高解析方法之巨觀車流模式數值解

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摘　要

在智慧型運輸系統的發展中，巨觀動態車流模式能描述車流現象且為其系統發展中關鍵重要的一環，而數值模擬在求解車流模式上乃是十分重要的工具。巨觀車流模式源於車流為連續流體的觀念，相關研究推演出一雙曲線型偏微分方程式或方程組，並給予適當的起始及邊界條件。由於雙曲線型偏微分方程求解不易，已有許多數值方法被提出以求得合理的數值解。其中一階準確的數值方法在解不連續處會有平滑化的情形，因而導致不精確的數值解；而較高階之方法卻會在解不平滑處附近發生震盪的現象。在頻譜法中，此種震盪稱為吉布斯(Gibbs)現象，其並不會隨著網格的更新加切而消失。本研究應用高解析數值方法求解一階及高階車流波動方程式，以期改進數值精確度及避免解不連續處之震盪。研究中應用高階加權基本不震盪(Weighted Essentially Non-Oscillatory)有限差分及有限體積法求解具黎曼問題(Riemann problems)性質之車流波動方程，並將結果與全變量消逝(Total Variation Diminishing)法及先前其他相關研究中用於求解車流模式之數值方法作比較。在一階車流模式的測試問題中，包括衝擊波、消散波、號誌及方形波，加權基本不震盪法具有極優勢的準確度。而在求解包括衝擊波及消散波例題的高階車流模式之黎曼問題時，加權基本不震盪法亦能產生合理有效的數值解。由研究結果顯示加權基本不震盪法具備模擬複雜車流現象的潛力，包括衝擊波、消散波、斷續性車流及區段擁擠性車流。未來藉由結合加權基本不震盪法與平行處裡，平行高解析數值方法將是一個可靠、有效率並精確的車流模擬數值解方法。

High Resolution Schemes for the Numerical Solutions of Macroscopic Continuum Traffic Flow Models

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Abstract

Numerical simulation is significant to solve macroscopic continuum traffic flow models, which describe various traffic phenomena and play an important role in the development of Intelligent Transport Systems (ITS). Continuum traffic flow models are often analyzed with systems of hyperbolic partial differential equations (PDEs) attended by suitable initial and boundary conditions. Due to difficulty in solving hyperbolic PDEs, numerous numerical methods have been presented to afford a considerable approach attaining the reasonable solution. The first order accurate method yields a numerical diffusion, which causes smoothing of shock fronts, and is inaccurate. However, higher order methods produce unrealistic oscillations close to steep gradients. Such oscillations, which are called the Gibbs phenomena in spectral methods, don't decay in magnitude when refining the mesh. The objective of the study was to simulate continuum traffic flow models with high resolution methods that improve the numerical precision and eliminate such spurious oscillations near discontinuities. High order Weighted Essentially Non-Oscillatory (WENO) finite difference and finite volume schemes were applied to solve the simple and high order continuum traffic flow models that involve the discontinuous initial conditions and thus are Riemann problems. The numerical solutions of WENO schemes were compared with results produced by Total Variation Diminishing (TVD) type scheme and other numerical methods previously used to solve traffic flow models. Test problems of the simple continuum model, including shock, rarefaction wave, traffic signal, and square wave cases, were shown to illustrate the dominant accuracy of WENO schemes. WENO schemes also exhibited the capability of presenting appropriate results in Riemann problems of high order continuum models with numerical examples, including shock and rarefaction wave problems, for Payne-Whitham (PW) and Jiang's improved models. The results indicate that WENO schemes can afford to be utilized in the simulation of complex traffic phenomena, such as shock, rarefaction waves, stop-and-go waves, and local cluster effects. In the future, with the implementation of parallel processing the WENO algorithm, parallel high resolution numerical scheme would be a reliable, fast, and robust method for traffic flow simulation.

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Nomenclature

k	: density (or concentration)
q	: traffic flow rate
u	: velocity (or speed)
ë	: character speed
k_l	: upstream density
k_r	: downstream density
Ät	: time interval
Äx	: space interval
I_i	: cell i
x_i	: center of cell I
Äx_i	: size of cell I
$S_r(i)$	: stencil r of cell i
w_r	: weight of stencil r
$\hat{a}_r$	: smooth indicator of stencil r
u_f	: free-flow speed
k_j	: jam density
L_1	: 1-norm error
L_∞	: infinity-norm error
k_d	: down-critical density
k_u	: up-critical density
M	: number of meshes in space
N	: number of time interval