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A TIME-SPACE NETWORK MODEL FOR ALLOCATING YARD STORAGE SPACE FOR EXPORT CONTAINERS

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ABSTRACT

To expedite the process of loading export containers onto vessels, it is common for port terminals to gather containers in advance and stack them in container yards according to their attributes, such as voyage, port of discharge, weight and size. As modern container vessels are getting larger, the limited storage space a container yard provides needs to accommodate more containers. Consequently, how to use storage space more efficiently has become an essential consideration of container yard management. In this article, a mathematical model with time-space network is developed to deal with the storage space allocation problem for export containers in a container yard. A real-world case extracted from one of the dedicated container terminals at Kaohsiung Port is studied utilizing this model.

Key Words: *Storage space allocation; Export container; Time-space network*

I. INTRODUCTION

Container yards are very precious resources in container port terminals. They provide

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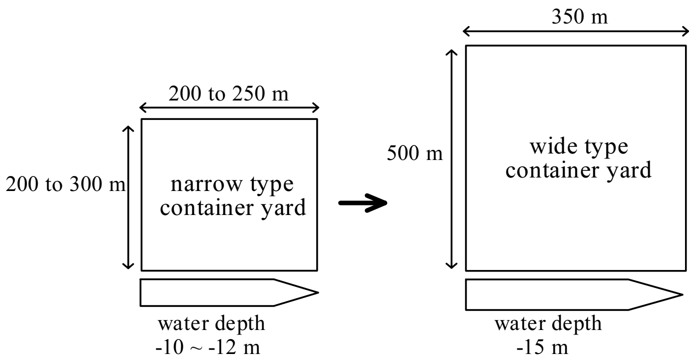
storage space for temporarily keeping containers to facilitate the process of loading/discharging containers onto/from vessels. In practice, to shorten vessel berthing time, the process has to be smooth. All export containers are going to be loaded onto incoming vessels. They are required to be delivered to the terminal before the arrival of vessels, which gives rise to a demand for storage space within which to keep containers. Storage space is needed for accommodating export containers, and import containers, which are discharged from vessels awaiting consignees to pick up.

To lower average unit cost, the capacities of modern container vessels are getting larger, resulting in more containers being discharged and loaded when a huge vessel is berthed. In other words, more storage space in a yard is needed in port terminals to serve huge vessels (Figure 1), but it may not be easy to extend container yards in most busy port terminals. Consequently, more sophisticated yard planning skill and stacking containers in higher tiers have become typical methods for dealing with the considerable amount of containers coming from huge vessels.



Source: [1]

Figure 1　Relationship between the Size of Container Yard and Vessel

Empirically, the operation of storing export containers in a container yard is composed of two stages (Figure 2). The first stage is called “yard planning”, which is performed before the arrival of containers. In the yard planning stage, the storage space of the yard in a two-dimensional (2D) aspect is allocated to serve export containers with various attributes. Yard planning is a very important issue in container terminal management, as it directly influences the usage of the limited storage space. The second stage is called “real-time assignment”, which must be performed when an export container is delivered to the yard. At this moment, a storage slot with a three-dimensional (3D) coordinate is real-time assigned according to the attributes of the arriving container. Choosing an appropriate three-dimensional storage slot depends on the outcome of the first stage, and the availability of yard cranes in the container yard. This paper focuses on the issue of "yard planning".

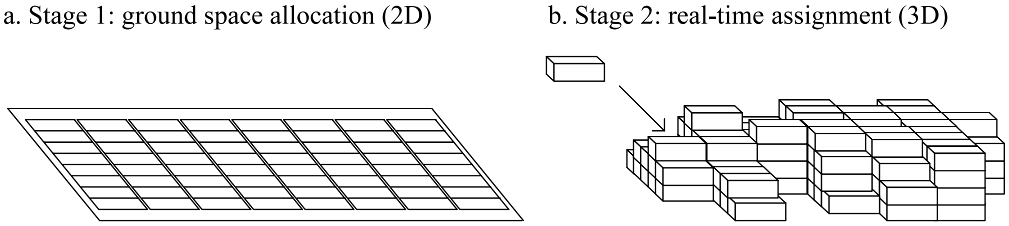


Figure 2 Two-Stage Procedure for Allocating Storage Space to Export Containers

In the yard planning, how the ground storage space of a yard is allocated to containers will influence the efficiency of loading containers onto vessels, and receiving export containers from shippers for future loading. There are two main strategies that have been popularly used in yard planning, namely, "premarshalling" and "sort and store" [2]. When the former is applied, the storage space of a container yard has to be segmented into a pooling area and a marshalling area. The pooling area is used to receive containers from shippers without precise categorization. In this area, containers can be stacked closer and higher to secure more yard capacity. Before vessel's arrival, those containers that are going to be loaded onto the coming vessel have to be moved to the marshalling area, and stacked in accordance with the planned loading sequence.

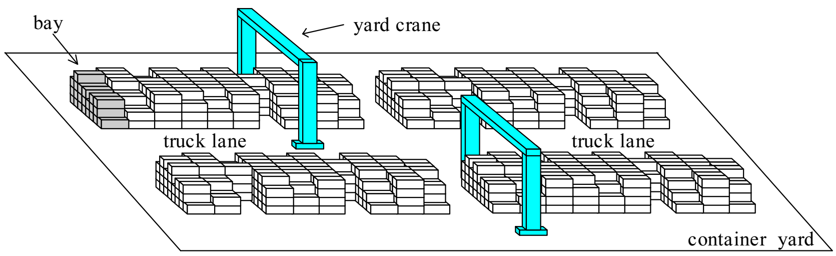
To avoid the burden of moving containers from the pooling area to the marshalling area, the "sort and store" strategy is usually adopted. Its main idea is that the storage space in a container yard is respectively reserved in advance according to the calling schedule of vessels and the attributes of containers. A reserved storage space is called a "preferred area" to a specified group of container, like voyages. Once an export container is delivered to the terminal, its attributes must first be identified to decide to which group it belongs, and then it will be moved to its preferred area for storage. The main advantage of this strategy is that the loading operation can be launched directly from the container yard to shipside, without moving containers from the pooling area to the marshalling area beforehand. However, utilization of the yard's storage space tends to be lower since containers must be stored in the preferred area instead of being pooled. Further, more complicated planning for storage space allocation of container yards is also critically needed. The purpose of this article is to develop a mathematical model with a time-space network to deal with yard planning with the "sort and store" strategy.

The rest of this article is organized as follows. In section 2, the storage space allocation problem of container yards is described in detail. Section 3 presents the conceptual model and mathematical formulation of the storage space allocation problem of a container yard. The case study is conducted in section 4, which is composed of a test case and a real-world case. The conclusions are presented in the last section.

II. THE STORAGE SPACE ALLOCATION PROBLEM

Container yards provide temporary storage for containers to facilitate the loading/discharging procedures at the shipside of the terminal. Usually, to secure more storage capacity, containers are stacked vertically in a yard, and yard cranes are set up to straddle across containers for handling lift-in and lift-out operations (Figure 3a). For container yards that adopt the transfer crane system, some belt space must be reserved for trucks to shuttle containers, and the remaining space is divided into several zones to store containers. For example, in Figures 3a and 3b, four zones are separated by truck lanes. Each zone is composed of ten bays; a bay is composed of eight ground slots that are the basic units for container storage. It is common to allocate a set of adjacent ground slots, called a ground-slot set (Figure 3b), to store containers with the same attributes because this can enable a yard crane to work more continuously when moving containers to shipside for loading. Therefore, as far as the supply of a yard space is concerned, the ground-slot set is commonly used as a basic spatial unit to be reserved as a preferred area in the yard planning stage.

a. A typical container yard with storage space and yard cranes



b. Ground slots and ground-slot sets in a container yard

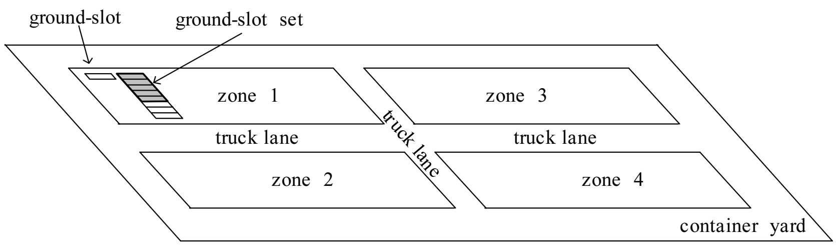


Figure 3　Configurations of a Container Yard

As for the demand of a yard space, there are four properties that should be taken into account. First, to facilitate movement of containers to shipside for loading, containers that will enter the yard should be grouped in advance. For instance, containers that are going to be loaded onto the same vessel are usually deemed as a group, and stored in adjacent ground slots. In practice, therefore, it is appropriate to use ground-slot sets for measuring and estimating the demands of containers for yard space.

Moreover, for a container yard that adopts the “sort and store” strategy to deal with export containers, the main purpose of yard planning is to decide which ground-slot sets should be reserved as the preferred area for certain container groups, and how long each ground-slot set should be occupied. For a specific container group, the ground-slot sets reserved as its preferred area are those the yard planner wishes them to be stored in. For example, in Figure 4, if vessel *i* is expected to berth at berth 1, then those ground-slot sets behind berth 1 are more suitable for storing the containers that are going to be loaded onto vessel *i*. To achieve this purpose, ground-slot sets behind berth 1 are usually reserved in advance as a specified preferred area, and containers will be stored in adjacent ground-slot sets 2-5 before the vessel’s arrival.

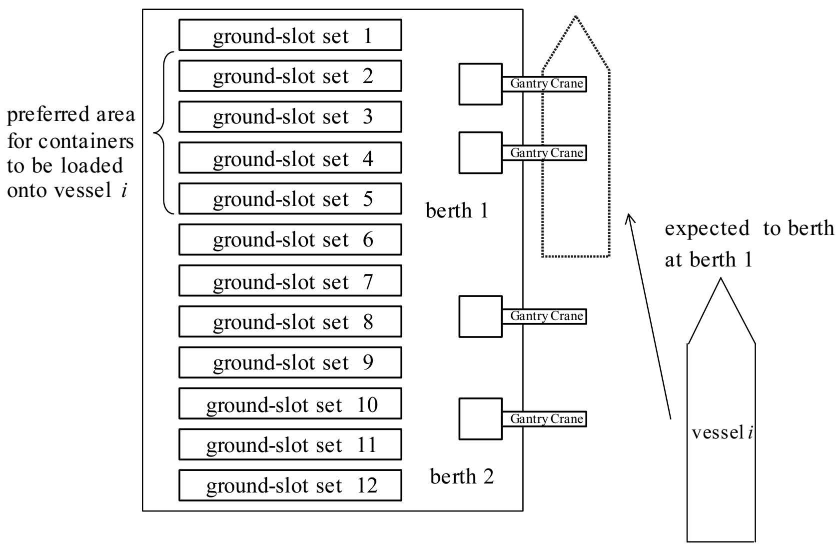


Figure 4 An Example of Deciding Preferred Areas

Second, while reserving ground set-sets as a specific preferred area, the time and space aspects should be taken into consideration. For example, if a ground-slot set is suitable for a container group, but it has already been occupied by another early-came container group, then the occupied ground-slot set cannot be allocated again. Instead, another ground-slot set in the yard must be found and reserved as the corresponding preferred area. The occupied ground-slot set will not be available until the containers that occupy it have been moved away. Third, the arrival pattern of export containers should be considered. In general, for a specific group, most export containers arrive at the terminal several days before the vessel's arrival, instead of arriving all together at the terminal on the same day. In addition, the number of containers in a specific group could be further divided into several subgroups, which have the same assignment cost but arrive at the yard at different time. Figure 5 shows a typical arrival rate curve of a specific group. The numbers of ground-slot sets required by each subgroup are not the same. For instance, subgroup 1 needs 1 ground-slot set and subgroup 4 requires 3 ground-slot sets in Figure 5. It is clear that the closer to the vessel’s arrival date, the more ground-slot sets are needed for container subgroups. Fourth, yard space demand is controllable to a certain extent. Once there are too many containers that need the yard space, terminal operators will accept only part of the total quantity of containers that is below the yard capacity, and shift the surplus to other yards near the terminal for accommodation. In this article, only the demand within the capacity of container yard is discussed.

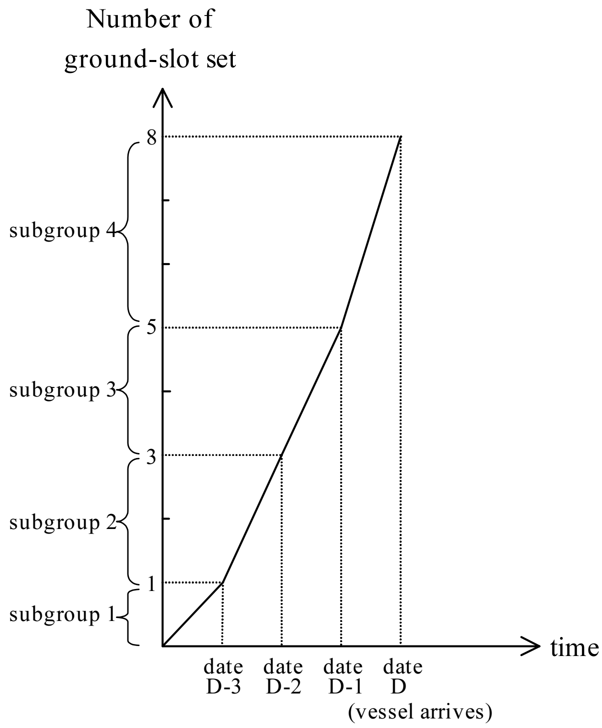


Figure 5　A Typical Arrival Rate Curve of a Specific Group of Export Containers

Taleb-Ibrahimi, Castilho and Daganzo formulated a queuing model to estimate storage area demand for keeping export containers [3]. They showed that reshuffling might be effectively reduced if the yard space could be divided into a pooling area and a marshalling area, conceptually similar to the above-mentioned "premarshalling" strategy. As regards allocation of the storage space of a container yard, Chen [2] examined the "premarshalling" and "sort and store" strategies, mainly adopted by most busy port terminals in the yard planning stage. In general, if the attributes of export containers are clear and not changeable, the "sort and store" strategy is more suitable for expediting the operation of loading containers onto vessels. However, if the uncertainty of container attributes is high, for example, changes in relation to the POD (port of discharging) or the target vessel, it is better to adopt a "premarshalling" strategy in order to reduce unproductive movements while loading containers onto vessels. The issue of how to real-time locate an export container in a yard, the second stage of allocating storage space (Figure 2b), was studied by Lan and Kao [4]. They focused on how to assign appropriate slots to incoming containers in order to minimize the unproductive moves when picking them out. Slots used to store containers were considered in a three dimensional (3D) aspect. An integer programming model had been formulated to deal with the assignment problem. When the number of containers is not large, and the arriving/departing sequence of each container is known, the model can be employed to find an optimal allocation of the slots in the yard. Kim [5] also studied the slots assignment problem. The weight of the container was the only factor used to decide the storage location. A dynamic programming model was formulated to minimize the number of relocation movements anticipated in relation to the loading operation.

With regard to resource allocation problems, especially in space resource, Chen *et al.* [6] formulated a time-space network model for the airport gate assignment problem. Essentially, it was a minimum cost flow problem with some side constraints. Chen and Hsieh [7] presented a study focused on the resource allocation problem in seaports. They made use of a time-space network model to allocate berth length in a container terminal. It was also a minimum cost flow problem with some side constraints. Since a container yard is also a space resource which time and space aspects are taken into account, the time-space network is employed as well to develop a model to help find the best allocation pattern of ground-slot sets, called preferred areas, to each container subgroup in the planning period.

III. FORMULATION OF STORAGE SPACE ALLOCATION

This section is composed of three subsections that devise a time-space network model to deal with the storage space allocation problem of export container yards. First, some assumptions are made to facilitate model development. Second, the structure of the time-space network is clearly defined, which is the core of the model. Third, the mathematical formulation of the model is presented.

3.1 The Assumptions

There are many factors that influence the allocation of ground-slot sets to export containers, for instance, the uncertainty of the containers' arrival and vessel calling schedules. In order to focus on the usage of ground-slot sets in terms of time and space, the following assumptions are made to simplify the problem and facilitate model formulation.

1. In this study, six adjacent ground slots are assembled as a *ground-slot set*, which can store thirty containers.
2. The arrival rate of export containers in each group (Figure 5) is given, that is, each day in the planning period, the demand quantity of ground-slot sets for each subgroup can be estimated.
3. Each vessel departs on schedule, and takes away all the containers that are waiting to be loaded onto it from the ground-slot sets of the yard. The released ground-slot sets then become available again.
4. In practice, there are several factors influencing the allocation of yard space. For example, reefer, hazardous and oversize containers must be store in specific reserved areas in the yard. In this article, however, only the assignment of standard dry containers is studied. Thus, the assignment cost is used as the only decision factor for allocating yard space.
5. The assignment costs are designed in terms of ranking, which take the aspect of "distance to the quayside" and "number of yard cranes" into account. The ground-slot sets located near the berth at which the target vessel expected to berth should have lower assignment costs. However, some farther ground-slot sets may have lower assignment costs to let yard cranes in different zones (Figure 3) be able to work simultaneously while picking containers for loading onto the vessel.
6. Containers will not be removed once they have been stacked in a ground-slot set.
7. No attention is paid to the possible need to handle different sizes of container. It is only noted that a ground slot for a forty-foot container is twice as long as that of a twenty-foot container.

3.2 The Structure of the Time-Space Network

Essentially, the storage space allocation problem of a container yard is an assignment problem that assigns export containers to appropriate ground-slot sets to minimize total assignment costs (Figure 6). It must be pointed out that, as time goes on, some ground-slot sets are reserved as preferred areas, and some ground-slot sets are released to become usable again. Thus, the assignment problem should take time and space aspects into consideration. In this study, the time-space network that can represent entities moving in both the time and the space dimensions is employed. Figure 7 presents the modified time-space network adopted to deal with the reusable resource aspect of ground-slot set allocation. There are only two ground-slot sets in Figure 7. Containers in subgroup *i*, which are the first to arrive, can be assigned to ground-slot set 1 or ground-slot set 2, depending on the assignment cost. If ground-slot set 1 is allocated to container subgroup *i*, then containers belonging to subgroup *j*, which are the second group to arrive, must be assigned to ground-slot set 2 because ground-slot set 1 is occupied and cannot be re-allocated. After the departure of container subgroup *i*, ground-slot set 1 becomes available again. Thus, the reusable resource aspect can be handled with the time-space network.

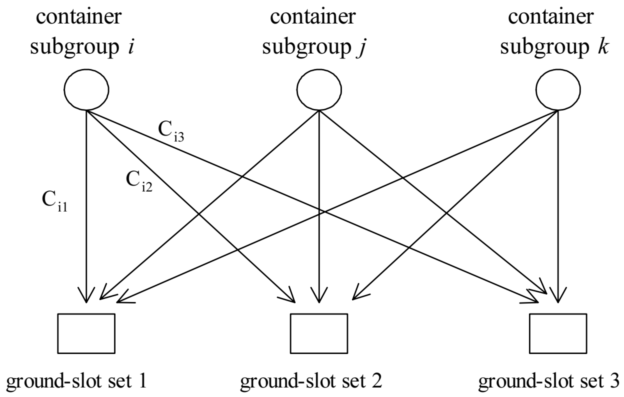
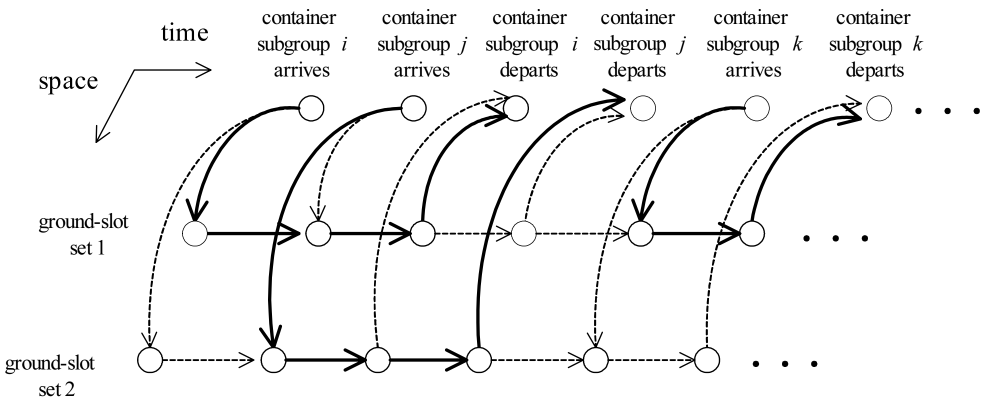


Figure 6 The Assignment Aspect in the Storage Space Allocation Problem



Note: solid lines represent the paths of flows.

Figure 7 The Reusable Resource Aspect of Ground-Slot Set Allocation

The concept of employing a time-space network to allocate ground-slot sets to export containers is illustrated with Figure 8. Definitions of nodes and arcs are presented below.

1. Nodes
   (1) The *arrival node* represents the time that containers arrive at the container yard. In Figure 8, node A shows the time that containers belonging to subgroup *i* arrive at the container yard.
   (2) The *departure node* represents the time that containers depart from a ground-slot set in the container yard. In Figure 8, node B shows the time when containers in subgroup *i* depart from the container yard, as they are taken away by a vessel.
   (3) The *ground-slot set node* is a node located on the time axis, which corresponds to a specific *arrival* or *departure node*, In Figure 8, nodes C and D are *ground-slot set nodes* corresponding to nodes A and B, respectively.
   (4) The *super node* (node S in Figure 8) shows the end of a planning period.
2. Arcs
   (1) The *arrival arc* links *an arrival node* with a *ground-slot set node*, e.g. arc (A,C) in Figure 8.
   (2) The *departure arc* links a departure *node* with a *ground-slot set node*, e.g. arc (D,B) in Figure 8.
   (3) The *ground-slot set arc* is an arc between two adjacent *ground-slot set nodes*, (e.g. arc (C,E) in Figure 8.
   (4) The super *arc* is an arc connecting the last *ground-slot set node* on a time axis with the *super node*, e.g. arc (G,S) in Figure 8.

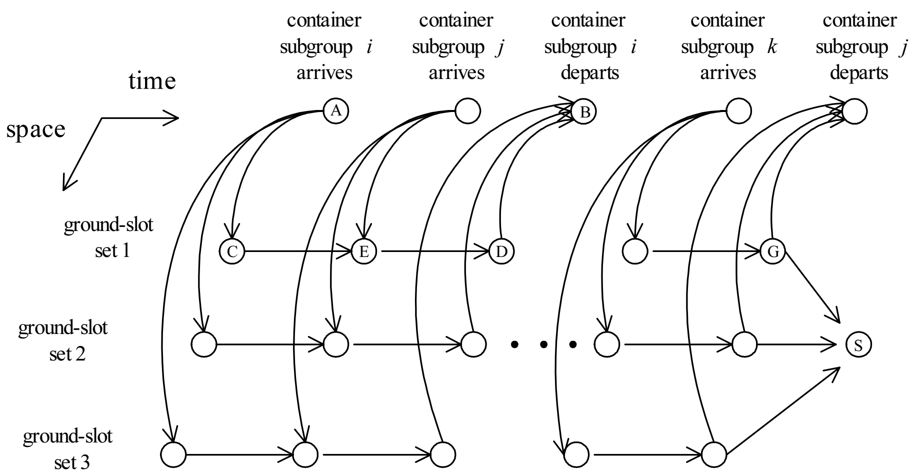


Figure 8　A Time-Space Network for Allocating Storage Space to Export Containers

3.3 The Mathematical Formulation

According to the assumptions and illustration of the time-space network described above, a model for allocating ground-slot sets in a container yard to export containers can be formulated as follows:

$$Min \quad \sum_{(i,j)\in AA} C_{ij}X_{ij} \tag{1}$$

$$s.t. \quad \sum_{(i,j)\in AA} X_{ij} = S_i \qquad \forall i \in AN \tag{2}$$

$$\sum_{(j,i)\in DA} X_{ji} = S_i \qquad \forall i \in DN \tag{3}$$

$$\sum_{\{j:(i,j)\in A\}} X_{ij} - \sum_{\{j:(j,i)\in A\}} X_{ji} = b_i \qquad \forall i \in CN \tag{4}$$

$$\sum_{(i,j)\in GA} X_{ij} = b_s \tag{5}$$

$$X_{ij} = X_{pq} \qquad \forall (i,j)\in AA,\ (p,q) = paired(i,j) \tag{6}$$

$$X_{ij} \in \{0,1\} \qquad \forall (i,j)\in A \tag{7}$$

where:

N：the set of all nodes,
A：the set of all arcs,
AN：the set of *arrival nodes*,
DN：the set of *departure nodes*,
CN：the set of *ground-slot set nodes*,
AA：the set of *arrival arcs*,
DA：the set of *departure arcs*,
CA：the set of *ground-slot set arcs*,
GA：the set of *collective arcs*,
i,j,p,q：the indices for nodes,
X_{ij} ：arc flow of arc (i, j),
C_{ij} ：cost coefficient of arc (i, j) ,
S_i ：container subgroup i's demand for ground-slot sets,
b_i ：supply quantity of *ground-slot set node* i; if a *ground-slot set* has been occupied at the beginning of the planning period, $b_i = 1$; otherwise $b_i = 0$,

b_s ： $\sum_{i \in AN} S_i - \sum_{j \in DN} S_j$ (number of occupied ground-slot sets at the end of the planning perion),

paired (*i, j*)：the *departure arc* paired with the *arrival ar*c (*i, j*), and corresponding to the same container subgroup.

The objective function of the model is to minimize the sum of the assignment costs. Constraint (2) and (3) state that the demand of container subgroup *i* be satisfied. Constraint (4) and (5) are the flow conservation constraint for all *ground-slot set nodes* and the *super node*, respectively. Constraint (6) states that the flow of each container subgroup on a *departure arc* should be equal to the arc flow on the corresponding *arrival arc*. Constraint (7) limits all decision variables, X_{ij} , to being binary. However, if arc flows on the *arrival arcs* and the *departure arcs* are binary, then arc flows of others should be binary, according to the unimodularity property of the network flow problem [8]. Therefore, it is not necessary to restrict all decision variables to being binary. Furthermore, due to the equality constraint (6), only the arc flows on the *arrival arcs* (or *departure arcs*) need to be restricted to being binary. Thus, constraint (7) can be substituted as follows:

$$X_{ij} \in \{0,1\}, \quad \forall (i,j) \in AA \tag{8}$$

$$0 \le X_{ij} \le 1 \quad \forall (i,j) \in A / AA \tag{9}$$

It should be noted that, the *arrival nodes* and *departure nodes* are corresponding to container subgroups, which have taken voyages into account. Therefore, all decision variables and parameters in the model also correspond to respective voyages. Essentially, the ground-slot set allocation model is a network flow problem with the side constraints, constraint (5). In the following section, this allocation model will be solved as an integer-programming problem with a direct approach, the branch and bound algorithm. However, if a large-scale problem is faced, it may be necessary to develop an algorithm taking the underlying network flow problem into account to solve it.

IV. CASE STUDY

In this section, the storage space allocation problem at the container yard of a dedicated container terminal in the port of Kaohsiung is studied. Due to the scale of the problem is large, in order to clearly observe computation results from the time-space model, the case study is divided into two stages. First, a test case relating to containers for five vessel voyages is tested, which is followed by sensitivity analyses on the assignment costs, arrival dates and the demand for ground-slot set, respectively. Second, the real-world case is studied. Furthermore, when applying this allocation model for the yard space allocation problem, the arrival rate curve of each specific container group should be estimated as accurately as possible because they give rise to demands for ground-slot sets in each day in the planning period. In addition, this model has the dynamic aspect. Each day the optimal solution can be found with this model. The allocation solution of the first day in the planning period can be used for allocating ground-slot sets. The rest part of the solution can be used for reference. Next day, another optimal solution should be found with latest information for tuning the allocation of yard space. That is, this model is continuously executed every day or twice a day, to have dynamic allocation results.

4.1 The Test Case

The data needed for the test case are listed below:

1. Planning period: from April 1st through April 9th.
2. Yard space supply: there are 12 ground-slot sets in the yard for keeping export containers.
3. Yard space demand: there are two kinds of export containers that need ground-slot sets in the yard. One is the containers that have already been stacked in the ground-slot sets at the beginning of the planning period (Table 1). The other is those containers that will arrive in the yard during the planning period (Table 2).
4. Assignment costs: considering the factors that directly influence the loading operation, such as distances to the quayside, and number of yard cranes used, the assumed assignment costs for newly arriving containers are presented in Table 3.

Table 1 Ground-Slot Sets Occupied at the Beginning of the Planning Period (Test Case)

Voyage	ground-slot sets needed	departure date	ground-slot sets occupied
1	1	April 5th	1,2,3
2	2	April 4th	11,12

Table 2 Demand for Ground-Slot Sets in the Planning Period (Test Case)

Date	April 3rd	April 4th	April 5th	April 6th	April 7th	April 8th	April 9th	April 10th
Voyage 3	1	2	2	3	departure	-	-	-
Voyage 4	-	-	1	1	2	2	3	departure

Table 3　Assignment Costs for Newly Arriving Containers (Test Case)

Voyage	ground-slot set 1	ground-slot set 2	ground-slot set 3	ground-slot set 4	ground-slot set 5	ground-slot set 6	ground-slot set 7	ground-slot set 8	ground-slot set 9	ground-slot set 10	ground-slot set 11	ground-slot set 12
3	1	2	3	4	2	2	3	4	3	3	3	4
4	2	2	3	4	1	2	3	4	2	2	3	4

4.2　Computation Results of the Test Case

The test case is solved by solver CPLEX and the minimum total assignment cost is 57. The result is displayed in Figure 9. In order to observe the result more clearly, Figure 9 has been transformed into Figure 10, which utilizes horizontal bars to show the time periods and ground-slot sets occupied by containers. Figure 10 shows that 5 ground-slot sets were occupied at the beginning of the planning period by containers going to be loaded onto vessels for voyages 1 and 2. Therefore, ground-slot set 1, 2, 3, 11 and 12 were not be available until the containers occupying these ground-slot sets had been taken away on the 4th and 5th of April. Table 2 shows containers for voyages 3 and 4 were arriving at the terminal from April 3rd through April 6th, and April 5th through April 9th, respectively. Both groups of containers needed ground-slot sets for storage on their arrival.

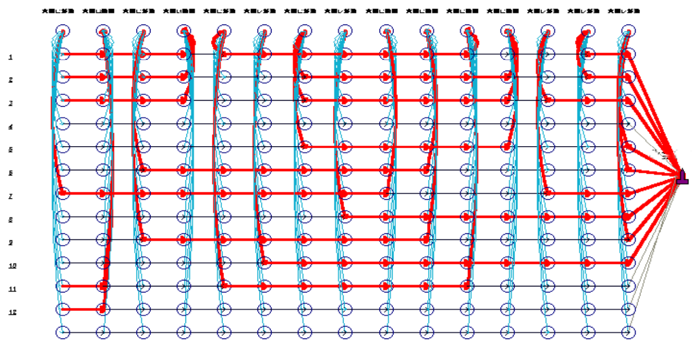


Figure 9　Computation Result with the Time-Space Network (Test Case)

Figure 10 indicates that containers for voyage 3 on April 3rd were assigned to ground-slot set 7 instead of ground-slot set 1, to which they should have been assigned according to the lowest cost given in Table 3, because ground-slot set 1 was already occupied. However, ground-slot set 1 was available again from April 5th, the date on which the containers occupying slot-set 1, 2, and 3 left. Therefore, on April 5th, some containers for voyage 3 were assigned to ground-slot set 1. The occupations shown in Figure 10 are the optimal combination for allocating ground-slot sets to all containers of the voyages in the planning period, with the lowest total assignment cost. The allocation result of any day in the planning period can be displayed in terms of actual location of ground-slot set. For example, Figure 11 presents the optimal allocation of each ground-slot set with its actual location in the container yard on April 4^{th}.

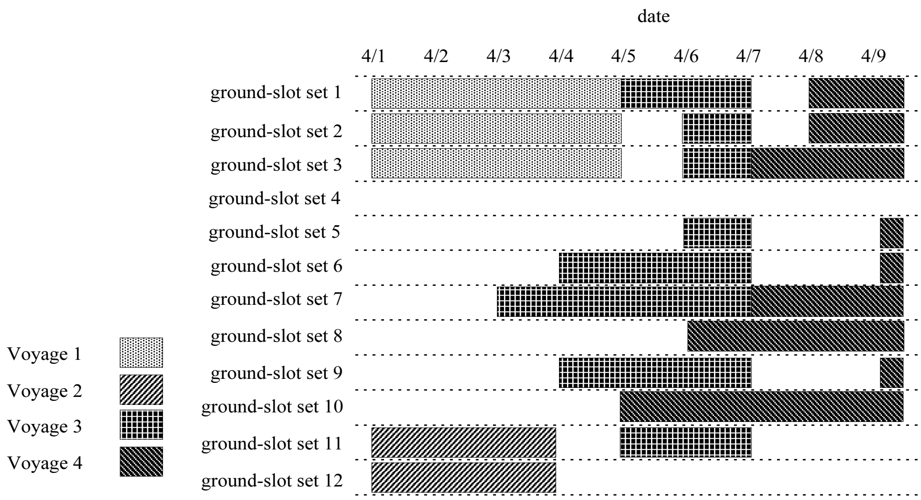


Figure 10　Computation Result with Time-Space Bars (Test Case)

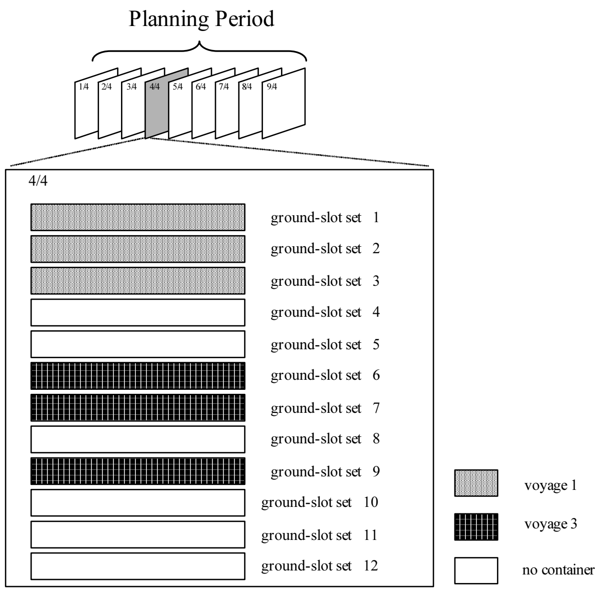


Figure 11　Computation Result of April 4^{th} (Test Case)

4.3 Sensitivity Analysis of the Test Case

To examine the validity and rationality of the above time-space network model, several changes are made to assignment costs, arrival dates of containers and demands for ground-slot sets. The optimal solution using the time-space network model is presented, respectively. The sensitivity analysis are conducted as follows:

4.3.1 Adjustment in Assignment Cost

The sensitivity analysis begins with an adjustment in assignment cost. In Figure 10, the utilization of ground-slot sets 4 and 12 is much lower than that of others due to their higher assignment cost. We now reduce the cost to 0 for assigning containers for voyage 3 to ground-slot set 12, and using the time-space network model endeavor to find the optimal solution. The optimal solution is indicated in Figure 12, which shows a total assignment cost of 36. Comparing Figure 10 with Figure 12, it is obvious that ground-slot set 12 is now also used by containers for voyage 3, as the related assignment has been reduced to 0.

Table 4　Adjustment in Assignment Cost (Test Case)

Voyage	ground-slot set 1	Ground-slot set 2	ground-slot set 3	ground-slot set 4	ground-slot set 5	ground-slot set 6	ground-slot set 7	ground-slot set 8	ground-slot set 9	ground-slot set 10	ground-slot set 11	ground-slot set 12
3	1	2	3	4	2	2	3	4	3	3	3	0*
4	2	2	3	4	1	2	3	4	2	2	3	4

*: Comparing Table 3, “3” has been changed to “0”.

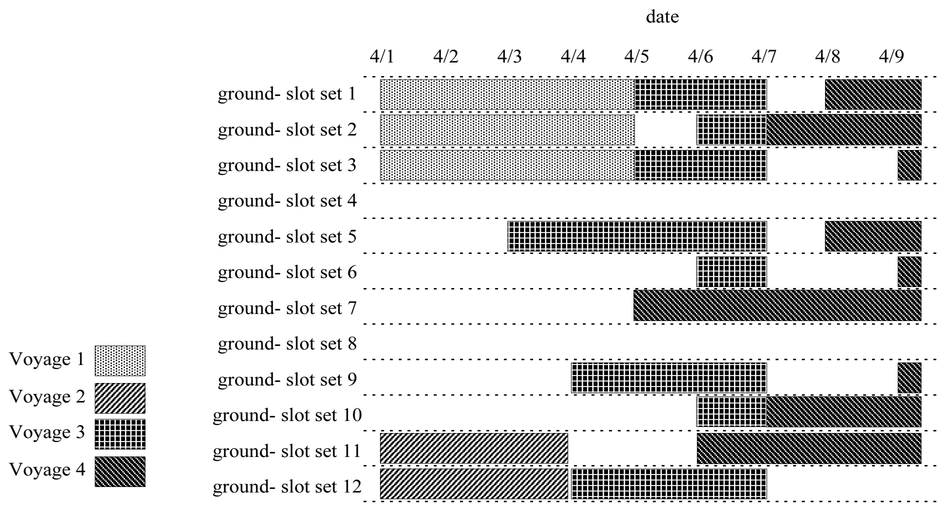


Figure 12　Computation Result after Adjustment in Assignment Costs (Test Case)

4.3.2 Adjustment in Arrival Date

Essentially, a ground-slot set can be viewed as a reusable space resource just like a parking space. The arrival sequence of containers will affect how the ground-slot sets are allocated. To ascertain the ability of the above model to find the optimal solution if the arrival dates are changed, we advance each arrival date of containers for voyage 3 by one day. The adjusted arrival dates are shown in Table 5. The optimal solution is shown in Figure 13, and the total assignment cost is 39.

Figure 10 shows that in the optimal solution for the original test case, the usage of ground-slot sets is not that tight before the arrival of containers for voyage 3, which means that even if each arrival date of containers for voyage 3 is advanced by one day, the total cost for assigning all containers for voyage 3 remains unchanged. For containers for voyage 3, the main difference between Figure 10 and 13 is that one time-space bar of containers for voyage 3 is moved from ground-slot set 3 to ground-slot set 10, but their assignment costs are both 3.

Table 5 Adjustment in Arrival Dates (Test Case)

Date	April 2nd	April 3rd	April 4th	April 5th	April 6th	April 7th	April 8th	April 9th	April 10th
Voyage 3	1*	2*	2*	3*	departure*	-	-	-	-
Voyage 4		-	-	1	1	2	2	3	departure

*: Comparing Table 2, the arrival dates are advanced by one day.

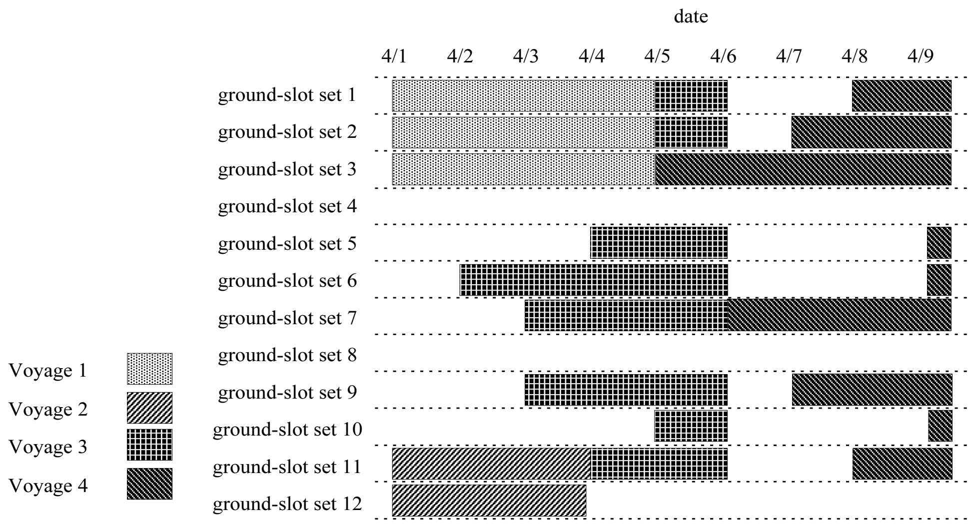


Figure 13 Computation Result after the Arrival Dates are Advanced (Test Case)

Taking advantage of the adjustment in arrival dates, when containers for voyage 4 start to arrive at the terminal, the possibility of finding less costly ground-slot sets for them is increased, and this is the main reason why there is a lower total assignment cost than in the original test case.

4.3.3 Adjustment in Demand for Ground-Slot Sets

It is clear from the results shown in Figures 10, 12, and 13 that there are still some time-spaces not covered by marked horizontal bars, that is, some time-space slots are still available during the planning period. Therefore, we will increase storage space demand to further test the time-space model by adding export containers to be loaded onto the vessel for voyage 5. The related data for containers for voyage 5 are inserted into Tables 6 and 7. The data for containers of voyages 3 and 4 remain the same as the original test case.

Figure 14 indicates the result of the optimal allocation of the ground-slot sets after demand is increased. The optimal total assignment cost is 46. All additional containers can be smoothly arranged to suitable time-space slots. However, in order to minimize the total assignment cost, assignments to ground-slot sets of containers for voyages 1, 2, 3, and 4 are slightly different from what they were in the optimal solution for the original test case.

Table 6　Adjustment in Demand for Ground-Slot Sets (Test Case)

Date	April 3rd	April 4th	April 5th	April 6th	April 7th	April 8th	April 9th	April 10th
Voyage 3	1	2	2	3	departure	-	-	-
Voyage 4	-	-	1	1	2	2	3	departure
Voyage 5						1*	1*	departure*

*: Comparing Table 2, additional demand for ground-slot set.

Table 7　Assignment Costs of the New Added Voyage (Test Case)

Voyage	ground-slot set 1	ground-slot set 2	ground-slot set 3	ground-slot set 4	ground-slot set 5	ground-slot set 6	ground-slot set 7	ground-slot set 8	ground-slot set 9	ground-slot set 10	ground-slot set 11	ground-slot set 12
5	3	3	3	4	2	2	3	4	1	2	3	4

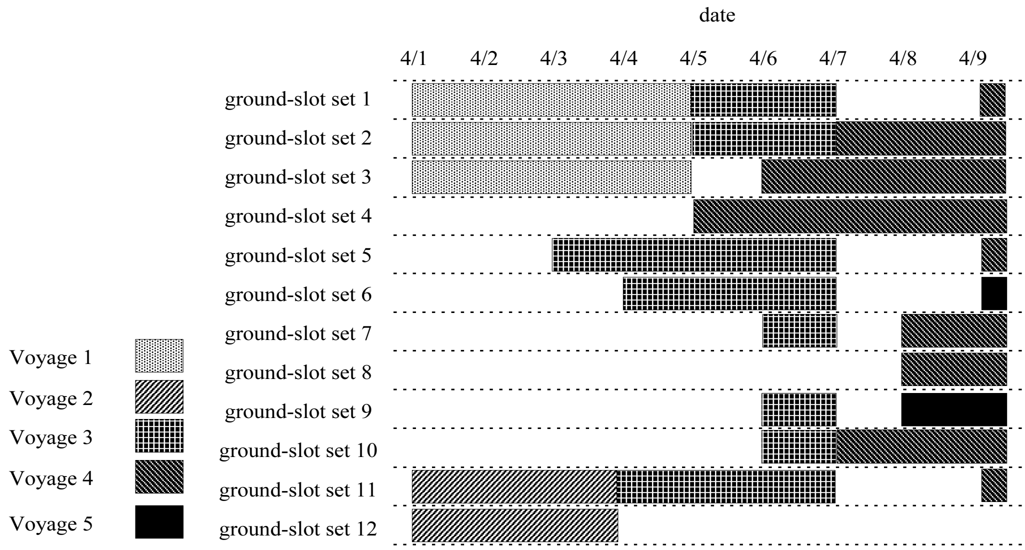


Figure 14 Computation Result after Demand for Ground-Slot Sets is Increased (Test Case)

4.4 A Real-World Case

In the last part of this section, the problem of container yard storage space allocation in a container terminal located in the port of Kaohsiung is studied using the above time-space network model. There are two container yards in the terminal for storing import and export containers, respectively.

The export container yard is composed of 160 ground-slot sets, each ground-slot set contains six adjacent ground slots. Transfer cranes in the yard can stack containers in five tiers at each ground slot. The planning period was between December 1^{st} and December 14^{th}. Ground-slot sets in the yard had to accommodate containers that were to be loaded onto vessels going on 51 different voyages. This case was assessed using the time-space network model constructed above with 30,174 links (decision variables), 15,778 nodes and 20,927 constraints. It took approximately 40 seconds on an IBM 586 PC to find the optimal solution with solver CPLEX.

Because too many decision variables were involved, it was necessary to transform the solution into graphics for easier observation. Figure 15 shows the optimal solution for the real-world case with bar charts. Using the vertical scroll bar, the occupation status of all ground-slot sets during the planning period could be checked one by one. In addition, for an arbitrary date in the planning period, the occupation status and the actual location of each ground-slot set could be browsed by another form of display. Figure 16 shows the status of all

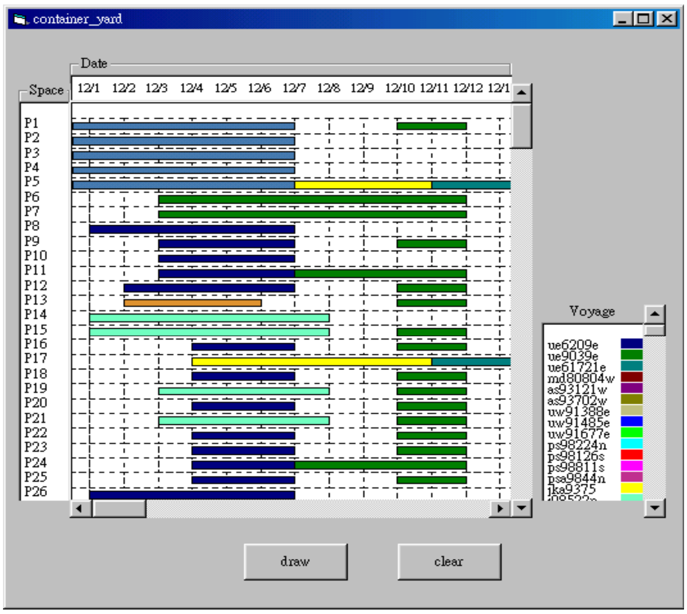


Figure 15　Computation Result with Time-Space Bars (Real-World Case)

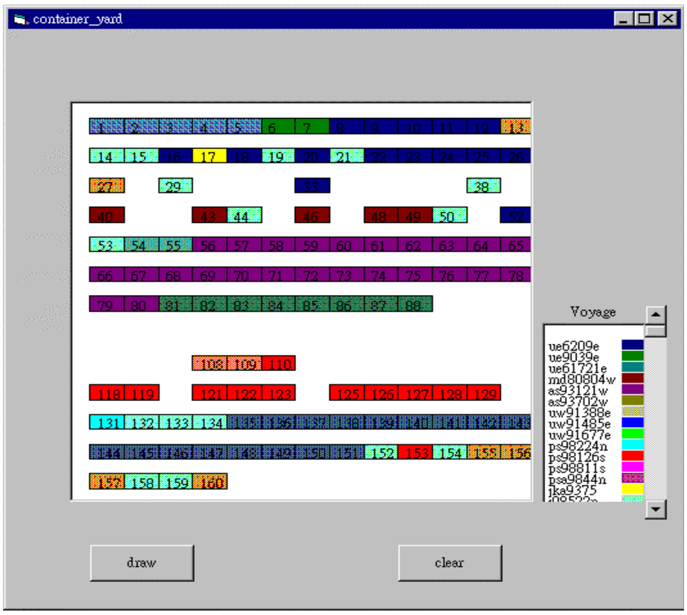


Figure 16　Computation Result of December 5th (Real-World Case)

ground-slot sets in the export container yard on December 5th. Most of the containers belonging to the same voyage are stored in adjacent ground-slot sets, which expedites the ship planning procedure, and shortens the distance yard cranes have to move when picking up these containers for loading onto a vessel.

V. CONCLUSIONS

One of the major problems container terminal operators facing is the yard space allocation problem. As vessels capacities continue to expand, the efficient utilization of existing storage spaces has become a vital issue in container terminal management. In this article, the problem of allocating ground-slot sets to export containers has been studied with a time-space network model. The computation results seem promising, in terms of computation time and model output. The following are our conclusions and suggestions drawn from the study:

1. Container yard space is a reusable spatial resource; this paper has made use of the time-space network to find the optimal allocation pattern. The study shows that an optimal solution is easily obtained through application of the network framework. Possible applications of the time-space network model include other resource allocation problems that touch on time and space aspects, for example, the room reservation problem in a hotel.
2. Container grouping plays an important role in yard planning. In this article, vessel voyage is used to group containers and allocate yard space. Criteria for grouping containers can be increased accordingly. Attributes such as size and port of discharging (POD) are common grouping criteria. One important consideration is that, as the number of groups as well as subgroups increases, the flexibility of yard space allocation decreases, which can in turn lead to delays in receiving export containers when they arrive at the yard.
3. In this article, the problem of allocating yard space to export containers is formulated as an assignment problem, meaning that the solution is critically influenced by the assignment costs. Because all preferred areas for each container subgroup are decided based on the model, assignment costs must, therefore, be studied further to be truly reflective of the preferences of yard planners.
4. Before allocating yard space to export containers, it is important to decide how many adjacent ground slots should be assembled as a ground-slot set. The range is from one to the maximum width within a yard crane. The more ground slots are assembled, the more efficiency is obtained when picking containers from the yard. However, the more distances yard cranes may travel when receiving export containers, which usually arrive in the terminal at different time. Therefore, the issue for deciding how many adjacent ground slots should be assembled

as a ground-slot set should be further studied.

5. The comparison between the allocation results from the time-space model and from the method used in practice is not done in this paper due to lack of the actual allocation data. However, in order to evaluate the performance of this time-space model, the actual allocation data should be obtained and this comparison should be done in the further study.

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