

# 高解析方法之巨觀車流模式數值解

學生：黃鋒樟

指導教授：卓訓榮

國立交通大學運輸科技與管理學系碩士班

## 摘 要

在智慧型運輸系統的發展中，巨觀動態車流模式能描述車流現象且為其系統發展中關鍵重要的一環，而數值模擬在求解車流模式上乃是十分重要的工具。巨觀車流模式源於車流為連續流體的觀念，相關研究推演出一雙曲線型偏微分方程式或方程組，並給予適當的起始及邊界條件。由於雙曲線型偏微分方程求解不易，已有許多數值方法被提出以求得合理的數值解。其中一階準確的數值方法在解不連續處會有平滑化的情形，因而導致不精確的數值解；而較高階之方法卻會在解不平滑處附近發生震盪的現象。在頻譜法中，此種震盪稱為吉布斯(Gibbs)現象，其並不會隨著網格的更新加切而消失。本研究應用高解析數值方法求解一階及高階車流波動方程式，以期改進數值精確度及避免解不連續處之震盪。研究中應用高階加權基本不震盪(Weighted Essentially Non-Oscillatory)有限差分及有限體積法求解具黎曼問題(Riemann problems)性質之車流波動方程，並將結果與全變量消逝(Total Variation Diminishing)法及先前其他相關研究中用於求解車流模式之數值方法作比較。在一階車流模式的測試問題中，包括衝擊波、消散波、號誌及方形波，加權基本不震盪法具有極優勢的準確度。而在求解包括衝擊波及消散波例題的高階車流模式之黎曼問題時，加權基本不震盪法亦能產生合理有效的數值解。由研究結果顯示加權基本不震盪法具備模擬複雜車流現象的潛力，包括衝擊波、消散波、斷續性車流及區段擁擠性車流。未來藉由結合加權基本不震盪法與平行處理，平行高解析數值方法將是一個可靠、有效率並精確的車流模擬數值解方法。

# **High Resolution Schemes for the Numerical Solutions of Macroscopic Continuum Traffic Flow Models**

Student : Feng-Jang Hwang

Advisor : Hsun-Jung Cho

Department of Transportation Technology and Management  
National Chiao Tung University

## **Abstract**

Numerical simulation is significant to solve macroscopic continuum traffic flow models, which describe various traffic phenomena and play an important role in the development of Intelligent Transport Systems (ITS). Continuum traffic flow models are often analyzed with systems of hyperbolic partial differential equations (PDEs) attended by suitable initial and boundary conditions. Due to difficulty in solving hyperbolic PDEs, numerous numerical methods have been presented to afford a considerable approach attaining the reasonable solution. The first order accurate method yields a numerical diffusion, which causes smoothing of shock fronts, and is inaccurate. However, higher order methods produce unrealistic oscillations close to steep gradients. Such oscillations, which are called the Gibbs phenomena in spectral methods, don't decay in magnitude when refining the mesh. The objective of the study was to simulate continuum traffic flow models with high resolution methods that improve the numerical precision and eliminate such spurious oscillations near discontinuities. High order Weighted Essentially Non-Oscillatory (WENO) finite difference and finite volume schemes were applied to solve the simple and high order continuum traffic flow models that involve the discontinuous initial conditions and thus are Riemann problems. The numerical solutions of WENO schemes were compared with results produced by Total Variation Diminishing (TVD) type scheme and other numerical methods previously used to solve traffic flow models. Test problems of the simple continuum model, including shock, rarefaction wave, traffic signal, and square wave cases, were shown to illustrate the dominant accuracy of WENO schemes. WENO schemes also exhibited the capability of presenting appropriate results in Riemann problems of high order continuum models with numerical examples, including shock and rarefaction wave problems, for Payne-Whitham (PW) and Jiang's improved models. The results indicate that WENO schemes can afford to be utilized in the simulation of complex traffic phenomena, such as shock, rarefaction waves, stop-and-go waves, and local cluster effects. In the future, with the implementation of parallel processing the WENO algorithm, parallel high resolution numerical scheme would be a reliable, fast, and robust method for traffic flow simulation.

# Table of Contents

<b>Dedication.....</b>	<b>i</b>
<b>Acknowledgements .....</b>	<b>ii</b>
<b>Chinese Abstract.....</b>	<b>iii</b>
<b>English Abstract.....</b>	<b>iv</b>
<b>Table of Contents .....</b>	<b>v</b>
<b>List of Tables .....</b>	<b>viii</b>
<b>List of Figures.....</b>	<b>ix</b>
<b>Nomenclature .....</b>	<b>xviii</b>
<b>1 INTRODUCTION .....</b>	<b>1</b>
1.1 Background .....	1
1.2 Motivation.....	3
1.3 Research Objectives.....	4
1.4 Thesis Organization.....	5
<b>2 LITERATURE REVIEW.....</b>	<b>8</b>
2.1 Review of Macroscopic Continuum Traffic Flow Models .....	8
2.1.1 The Simple Continuum Model .....	8
2.1.2 The High Order Continuum Models .....	10
2.2 Review of Numerical Simulation of Continuum Traffic Flow Models .....	15
2.3 Brief Review of Numerical Methods for Hyperbolic PDEs .....	16
<b>3 DESCRIPTION OF CONTINUUM TRAFFIC FLOW MODELS AND RIEMANN PROBLEMS .....</b>	<b>19</b>
3.1 Derivation of Conservation Equation of Traffic Flow.....	19
3.2 Formulation of Continuum Traffic Flow Models for Numerical Discretization.....	21
3.2.1 LWR Model .....	21
3.2.2 High Order Continuum Models .....	22
3.2.2.1 PW Model.....	22
3.2.2.2 Jiang' s Improved Model .....	23

3.3 Riemann Problems in Continuum Traffic Flow Models .....	24
<b>4 WEIGHTED ESSENTIALLY NON-OSCILLATORY FINITE DIFFERENCE AND FINITE VOLUME SCHEMES .....</b>	<b>25</b>
4.1 TVD Runge-Kutta Time Discretization.....	26
4.2 Weighted Essentially Non-Oscillatory Schemes .....	26
4.2.1 WENO Reconstruction and Approximation.....	26
4.2.2 Finite Difference Formulation.....	30
4.2.3 Finite Volume Formulation.....	31
4.3 WENO Schemes for High Order Continuum Traffic Flow Models .....	33
4.3.1 Component-wise Finite Difference Formulation.....	34
4.3.2 Component-wise Finite Volume Formulation.....	36
<b>5 TEST PROBLEMS AND NUMERICAL RESULTS .....</b>	<b>37</b>
5.1 Numerical Examples for LWR Model.....	37
5.1.1 Shock Problems .....	37
5.1.1.1 Case I .....	38
5.1.1.2 Case II.....	44
5.1.2 Rarefaction Wave Problems .....	47
5.1.2.1 Case I .....	47
5.1.2.2 Case II.....	54
5.1.3 Traffic Signal Switching from Red to Green.....	57
5.1.4 Square Wave Problem.....	60
5.2 Numerical Examples for PW and Jiang's Improved Models .....	70
5.2.1 Shock Problems .....	72
5.2.1.1 Case I .....	72
5.2.1.2 Case II.....	77
5.2.1.3 Case III.....	82
5.2.2 Rarefaction Wave Problems .....	84
5.2.2.1 Case I .....	84
5.2.2.2 Case II.....	90

5.2.2.3 Case III.....	95
5.2.3 Local Cluster Effect.....	97
<b>6 CONCLUDING REMARKS .....</b>	<b>105</b>
6.1 Conclusions .....	105
6.2 Recommendations for Further Research .....	105
<b>References.....</b>	<b>107</b>
<b>Bibliography .....</b>	<b>111</b>

# List of Tables

2.1	Finite Difference Methods for hyperbolic conservation laws $k_t + f(k)_x = 0$ .....	18
5.1	Numerical accuracy in density at $t = 0.05\text{hr}$ for traffic signal problem with infinity-norm and one-norm errors (unit: veh/km) .....	60
5.2	Execution times for simulation of traffic signal problem (unit: CPU sec.) .....	60
5.3	Numerical accuracy in density at $t = 0.4\text{ hr}$ for square wave problem with infinity-norm and one-norm errors (unit: veh/km) .....	70
5.4	Execution times for simulation of square wave problem (unit: CPU sec.) .....	70
5.5	Comparison of down-critical density $k_d$ and up-critical density $k_u$ between Jiang' s FD and WENO FV solutions .....	104

# List of Figures

1.1	Solutions of the simple continuum traffic flow model simulated by (a) Godunov's method (1st order) and (b) Lax-Wendroff method (2nd order).....	4
1.2	The procedure of the study .....	7
3.1	An interval of roadway without entrances or exits .....	19
4.1	The candidate stencils for $k = 3$ .....	28
4.2	WENO Finite Difference Grid Structure.....	31
4.3	Procedure of WENO scheme .....	33
5.1	The density behavior for case I of shocks simulated by the Upwind scheme .....	38
5.2	The density behavior for case I of shocks simulated by the Lax-Friedrichs scheme .....	39
5.3	The density behavior for case I of shocks simulated by the Leapfrog scheme .....	39
5.4	The density behavior for case I of shocks simulated by the Beam-Warming scheme .....	39
5.5	The density behavior for case I of shocks simulated by the Lax-Wendroff scheme	40
5.6	The density behavior for case I of shocks simulated by the MacCormack scheme	40
5.7	The density behavior for case I of shocks simulated by the Godunov scheme .....	40
5.8	The density behavior for case I of shocks simulated by TVD Slope-limiter scheme .....	41
5.9	The density behavior for case I of shocks simulated by WENO FV scheme .....	41
5.10	The density profile at $t = 0.05\text{hr}$ for case I of shocks simulated by the Upwind scheme .....	41
5.11	The density profile at $t = 0.05\text{hr}$ for case I of shocks simulated by the Lax-Friedrichs scheme .....	42
5.12	The density profile at $t = 0.05\text{hr}$ for case I of shocks simulated by the Leapfrog scheme .....	42
5.13	The density profile at $t = 0.05\text{hr}$ for case I of shocks simulated by the Beam-Warming scheme .....	42
5.14	The density profile at $t = 0.05\text{hr}$ for case I of shocks simulated by the Lax-Wendroff scheme.....	43

5.15	The density profile at $t = 0.05\text{hr}$ for case I of shocks simulated by the MacCormack scheme .....	43
5.16	The density profile at $t = 0.05\text{hr}$ for case I of shocks simulated by the Godunov scheme .....	43
5.17	The density profile at $t = 0.05\text{hr}$ for case I of shocks simulated by TVD Slope-limiter scheme .....	44
5.18	The density profile at $t = 0.05\text{hr}$ for case I of shocks simulated by WENO FV scheme .....	44
5.19	The density behavior for case II of shocks simulated by the Upwind scheme .....	45
5.20	The density behavior for case II of shocks simulated by the Godunov scheme .....	45
5.21	The density behavior for case II of shocks simulated by TVD Slope-limiter scheme .....	45
5.22	The density behavior for case II of shocks simulated by WENO FV scheme .....	46
5.23	The density profile at $t = 0.05\text{hr}$ for case II of shocks simulated by the Upwind scheme .....	46
5.24	The density profile at $t = 0.05\text{hr}$ for case II of shocks simulated by the Godunov scheme .....	46
5.25	The density profile at $t = 0.05\text{hr}$ for case II of shocks simulated by TVD Slope-limiter scheme .....	47
5.26	The density profile at $t = 0.05\text{hr}$ for case II of shocks simulated by WENO FV scheme .....	47
5.27	The density behavior for case I of rarefaction waves simulated by the Upwind scheme .....	48
5.28	The density behavior for case I of rarefaction waves simulated by the Lax-Friedrichs scheme .....	48
5.29	The density behavior for case I of rarefaction waves simulated by the Leapfrog scheme .....	49
5.30	The density behavior for case I of rarefaction waves simulated by the Beam-Warming scheme .....	49
5.31	The density behavior for case I of rarefaction waves simulated by the Lax-Wendroff scheme .....	49
5.32	The density behavior for case I of rarefaction waves simulated by the MacCormack scheme .....	50



5.33	The density behavior for case I of rarefaction waves simulated by the Godunov scheme .....	50
5.34	The density behavior for case I of rarefaction waves simulated by TVD Slope-limiter scheme .....	50
5.35	The density behavior for case I of rarefaction waves simulated by WENO FV scheme .....	51
5.36	The density profile at $t = 0.05\text{hr}$ for case I of rarefaction waves simulated by the Upwind scheme .....	51
5.37	The density profile at $t = 0.05\text{hr}$ for case I of rarefaction waves simulated by the Lax-Friedrichs scheme .....	51
5.38	The density profile at $t = 0.05\text{hr}$ for case I of rarefaction waves simulated by the Leapfrog scheme.....	52
5.39	The density profile at $t = 0.05\text{hr}$ for case I of rarefaction waves simulated by the Beam-Warming scheme .....	52
5.40	The density profile at $t = 0.05\text{hr}$ for case I of rarefaction waves simulated by the Lax-Wendroff scheme.....	52
5.41	The density profile at $t = 0.05\text{hr}$ for case I of rarefaction waves simulated by the MacCormack scheme .....	53
5.42	The density profile at $t = 0.05\text{hr}$ for case I of rarefaction waves simulated by the Godunov scheme .....	53
5.43	The density profile at $t = 0.05\text{hr}$ for case I of rarefaction waves simulated by TVD Slope-limiter scheme .....	53
5.44	The density profile at $t = 0.05\text{hr}$ for case I of rarefaction waves simulated by WENO FV scheme .....	54
5.45	The density behavior for case II of rarefaction waves simulated by the Upwind scheme .....	54
5.46	The density behavior for case II of rarefaction waves simulated by the Godunov scheme .....	55
5.47	The density behavior for case II of rarefaction waves simulated by TVD Slope-limiter scheme .....	55
5.48	The density behavior for case II of rarefaction waves simulated by WENO FV scheme .....	55
5.49	The density profile at $t = 0.05\text{hr}$ for case II of rarefaction waves simulated by the	

Upwind scheme .....	56
5.50 The density profile at $t = 0.05\text{hr}$ for case II of rarefaction waves simulated by the Godunov scheme .....	56
5.51 The density profile at $t = 0.05\text{hr}$ for case II of rarefaction waves simulated by TVD Slope-limiter scheme .....	56
5.52 The density profile at $t = 0.05\text{hr}$ for case II of rarefaction waves simulated by WENO FV scheme .....	57
5.53 A road section with a traffic signal.....	58
5.54 The density behavior with a traffic signal turning from red to green simulated by WENO FV scheme .....	59
5.55 The density profile at $t = 0.05$ hr for traffic signal problem (Solid line: exact solution; Dashes: solution of Godunov-type TVD Slope-limiter FD scheme; Dots: solution of WENO FV scheme) .....	59
5.56 The flow-density relationship used in square wave problem .....	61
5.57 Characteristics diagram for square wave problem.....	62
5.58 Characteristics diagram for square wave problem.....	66
5.59 Characteristics for square wave problem simulated by WENO FV scheme .....	66
5.60 Contours of density for square wave problem simulated by WENO FV scheme ...	67
5.61 The density behavior for square wave problem simulated by WENO FV scheme .	67
5.62 The density profile at $t = 0.4$ hr for square wave problem (Mesh: $100 \times 100$ ; Solid line: exact solution; Dashes: solution of Godunov-type TVD Slope-limiter FD scheme; Dots: solution of WENO FV scheme) .....	68
5.63 The density profile at $t = 0.4$ hr for square wave problem (Mesh: $160 \times 160$ ; Solid line: exact solution; Dashes: solution of Godunov-type TVD Slope-limiter FD scheme; Dots: solution of WENO FV scheme) .....	68
5.64 The density profile at $t = 0.4$ hr for square wave problem (Mesh: $200 \times 200$ ; Solid line: exact solution; Dashes: solution of Godunov-type TVD Slope-limiter FD scheme; Dots: solution of WENO FV scheme) .....	69
5.65 The density profile at $t = 0.4$ hr for square wave problem (Mesh: $400 \times 400$ ; Solid line: exact solution; Dashes: solution of Godunov-type TVD Slope-limiter FD scheme; Dots: solution of WENO FV scheme) .....	69
5.66 The equilibrium speed-density and flow-density relationships developed by Del Castillo and Benitez.....	71

5.67	Two conditions in case I of shock problems: (a) $k_l = 0.03$ veh/m and $k_r = 0.14$ veh/m; (b) $k_l = 0.1$ veh/m and $k_r = 0.14$ veh/m.....	72
5.68	The density and velocity behaviors for condition (a) in case I of shock problems simulated by Jiang's FD scheme with PW model .....	72
5.69	The density and velocity behaviors for condition (a) in case I of shock problems simulated by WENO FV scheme with PW model.....	73
5.70	The density and velocity profiles at $t = 2$ min for condition (a) in case I of shock problems simulated with PW model (Dashes: solution of Jiang's FD scheme; Dots: solution of WENO FV scheme) .....	73
5.71	The density and velocity behaviors for condition (a) in case I of shock problems simulated by Jiang's FD scheme with Jiang's improved model.....	73
5.72	The density and velocity behaviors for condition (a) in case I of shock problems simulated by WENO FV scheme with Jiang's improved model.....	74
5.73	The density and velocity profiles at $t = 2$ min for condition (a) in case I of shock problems simulated with Jiang's improved model (Dashes: solution of Jiang's FD scheme; Dots: solution of WENO FV scheme) .....	74
5.74	The density and velocity behaviors for condition (b) in case I of shock problems simulated by Jiang's FD scheme with PW model .....	75
5.75	The density and velocity behaviors for condition (b) in case I of shock problems simulated by WENO FV scheme with PW model.....	75
5.76	The density and velocity profiles at $t = 2$ min for condition (b) in case I of shock problems simulated with PW model (Dashes: solution of Jiang's FD scheme; Dots: solution of WENO FV scheme) .....	75
5.77	The density and velocity behaviors for condition (b) in case I of shock problems simulated by Jiang's FD scheme with Jiang's improved model.....	76
5.78	The density and velocity behaviors for condition (b) in case I of shock problems simulated by WENO FV scheme with Jiang's improved model.....	76
5.79	The density and velocity profiles at $t = 2$ min for condition (b) in case I of shock problems simulated with Jiang's improved model (Dashes: solution of Jiang's FD scheme; Dots: solution of WENO FV scheme) .....	76
5.80	Two conditions in case II of shock problems: (a) $k_l = 0.02$ veh/m and $k_r = 0.1$ veh/m; (b) $k_l = 0.02$ veh/m and $k_r = 0.04$ veh/m.....	77
5.81	The density and velocity behaviors for condition (a) in case II of shock problems simulated by Jiang's FD scheme with PW model .....	77

5.82	The density and velocity behaviors for condition (a) in case II of shock problems simulated by WENO FV scheme with PW model.....	78
5.83	The density and velocity profiles at $t = 2$ min for condition (a) in case II of shock problems simulated with PW model (Dashes: solution of Jiang's FD scheme; Dots: solution of WENO FV scheme) .....	78
5.84	The density and velocity behaviors for condition (a) in case II of shock problems simulated by Jiang's FD scheme with Jiang's improved model.....	78
5.85	The density and velocity behaviors for condition (a) in case II of shock problems simulated by WENO FV scheme with Jiang's improved model.....	79
5.86	The density and velocity profiles at $t = 2$ min for condition (a) in case II of shock problems simulated with Jiang's improved model (Dashes: solution of Jiang's FD scheme; Dots: solution of WENO FV scheme) .....	79
5.87	The density and velocity behaviors for condition (b) in case II of shock problems simulated by Jiang's FD scheme with PW model .....	80
5.88	The density and velocity behaviors for condition (b) in case II of shock problems simulated by WENO FV scheme with PW model.....	80
5.89	The density and velocity profiles at $t = 2$ min for condition (b) in case II of shock problems simulated with PW model (Dashes: solution of Jiang's FD scheme; Dots: solution of WENO FV scheme) .....	80
5.90	The density and velocity behaviors for condition (b) in case II of shock problems simulated by Jiang's FD scheme with Jiang's improved model.....	81
5.91	The density and velocity behaviors for condition (b) in case II of shock problems simulated by WENO FV scheme with Jiang's improved model.....	81
5.92	The density and velocity profiles at $t = 2$ min for condition (b) in case II of shock problems simulated with Jiang's improved model (Dashes: solution of Jiang's FD scheme; Dots: solution of WENO FV scheme) .....	81
5.93	Case III of shock problems: $k_l = 0.0133$ veh/m and $k_r = 0.1636$ veh/m .....	82
5.94	The density and velocity behaviors for case III of shock problems simulated by Jiang's FD scheme with PW model.....	82
5.95	The density and velocity behaviors for case III of shock problems simulated by WENO FV scheme with PW model.....	83
5.96	The density and velocity profiles at $t = 2$ min for case III of shock problems simulated with PW model (Dashes: solution of Jiang's FD scheme; Dots: solution of WENO FV scheme).....	83

5.97	The density and velocity behaviors for case III of shock problems simulated by Jiang' s FD scheme with Jiang' s improved model.....	83
5.98	The density and velocity behaviors for case III of shock problems simulated by WENO FV scheme with Jiang' s improved model.....	84
5.99	The density and velocity profiles at $t = 2$ min for case III of shock problems simulated with Jiang' s improved model (Dashes: solution of Jiang' s FD scheme; Dots: solution of WENO FV scheme) .....	84
5.100	Two conditions in case I of rarefaction wave problems: (a) $k_l = 0.1$ veh/m and $k_r = 0.02$ veh/m; (b) $k_l = 0.04$ veh/m and $k_r = 0.02$ veh/m.....	85
5.101	The density and velocity behaviors for condition (a) in case I of rarefaction wave problems simulated by Jiang' s FD scheme with PW model.....	85
5.102	The density and velocity behaviors for condition (a) in case I of rarefaction wave problems simulated by WENO FV scheme with PW model.....	86
5.103	The density and velocity profiles at $t = 2$ min for condition (a) in case I of rarefaction wave problems simulated with PW model (Dashes: solution of Jiang' s FD scheme; Dots: solution of WENO FV scheme).....	86
5.104	The density and velocity behaviors for condition (a) in case I of rarefaction wave problems simulated by Jiang' s FD scheme with Jiang' s improved model.....	86
5.105	The density and velocity behaviors for condition (a) in case I of rarefaction wave problems simulated by WENO FV scheme with Jiang' s improved model .....	87
5.106	The density and velocity profiles at $t = 2$ min for condition (a) in case I of rarefaction wave problems simulated with Jiang' s improved model (Dashes: solution of Jiang' s FD scheme; Dots: solution of WENO FV scheme) .....	87
5.107	The density and velocity behaviors for condition (b) in case I of rarefaction wave problems simulated by Jiang' s FD scheme with PW model.....	88
5.108	The density and velocity behaviors for condition (b) in case I of rarefaction wave problems simulated by WENO FV scheme with PW model.....	88
5.109	The density and velocity profiles at $t = 2$ min for condition (b) in case I of rarefaction wave problems simulated with PW model (Dashes: solution of Jiang' s FD scheme; Dots: solution of WENO FV scheme).....	88
5.110	The density and velocity behaviors for condition (b) in case I of rarefaction wave problems simulated by Jiang' s FD scheme with Jiang' s improved model.....	89
5.111	The density and velocity behaviors for condition (b) in case I of rarefaction wave problems simulated by WENO FV scheme with Jiang' s improved model .....	89

5.112 The density and velocity profiles at $t = 2$ min for condition (b) in case I of rarefaction wave problems simulated with Jiang's improved model (Dashes: solution of Jiang's FD scheme; Dots: solution of WENO FV scheme) .....	89
5.113 Two conditions in case II of rarefaction wave problems: (a) $k_l = 0.14$ veh/m and $k_r = 0.03$ veh/m; (b) $k_l = 0.14$ veh/m and $k_r = 0.1$ veh/m.....	90
5.114 The density and velocity behaviors for condition (a) in case II of rarefaction wave problems simulated by Jiang's FD scheme with PW model.....	90
5.115 The density and velocity behaviors for condition (a) in case II of rarefaction wave problems simulated by WENO FV scheme with PW model .....	91
5.116 The density and velocity profiles at $t = 2$ min for condition (a) in case II of rarefaction wave problems simulated with PW model (Dashes: solution of Jiang's FD scheme; Dots: solution of WENO FV scheme).....	91
5.117 The density and velocity behaviors for condition (a) in case II of rarefaction wave problems simulated by Jiang's FD scheme with Jiang's improved model.....	91
5.118 The density and velocity behaviors for condition (a) in case II of rarefaction wave problems simulated by WENO FV scheme with Jiang's improved model....	92
5.119 The density and velocity profiles at $t = 2$ min for condition (a) in case II of rarefaction wave problems simulated with Jiang's improved model (Dashes: solution of Jiang's FD scheme; Dots: solution of WENO FV scheme) .....	92
5.120 The density and velocity behaviors for condition (b) in case II of rarefaction wave problems simulated by Jiang's FD scheme with PW model.....	93
5.121 The density and velocity behaviors for condition (b) in case II of rarefaction wave problems simulated by WENO FV scheme with PW model .....	93
5.122 The density and velocity profiles at $t = 2$ min for condition (b) in case II of rarefaction wave problems simulated with PW model (Dashes: solution of Jiang's FD scheme; Dots: solution of WENO FV scheme).....	93
5.123 The density and velocity behaviors for condition (b) in case II of rarefaction wave problems simulated by Jiang's FD scheme with Jiang's improved model.....	94
5.124 The density and velocity behaviors for condition (b) in case II of rarefaction wave problems simulated by WENO FV scheme with Jiang's improved model....	94
5.125 The density and velocity profiles at $t = 2$ min for condition (b) in case II of rarefaction wave problems simulated with Jiang's improved model (Dashes: solution of Jiang's FD scheme; Dots: solution of WENO FV scheme) .....	94
5.126 Case III of rarefaction wave problems: $k_l = 0.1636$ veh/m and $k_r = 0.0133$ veh/m.	95

5.127 The density and velocity behaviors for case III of rarefaction wave problems simulated by Jiang's FD scheme with PW model .....	95
5.128 The density and velocity behaviors for case III of rarefaction wave problems simulated by WENO FV scheme with PW model.....	96
5.129 The density and velocity profiles at $t = 2$ min for case III of rarefaction wave problems simulated with PW model (Dashes: solution of Jiang's FD scheme; Dots: solution of WENO FV scheme) .....	96
5.130 The density and velocity behaviors for case III of rarefaction wave problems simulated by Jiang's FD scheme with Jiang's improved model.....	96
5.131 The density and velocity behaviors for case III of rarefaction wave problems simulated by WENO FV scheme with Jiang's improved model.....	97
5.132 The density and velocity profiles at $t = 2$ min for case III of rarefaction wave problems simulated with Jiang's improved model (Dashes: solution of Jiang's FD scheme; Dots: solution of WENO FV scheme) .....	97
5.133 The equilibrium speed-density and flow-density relationship proposed by Kerner and Konhäuser.....	99
5.134 The initial variation of the average density $k_0 = 0.035$ veh/m.....	99
5.135 Temporal evolution of traffic on a ring of 32.2 km circumference with a homogeneous initial traffic and a localized perturbation of amplitude $\tilde{A}_0 = 0.01$ veh/m simulated by Jiang's FD scheme for: (a) $k_0 = 0.03$ veh/m; (b) $k_0 = 0.035$ veh/m; (c) $k_0 = 0.04$ veh/m; (d) $k_0 = 0.042$ veh/m; (e) $k_0 = 0.046$ veh/m; (f) $k_0 = 0.07$ veh/m; (g) $k_0 = 0.077$ veh/m; (h) $k_0 = 0.08$ veh/m; (i) $k_0 = 0.082$ veh/m; (j) $k_0 = 0.085$ veh/m.....	101
5.136 Temporal evolution of traffic on a ring of 32.2 km circumference with a homogeneous initial traffic and a localized perturbation of amplitude $\tilde{A}_0 = 0.01$ veh/m simulated by WENO FV scheme for: (a) $k_0 = 0.03$ veh/m; (b) $k_0 = 0.035$ veh/m; (c) $k_0 = 0.04$ veh/m; (d) $k_0 = 0.042$ veh/m; (e) $k_0 = 0.046$ veh/m; (f) $k_0 = 0.07$ veh/m; (g) $k_0 = 0.077$ veh/m; (h) $k_0 = 0.08$ veh/m; (i) $k_0 = 0.082$ veh/m; (j) $k_0 = 0.085$ veh/m.....	103
5.137 Comparison of down-critical density $k_d$ and up-critical density $k_u$ between Jiang's FD and WENO FV solutions .....	104

# Nomenclature

$k$	: density (or concentration)
$q$	: traffic flow rate
$u$	: velocity (or speed)
$\bar{e}$	: character speed
$k_l$	: upstream density
$k_r$	: downstream density
$\Delta t$	: time interval
$\Delta x$	: space interval
$I_i$	: cell $i$
$x_i$	: center of cell $I$
$\Delta x_i$	: size of cell $I$
$S_r(i)$	: stencil $r$ of cell $i$
$w_r$	: weight of stencil $r$
$\hat{a}_r$	: smooth indicator of stencil $r$
$u_f$	: free-flow speed
$k_j$	: jam density
$L_1$	: 1-norm error
$L_\infty$	: infinity-norm error
$k_d$	: down-critical density
$k_u$	: up-critical density
$M$	: number of meshes in space
$N$	: number of time interval