Chapter 1
INTRODUCTION

1.1 Background

With the rising demand of automobile and highway usage in recent years, highway driving has become the prominent mode of urban transportation in most metropolitan areas around the world. The need for transport services has grown significantly while the general capacity of our transport infrastructure has not. This has led to a great deal of traffic congestion that persists to be one of the momentous social and economic problems related to transportation nowadays. For this reason, the definite cognition of traffic flow operations is required to devise efficient traffic control strategies.

Traffic flow dynamics describing traffic phenomena provides insights into why congestion occurs, what governs the time and location of traffic breakdown, how the congestion propagates through the road, etc. Hence, traffic flow dynamics is essential to the development of Intelligent Transport Systems (ITS), which was proposed to ameliorate the recurrent problems of growing traffic demand and limited capacity. Simulation of traffic flow using macroscopic continuum models is an adequate tool to comprehending traffic flow dynamics because such models contain both space and time in the state equations and take compressibility into consideration.

Macroscopic continuum traffic flow models that involve the partial differential equations (PDEs) with suitable initial and boundary conditions have been proposed and analyzed in the past. During the past fifty years, a wide variety of traffic flow theories and models have been developed to answer these research questions in traffic flow. Since Lighthill and Whitham (1955) as well as Richards (1956) first presented a simple continuum model based on the analogy of vehicles in traffic flow and particles in a fluid to describe the characteristics of traffic flow, mathematical description of traffic flow has been a lively subject of research and debate for researchers in traffic science. This has resulted in a broad range of models describing different respects of traffic flow operations, either by considering the time-space behavior of individual drivers under the influence of vehicles in their proximity (microscopic), or from the viewpoint of the collective vehicular flow (macroscopic). Besides the controversy between these microscopic and macroscopic modeling streams, much progress has been made in the development and application of macroscopic modeling approach, which is most appropriate for a correct description of traffic flow, especially with the introduction of high order continuum models. Payne (1971) presented the first high order continuum model that includes the effects of momentum, inertia, and acceleration. Hereafter a lot of new high order continuum models have been developed by several researchers in traffic flow theory. Some examples are Papageorgiou's improved high-order models (Papageorgiou et al., 1989), the semi-viscous and viscous high-order models (Michalopoulos et al., 1991b, 1993a; Lyrintzis et al., 1994a) and others (Phillip, 1977; Kühne, 1991; Del Castillo et al., 1993; Lyrintzis et al., 1994b; Helbing, 1995, 1996b, 1997b, 2001; Liu et al., 1997; Zhang, 1998, 1999, 2000, 2001; Jiang et al., 2002).

Traffic flow Simulation using macroscopic continuum models would provide an effective tool to examine and access real-time traffic control strategies in practice. However, the more realistic the traffic flow models are, the more complex the PDEs will be. Due to the difficulty in the analytic solutions of the PDEs, numerous numerical methods have been presented to afford a considerable approach to the numerical solutions. The famous FREFLO (Payne, 1979) and KRONOS (Michalopoulos et al., 1987) simulation programs are two examples which contain macroscopic continuum traffic flow models. Both programs make use of explicit finite difference methods to solve a single or system of PDEs. Payne's model used in FREELO is a system of two PDEs that doesn't have known analytical solutions and has to be solved by numerical methods. However, the equations are difficult to solve even numerically.

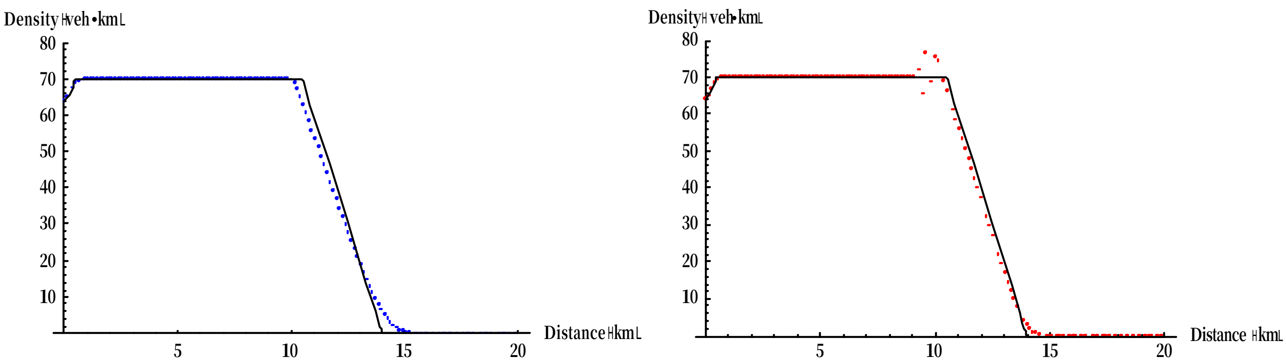
1.2 Motivation

The increasing need for efficient traffic optimization measures is making reliable, fast, and robust methods for traffic simulation more and more important (Helbing and Treiber, 1999). In addition to the in-depth understanding of the macroscopic traffic mechanisms, appropriate numerical methods have to be provided. Helbing and Treiber (1999) demonstrated that traffic flows are characterized by the occurrence of congested regions, jams, and stop-and-go waves which are associated with large gradients of the traffic density and average velocity.

Studying, analyzing and simulating discontinuous solutions of traffic flow problems have important meaning in theory and practice (Liu and colleagues, 2001). Most practical traffic flow situations contain the complicated initial and boundary conditions, e.g. a traffic signal turning from red to green at that time the traffic is lined up behind the red light. In this situation, the initial traffic density distribution is a discontinuous function and solving this continuum model with simple numerical methods might cause numerical inaccuracy.

The first order accurate method yields a numerical diffusion, which causes smoothing of shock fronts, and is inaccurate. However, the second and higher order methods produce unrealistic oscillations close to steep gradients. Such oscillations, which are called the Gibbs phenomena in spectral methods, don't decay in magnitude when refining the mesh. Figure 1.1 shows the poor performance results of the simple numerical methods.

Furthermore, simple numerical schemes were often used to compute approximate solutions of certain higher order models such as the Payne-Whitham (PW) model (Payne, 1971; Whitham, 1974), and caused confusion over the properties of these models because the mis-behavior of the finite difference schemes was sometimes mistaken as the behavior of the models themselves (Zhang, 2001). Due to the deficiency of simple numerical methods, high accuracy numerical schemes, which have been widely used in computational fluid dynamics (CFD), could be explored to obtain the satisfactory solution of continuum traffic flow models.



(a) (b)

Figure 1.1. Solutions of the simple continuum traffic flow model simulated by (a) Godunov's method (1st order) and (b) Lax-Wendroff method (2nd order) (Solid line: exact solution; Dots: computed solution).

1.3 Research Objectives

In this thesis, simulations of the simple and high order continuum traffic flow models with high resolution schemes were presented. The main objective of this research is to carry out numerical solutions for continuum traffic flow equations by high resolution schemes that improve the numerical precision and eliminate spurious oscillations near discontinuities. These high resolution methods, such as high order Weighted Essentially Non-Oscillatory (WENO) finite difference and finite volume schemes were applied to solve continuum traffic flow models that involve the discontinuous initial conditions and thus are Riemann problems.

The numerical solutions of WENO schemes were compared with results from Total Variation Diminishing (TVD) type scheme and other numerical methods previously used to solve traffic flow models.

Moreover, the produced results with higher computational accuracy would be beneficial to treat realistic traffic flow simulation. Such realistic traffic flow conditions often described by high order models are quite intricate. For example, PW model, which contains the moment equation in order to account for the dynamic speed-density relationship observed in real traffic flow, is a nonlinear system of PDEs. A major hindrance to the development of accurate finite difference approximations of higher order traffic flow models is the lack of a complete understanding of various kinds of waves present in these models (Zhang, 2001). Hence the solutions of Riemann problems, which consist of all patterns of shocks and rarefaction waves, for high order continuum models were endeavored to establish in this study.

1.4 Thesis Organization

This thesis is divided into six chapters.

After Chapter 1 presenting background information, the motivation of this research, and research objectives, the review of literature is provided in Chapter 2. Numerical simulation of macroscopic continuum traffic flow models and related subjects, including the simple continuum, original high order, and improved high order models, are reviewed. The numerical methods recently applied to solve hyperbolic PDEs are also surveyed in Chapter 2.

Chapter 3 describes continuum traffic flow models by deriving the conservation equation of traffic flow. Then the simple and high order continuum models are formulated for numerical discretization. Finally, Riemann problems in continuum traffic flow models are described since it is the main problem to solve in this study.

In Chapter 4, WENO finite difference and finite volume schemes are introduced, including a detailed description of the temporal and spatial discretization algorithms. For the sake of solving high order models, WENO schemes for high order continuum traffic flow models are provided in this chapter.

Chapter 5 presents test problems and numerical results of the selected continuum traffic flow models. Four numerical examples for LWR model and three test problems for PW and Jiang's improved models are contained to demonstrate the performance results of WENO schemes. The numerical solutions of WENO schemes were compared with those of TVD type scheme and other numerical methods previously used to solve continuum traffic flow models.

Concluding remarks are given in Chapter 6. This chapter presents some conclusions of this study. Finally, several recommendations for further research are also offered.

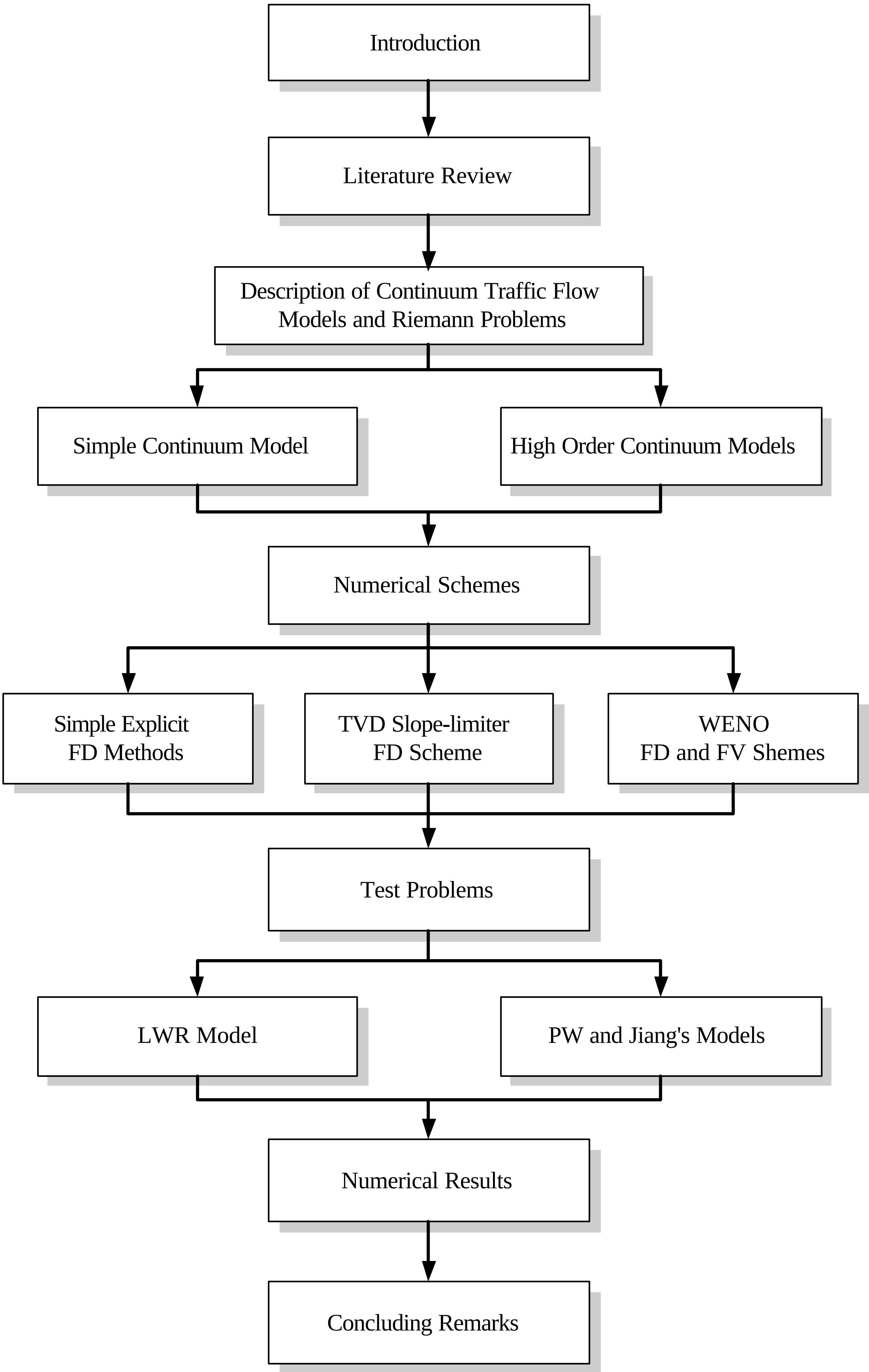


Figure 1.2. The procedure of the study.