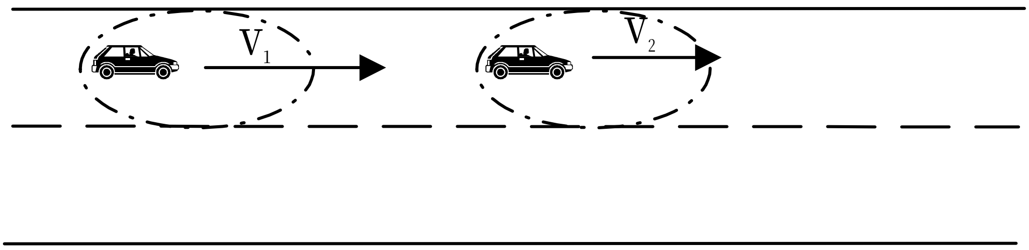


Fig. 3.3 (a) Vehicles with $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_1 > \mathbf{v}_2$.

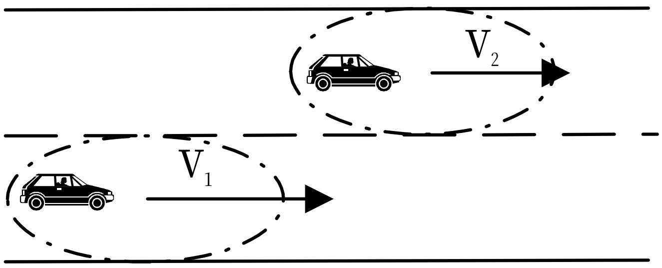


Fig. 3.3 (b) Scattering mechanisms between vehicles: lane changing scattering.

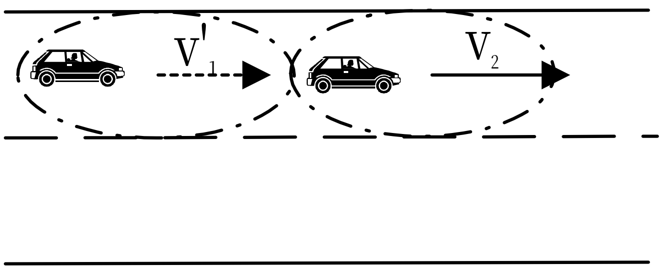


Fig. 3.3 (c) Scattering mechanisms between vehicles: deceleration scattering.

We will generate a random number to determine which scattering mechanism is happened. And the two types of scattering mechanisms that listed as following are taken into account:

- Lane Changing Scattering

Lane changing can be classified into mandatory and discretionary [43].

Mandatory lane changing occurs when drivers have to change lanes in order to: (1) connect to the next link on their path; (2) bypass a lane blockage downstream; (3) avoid entering a restricted use lane, etc. Discretionary lane changing refers to cases in which drivers change lane in order to bypass a slower vehicle. To simplify the complication of the lane-changing problem, we only consider the discretionary lane changing.

For discretionary lane changing, the decision to change is based on the traffic condition of both the current lane and adjacent lanes. If a vehicle has a speed lower than the driver's desired speed or the maximum velocity of road due to a slow vehicle in front, it checks the neighboring lanes for opportunities to keep its speed. Firstly we generate a random number to determine which scattering mechanism is occurred. Once a vehicle has decided to change lane, we will generate a random number to determine a desired lane after the scattering. Then it examines the gap in the target lane to determine whether the desired lane can be executed. If the gap is acceptable, the desired lane is executed instantaneously. If no, it slows down to the velocity of the slow vehicle in front. Thus the state of the vehicle after lane changing scattering is given by

$$\mathbf{v}_1' = \mathbf{v}_1 , \tag{3.12}$$

and

$$\mathbf{r}'(x, y) = \mathbf{r}(x, y \pm 1). \tag{3.13}$$

- Deceleration Scattering

The state after deceleration scattering, deceleration caused by interactions, could be described by

$$\mathbf{v}_1' = \beta \cdot \mathbf{v}_2 , \tag{3.14}$$

and

$$\mathbf{r}'(x,y) = \mathbf{r}(x,y), \tag{3.15}$$

where $\mathbf{v}'$ and $\mathbf{r}'$ are the state of vehicle after the scattering. From equation (3.14), the velocity of vehicle changes to β times of the velocity of former slow vehicle due to deceleration scattering. It causes that the distance between two vehicles becomes larger then the virtual length of vehicle in the rear again.

We can write the traffic Boltzmann transport equation as

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{r}} f + \dot{\mathbf{v}} \cdot \nabla_{\mathbf{v}} f = \left(\frac{\partial f}{\partial t} \right)_{scattering-lanechanging} + \left(\frac{\partial f}{\partial t} \right)_{scattering-deceleration} . \tag{3.16}$$

We will simulate the traffic Boltzmann transport equation (3.16) by using Monte Carlo simulation technique in the next chapter.

Chapter 4 Monte Carlo Simulation Technique

The Monte Carlo simulation technique is a stochastic method, which implies that is employing a stochastic process to simulate a system. We would like to know whether Monte Carlo calculations are in fact worth carrying out. This can be answered in a very pragmatic way: many people do them and they have become an accepted part of scientific practice in many fields. The reasons do not always depend on pure computational economy. Convenience, ease, directness, and expressiveness of the method are important assets. As we discuss above, the Monte Carlo simulation technique is an important scientific tool, which will help to develop an understanding of transport phenomena in traffic Boltzmann transport equation. The principle of the Monte Carlo simulation technique applied to the transport analysis is to simulate the motion of a single particle on the road. And we will improve the drawback of Monte Carlo simulation technique with MPI Library.

4.1 A Short History of Monte Carlo

Monte Carlo method is called after the city in the Monaco principality, because of roulette, a simple random number generator. The name and the systematic development of Monte Carlo methods date from about 1944.

Perhaps the earliest documented use of random sampling to find the solution to an integral is that of Comte de Buffon [44]. In 1777 he described the following experiment. A needle of length L is thrown at random onto a horizontal plane ruled with straight lines a distance d ($d > L$) apart as shown in Figure 4.1. What is the probability P that the needle will intersect one of these lines? Comte de Buffon performed the experiment of throwing the needle many times to determine P. He also carried out the mathematical analysis of the problem and showed that

$$P = \frac{2L}{\pi d}. \tag{4.1}$$

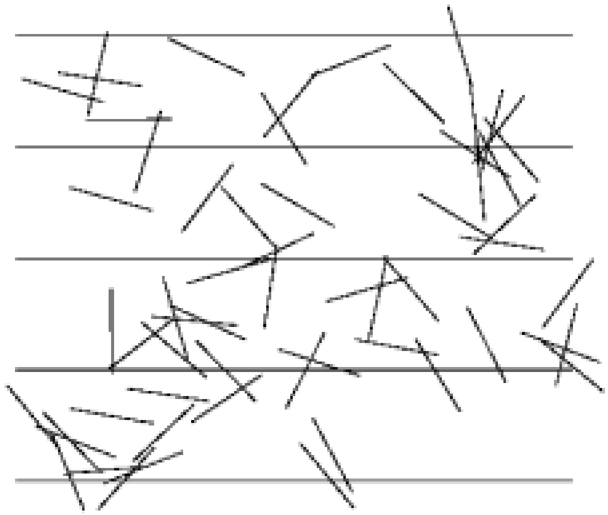


Fig. 4.1 The experiment of Comte de Buffon.

If we drop the needle N times and count R intersections we obtain

$$P = R / N,$$

$$\pi = 2LN / Rd. \tag{4.2}$$

Larger values of the parameter N will give us more accurate approximations of π.

4.2 Pseudorandom number generator

The Monte Carlo is a fundamental tool of computational statistics. At the kernel of a Monte Carlo or simulation method is random number generation. Nowadays most computers contain routines that generate random numbers evenly distributed between 0 and 1. A solution is determined by random sampling of the relationships, or the microscopic interactions, until the result converges. Thus, the mechanics of executing a solution involves repetitive action or calculation. To the extent that many microscopic interactions can be modeled mathematically, the repetitive solution can be executed on a computer. However, the Monte Carlo simulation technique predates the computer and is not essential to carry out a solution although in most cases computers make the determination of a solution much faster.

Various methods for generation of random numbers have been used. Sometimes processes that are considered random are used, but for Monte Carlo simulation techniques, which depend on millions of random numbers, a physical process as a source of random numbers is generally cumbersome. Instead of “random” numbers, most applications use “pseudorandom” numbers, which are deterministic but “look like” they were generated randomly. A simple pseudorandom number generator is given as follows:

- Program

```
/* a very simple random number generator */
#include   <stdio.h>
#include   <time.h>
#include   <stdlib.h>
#define   max   100                /* number of numbers generated */

void   main()
{
     time_t   t;
     double   x;
```

```
    int    y;
    srand((float) time(&t));        /* seed for number generator */


  /* generating random numbers */
  for (int i = 0; i < max; i++)
  {
    y = rand();
    y = y%987;
    x = y/986.0;
    printf ( "%f\n",x );
  }
}
```

4.3 Procedure of Monte Carlo Simulation Technique

Monte Carlo simulation technique is an important method for solving the traffic Boltzmann transport equation. The two main principles of a Monte Carlo procedure are that these vehicles are accelerated through the simulated system with the traffic field and that these vehicles are scattered due to some random scattering mechanism. The detail of Monte Carlo algorithm is stated as below and the flowchart is drawn in Figure 4.3.

Firstly, the simulated region is divided into a network of spatial cells with dimensions $\Delta x_i \Delta y$. Time is advanced by discrete steps of magnitude Δt small compared with the free time between scatterings.

Step1. Set the initial density ρ_0 each section of the highway.

Step2. From the initial density and a random number r_1, we obtain the initial position of the vehicle.

The distance between two vehicles ≧ 7.5m (the assumptive length of vehicle).

Step3. From the initial density and speed-density relation model, we obtain the initial velocity $\mathbf{v}_0(x,y)$ of the vehicle. We assume that the initial velocity is the same in one section.

May [45] suggested that a bell-shaped curve, which is presented in Figure 4.2, might fit some speed-density data very well, based upon empiric observations in several studies. The curve would be of the form.

$$v_e = v_f e^{-0.5(\rho/\rho_m)^2} \tag{4.3}$$

v_e : equilibrium velocity of the section with density ρ

v_f : free velocity

ρ_m : concentration at maximum flow

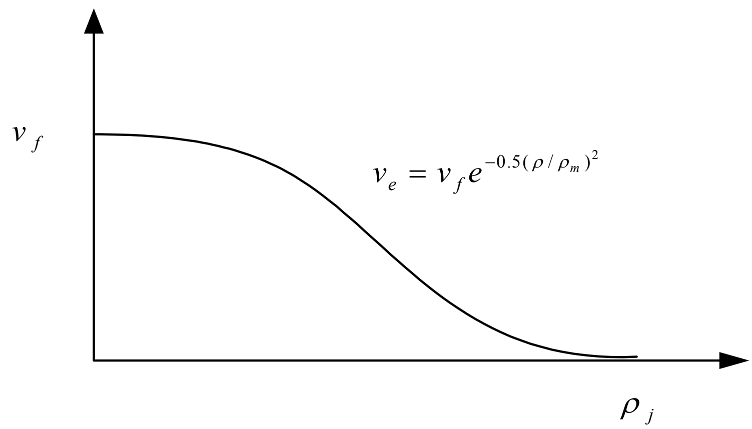


Fig. 4.2 Bell-shaped curve

Step4. If $\mathbf{r}_{ij} < h$, go to step 5.

$\mathbf{r}_{ij}$: the distance between vehicle i and vehicle j.

h: the virtual length of vehicle has defined in section 3.2.

Else, a virtual scattering mechanism called self-scattering is happened. The self-scattering does not affect the state of the vehicle, which maintains the same position and velocity as it had before. Go to step 7.

Step5. The scattering mechanism is selected by generating a random number r_2 and lane changing scattering probability. To simplify the complication of the lane-changing problem, we only consider that lane-changing scattering probability is equal to $n/(n+1)$, and n is the number of lanes that can be arrived. Therefore, it is necessary for executing the adjacent lanes if the gap distance is acceptable. The probability of deceleration scattering is fixed $1/(n+1)$. Decide the scattering process by generating a random number $r_2 \in [0,1]$. For example, if the adjacent lanes can be arrived all. Then the probability of changing to left lane, changing to right lane and the deceleration scattering is all 1/3.

Step6. Determine the state of the vehicle after the scattering.

Lane-changing scattering:

$$\mathbf{v}_1' = \mathbf{v}_1, \tag{4.4}$$

$$\mathbf{r}'(x, y) = \mathbf{r}(x, y \pm 1). \tag{4.5}$$

Deceleration scattering:

$$\mathbf{v}_1' = \beta \cdot \mathbf{v}_2, \tag{4.6}$$

$$\mathbf{r}'(x, y) = \mathbf{r}(x, y), \tag{4.7}$$

where $\mathbf{v}'$ and $\mathbf{r}'$ are the state of vehicle after the scattering. β is equal to 0.95 in this study.

Step7. After duration of Δt, obtain the position and velocity of vehicles from drift and field term. The acceleration rate of the vehicle caused by external traffic field is given by

$$a_e = e\mathbf{E} \tag{4.8}$$

In this study, to simplify matters choose e equal to 1, it denotes that only single-class vehicles are considered. And we also suggest that the external traffic field only depends on the equilibrium speed in the prior section and the average length of the vehicles moving to their desired speed. It is an observable fact that drivers increase or decease their speed to the equilibrium speed as their desired speed in the prior section.

From equation (4.9), it could make observation on drivers increasing their speed as the number of vehicles ahead of them decreases, and contrariwise. Moreover, the behavior of vehicle's acceleration in y direction is not obvious in actual phenomenon. Following above derivation we obtain the acceleration rate of vehicle with velocity v in section i at time t:

$$a_e(t) = \mathbf{E}(t) = \frac{(v_{e,i+1}(t))^2 - v(t)^2}{2L} \tag{4.9}$$

where $v_{e,i+1}$ is the equilibrium speed in the prior section, v is the velocity itself, L is the average moving length of each vehicle to arrive their desired velocity. The external force changes the velocity and position of vehicle by

$$\mathbf{v}(t+\Delta t) = \mathbf{v}(t) + a_e(t)\Delta t$$

$$\mathbf{r}(t+\Delta t) = \mathbf{r}(t) + \mathbf{v}(t)\Delta t + \frac{1}{2}a_e(t)\Delta t^2 \tag{4.10}$$

Step8. $t = t + \Delta t$ > the pre-assigned time of simulation?

If yes, stop.

Else, return to step 4.

A flowchart of Monte Carlo simulation technique is illustrated in Figure 4.3.

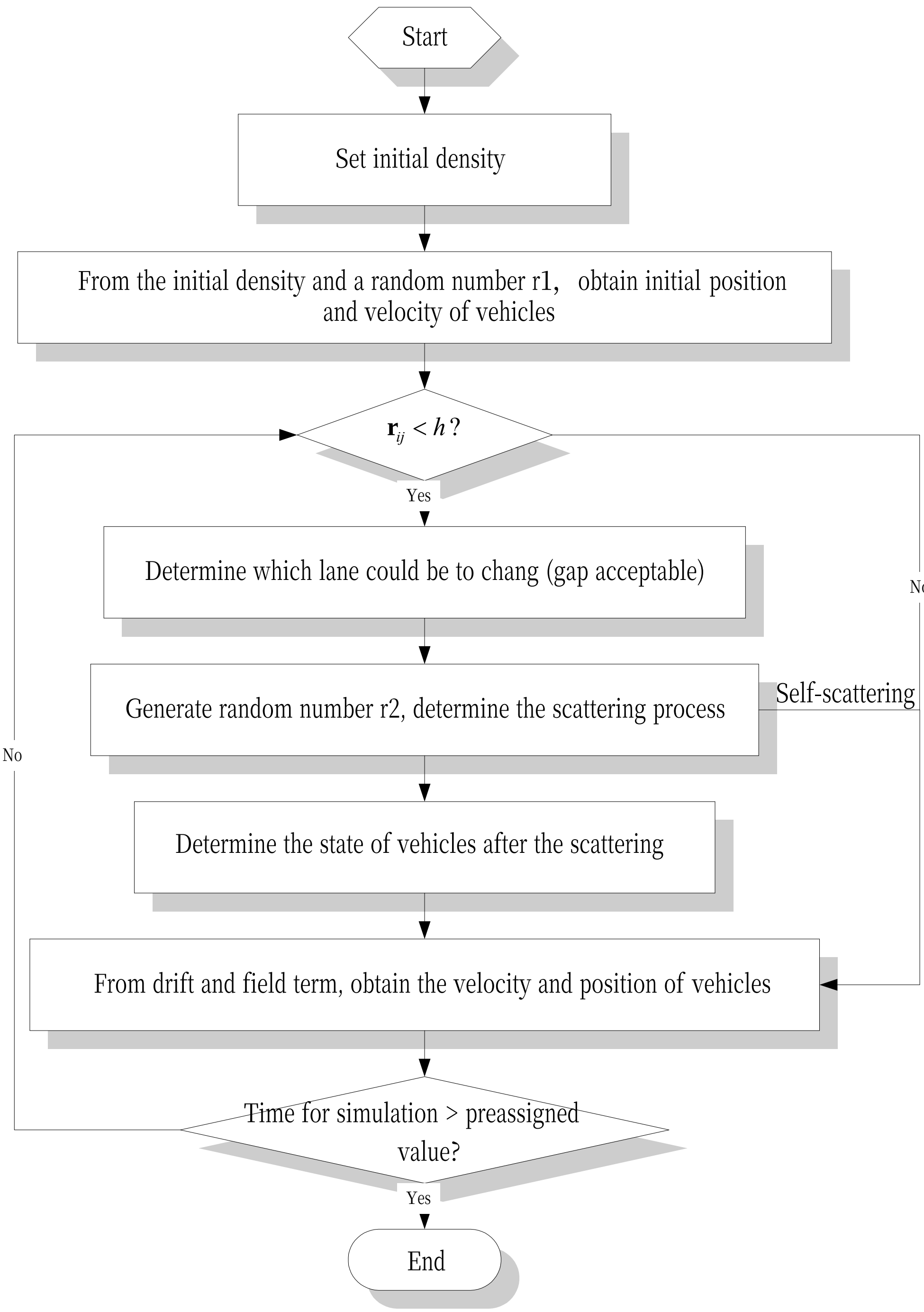


Fig. 4.3 A flowchart of Monte Carlo simulation technique

4.4 Technique in Parallel Computing

The limiting factor for Monte Carlo simulation is, of course, the number of simulation samples acquired, as the uncertainty in a Monte Carlo integral scales as the inverse of the square root of the number of simulation samples. Therefore, the only way to improve the results of a Monte Carlo simulation is to increase the number of simulation samples taken. Note, however, that because of the square root dependence, a factor of four more simulation samples is required in order to achieve a factor of two improvements in the accuracy of the integral.

There are two straightfoward approaches to increasing the number Monte Carlo samples:

- Running the calculation longer
- Running the calculation on many separate nodes concurrently

Of the two choices, the second is by far the most preferable. Indeed, the default mode of operation on many parallel (or concurrent) computers is that each node performs the same task unless explicitly instructed otherwise. As such, Monte Carlo simulations are perfectly suited for parallel computation. All we need to do is run independent copies of the same program on many nodes, and collect our final samples off of each node. This is an operation that is frequently referred to as trivially parallelizable. Note, however, that it is imperative that each node be made independent by selecting a different random number for each node. Otherwise, we may have N copies of the same calculation.

In this study, our experimental environment, a platform used for parallel computing is PC cluster with MPICH 1.2.2 (shown in Figure 4.4 and Table 4.1). These machines are essentially a collection of N computers (nodes) with a relatively fast communications system between the nodes. The default mode of operation for these computers is that the program is loaded onto each node, and executed independently, unless they are instructed otherwise.

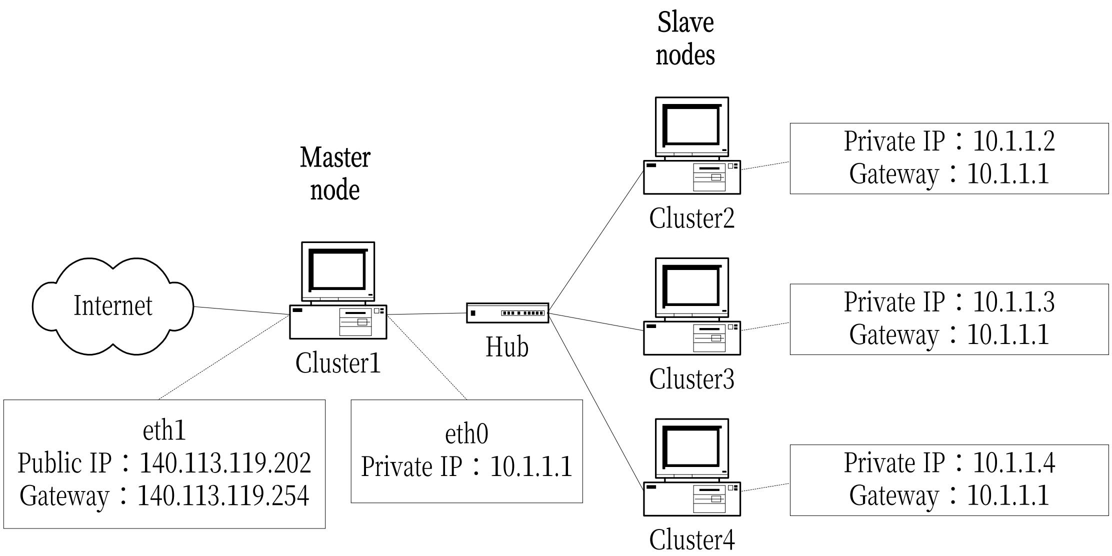


Fig. 4.4 The experimental environment in this study

Table 4.1 The OS and hardware of the PC cluster

OS	Red Hat Linux 8.0 with Kernel ver. 2.4.18-14
MPI	MPICH-1.2.4
CPU	AMD K7-1.6G XP CPU
MB	GA-7VKML
VGA	S3 Savage4 (generic)
RAM	512MB DDRAM PC-266
HDD	80GB 7200rpm
POWER	ATX 300W
Lan	10/100 Fast Ethernet

PC clusters are more general compared with other parallel systems. Researchers can easily own a high-performance, low-cost parallel computer system with clusters. At a basic level a cluster is a collection of workstations or PCs that are interconnected via some network technology. A cluster is a type of parallel or distributed processing system, which consists of a collection of interconnected stand-alone computers working together as a single, integrated computing resource.

A computer node can be a single or multiprocessor system (PCs, workstations) with memory, I/O facilities, and an operating system. A cluster generally refers to two or more computers (nodes) connected together. The nodes can exist in a single cabinet or be physically separated and connected via a LAN. An interconnected (LAN-based) cluster of computers can appear as a single system to users and applications. Such a system can provide a cost-effective way to gain features more expensive proprietary shared memory systems. Thus in this study, we will study clusters with emphasis on analyzing their performance.

The most important MPI implementation is MPICH, developed at Argonne National Laboratory and Mississippi State University. MPICH is a freely available, complete implementation of the MPI specification, designed to be both portable and efficient. The "CH" in MPICH stands for "Chameleon," symbol of adaptability to one's environment and thus of portability. Chameleons are fast, and from the beginning a secondary goal was to give up as little efficiency as possible for the portability. The current version of MPICH is 1.2.4 and was released on May 7, 2002. There are six indispensable functions (shown in Table 4.2), the ones that the programmer really cannot do without.

The most important functions which exist within the MPI standard are two fundamental for sending and receiving messages. MPI_Send() is called at the side of the message sender. The corresponding MPI_Recv() is placed at the destination

Table 4.2 The basic six-function version of MPI

MPI_INIT	Initialize MPI
MPI_COMM_SIZE	Find out how many processes there are
MPI_COMM_RANK	Find out which process I am
MPI_SEND	Send a message
MPI_RECV	Receive a message
MPI_FINALIZE	Terminate MPI

process. Both functions are blocking, but they also have pendants for a non-blocking transmission. Besides these two basic functions, many others like broadcast, gather, or scatter exist in the MPI standard which are not explained here.

Shared-variable and message-passing programming operate one a higher level of abstraction and provide facilities for process communication. Even on this, we can distinguish two different ways, how the data is processed. The first idea is write one program and execute it on some processors at the same time. Therefore, the data is divided into different parts processed by multiple incarnations of the program in parallel. This approach is called data parallelism or SPMD (Single Program Multiple Data).

The second idea is to write multiple programs, where each one is responsible for a special task. These different programs are then executed in parallel. We also call this approach task parallelism or MPMD (Multiple Program Multiple Data). In this study, we will use the first idea to parallel Monte Carlo simulation technique.

Chapter 5 Numerical Simulation of Freeway Traffic Flow

In this chapter a simulation study is undertaken of the traffic Boltzmann transport equation that has been developed in Chapter 3. And in the Chapter 4 the simulation procedure and environment are described. Once initial values have been chosen for density and equilibrium speed and boundary conditions are defined the set of equations can be solved numerically, simulating the Boltzmann transport equation by Monte Carlo simulation technique (some random number generation procedure). To simulate the model a computer program was written in C language. Furthermore, performance of results for parallel Monte Carlo simulation technique will be displayed in section 5.3. In the last section, comparison with real data will be discussed.

5.1 Numerical Results of Some Examples

In the study of traffic flow, we are more interesting in the discussion of the vehicle density then the velocity distribution. Since Monte Carlo simulation technique is introduced to directly solve the traffic Boltzmann transport equation by direct physical simulation. The position, velocity of every vehicle and the density of sections could be conveniently obtained by Direct Monte Carlo Simulation.

A three-lane highway that consists of 45 sections of 100m (=Δx) is considered. In the following contents, numerical simulations of some examples with the different initial traffic densities are considered. We will discuss if the model does show a realistic behavior of vehicles. And as stated in section 4.4, we improve the results of a Monte Carlo simulation by increasing the number of simulation samples taken. Nowadays, not only during one trial, 100 simulation samples are taken to improve the accuracy of the later simulation results.

To obtain a first impression of the validity of the model, we firstly start by simulating example I traffic situation causing by an accident. Suppose that traffic is lined up behind up behind a traffic accident in the highway. We call the position of the traffic accident x = 2000(m). Since the cars are bumper to bumper behind the traffic accident, ρ = 100(veh/km/lane). If the accident traffic long enough, then we may also assume that there is no traffic ahead of the accident, ρ=0(veh/km/lane) for x > 2000(m). Thus the initial traffic density distribution at t = 0s is as sketched in Figure 5.1(a).

In addition, the model parameters are chosen according to Table 5.1. Suppose that at t = 0; the traffic accident is eliminated. The initial velocity-location distribution is presented in Figure 5.2(a). In this example, we assume the entrance flow is equal to 0. The results of the numerical simulation are taken at one-minute intervals, and are as shown in Figure 5.1(a) and (b). The results of the velocity and position distribution at 300s are shown in Figure 5.2(b) during one trial.

Form the density distribution shown in Figure 5.1(a) and (b), we know that as soon as the traffic accident is eliminated, the traffic starts to thin out, but sufficiently far behind the accident position, the traffic density doesn’t change even after the accident is eliminated. The heavier traffic in this example is almost dispersed at about t = 240s. Since there is not entrance flow, the density of the front sections become to zero gradually. From the velocity and position distribution status Figure 5.2(b), there are obvious differences in velocity between the time ranges from 0s to 300s. The vehicles are very slow in the initial situation. When the heavier traffic is dispersed, the vehicles are moving at faster velocity.

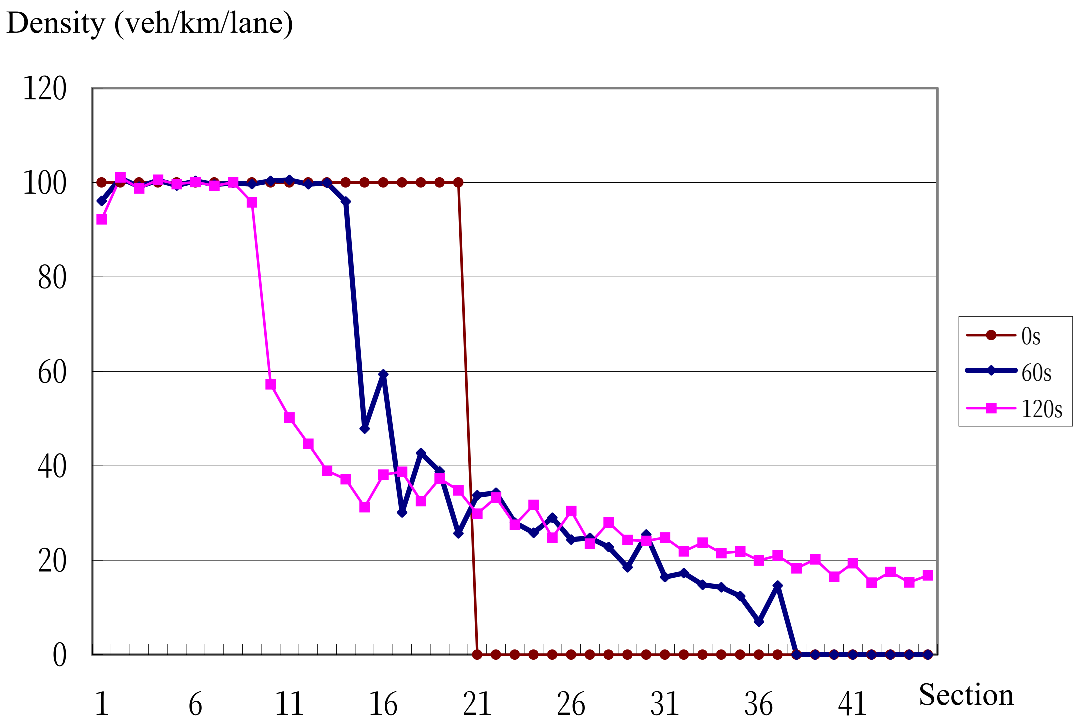


Fig. 5.1(a) Example I density distribution: the initial density and the results of simulation during 0s to 120s.

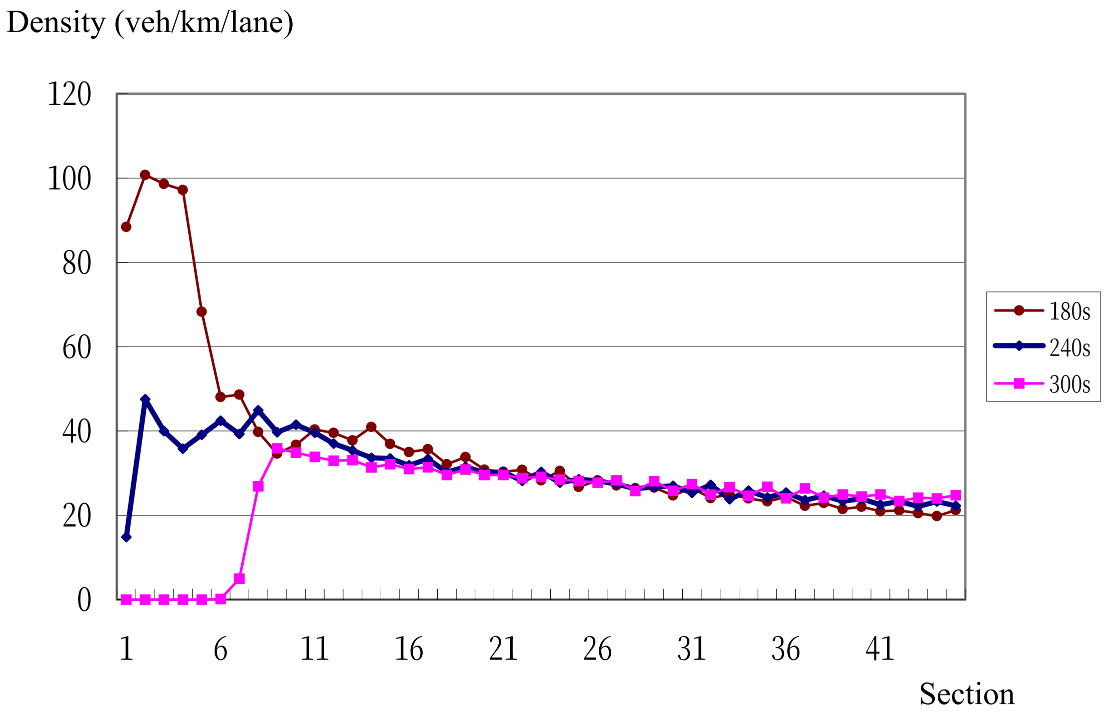


Fig. 5.1(b) Example I density distribution: the initial density and the results of simulation during 180s to 300s.

Table 5.1 The model parameters.

Parameter	Value	Unit
v_f： free velocity	28.89 (104)	m/s (km/hr)
ρ_m：concentration at maximum flow	28.75	veh/km/lane
Δt： time step	1	Second
L	100	m
α	0.7	1
β	0.95	1

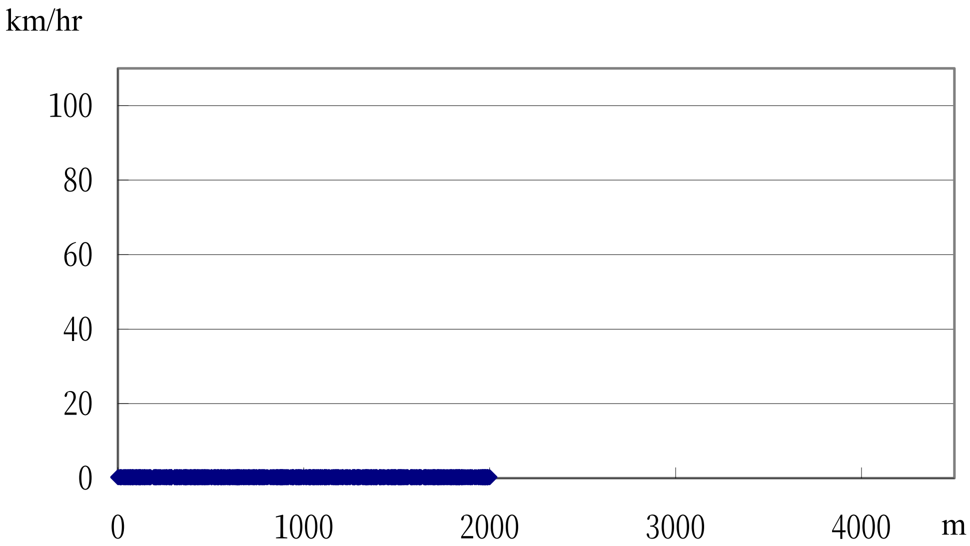


Fig. 5.2 (a) Example I: the velocity and position distribution; t=0s.

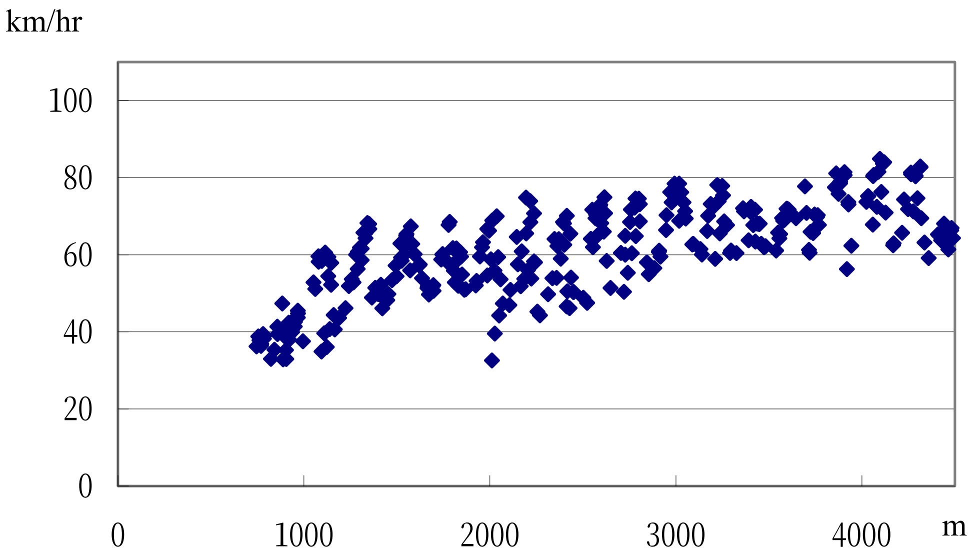


Fig. 5.2 (b) Example I: the velocity and position distribution; t=300s.

The validity of the model at another example is now tested. Again a three-lane highway of 45 sections with a length of 100m each is considered. We imagine a traffic situation in which traffic initially becomes heavier as we go further along the road. The traffic becomes denser or compressed, as shown in Figure 5.3(a). Parameters are chosen as in the simulation of the previous example. The entrance flow constant in time is equal to the equilibrium value of 1000 veh/hr/lane. Vehicle generation takes place on an entry section. In this study, we consider a dirichlet boundary program. The entrance velocity is equal to the velocity the prior vehicle.

As the simulation of previous example, the numerical results (density and velocity-position distribution) are illustrated in Figure 5.3(a), (b) and 5.4 (b). Obviously, it shows that the lighter traffic with faster velocity reaches the heavier traffic with slower velocity at t = 60s, and the vehicles with faster velocity will slow down to avoid the accident happen. From Figure 5.3(a) and (b), it reveals that the density wave appears to be moving backwards.

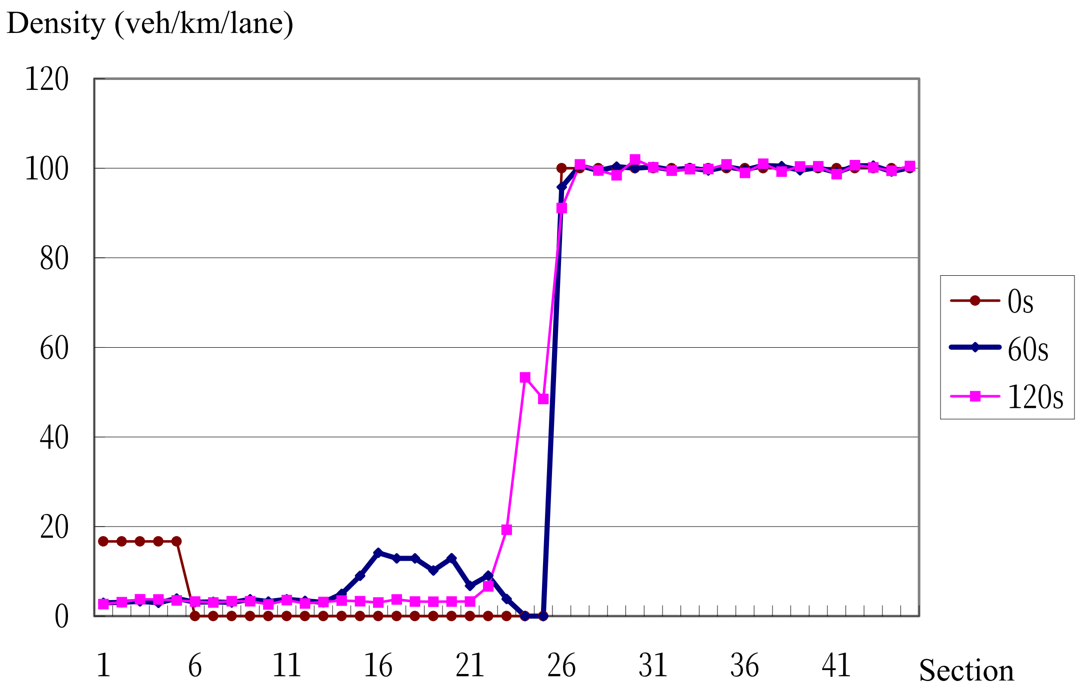


Fig. 5.3(a) Example II density distribution: the initial density and the results of simulation during 0s to 120s.

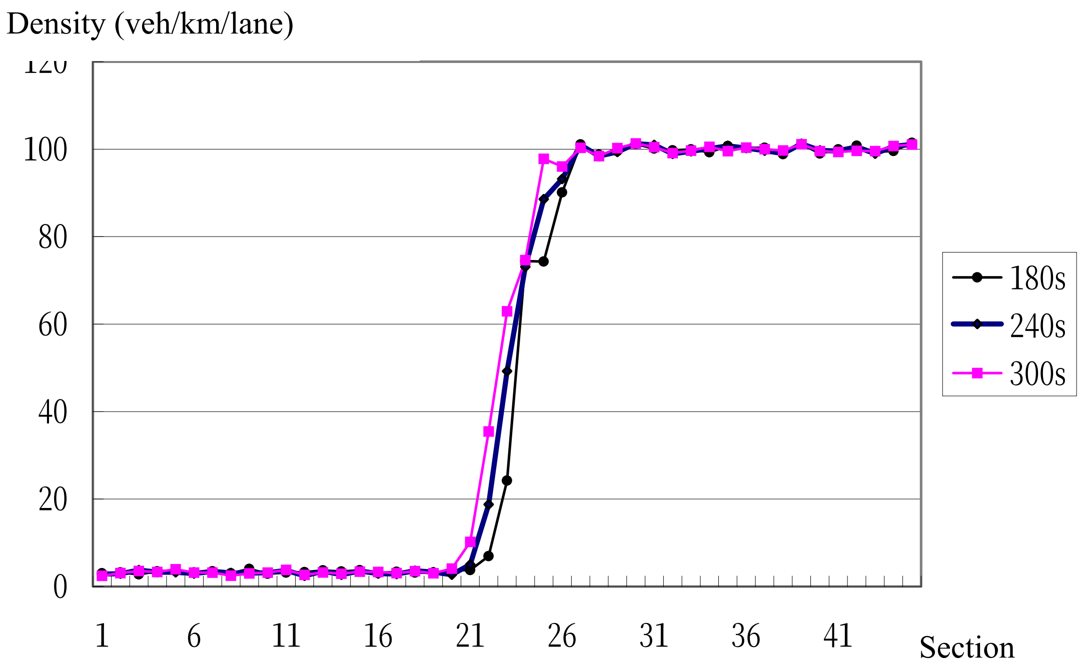


Fig. 5.3(b) Example II density distribution: the initial density and the results of simulation during 180s to 300s.

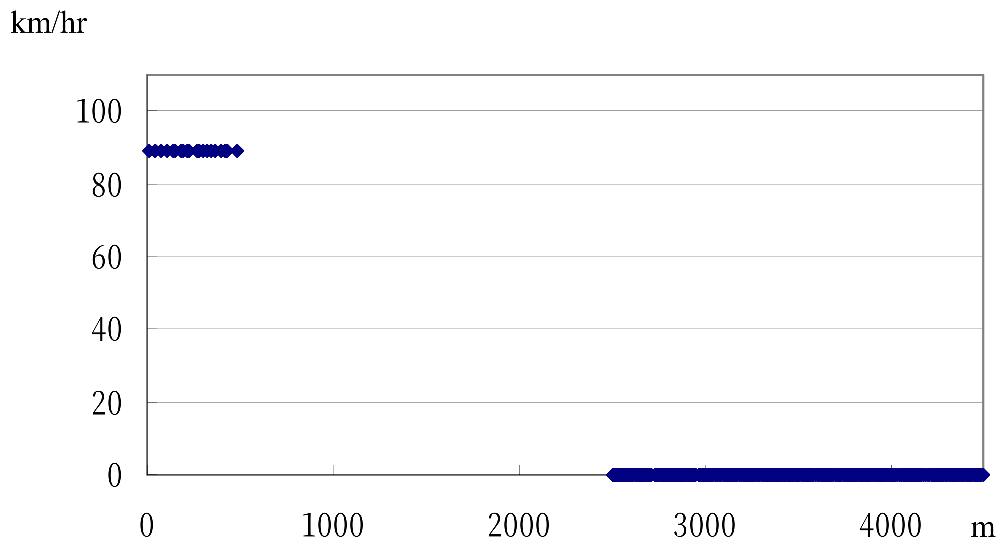


Fig. 5.4 (a) Example II: the velocity and position distribution; t=0s.

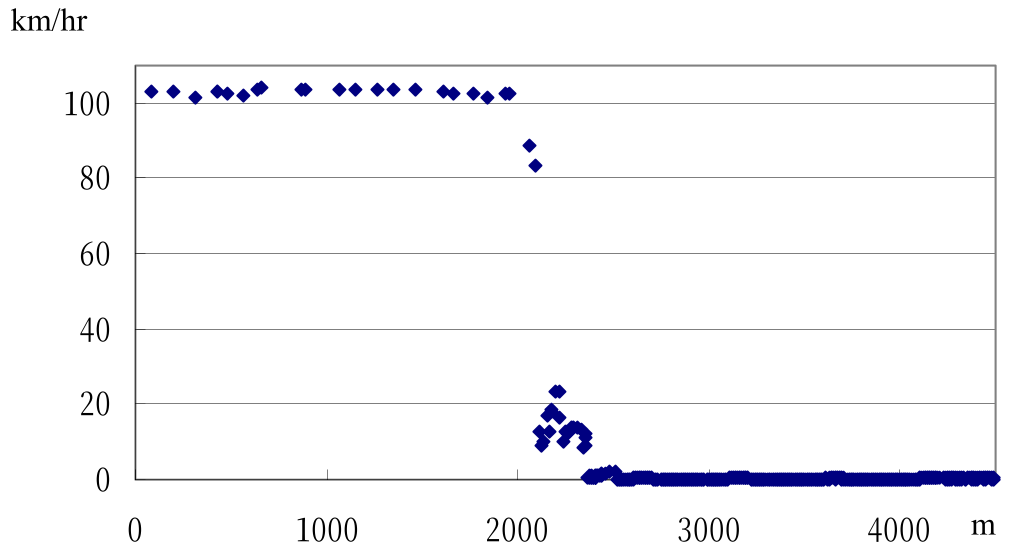


Fig. 5.4 (b) Example II: the velocity and position distribution; t=300s.

5.2 Comparison with Real Data

In this section, the comparison with real data is required to obtain accuracy of the model and numerical scheme. Vertex Detector data at various locations on the National Freeway No.1 (Shijr-Wuku Overpass- Southbound) are used. From HuanPei (26km) to WuKu (33km) interchange, there is a VD approximately every 500-meter interval. Firstly, space mean speed and time mean speed should be discussed.

5.2.1 Space Mean and Time Mean Speed

Mean speed can be computed in two different way, the time mean speed and the space mean speed.

- Time mean speed (Spot speed) is defined as the average speed of all vehicles passing a point on a highway over some specified time period.
- Space mean speed (Harmonic mean speed) is defined as the average speed of all vehicles occupying a given section of a highway over some specified time period.

Because space mean speed is applied to ours traffic flow models, the relationships between time and space mean speed is given by [46]

$$u_s = u_t - \frac{\sigma_t^2}{u_t} \tag{5.3}$$

as an approximate method for use in traffic engineering practice. From the aggregated VDs data, the variance about the time mean speed, σ_t^2, is assumed small enough, we strongly suppose that the space mean speed is equal to time mean speed.

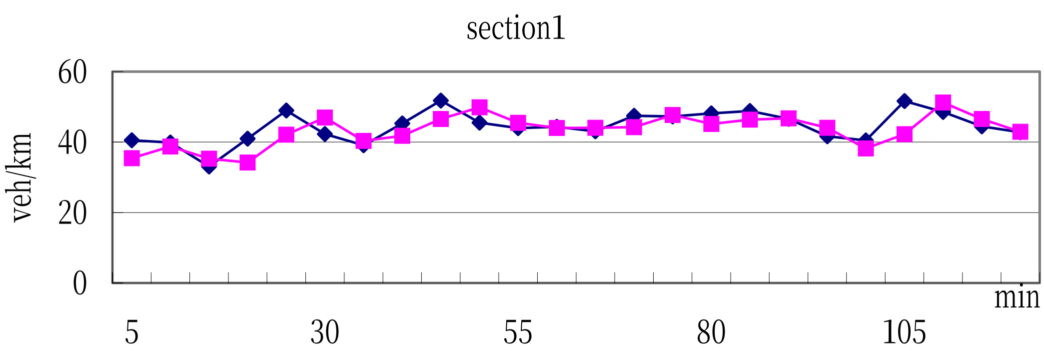
5.2.2 Simulation Results

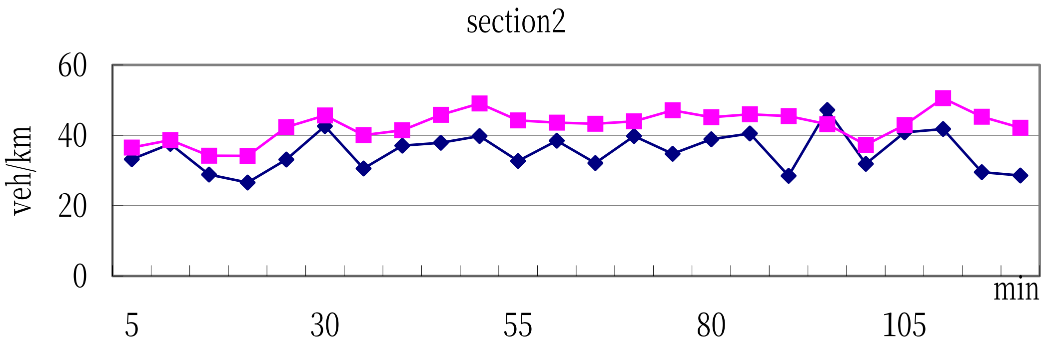
Traffic counts and speeds, aggregated and exponentially smoothed for 5-minute intervals, were recorded in files for a period of 48 hours between June 19th and 20th, 2002. Using these data of 8 VDs in the segment (stated in Table 5.2), traffic flow during half hour from 5:00 to 7:00pm are simulated. In addition, some VDs are omitted showing incomplete data or revealing the exact same value across all the time intervals (a strong indication of malfuntion of the VDs). And there is no VD in about 28.5 kilometer. Linear interpolation allows us to predict those unknown values. The initial and boundary conditions are also given from the real data for VD data. The model parameters are also chosen according to Table 5.1.

Table 5.2 VDs' Location on the Shijr-Wugu Section Viaduct

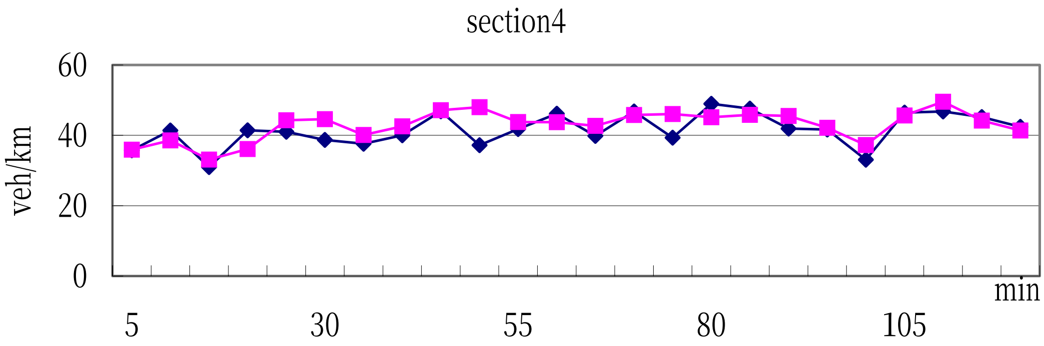
VD	IBS27	IBS28	IBS29	IBS29.5	IBS30	IBS30.5	IBS31	IBS31.4
Kilometer (km)	27.507	28.004	29	29.203	30.009	30.518	31.009	31.498
Section	1	2	4	5	6	7	8	9

According to the computation experience, we have got that simulating the case with 100 times could obtain satisfied results. The computing performance is discussed in next section. The Monte Carlo simulation result and real VD data in all sections of the freeway are illustrated in Figure 5.5. As seen in Figure 5.5, the some of results of Monte Carlo simulation are almost approximate to the real data exclude from some heavy variation of real VD data. The main reason is the real traffic flow certainly can't be described by two scattering mechanisms. In the future, we will discuss more scattering mechanisms to obtain more the accuracy of simulation result.

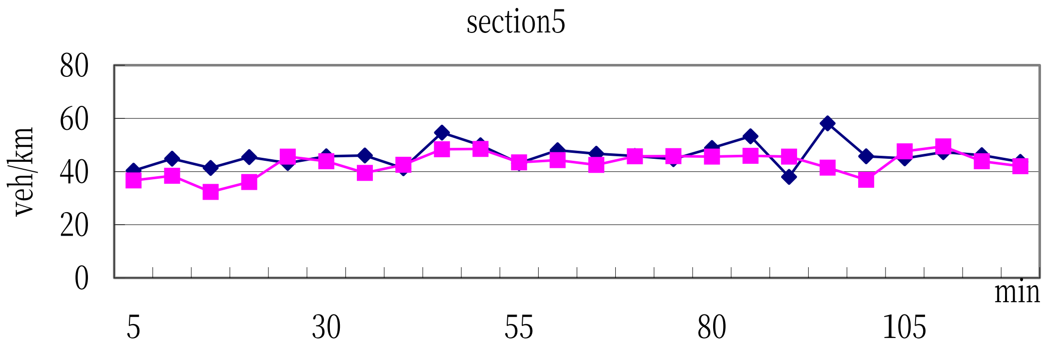




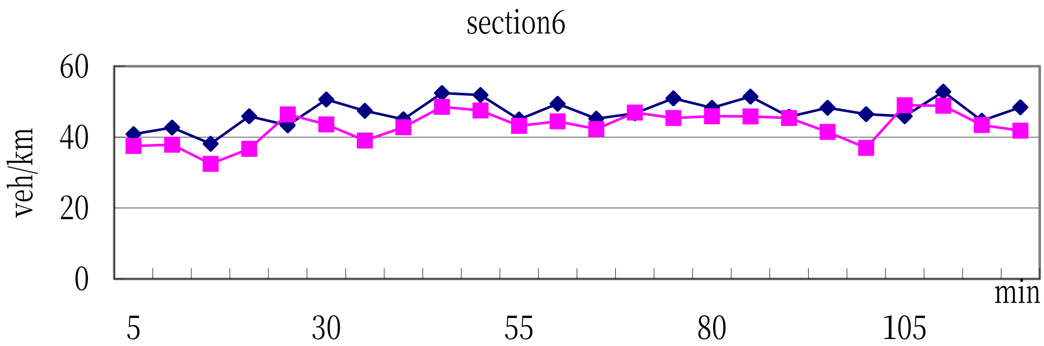
section2
60
40
20
0
veh/km
min
5
30
55
80
105



section4
60
40
20
0
veh/km
min
5
30
55
80
105



section5
80
60
40
20
0
veh/km
min
5
30
55
80
105



section6
60
40
20
0
veh/km
min
5
30
55
80
105

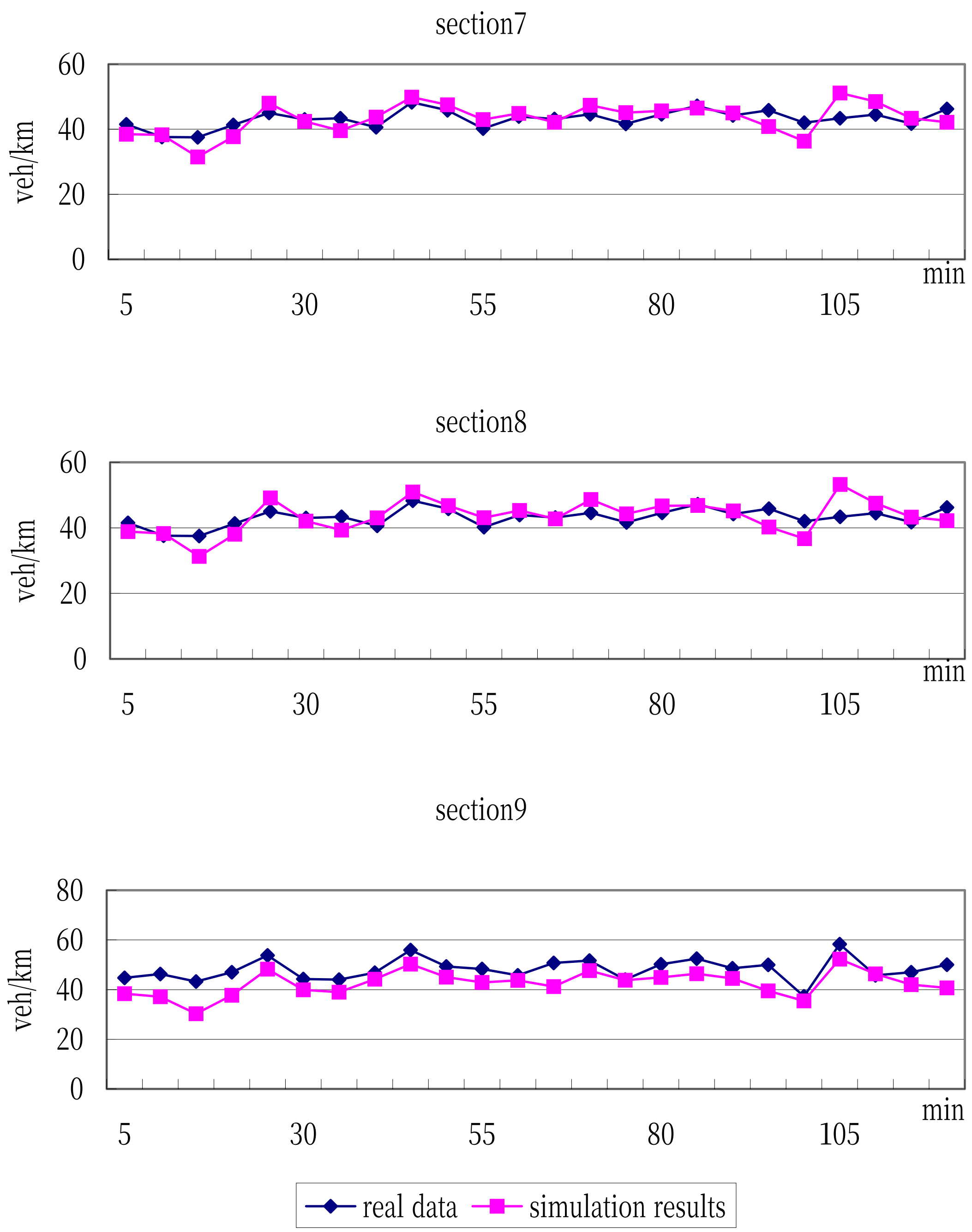


Fig. 5.5 The simulation result and real VD data in some sections of the freeway.

5.3 Result of Parallel Monte Carlo Simulation Technique

The main disadvantage of Monte Carlo simulation technique is time-consuming. Hence we will improve the drawback of Monte Carlo simulation technique with MPI Library. We already briefly introduce the development of MPI in section 2.3 and 4.4.

In the data parallelism method, the simulated samples are divided into several groups, which are performed simultaneously. In this study, we execute our programs on one, two and four machines respectively and then compare the time consumed and the performance of PCs cluster in each execution.

The performance gain that can be obtained by parallel can be calculated using Amdahl's Law. Amdahl's Law defines the speedup that can be gained by using a particular feature. Speedup for n processors is defined as

$$Speedup = \frac{\text{Execution time for 1 Process}}{\text{Execution time for } n \text{ Processes}} \tag{5.1}$$

With varying numbers of processes, we can measure speedup.
And the efficiency is defined as

$$Efficiency = \frac{Speedup}{n} \tag{5.2}$$

Numerical results of the real VD data in section 5.2 are stated in Table 5.3 by using parallel Monte Carlo simulation technique. Since $Speedup = n$, we have $Efficiency = 1$. When perfect speedup of $Speedup = n$, referred to as linear speed-up, is achieved $Efficiency = 1$. In almost any algorithm there are operations that must be executed on one processor at a time, thereby decreasing the efficiency. The fraction of such operations is referred to as the "serial fraction". Other factors that degrade speed-up include synchronization of tasks, and communication between processors and are referred to as the "parallelization overhead".

The results of Table 5.3 indicate near linear speed-up. This is to be expected because the present implementation, both the serial fraction and inter-processor communication is in minimal. There is a very small degradation in speed-up and efficiency with the number of processors because of increased parallelization overhead.

Table 5.3 Performance results for parallel Monte Carlo simulation technique.

Processors	1	2	4
Time (seconds)	1201	606	305
Speedup	1	1.982	3.938
Efficiency	100%	99.10%	98.45%

Chapter 6 Conclusions and Future Works

6.1 Conclusions

The main work of this study lies in three areas: (1) a mesoscopic traffic simulator with Monte Carlo simulation technique is used to predict the traffic condition; (2) using MPI parallel computing technique to improve the drawback of Monte Carlo simulation technique; and, (3) the demonstration of the use of the developed simulation model.

6.2 Future Works

The research presented in this thesis can be extended in the following directions:

1. In this study, we use and assume a set of parameters or established model to represent drivers' behavior, vehicle performance. These parameters provide the flexibility to customize the mesoscopic traffic flow model for use in various environments. However, many of the parameters should be calibrated based on field traffic data in Taiwan to improve this model.

2. As discuss above in section 3.2, Scattering mechanisms are very "flexible". With more knowledge of scattering mechanisms, more the accuracy of simulation result will be obtained.

3. This distribution function obtained from great quality simulation of Monte Carlo method can be used to obtain various macroscopic quantities interested and some of the important macroscopic variables.

4. The efficiency of parallel Monte Carlo simulation technique is excellent. As a result, as simulation in traffic network, using parallel Monte Carlo simulation technique to characterize vehicles transport behavior will be the most accurate and popular technique.
5. Expect to use the simulation solution of traffic Boltzmann transport equation in dynamic traffic flow for drafting the analytic tool of traffic real-time control in ITS.

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