兩波交會作用之研究

Investigation on Wave-Wave Interactions

計畫主持人：陳　陽　益

　　工　作：整個理論架構之建立，模式之推導解析與運算，試驗規劃督導、結果整理，及報告之撰寫。

參與人員　：何　良　勝

　　工　作：試驗之規劃、佈置與執行及資料之整理分析。

張　憲　國

：試驗之執行及資料之整理分析。

莊　文　傑

：理論解析結果之圖形繪製。

陳　明　宗

：儀器之維修與調適。

江　中　權

：資料整理與校稿。

曾　文　傑

：協助繪圖

陳　毓　清

：資料整理及圖形編排。

兩波交會作用之研究

目錄

Investigation on Wave-Wave Interactions

By

Chen Yang-Yih

ABSTRACT

For describing the characteristics and the mechanism of the resultant waves-motion system due to the interaction between two arbitrary trains of free-surface progressive gravity waves in water of any uniform depth, the obstruction of so-called secular term occurred in the previous investigations has effectively been avoided in this paper. In suitably taking account of the nonlinear essences of original waves trains and the effects resulted from their interactions, the mechanism of the resultant waves-motion system has clearly been stated, and an available perturbation expansion procedure with an appropriate differentiation transformation by chain rule has also been developped to analyse it. So, all the governing equations required for solving the solution of the resultant waves-motion system are directly and systematically formulated in a more obvious form orderly. From the entire expansion to the flow field, the obtained third-order approximation solution is theoretically verified to be valid under the checks of two special cases as the resultant waves-motion system is degenerated into a simple free-surface progressive gravity waves train and the standing waves, respectively. The mechanism relative to the nonlinear interactions

between two free-surface progressive gravity waves trains has also been clearly demonstrated in the relation among the essential characteristics of the original waves trains, i.e. the amplitude and the wavelength of each one(or say the individual wave-steepnesses), and their intersecting angle for the case in any uniform water depth, such as the resultant surface elevation, pressure in field, and the variation of frequency of each waves train after their interaction in which the possibility of so-called frequency reversal feature is also interpreted. As to the special case of the short-crested waves motion, which is produced by the intersection of two free-surface progressive gravity waves trains having same properties, it is also described by degenerating the general waves-motion system considered into it, and its characteristics in whole flow field are also apparently exposed consequently. The phenomenon so-called resonance which may occur in the resultant waves-motion system is more definitely proved in the analysis of present paper too, and extend it to the case of finite water depth from deep water in the previous investigations. To express the validities and adequacies of the theoretical results obtained in this paper, the experimental studies on the interactions between two free-surface progressive gravity waves trains are carried out in a large scale basin. The spatial and temporal distributions of surface elevation and pressure in the resultant waves-motion system are detected by wave gagues and pressure transducers, respectively. From the comparisons of the theoretical results and the experimental measurements, the good agreements between them reveal that the analytical formulation developped in this paper can sufficiently describe the resultant waves-motion system considered in the present topic.

摘 要

針對任一均勻等深水中，任兩自由表面規則前進重力波列相交會所構成之波動系統，其波動流場結構的解析與闡論，在避免往昔對其解析會產生庸凡項(secular term)而無法得其合理之解下，本文於符合波動之非線性本質，適足地考量入兩來源成份波列原具有的脈動本質特性，及因它們交會而生相互作用所衍生的影響效應特性，清楚地描述出其波動流場整體的脈動機構，並於利用攝動展開法對其解析中引入chain rule做適當的微分轉換處理下，給予其解析直接系統化地逐階次之明確的展開。對所考慮的波動系統，本文已求至第三階次的整體波動流場結構解，並於其退化成單一波列（即其中一來源成份波列消失情況）與駐波（即兩完全相同性質但相反方向前進之波列交會的情形）時之二特例情況的檢驗下為正確。等深水中兩自由表面規則前進重力波列相交會後，其間之相互作用隨所在的水深、各來源成份波列之本質特性（即其波長與振幅或言波浪尖銳度）與它們前進方向夾角間之互動關係的力學機構，因此被清楚地瞭解與描述；而其波動流場之整體的脈動特性，如交會後各波列之週波率的變化（包括會發生所謂的逆變現象之情形）及其表面脈動水位與波動場內之波壓等，因而亦被了然的得知與掌握。至於短峯波情況其波動流場之結構，依所得之所考慮的波動系統之（通案性）波動流場結構解，於波動系統退化成短峯波之特例情況時而獲得之，而短峯波之波動流場的整體脈動特性亦因此被明確地闡述。所考慮的波動系統，其會產生所謂的共振現象的可能情況，由本文之解亦給予往昔研究所得之此一結果進一步地確證。對所考慮的波動系統，其波動流場結構之本文所得的理論解析結果進行具體的驗證，本研究於大尺度之平面水槽中進行試驗，由兩來源成份波列相交會後量測得之其表面脈動水位與波動場內之波壓的結果，顯示本文理論解析結果與試驗所得者間頗為良好的脗合。因此，可具體的確知，本文對所考慮的波動系統之解析，具正確性與適足性。

一、前　　言

對於水波波動現象，在近二百年來眾多學者的研究探討下，可斷言的是：關於其中最爲基本問題的“自由表面單一規則前進重力波列”，其波動特性已被相當充分地解析與掌握；即於自然界之波動現象中佔爲基本成份的此種規則波動，由淺水至深海、由微小振幅至近於極端碎波之整個可存在的領域裡所有的各種情況，其皆已被相當詳盡的闡述，如詳見Cokelet（1977）、Fenton（1981）、Williams（1981）、Chen（1983，1985）與Lin（1987，1988）等之論述。因此，就純理論解析的觀點而言，描述單一規則前進重力波之可行性的解析法，似應可被適當地延伸推廣來探討由此種波列所相交會組合而成的重複波，或言對由多個任意之此種波列所組合成的波動系統；當然其間以任意二波列之相交會合成者最爲單純。再者，就實際狀況而言，任一海域上之波動常是由多個成份波列之組合而整體呈現的；即使在近岸處，亦常有遠處海面傳來的長波與區域性短波之組合；或者甚至最爲單純者之由單一規則前進的重力波列經岸壁或結構物的反射而形成之重複波現象，諸如常見的短峯波（short-crested waves）與駐波（standing waves）者等。是故欲對海洋上之波動現象做較全面性的考慮與描述，則對構成它的成份波列間相交會後所造成的組合機構，應是有必要事先給予探討與瞭解；當然地，這是需從最爲單純情況之任意二波列的交會現象做起。

任意二自由表面規則前進重力波列的交會研究最先由Phillips（1960）給予考慮，其主旨在於尋求海面上紛紜波場中各成份波間之能量轉移的基本力學機構，却因而發現二波列間的交互作用至第三階解時會出現共振現象的情況。此一重大發現即刻促使了此課題的研究受到極大的關注與展開，且幾皆着重在共振現象部份的探討；如見Longuet-Higgins（1962）、Benney（1962）、Bretherton（1964）、Longuet-Higgins & Smith（1966）、McGoldrick et al（1966）、Phillips（1967）與Simmon（1969）等之各所從事的理論解析與試驗說明。此外，Hasselmann（1962，1963，1967）於探討海洋紛紜波場之波譜內的各成份波能量間之相互轉移時，亦獨自地發現到，成份波間存有非線性的交互作用且扮演此種轉移力學機構的主角，並推導出更一般化的共振現象發生之情況。至於其他有關於波列間交互作用的研究，如重力波與毛細波（capillary waves）、或表面波（surface waves）與內波（internal waves）及內波與內波等之情況，可參見

Phillips（1981）對此等問題之研究發展給予一總結性的論述。

從上述這些關於波列間交互作用研究具有重大貢獻之文獻的學習與探討中，有幾點重要的有關訊息是可被獲致的；即是當吾人檢視這些前人們的論著時，可容易地發現到：對此問題的探求迄今皆是以深海情況來進行考慮（除單一規則前進重力波列經全反射所形成的短峯波（Hsu，1979）與駐波（Chen，1988，1989）之特例外，而所展開解析的方法皆是基於傳統攝動法爲之；至於對所考慮的波動場之描述顯然地皆着重在共振現象部份，尙無全盤地解析出所造致之整個波動流場的結構。更甚者，於他們的解析中皆無明確地對高階解時（3階以上）所會產生的庸凡項（secular terms）有所交待。然這是不可疏漏的，蓋因此庸凡項將會造致分散關係（dispersion relation）的高階非線性量修正，及進而嚴重地影響解析解的正確性或存在與否。這種情況乃是以攝動法來解析描述波動之非線性時一道必存的過程，亦因如此，才能全面性地展現出波動非線性本質的全貌，否則則否，此乃衆所熟知的事實。這是明顯可得知的，前人們的此種疏漏是在於他們似都直接以線性的分散關係做爲界定，而不顧波動於此所原具有的非線性本質，即使甚至有之如Benney（1962）者亦無合乎此一本質的結果，此將被澈底地評論於下文的解析結果之比較中而確知之。由以上之檢視，這是可明顯地確定，迄今對波列間交互作用的研究，尙無全面地給予其整個流場結構做較完整妥當的描述；當然其中之共振現象，僅是被包含在所得的整體流場結構中之一特例而已。

針對前人們對波列間交互作用之描述上尙遺留的不完整點的滿足，及基於求解出所考慮的波動系統之整體流場結構解的目的下，本文考量入各來源成份波本身原具有非線性因素之本質，描述出兩來源成份波列相交會因其間相互作用所造致之整體的波動流場機構，且於攝動法解析中稍加一些微分過程的適當轉換並使之系統化展開，及進而延伸擴展至任一均勻等深水中情況解出其至第三階的整個波動流場滿足解。當然於其間所出現的庸凡項亦一併給予完全地解決，因而可適足地描述出二波列交會後，各成份波所特有的分散關係受到它們間交互作用而造致的影響情況。同時，對常見之單一規則前進重力波列經直立岸壁全反射所形成之短峯波情況，在本文之解退化成此特例下，描述出其波動流場的結構特性。接著，爲檢核本文所得之解析解的正確性，吾人以二種特例情況來對它進行理論印證，即當一來源成份波列消失時，則波動系統變成爲僅一單一規則前進重力波列者；另一者爲當二完全相同性質（即振幅、波長或週期全爲相同）但前進方向却相反之規則重力波列組合而成的重力駐波情況；至於試驗方

面，則於大型平面水池中，量測出兩來源波列相交會後之波動流場內的水位脈動與壓力變化，來與本文解析結果進行比較之。基此驗證而得到理論上完全正確與試驗上吻合的事實下，則似可言本文所展開的攝動解析法其普遍適足性應可被確定，及所考慮的波動場其整體結構的特性因而可全面性地被充分描述。如此，前人們對此課題研究尚餘有的一些不足與障礙似皆可被適切地給予彌補與疏解。

本文之論述是由筆者近三年來（Chen et al，1988，1989，1990），針對兩波交會研究所提出的七份掘作匯集而成。文中第一節爲於此所述之前言。第二節，對所考慮的均勻等深水中，任二自由表面規則前進重力波列相交會組合而成的波動系統，給予其必要滿足之公式化的基本陳述。第三節，則對前人所用的攝動法解析型式進一步地展開，以說明其無法滿足地解決其中所會出現的庸凡項，及因而無法定出交會後之二波列各自適足的分散關係與整個流場的合理解。第四節，則於波動之非線性本質考量下，對相交會的兩來源波列，因其間各階次脈動成份量間的相互作用，所形成的整個波動流場機構給予清楚地描述，並系統化地列出其逐階次的控制方程式。基於攝動展開法之原則下，於解析過程中稍加做些適當的微分轉換處理，以求解出所考慮的波動系統滿足至第三階的整體流場結構解之詳細推演，及論述波場中會發生的一些主要特性的概貌，則於第五節中被陳述。第六節，對兩波列交會中之一例短峯波者，依第五節所得之解直接退化成短峯波之流場解，並描述其波動特性。第七節，說明所得之解析解的正確性與適足性，則以單一前進波列與駐波兩個特例情況對它進行理論上之印證，隨後再引於大型平面水池中，對兩波列交會之試驗所量測得的水位脈動與壓力變化，來與解析結果進行比較驗證之。最後，歸結本文之整個論述，則於第八節中給予一些當有的結語與討論。

二、系統之描述

於三度空間中，任一均勻等深 d 之廣大水域上，考慮波數向量各爲 $\vec{k}_1$ 與 $\vec{k}_2$ 之二自由表面規則前進重力波列相交會所構成的波動系統，如圖 1 所示；$k_1 = |\vec{k}_1| = 2\pi / L_1$，$k_2 = |\vec{k}_2| = 2\pi / L_2$，$L_1$ 與 L_2 各爲二來源成份波列之波長。描述此波動系統，吾人取用三度空間的卡氏直角座標（x，y，z）爲參考架構，其中 x－y 平面恰位於平均靜止之水平面處，而 z 軸垂直於 x－y 平面且向上取正；在此參考座標下，指示出二來源成份波列之前進方向的波數向量 $\vec{k}_1$ 與 $\vec{k}_2$ 分別與 x 軸成 θ_1 與 θ_2 之夾角，即表示此二波列之前進方向間有 $\theta = \theta_2 - \theta_1$ 之交角，如圖 1 所示。

今假設流體爲無黏性與不可壓縮性的（ inviscid and incompressible ），且其運動爲非旋轉性的（ irrotational ），因而吾人可定義出一流速勢函數 velocity potential ）ϕ，使得所考慮的波動流場內之水粒子的速度與其分量可被表示爲

$$\vec{V} = \nabla\phi = (\phi_x, \phi_y, \phi_z) = (u, v, w) \tag{2.1}$$

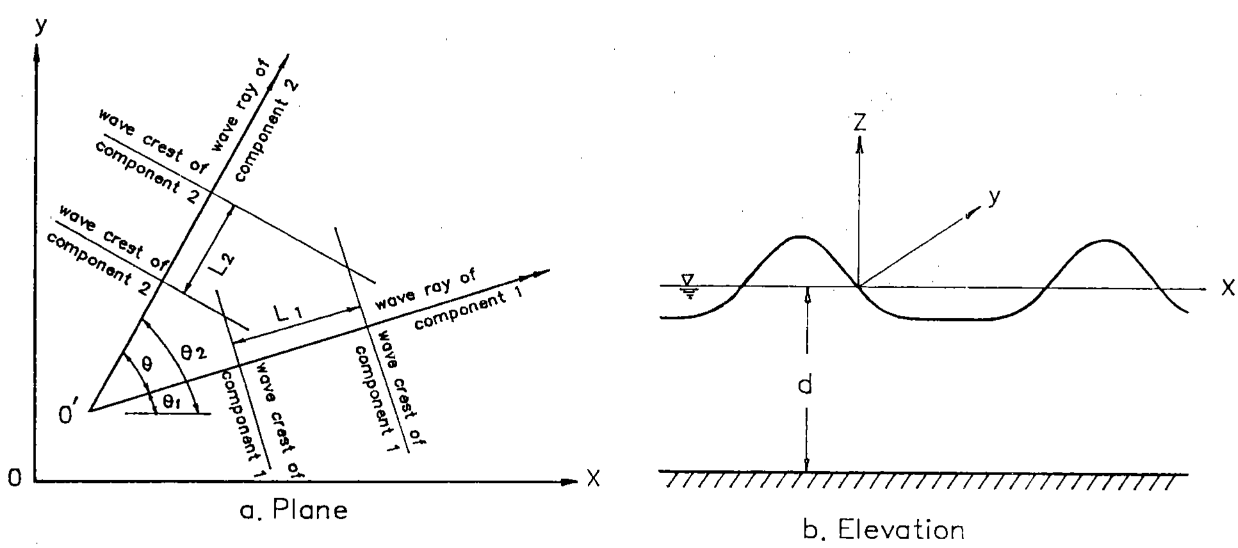


圖 1　等深d水中，二自由表面規則前進重力波列相交會所構成之波動系統示意圖
Fig. 1　Definition sketch for two wave-trains interaction in uniform water depth d.

此處流速勢函數 $\phi(x, y, z, t)$ 須滿足Laplace's equation ，而爲波動流場之基本控制方程式，爲

$$\nabla^2\phi = \phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \tag{2.2}$$

至於波動流場所必要滿足的邊界條件有：

(A) 在底部 $z=-d$ 處

$$w = \phi_z = 0, \quad z=-d \tag{2.3}$$

(B) 在波動表面處，有

(a)運動邊界條件

$$\phi_z = \frac{d\eta}{dt} = \eta_t + \phi_x\eta_x + \phi_y\eta_y, \quad z=\eta \tag{2.4}$$

(b)動力邊界條件（假設處於常壓下之自由表面者＊）

$$\phi_t + g\eta + \frac{1}{2}(\phi_x^2 + \phi_y^2 + \phi_z^2) = 0, \quad z=\eta \tag{2.5}$$

上式中 $\eta=\eta(x, y, t)$ 爲波動表面水位，g 爲重力加速度，t 爲時間。注意，關於波動流場解中僅涉及到時間 t 的函數（包括任意常數）已被併入在 ϕ 中。

另，由於 $x-y$ 平面是被取爲平均靜止的水平面，因此，依質量守恆及波動系統的空間週期性，再有一必要的條件式爲

$$\int_0^{2\pi}\int_0^{2\pi} \eta(x, y, t)\, d(\vec{k_1}\cdot\vec{X})\, d(\vec{k_2}\cdot\vec{X}) = 0 \tag{2.6}$$

此處 $\vec{X}=\vec{i}x+\vec{j}y$ 爲 $x-y$ 平面上的位置向量。注意：(2.6) 式之積分已被轉換對無因次量 $\vec{k_1}\cdot\vec{X}$ 與 $\vec{k_2}\cdot\vec{X}$ 之積分形式。

這是如同所有相關的水波問題之處理般，對本文所考慮的波動系統之求解，旨在解析出滿足 (2.2) 至 (2.6) 式之所有控制方程式下的流場結構解 ϕ 與 η 者，然其關連到非線性邊界條件的處理是爲眾所皆知的主要障礙。今爲便於其被逐次展開解析起見，於此先將非線性的自由表面運動與動力邊界條件 (2.4) 與 (2.5) 式做適當的處理，如下。對 (2.5) 式進行 d/dt 總微分後減去 g 倍的 (2.4) 式，可得合併後的自由表面邊界條件爲

$$\left(\frac{\partial^2\phi}{\partial t^2}+g\frac{\partial\phi}{\partial z}\right)+\frac{\partial}{\partial t}(\vec{V}^2)+\vec{V}\cdot\nabla\left(\frac{1}{2}\vec{V}^2\right)=0\text{，}z=\eta \qquad (2.7)$$

接著，應用Taylor 級數展開在 $z=0$ 處以取代在未知的波動表面 $z=\eta$ 處，則

* 這是包括本文所考慮者在內的處理一般水波問題的通常假設，其至第三階解下為正確，至於駐波情況可參見作者（1988）。

可得自由表面運動與動力邊界條件（2.4）與（2.5）兩式被轉換為

$$g\eta+\left[\frac{\partial\phi}{\partial t}+\eta\frac{\partial^2\phi}{\partial z\partial t}+\frac{1}{2}\eta^2\frac{\partial^3\phi}{\partial z^2\partial t}+\cdots\right]+\left[\frac{1}{2}\vec{V}^2+\eta\frac{\partial}{\partial z}\left(\frac{1}{2}\vec{V}^2\right)+\cdots\right]$$
$$=0\text{，}z=0 \qquad (2.8)$$

$$\left[\left(\frac{\partial^2\phi}{\partial t^2}+g\frac{\partial\phi}{\partial z}\right)+\eta\frac{\partial}{\partial z}\left(\frac{\partial^2\phi}{\partial t^2}+g\frac{\partial\phi}{\partial z}\right)+\frac{1}{2}\eta^2\frac{\partial^2}{\partial z^2}\left(\frac{\partial^2\phi}{\partial t^2}+g\frac{\partial\phi}{\partial z}\right)+\cdots\right]$$
$$+\left[\frac{\partial}{\partial t}(\vec{V}^2)+\eta\frac{\partial^2}{\partial z\partial t}(\vec{V}^2)+\cdots\right]+\left[\vec{V}\cdot\nabla\left(\frac{1}{2}\vec{V}^2\right)+\cdots\right]=0\text{，}z=0$$
$$(2.9)$$

由以上之處理可知，這是如同攝動法應用到一般波動問題之解析般的，如今對整個問題的求解，則已被轉換至於滿足以 $z=0$ 處之展開下的各對應條件而進行逐次解析者；當然亦因而使吾人能明確地予之考量。有關於它被系統化地直接展開與解析處理，其詳情緊接著被列述於下文中。

三、往昔解析

基於對所考慮的二自由表面規則前進重力波列相交會的問題，能有較全盤性的完整解決，則往昔之對它研究具有啟發性之重大貢獻的典範論述，當然是有必要給予詳加探究的；蓋因如此，方能由其間的學習蒐索出正確的觀點與良好的處理方式，以便進一步地因而擴展延伸做深入的探討，或甚者亦可由此知其是否具足普遍的圓通性，俾再給予一些適當的必要修飾以致使問題的解決更趨圓滿。基此之故，於本節將對往昔對此問題的研究論述，最具明確解析的 Longuet-Higgins（1962）者進行深一點的探究，以做爲吾人對此問題尋求更完整性解決的基石。

Longuet-Higgins（1962）於求解所考慮的深海中二波列相交會所生之共振現象問題中，令流速勢函數 ϕ、水粒子速度 $\vec{V}$ 及波動水位 η 各爲

$$
\begin{aligned}
\phi &= (\alpha\phi_{10}+\beta\phi_{01}) + (\alpha^2\phi_{20}+\alpha\beta\phi_{11}+\beta^2\phi_{02}) + \cdots\cdots \\
\vec{V} &= (\alpha\vec{V}_{10}+\beta\vec{V}_{01}) + (\alpha^2\vec{V}_{20}+\alpha\beta\vec{V}_{11}+\beta^2\vec{V}_{02}) + \cdots\cdots \qquad (3.1) \\
\eta &= (\alpha\eta_{10}+\beta\eta_{01}) + (\alpha^2\eta_{20}+\alpha\beta\eta_{11}+\beta^2\eta_{02}) + \cdots\cdots
\end{aligned}
$$

此處 α、β 是正比於波動表面斜率之二獨立小量，而 $\alpha\phi_{10}$，$\beta\phi_{01}$，$\alpha\vec{V}_{10}$，$\beta\vec{V}_{01}$ 與 $\alpha\eta_{10}$，$\beta\eta_{01}$ 各表示爲此二相交波列之第一階近似流場解。至於其他項則皆被認定是由此二相交波列間之相互作用所衍生者，且被認爲可由控制方程式（2.2）、（2.3）式與展開後之（2.7）至（2.9）式的應用，而依 $\alpha^i\beta^i$ 階次量之係數關係的考量逐次求解之，如 Longuet-Higgins（1962）所解析者。爲洞察 **Longuet-Higgins** 之此良好的觀點可被適當地稍加修飾，以便推廣展開解析出所考慮的波動系統之整個流場逐階滿足解，此處簡列出依其方法所得之解，並再給予更進一點的解析以說明其仍有必要稍作改良的理由；至於更詳盡的解析過程則見於下節之論述。

依 Longuet-Higgins（1962）之方式下，可得至第二階的波動流場解，其於深海中之型式爲

$$\phi_{10} = a_1\sigma_1 k_1^{-1} e^{k_1 z} \sin S_1 \text{，} \eta_{10} = a_1 \cos S_1 \qquad (3.2)$$

$$\phi_{01} = a_2\sigma_2 k_2^{-1} e^{k_2 z} \sin S_2 \text{，} \eta_{01} = a_2 \cos S_2 \qquad (3.3)$$

$$\phi_{20}=0\ ,\ \eta_{20}=\frac{1}{2}a_1^2k_1\cos 2S_1 \tag{3.4}$$

$$\phi_{02}=0\ ,\ \eta_{02}=\frac{1}{2}a_2^2k_2\cos 2S_2 \tag{3.5}$$

$$\left.\begin{aligned}
&\phi_{11}=Ae^{|\vec{k}_1-\vec{k}_2|z}\sin(S_1-S_2)-Be^{|\vec{k}_1+\vec{k}_2|z}\sin(S_1+S_2)\\
&A=\frac{2a_1a_2\sigma_1\sigma_2(\sigma_1-\sigma_2)\cos^2\frac{1}{2}\theta}{(\sigma_1-\sigma_2)^2-g\,|\vec{k}_1-\vec{k}_2|}\ ,\ B=\frac{2a_1a_2\sigma_1\sigma_2(\sigma_1+\sigma_2)\sin^2\frac{1}{2}\theta}{(\sigma_1+\sigma_2)^2-g\,|\vec{k}_1+\vec{k}_2|}\\
&\eta_{11}=\frac{1}{g}\left[(\sigma_1-\sigma_2)A+\frac{1}{2}a_1a_2(\sigma_1^2+\sigma_2^2)-a_1a_2\sigma_1\sigma_2\cos^2\frac{1}{2}\theta\right]\\
&\qquad\times\cos(S_1-S_2)+\frac{1}{g}\left[-(\sigma_1+\sigma_2)B+\frac{1}{2}a_1a_2(\sigma_1^2+\sigma_2^2)\right.\\
&\qquad\left.+a_1a_2\sigma_1\sigma_2\sin^2\frac{1}{2}\theta\right]\cos(S_1+S_2)
\end{aligned}\right\}(3.6)$$

上式中 a_1 、 a_2 各爲正比於相交之二來源波列振幅的長度因次量；而 S_1 、 S_2 各爲其變動相位， σ_1 、 σ_2 各是其對應的週波率（frequency），此在Longuet-Higgins（1962）之解析方式下，它們被給定爲

$$S_i=\vec{k}_i\cdot\vec{X}-\sigma_i t\ ,\ \sigma_i^2=gk_i\ (\text{dispersion relation})\ ,\ i=1,2 \tag{3.7}$$

此處 $\vec{X}=\vec{i}x+\vec{j}y$ 表示水平面上之位置向量，如上節所定義者。

至於求解第三階之解（即 $\alpha^i\beta^j$ ， $i+j=3$ 者），雖然可由（3.2）－（3.7）式之型式的解代入於展開在 $\alpha^i\beta^j$ ， $i+j=3$ 階次下之控制方程式中求解，然其間之一些不完全滿足性是會被呈現的。今爲詳細說明此現象起見，於此將針對這會產生此情況的解析項深入地陳述之，即求解 ϕ_{30} （與 ϕ_{03} ）者。依本節所述之Longuet-Higgins（1962）方式下，求解 ϕ_{30} ，則其對應的控制方程式是被展開在 α^3 之階次裡（至於 ϕ_{03} 者則在 β^3 裡），可被寫出爲

$$\nabla^2\phi_{30}=0\ ;\ \nabla\phi_{30}\to 0\ ,\ z=-\infty \tag{3.8a}$$

$$\left(\frac{\partial^2\phi_{30}}{\partial t^2}+g\frac{\partial\phi_{30}}{\partial z}\right)+\eta_{10}\frac{\partial}{\partial z}\left(\frac{\partial^2\phi_{20}}{\partial t^2}+g\frac{\partial\phi_{20}}{\partial z}\right)+\eta_{20}\frac{\partial}{\partial z}\left(\frac{\partial^2\phi_{10}}{\partial t^2}+g\frac{\partial\phi_{10}}{\partial z}\right)$$

$$+\frac{1}{2}\eta_{10}^2\frac{\partial^2}{\partial z^2}\left(\frac{\partial^2\phi_{10}}{\partial t^2}+g\frac{\partial\phi_{10}}{\partial z}\right)+\frac{\partial}{\partial t}\left(2\vec{V}_{10}\cdot\vec{V}_{20}\right)$$

$$+\eta_{10}\frac{\partial^2}{\partial z\partial t}\left(\vec{V}_{10}^2\right)+\vec{V}_{10}\cdot\nabla\left(\frac{1}{2}\vec{V}_{10}^2\right)=0，z=0 \qquad (3.8b)$$

因待解之ϕ_{30}，除了其中所含有的時間 t 之函數外（其實此於最後結果中爲零，見下節），其型式可依（3.8a，b）式之應用來探索之。因此，避免煩瑣，不必列出其他的控制式，就足以說明吾人所欲言的依Longuet-Higgins 之此方式來求解是不完整性的，如下。

以（3.2）至（3.7）式之Longuet-Higgins 方式所得之解代入（3.8b）式中，則可計算出其中有關之項爲

$$\left.\begin{aligned}
&\eta_{10}\left[\frac{\partial}{\partial z}\left(\frac{\partial^2\phi_{20}}{\partial t^2}+g\frac{\partial\phi_{20}}{\partial z}\right)\right]_{z=0}=0\\
&\eta_{20}\left[\frac{\partial}{\partial z}\left(\frac{\partial^2\phi_{10}}{\partial t^2}+g\frac{\partial\phi_{10}}{\partial z}\right)\right]_{z=0}\\
&=\frac{1}{2}\eta_{10}^2\left[\frac{\partial^2}{\partial z^2}\left(\frac{\partial^2\phi_{10}}{\partial t^2}+g\frac{\partial\phi_{10}}{\partial z}\right)\right]_{z=0}=0\\
&\left[\frac{\partial}{\partial t}\left(2\vec{V}_{10}\cdot\vec{V}_{20}\right)\right]_{z=0}=\left[\frac{\partial}{\partial t}\left(2\nabla\phi_{10}\cdot\nabla\phi_{20}\right)\right]_{z=0}=0\\
&\eta_{10}\left[\frac{\partial^2}{\partial z\partial t}\left(\vec{V}_{10}^2\right)\right]_{z=0}=\eta_{10}\left[\frac{\partial^2}{\partial z\partial t}\left(\nabla\phi_{10}\cdot\nabla\phi_{10}\right)\right]_{z=0}=0\\
&\left[\vec{V}_{10}\cdot\nabla\left(\frac{1}{2}\vec{V}_{10}^2\right)\right]_{z=0}=\left\{\nabla\phi_{10}\cdot\nabla\left[\frac{1}{2}(\nabla\phi_{10})^2\right]\right\}_{z=0}\\
&\qquad=a_1^3\sigma_1^3k_1\sin S_1=gk_1^2a_1^3\sigma_1\sin S_1
\end{aligned}\right\} \qquad (3.9)$$

因此，（3.8b）式可得之爲

$$\left(\frac{\partial^2\phi_{30}}{\partial t^2}+g\frac{\partial\phi_{30}}{\partial z}\right)_{z=0}=-gk_1^2a_1^3\sigma_1\sin S_1 \qquad (3.10)$$

依（3.8a）與（3.10）式來求解ϕ_{30}，這是明顯地，於ϕ_{30}解中需含有 $\sin S_1$ 項，然在$\sigma_1^2=gk_1$之線性分散關係給定下（3.10）式爲共振方程式，則必然造致其趨於

無限大而使波動系統本質上是不可能存在的矛盾（不論其於何種情況下是有共振現象發生否）；蓋因此時已全面性地使得整個系統的解析在任何情況下皆趨於無限大者，故無法得其實質解。這就是爲何攝動展開法解析一般的波動問題時，必需要對所謂的庸凡項（ secular terms）有所處理的因由，否則與其描述波動中分散關係之高階非線性近似量的本質精神違背之（其實，此處無論如何是無法求得此種近似量的）。

因此，這是可確知的，依往昔之對二前進重力波列相交會所構成的波動系統之研究，其最具代表性的Longuet-Higgins（ 1962 ）的良好解析法，是無法全面性地逐階求解出整個波動流場之滿足解。至於其他者亦復如是，如見Phillips（ **1960** ）中之庸凡項已有共振現象的出現，見其(5.7)式，此即如上所述的，表示在任何情況皆是共振的；Benney（ 1962 ）雖有稍不同的處理，然亦僅能對退化至線性量時有所符合而已，如見其(2.14)式及說明等。

四、流場機構與展開處理

針對上節所述之往昔對二自由表面規則前進重力波列相交會，所構成之波動系統的解析之非完全適足性，因此，基於求得此波動系統之整個流場的逐階滿足解的目的下，本文將陳述一可全面性適足展開的解析法，來對它做較整體性的充分描述，至第三階解。

由上節對往昔解析法的探討中，這是顯然地可知，其對所考慮的波動場之描述主要的不足，是在於其無適足地考量入各來源成份波列之原具有非線性的本質，以給予所會於攝動法解析中產生之庸凡項做妥當地處理。避免往昔之此種疏忽及給予問題全盤地考慮，則在攝動展開法的精神下，有必要對兩波列相交會所形成之波動系統，其波動流場的整體機構，包括兩來源波列本身之非線性量及因其間相互作用所衍生者，事先給予清楚地探究與描述，然後進而予之明顯系統化地條次分明的展開解析之，對此將於本節闡論之。

顯然地，就外觀上，這常是為人所共會感受到的事實，當檢視往昔至今對所考慮的兩波列相交會問題的研究時，一個足以令人燎起的直覺是：對兩波交會所構成之波動系統的解析，似乎非經過一道非常繁雜冗長的處理過程不可，因而於先覺上，使吾人對其流場機構有奧妙難測之感。當然，兩波交會所形成的波動流場機構，其複雜性較甚於單一波列者自不在話下，此蓋因兩波列間的相互作用涉及於其中之故。然而，由於對兩來源波列自身而言，不論就其波動之物理特性或是其流場之數學解析，皆有其系統化的脈絡可尋，因此，若能對它們相交後而發生於其間的相互作用情況事先有所瞭解，則兩波交會所形成的整個波動流場的機構當可被事先全然清楚地描述，如此，對其解析亦可系統化地展開，而不再重蹈如前人般的混雜處理了。對此論述謹將詳述於下。

4-1 流場機構之描述

由於波動具有非線性的本質特性，因此，對兩波的交會，其間當會產生非線性的相互作用量，而存於其造成的流場機構中，這顯然可由波動系統受到非線性的表面邊界條件的限制而確知，如見(2.4)與(2.5)兩式或更清楚地其經處理展開的(2.8)

與(2.9)兩替代式。是故，爲直接洞察兩波交會所形成的波動流場機構，則因兩來源波列各有的脈動本質而造致的相互作用機制，即因而出現於波場中的相位脈動，首先需要被清楚地掌握。做此解析，今令兩來源波列脈動基本相位各爲 $S_1 = (\overrightarrow{k_1} \cdot \overrightarrow{X} - \sigma_1 t + \epsilon_1)$ 與 $S_2 = (\overrightarrow{k_2} \cdot \overrightarrow{X} - \sigma_2 t + \epsilon_2)$，此處 $\overrightarrow{X} = \overrightarrow{i}x + \overrightarrow{j}y$ 表示水平面上之位置向量，σ_1 與 σ_2 爲兩波列各對應的週波率，如上節者，而 ϵ_1 與 ϵ_2 爲此兩波列交會時在座標原點處的各相位值。因此，依各波列本身之波動流場結構，即其流場的攝動解析解，則由它們的交會而形成的整個波動流場之脈動相位的機構，包括因其間的相互作用而衍生者，可依它們間的波動相位之交錯而被清楚地描述，如圖2所示。

依圖2所示，即刻可很清楚地直接看出，兩波列交會，由其間之相互作用而造成的整個波動流場的機構，只要兩來源波列自身者事先已被徹底地掌握，則其整體脈動的特性當可被完整性地逐階次描述出。爲詳述起見，今對脈動基本相位各爲 S_1 與 S_2 之兩前進規則波列交會後，其所形成的波動流場之流速勢 ϕ，表面波動水位 η、與交

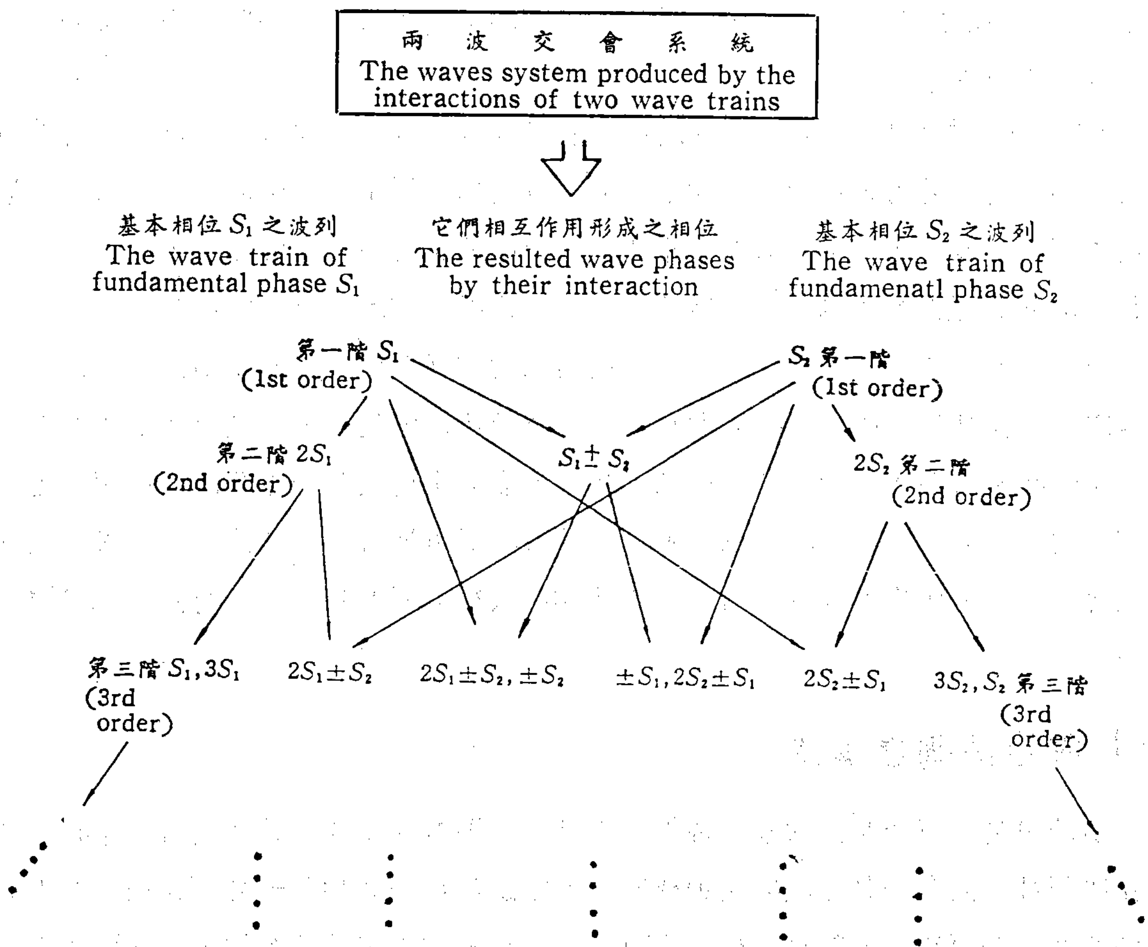


圖 2　兩波列交會，由其間之相互作用所造成之流場中，其逐階次量對應的脈動相位
Fig. 2　In the waves system produced by the interaction of two progressive wave trains, the resulted variational phases in each corresponding order.

會相互作用後兩波列的脈動週波率（angular frequency）σ_1、σ_2，直接給予明顯地表示爲

$$\left.\begin{aligned}
&\phi=\phi_{10}+\phi_{01}+\phi_{20}+\phi_{11}^{+}+\phi_{11}^{-}+\phi_{02}+\phi_{30}+\phi_{21}^{+}+\phi_{21}^{-}+\phi_{12}^{+}+\phi_{12}^{-}\\
&\qquad+\phi_{03}+\cdots\\
&\eta=\eta_{10}+\eta_{01}+\eta_{20}+\eta_{11}^{+}+\eta_{11}^{-}+\eta_{02}+\eta_{30}+\eta_{21}^{+}+\eta_{21}^{-}+\eta_{12}^{+}+\eta_{12}^{-}\\
&\qquad+\eta_{03}+\cdots\\
&\sigma_1=\sigma_0^{(1)}+\sigma_1^{(1)}+\sigma_2^{(1)}+\cdots,\ \sigma_2=\sigma_0^{(2)}+\sigma_1^{(2)}+\sigma_2^{(2)}+\cdots
\end{aligned}\right\}\tag{4.1}$$

上式所示之描述波動流場機構所必要的物理量，是依圖2所表示出的兩來源波列交會而生之相互作用結果逐階次列下的；其中 ϕ_{ij}、η_{ij}，$i+j\geq 1$，表示對應於（$i+j$）階之流速勢與水位，而 $\sigma_k^{(l)}$，$k\geq 0$，$l=1$，2，表示交會相互作用後的兩波列之週波率的第k階量。至此，這是顯然的，依圖2所示之兩來源波列交會相互作用所形成的整個波場脈動結構，即其逐階的相位脈動，則描述波動場機構的（4.1)式中之各階次的流速勢與水位量所對應的脈動相位可被明確地給定。換言之，由相位的對應，則 ϕ_{ij} 與 η_{ij} 對應的脈動相位間的函數關係可被淸楚地寫出，亦即所考慮的兩波交會所形成的波動流場機構特性可被明顯直接掌握，如至第三階次，它們有

$$\left.\begin{aligned}
&\phi_{10}=\phi_{10}(S_1),\ \phi_{01}=\phi_{01}(S_2);\ S_i=(\vec{k}_i\cdot\vec{X}-\sigma_i t+\epsilon_i),\ i=1,2,\\
&\phi_{20}=\phi_{20}(2S_1),\ \phi_{11}^{+}=\phi_{11}^{+}(S_1+S_2),\ \phi_{11}^{-}=\phi_{11}^{-}(S_1-S_2),\\
&\qquad\phi_{02}=\phi_{02}(2S_2),\\
&\phi_{30}=\phi_{30}(S_1,3S_1),\ \phi_{21}^{+}=\phi_{21}^{+}(2S_1+S_2),\ \phi_{21}^{-}=\phi_{21}^{-}(2S_1-S_2),\\
&\qquad\phi_{12}^{+}=\phi_{12}^{+}(S_1+2S_2),\ \phi_{12}^{-}=\phi_{12}^{-}(S_1-2S_2),\ \phi_{03}=\phi_{03}(S_2,3S_2)
\end{aligned}\right\}\tag{4.2}$$

與

$$\left.\begin{aligned}
&\eta_{10}=\eta_{10}(S_1),\ \eta_{01}=\eta_{01}(S_2);\ S_i=(\vec{k}_i\cdot\vec{X}-\sigma_i t+\epsilon_i),\ i=1,2,\\
&\eta_{20}=\eta_{20}(2S_1),\ \eta_{11}^{+}=\eta_{11}^{+}(S_1+S_2),\ \eta_{11}^{-}=\eta_{11}^{-}(S_1-S_2),\\
&\qquad\eta_{02}=\eta_{02}(2S_2),\\
&\eta_{30}=\eta_{30}(S_1,3S_1),\ \eta_{21}^{+}=\eta_{21}^{+}(2S_1+S_2),\\
&\eta_{21}^{-}=\eta_{21}^{-}(2S_1-S_2),\ \eta_{12}^{+}=\eta_{12}^{+}(S_1+2S_2),\\
&\eta_{12}^{-}=\eta_{12}^{-}(S_1-2S_2),\ \eta_{03}=\eta_{03}(S_2,3S_2)
\end{aligned}\right\}\tag{4.3}$$

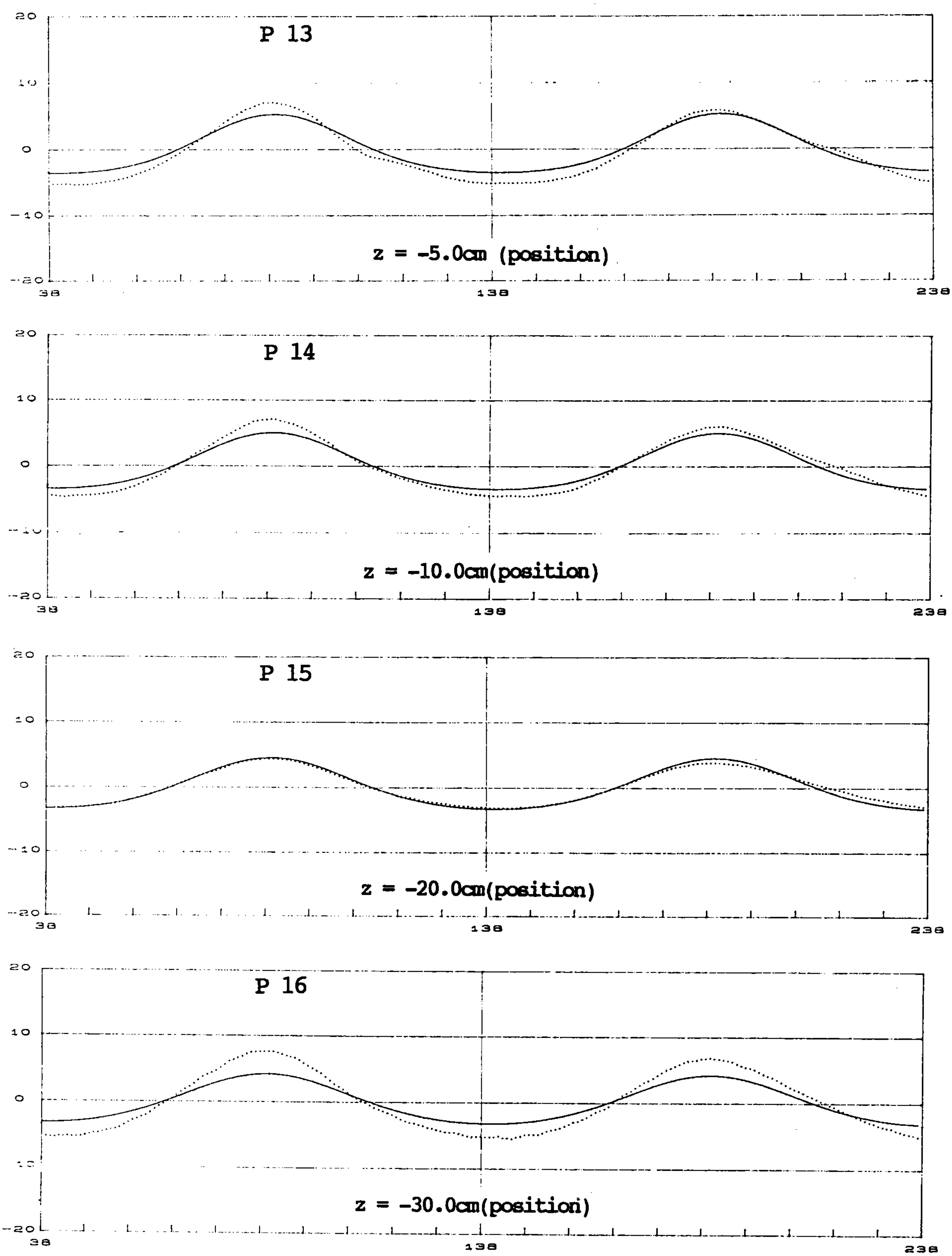

20
10
0
-10
-20
P 13
z = -5.0cm (position)
38
138
238
P 14
z = -10.0cm(position)
P 15
z = -20.0cm(position)
P 16
z = -30.0cm(position)

圖11-16

因此，(2.8)與(2.9)兩控制條件式中之$\partial\phi/\partial t$與$\partial^2\phi/\partial t^2$兩項，可依 chain rule 之微分轉換處理而逐階依序地展開，即於(4.1)、(4.2)與(4.4)式的應用下，其可被展開爲

$$
\begin{aligned}
\frac{\partial\phi}{\partial t} &= \frac{\partial}{\partial t}(\phi_{10}+\phi_{01}+\phi_{20}+\phi_{11}^{+}+\phi_{11}^{-}+\phi_{02}+\phi_{30}+\phi_{21}^{+}+\phi_{21}^{-}+\phi_{12}^{+}+\phi_{12}^{-} \\
&\quad +\phi_{03}+\cdots) \\
&= \sigma_1\frac{\partial}{\partial t_1}(\phi_{10}+\phi_{20}+\phi_{30})+\sigma_2\frac{\partial}{\partial t_2}(\phi_{01}+\phi_{02}+\phi_{03})+(\sigma_1+\sigma_2)\frac{\partial\phi_{11}^{+}}{\partial t_3} \\
&\quad +(\sigma_1-\sigma_2)\frac{\partial\phi_{11}^{-}}{\partial t_4}+(2\sigma_1+\sigma_2)\frac{\partial\phi_{21}^{+}}{\partial t_5}+(2\sigma_1-\sigma_2)\frac{\partial\phi_{21}^{-}}{\partial t_6} \\
&\quad +(\sigma_1+2\sigma_2)\frac{\partial\phi_{12}^{+}}{\partial t_7}+(\sigma_1-2\sigma_2)\frac{\partial\phi_{12}^{-}}{\partial t_8}+\cdots \\
&= \left(\sigma_0^{(1)}\frac{\partial\phi_{10}}{\partial t_1}+\sigma_0^{(2)}\frac{\partial\phi_{01}}{\partial t_2}\right)+\left[\sigma_1^{(1)}\frac{\partial\phi_{10}}{\partial t_1}+\sigma_1^{(2)}\frac{\partial\phi_{01}}{\partial t_2}+\sigma_0^{(1)}\frac{\partial\phi_{20}}{\partial t_1}\right. \\
&\quad \left.+\sigma_0^{(2)}\frac{\partial\phi_{02}}{\partial t_2}+(\sigma_0^{(1)}+\sigma_0^{(2)})\frac{\partial\phi_{11}^{+}}{\partial t_3}+(\sigma_0^{(1)}-\sigma_0^{(2)})\frac{\partial\phi_{11}^{-}}{\partial t_4}\right] \\
&\quad +\left[\sigma_2^{(1)}\frac{\partial\phi_{10}}{\partial t_1}+\sigma_2^{(2)}\frac{\partial\phi_{01}}{\partial t_2}+\sigma_1^{(1)}\frac{\partial\phi_{20}}{\partial t_1}+\sigma_1^{(2)}\frac{\partial\phi_{02}}{\partial t_2}\right. \\
&\quad +(\sigma_1^{(1)}+\sigma_1^{(2)})\frac{\partial\phi_{11}^{+}}{\partial t_3}+(\sigma_1^{(1)}-\sigma_1^{(2)})\frac{\partial\phi_{11}^{-}}{\partial t_4}+\sigma_0^{(1)}\frac{\partial\phi_{30}}{\partial t_1} \\
&\quad +\sigma_0^{(2)}\frac{\partial\phi_{03}}{\partial t_2}+(2\sigma_0^{(1)}+\sigma_0^{(2)})\frac{\partial\phi_{21}^{+}}{\partial t_5}+(2\sigma_0^{(1)}-\sigma_0^{(2)})\frac{\partial\phi_{21}^{-}}{\partial t_6} \\
&\quad \left.+(\sigma_0^{(1)}+2\sigma_0^{(2)})\frac{\partial\phi_{12}^{+}}{\partial t_7}+(\sigma_0^{(1)}-2\sigma_0^{(2)})\frac{\partial\phi_{12}^{-}}{\partial t_8}\right]+\text{higher order} \\
&\quad \text{terms} \qquad (4.5)
\end{aligned}
$$

與

$$
\frac{\partial^2\phi}{\partial t^2}=\sigma_1^2\frac{\partial^2}{\partial t_1^2}(\phi_{10}+\phi_{20}+\phi_{30})+\sigma_2^2\frac{\partial^2}{\partial t_2^2}(\phi_{01}+\phi_{02}+\phi_{03})
$$

$$+(\sigma_1+\sigma_2)^2\frac{\partial^2\phi_{11}^+}{\partial t_3^2}+(\sigma_1-\sigma_2)^2\frac{\partial^2\phi_{11}^-}{\partial t_4^2}+(2\sigma_1+\sigma_2)^2\frac{\partial^2\phi_{21}^+}{\partial t_5^2}$$

$$+(2\sigma_1-\sigma_2)^2\frac{\partial^2\phi_{21}^-}{\partial t_6^2}+(\sigma_1+2\sigma_2)^2\frac{\partial^2\phi_{12}^+}{\partial t_7^2}+(\sigma_1-2\sigma_2)^2\frac{\partial^2\phi_{12}^-}{\partial t_8^2}+\cdots$$

$$=(\sigma_0^{(1)2}\frac{\partial^2\phi_{10}}{\partial t_1^2}+\sigma_0^{(2)2}\frac{\partial^2\phi_{01}}{\partial t_2^2})+[2\sigma_0^{(1)}\sigma_1^{(1)}\frac{\partial^2\phi_{10}}{\partial t_1^2}$$

$$+2\sigma_0^{(2)}\sigma_1^{(2)}\frac{\partial^2\phi_{01}}{\partial t_2^2}+\sigma_0^{(1)2}\frac{\partial^2\phi_{20}}{\partial t_1^2}+\sigma_0^{(2)2}\frac{\partial^2\phi_{02}}{\partial t_2^2}+(\sigma_0^{(1)}+\sigma_0^{(2)})^2\frac{\partial^2\phi_{11}^+}{\partial t_3^2}$$

$$+(\sigma_0^{(1)}-\sigma_0^{(2)})^2\frac{\partial^2\phi_{11}^-}{\partial t_4^2}]+[\sigma_1^{(1)2}\frac{\partial^2\phi_{10}}{\partial t_1^2}+2\sigma_0^{(1)}\sigma_2^{(1)}\frac{\partial^2\phi_{10}}{\partial t_1^2}$$

$$+\sigma_1^{(2)2}\frac{\partial^2\phi_{01}}{\partial t_2^2}+2\sigma_0^{(2)}\sigma_2^{(2)}\frac{\partial^2\phi_{01}}{\partial t_2^2}+2\sigma_0^{(1)}\sigma_1^{(1)}\frac{\partial^2\phi_{20}}{\partial t_1^2}$$

$$+2\sigma_0^{(2)}\sigma_1^{(2)}\frac{\partial^2\phi_{02}}{\partial t_2^2}+2(\sigma_0^{(1)}+\sigma_0^{(2)})(\sigma_1^{(1)}+\sigma_1^{(2)})\frac{\partial^2\phi_{11}^+}{\partial t_3^2}$$

$$+2(\sigma_0^{(1)}-\sigma_0^{(2)})(\sigma_1^{(1)}-\sigma_1^{(2)})\frac{\partial^2\phi_{11}^-}{\partial t_4^2}+\sigma_0^{(1)2}\frac{\partial^2\phi_{30}}{\partial t_1^2}$$

$$+\sigma_0^{(2)2}\frac{\partial^2\phi_{03}}{\partial t_2^2}+(2\sigma_0^{(1)}+\sigma_0^{(2)})^2\frac{\partial^2\phi_{21}^+}{\partial t_5^2}+(2\sigma_0^{(1)}-\sigma_0^{(2)})^2\frac{\partial^2\phi_{21}^-}{\partial t_6^2}$$

$$+(\sigma_0^{(1)}+2\sigma_0^{(2)})^2\frac{\partial^2\phi_{12}^+}{\partial t_7^2}+(\sigma_0^{(1)}-2\sigma_0^{(2)})^2\frac{\partial^2\phi_{12}^-}{\partial t_8^2}]$$

$$+\text{higher order terms} \tag{4.6}$$

因此，依上二式的應用，則(2.8)與(2.9)兩控制條件式中之有關項可被運算逐階展開之，至第三階次量下，它們有

$$\eta\frac{\partial}{\partial z}(\frac{\partial^2\phi}{\partial t^2}+g\frac{\partial\phi}{\partial z})=(\eta_{10}+\eta_{01})\frac{\partial}{\partial z}(\sigma_0^{(1)2}\frac{\partial^2\phi_{10}}{\partial t_1^2}+g\frac{\partial\phi_{10}}{\partial z}$$

$$+\sigma_0^{(2)2}\frac{\partial^2\phi_{01}}{\partial t_2^2}+g\frac{\partial\phi_{01}}{\partial z})+\{(\eta_{10}+\eta_{01})\frac{\partial}{\partial z}[2\sigma_0^{(1)}\sigma_1^{(1)}\frac{\partial^2\phi_{10}}{\partial t_1^2}$$

$$+2\sigma_0^{(2)}\sigma_1^{(2)}\frac{\partial^2\phi_{01}}{\partial t_2^2}+\sigma_0^{(1)2}\frac{\partial^2\phi_{20}}{\partial t_1^2}+g\frac{\partial\phi_{20}}{\partial z}+\sigma_0^{(2)2}\frac{\partial^2\phi_{02}}{\partial t_2^2}$$

$$+g\frac{\partial\phi_{02}}{\partial z}+(\sigma_0^{(1)}+\sigma_0^{(2)})^2\frac{\partial^2\phi_{11}^+}{\partial t_3^2}+g\frac{\partial\phi_{11}^+}{\partial z}+(\sigma_0^{(1)}-\sigma_0^{(2)})^2\frac{\partial^2\phi_{11}^-}{\partial t_4^2}$$

$$+g\frac{\partial\phi_{11}^-}{\partial z}]+(\eta_{20}+\eta_{11}^++\eta_{11}^-+\eta_{02})\frac{\partial}{\partial z}(\sigma_0^{(1)2}\frac{\partial^2\phi_{10}}{\partial t_1^2}+g\frac{\partial\phi_{10}}{\partial z}$$

$$+\sigma_0^{(2)2}\frac{\partial^2\phi_{01}}{\partial t_2^2}+g\frac{\partial\phi_{01}}{\partial z})\}+\text{higer order terms} \tag{4.7}$$

$$\frac{1}{2}\eta^2\frac{\partial^2}{\partial z^2}(\frac{\partial^2\phi}{\partial t^2}+g\frac{\partial\phi}{\partial z})=\frac{1}{2}(\eta_{10}+\eta_{01})^2\frac{\partial^2}{\partial z^2}(\sigma_0^{(1)2}\frac{\partial^2\phi_{10}}{\partial t_1^2}+g\frac{\partial\phi_{10}}{\partial z}$$

$$+\sigma_0^{(2)2}\frac{\partial^2\phi_{01}}{\partial t_2^2}+g\frac{\partial\phi_{01}}{\partial z})+\text{higher order terms} \tag{4.8}$$

$$\eta\frac{\partial^2\phi}{\partial z\partial t}=(\eta_{10}+\eta_{01})\frac{\partial}{\partial z}(\sigma_0^{(1)}\frac{\partial\phi_{10}}{\partial t_1}+\sigma_0^{(2)}\frac{\partial\phi_{01}}{\partial t_2})+\{(\eta_{10}+\eta_{01})\frac{\partial}{\partial z}[\sigma_1^{(1)}$$

$$\times\frac{\partial\phi_{10}}{\partial t_1}+\sigma_1^{(2)}\frac{\partial\phi_{01}}{\partial t_2}+\sigma_0^{(1)}\frac{\partial\phi_{20}}{\partial t_1}+\sigma_0^{(2)}\frac{\partial\phi_{02}}{\partial t_2}+(\sigma_0^{(1)}+\sigma_0^{(2)})\frac{\partial\phi_{11}^+}{\partial t_3}$$

$$+(\sigma_0^{(1)}-\sigma_0^{(2)})\frac{\partial\phi_{11}^-}{\partial t_4}]+(\eta_{20}+\eta_{11}^++\eta_{11}^-+\eta_{02})\frac{\partial}{\partial z}(\sigma_0^{(1)}\frac{\partial\phi_{10}}{\partial t_1}$$

$$+\sigma_0^{(2)}\frac{\partial\phi_{01}}{\partial t_2})\}+\text{higher order terms} \tag{4.9}$$

$$\frac{1}{2}\eta^2\frac{\partial^3\phi}{\partial z^2\partial t}=\frac{1}{2}(\eta_{10}+\eta_{01})^2\frac{\partial^2}{\partial z^2}(\sigma_0^{(1)}\frac{\partial\phi_{10}}{\partial t_1}+\sigma_0^{(2)}\frac{\partial\phi_{01}}{\partial t_2})$$

$$+\text{higher order terms} \tag{4.10}$$

至於（2.8）與（2.9）兩式中之其餘項，可以（4.1）式之代入及（4.4）式之轉換變數的應用，並經 chain rule 之微分處理，得其逐階的展開，至第三階次量下，爲

$$\frac{\partial}{\partial t}(\vec{V}^2)=\frac{\partial}{\partial t}[(\nabla\phi)^2]=\frac{\partial}{\partial t}[(\nabla\phi_{10})^2+2(\nabla\phi_{10}\cdot\nabla\phi_{01})+(\nabla\phi_{01})^2$$

$$+2(\nabla\phi_{10}\cdot\nabla\phi_{20}+\nabla\phi_{10}\cdot\nabla\phi_{11}^++\nabla\phi_{10}\cdot\nabla\phi_{11}^-+\nabla\phi_{10}\cdot\nabla\phi_{02})$$

$$+2(\nabla\phi_{01}\cdot\nabla\phi_{20}+\nabla\phi_{01}\cdot\nabla\phi_{11}^++\nabla\phi_{01}\cdot\nabla\phi_{11}^-+\nabla\phi_{01}\cdot\nabla\phi_{02})+\cdots]$$

$$=2(\nabla\phi_{10}+\nabla\phi_{01})\cdot[\sigma_0{}^{(1)}\frac{\partial}{\partial t_1}(\nabla\phi_{10})+\sigma_0{}^{(2)}\frac{\partial}{\partial t_2}(\nabla\phi_{01})]$$

$$+\{2(\nabla\phi_{10}+\nabla\phi_{01})\cdot[\sigma_1{}^{(1)}\frac{\partial}{\partial t_1}(\nabla\phi_{10})+\sigma_1{}^{(2)}\frac{\partial}{\partial t_2}(\nabla\phi_{01})$$

$$+\sigma_0{}^{(1)}\frac{\partial}{\partial t_1}(\nabla\phi_{20})+(\sigma_0{}^{(1)}+\sigma_0{}^{(2)})\frac{\partial}{\partial t_3}(\nabla\phi_{11}{}^{+})$$

$$+(\sigma_0{}^{(1)}-\sigma_0{}^{(2)})\frac{\partial}{\partial t_4}(\nabla\phi_{11}{}^{-})+\sigma_0{}^{(2)}\frac{\partial}{\partial t_2}(\nabla\phi_{02})]+2(\nabla\phi_{20}$$

$$+\nabla\phi_{11}{}^{+}+\nabla\phi_{11}{}^{-}+\nabla\phi_{02})\cdot[\sigma_0{}^{(1)}\frac{\partial}{\partial t_1}(\nabla\phi_{10})+\sigma_0{}^{(2)}\frac{\partial}{\partial t_2}(\nabla\phi_{01})]\}$$

$$+\text{higer order terms} \qquad (4.11)$$

$$\eta\frac{\partial^2}{\partial z\partial t}(\vec{V}^2)=2(\eta_{10}+\eta_{01})\frac{\partial}{\partial z}\{(\nabla\phi_{10}+\nabla\phi_{01})\cdot[\sigma_0{}^{(1)}\frac{\partial}{\partial t_1}(\nabla\phi_{10})$$

$$+\sigma_0{}^{(2)}\frac{\partial}{\partial t_2}(\nabla\phi_{01})]\}+\text{higher order terms} \qquad (4.12)$$

$$\vec{V}\cdot\nabla(\frac{1}{2}\vec{V}^2)=\frac{1}{2}(\nabla\phi_{10}+\nabla\phi_{01})\cdot\nabla[(\nabla\phi_{10})^2+2\nabla\phi_{10}\cdot\nabla\phi_{01}+(\nabla\phi_{01})^2]$$

$$+\text{higher order terms} \qquad (4.13)$$

$$\frac{1}{2}(\vec{V}^2)=\frac{1}{2}(\nabla\phi_{10}+\nabla\phi_{01})^2+(\nabla\phi_{10}+\nabla\phi_{01})\cdot(\nabla\phi_{20}+\nabla\phi_{11}{}^{+}+\nabla\phi_{11}{}^{-}+\nabla\phi_{02})$$

$$+\text{higher order terms} \qquad (4.14)$$

$$\eta\frac{\partial}{\partial z}(\frac{1}{2}\vec{V}^2)=\frac{1}{2}(\eta_{10}+\eta_{01})\frac{\partial}{\partial z}[(\nabla\phi_{10}+\nabla\phi_{01})^2]+\text{higher order terms}$$

$$(4.15)$$

今將（4.5）～（4.15）式代入（2.8）與（2.9）兩控制條件式中之各對應項裡，則所考慮的波動系統其整個流場之所有必要的基本控制條件式，即（2.2）、（2.3）、（2.6）、（2.8）與（2.9）式，即刻可在（4.1）～（4.4）式之闡述下（或簡言之，在圖 2 所示之波動系統之整個流場脈動機構的說明下），把其逐階次的對應條件式列出，如在第三階次的考量下，它們可被依序地寫出為：

(1)波動流場之第一階次控制條件式（注意：已顯然直接以 S_1 與 S_2 之相位對應而列

出）

$$\nabla^2\phi_{10}=0\,,\quad \nabla^2\phi_{01}=0 \tag{4.16a}$$

$$\frac{\partial\phi_{10}}{\partial z}=0\,,\quad \frac{\partial\phi_{01}}{\partial z}=0\,,\quad z=-d \tag{4.16b}$$

$$\sigma_0^{(1)2}\frac{\partial^2\phi_{10}}{\partial t_1^2}+g\frac{\partial\phi_{10}}{\partial z}=0\,,\ \sigma_0^{(2)2}\frac{\partial^2\phi_{01}}{\partial t_2^2}+g\frac{\partial\phi_{01}}{\partial z}=0\,,\ z=0 \tag{4.16c}$$

$$\sigma_0^{(1)}\frac{\partial\phi_{10}}{\partial t_1}+g\eta_{10}=0\,,\ \sigma_0^{(2)}\frac{\partial\phi_{01}}{\partial t_2}+g\eta_{01}=0\,,\ z=0 \tag{4.16d}$$

$$\int_0^{2\pi}\int_0^{2\pi}\eta_{10}\,d(\vec{k}_1\cdot\vec{X})\,d(\vec{k}_2\cdot\vec{X})=0\,,\ \int_0^{2\pi}\int_0^{2\pi}\eta_{01}\,d(\vec{k}_1\cdot\vec{X})\,d(\vec{k}_2\cdot\vec{X})=0 \tag{4.16e}$$

(2)第二階次控制條件式

$$\nabla^2\phi_{20}=0\,,\quad \nabla^2\phi_{11}^{+}=0\,,\quad \nabla^2\phi_{11}^{-}=0\,,\quad \nabla^2\phi_{02}=0 \tag{4.17a}$$

$$\frac{\partial\phi_{20}}{\partial z}=0\,,\ \frac{\partial\phi_{11}^{+}}{\partial z}=0\,,\ \frac{\partial\phi_{11}^{-}}{\partial z}=0\,,\ \frac{\partial\phi_{02}}{\partial z}=0\,,\ z=-d \tag{4.17b}$$

$$\begin{aligned}&\left(\sigma_0^{(1)2}\frac{\partial^2\phi_{20}}{\partial t_1^2}+g\frac{\partial\phi_{20}}{\partial z}\right)+\left[(\sigma_0^{(1)}+\sigma_0^{(2)})^2\frac{\partial^2\phi_{11}^{+}}{\partial t_3^2}+g\frac{\partial\phi_{11}^{+}}{\partial z}\right]\\&+\left[(\sigma_0^{(1)}-\sigma_0^{(2)})^2\frac{\partial^2\phi_{11}^{-}}{\partial t_4^2}+g\frac{\partial\phi_{11}^{-}}{\partial z}\right]+\left(\sigma_0^{(2)2}\frac{\partial^2\phi_{02}}{\partial t_2^2}+g\frac{\partial\phi_{02}}{\partial z}\right)\\&+2\sigma_0^{(1)}\sigma_1^{(1)}\frac{\partial^2\phi_{10}}{\partial t_1^2}+2\sigma_0^{(2)}\sigma_1^{(2)}\frac{\partial^2\phi_{01}}{\partial t_2^2}+(\eta_{10}+\eta_{01})\frac{\partial}{\partial z}\left(\sigma_0^{(1)2}\frac{\partial^2\phi_{10}}{\partial t_1^2}\right.\\&\left.+g\frac{\partial\phi_{10}}{\partial z}+\sigma_0^{(2)2}\frac{\partial^2\phi_{01}}{\partial t_2^2}+g\frac{\partial\phi_{01}}{\partial z}\right)+2(\nabla\phi_{10}+\nabla\phi_{01})\cdot\left[\sigma_0^{(1)}\frac{\partial}{\partial t_1}(\nabla\phi_{10})\right.\\&\left.+\sigma_0^{(2)}\frac{\partial}{\partial t_2}(\nabla\phi_{01})\right]=0\,,\ z=0\end{aligned} \tag{4.17c}$$

$$\begin{aligned}&g(\eta_{20}+\eta_{11}^{+}+\eta_{11}^{-}+\eta_{02})+\sigma_0^{(1)}\frac{\partial\phi_{20}}{\partial t_1}+(\sigma_0^{(1)}+\sigma_0^{(2)})\frac{\partial\phi_{11}^{+}}{\partial t_3}+(\sigma_0^{(1)}\\&-\sigma_0^{(2)})\frac{\partial\phi_{11}^{-}}{\partial t_4}+\sigma_0^{(2)}\frac{\partial\phi_{02}}{\partial t_2}+\sigma_1^{(1)}\frac{\partial\phi_{10}}{\partial t_1}+\sigma_1^{(2)}\frac{\partial\phi_{01}}{\partial t_2}\end{aligned}$$

$$+(\eta_{10}+\eta_{01})\frac{\partial}{\partial z}\left(\sigma_0^{(1)}\frac{\partial\phi_{10}}{\partial t_1}+\sigma_0^{(2)}\frac{\partial\phi_{01}}{\partial t_2}\right)+\frac{1}{2}(\nabla\phi_{10}+\nabla\phi_{01})^2=0,$$

$$z=0 \tag{4.17d}$$

$$\int_0^{2\pi}\int_0^{2\pi}\eta_{20}\,d(\vec{k}_1\cdot\vec{X})\,d(\vec{k}_2\cdot\vec{X})=0,\quad \int_0^{2\pi}\int_0^{2\pi}\eta_{11}^{+}\,d(\vec{k}_1\cdot\vec{X})\,d(\vec{k}_2\cdot\vec{X})=0,$$

$$\int_0^{2\pi}\int_0^{2\pi}\eta_{11}^{-}\,d(\vec{k}_1\cdot\vec{X})\,d(\vec{k}_2\cdot\vec{X})=0,\quad \int_0^{2\pi}\int_0^{2\pi}\eta_{02}\,d(\vec{k}_1\cdot\vec{X})\,d(\vec{k}_2\cdot\vec{X})=0$$

$$\tag{4.17e}$$

(3)第三階次控制條件式

$$\nabla^2\phi_{30}=0,\quad \nabla^2\phi_{21}^{+}=0,\quad \nabla^2\phi_{21}^{-}=0,\quad \nabla^2\phi_{12}^{+}=0,$$

$$\nabla^2\phi_{12}^{-}=0,\quad \nabla^2\phi_{03}=0 \tag{4.18a}$$

$$\frac{\partial\phi_{03}}{\partial z}=0,\quad \frac{\partial\phi_{21}^{+}}{\partial z}=0,\quad \frac{\partial\phi_{21}^{-}}{\partial z}=0,\quad \frac{\partial\phi_{12}^{+}}{\partial z}=0,$$

$$\frac{\partial\phi_{12}^{-}}{\partial z}=0,\quad \frac{\partial\phi_{03}}{\partial z}=0,\quad z=-d \tag{4.18b}$$

$$\left(\sigma_0^{(1)2}\frac{\partial^2\phi_{30}}{\partial t_1^2}+g\frac{\partial\phi_{30}}{\partial z}\right)+\left[(2\sigma_0^{(1)}+\sigma_0^{(2)})^2\frac{\partial^2\phi_{21}^{+}}{\partial t_5^2}+g\frac{\partial\phi_{21}^{+}}{\partial z}\right]$$

$$+\left[(2\sigma_0^{(1)}-\sigma_0^{(2)})^2\frac{\partial^2\phi_{21}^{-}}{\partial t_6^2}+g\frac{\partial\phi_{21}^{-}}{\partial z}\right]+\left[(\sigma_0^{(1)}+2\sigma_0^{(2)})^2\frac{\partial^2\phi_{12}^{+}}{\partial t_7^2}+g\frac{\partial\phi_{12}^{+}}{\partial z}\right]$$

$$+\left[(\sigma_0^{(1)}-2\sigma_0^{(2)})^2\frac{\partial^2\phi_{12}^{-}}{\partial t_8^2}+g\frac{\partial\phi_{12}^{-}}{\partial z}\right]+\left(\sigma_0^{(2)2}\frac{\partial^2\phi_{03}}{\partial t_2^2}+g\frac{\partial\phi_{03}}{\partial z}\right)$$

$$+(\sigma_1^{(1)2}+2\sigma_0^{(1)}\sigma_2^{(1)})\frac{\partial^2\phi_{10}}{\partial t_1^2}+(\sigma_1^{(2)2}+2\sigma_0^{(2)}\sigma_2^{(2)})\frac{\partial^2\phi_{01}}{\partial t_2^2}$$

$$+2\sigma_0^{(1)}\sigma_1^{(1)}\frac{\partial^2\phi_{20}}{\partial t_1^2}+2(\sigma_0^{(1)}+\sigma_0^{(2)})(\sigma_1^{(1)}+\sigma_1^{(2)})\frac{\partial^2\phi_{11}^{+}}{\partial t_3^2}+2(\sigma_0^{(1)}$$

$$-\sigma_0^{(2)})(\sigma_1^{(1)}-\sigma_1^{(2)})\frac{\partial^2\phi_{11}^{-}}{\partial t_4^2}+2\sigma_0^{(2)}\sigma_1^{(2)}\frac{\partial^2\phi_{02}}{\partial t_2^2}+(\eta_{20}+\eta_{11}^{+}+\eta_{11}^{-}$$

$$+\eta_{02})\frac{\partial}{\partial z}\left(\sigma_0^{(1)2}\frac{\partial^2\phi_{10}}{\partial t_1^2}+g\frac{\partial\phi_{10}}{\partial z}+\sigma_0^{(2)2}\frac{\partial^2\phi_{01}}{\partial t_2^2}+g\frac{\partial\phi_{01}}{\partial z}\right)+(\eta_{10}$$

$$+\eta_{01})\frac{\partial}{\partial z}[2\sigma_0^{(1)}\sigma_1^{(1)}\frac{\partial^2\phi_{10}}{\partial t_1^2}+2\sigma_0^{(2)}\sigma_1^{(2)}\frac{\partial^2\phi_{01}}{\partial t_2^2}+\sigma_0^{(1)2}\frac{\partial^2\phi_{20}}{\partial t_1^2}+g\frac{\partial\phi_{20}}{\partial z}$$

$$+(\sigma_0^{(1)}+\sigma_0^{(2)})^2\frac{\partial^2\phi_{11}^+}{\partial t_3^2}+g\frac{\partial\phi_{11}^+}{\partial z}+(\sigma_0^{(1)}-\sigma_0^{(2)})^2\frac{\partial^2\phi_{11}^-}{\partial t_4^2}+g\frac{\partial\phi_{11}^-}{\partial z}$$

$$+\sigma_0^{(2)2}\frac{\partial^2\phi_{02}}{\partial t_2^2}+g\frac{\partial\phi_{02}}{\partial z}]+\frac{1}{2}(\eta_{10}+\eta_{01})^2\frac{\partial^2}{\partial z^2}(\sigma_0^{(1)2}\frac{\partial^2\phi_{10}}{\partial t_1^2}+g\frac{\partial\phi_{10}}{\partial z}$$

$$+\sigma_0^{(2)2}\frac{\partial^2\phi_{01}}{\partial t_2^2}+g\frac{\partial\phi_{01}}{\partial z})+2(\nabla\phi_{10}+\nabla\phi_{01})\cdot[\sigma_1^{(1)}\frac{\partial}{\partial t_1}(\nabla\phi_{10})$$

$$+\sigma_1^{(2)}\frac{\partial}{\partial t_2}(\nabla\phi_{01})+\sigma_0^{(1)}\frac{\partial}{\partial t_1}(\nabla\phi_{20})+\sigma_0^{(2)}\frac{\partial}{\partial t_2}(\nabla\phi_{02})+(\sigma_0^{(1)}$$

$$+\sigma_0^{(2)})\frac{\partial}{\partial t_3}(\nabla\phi_{11}^+)+(\sigma_0^{(1)}-\sigma_0^{(2)})\frac{\partial}{\partial t_4}(\nabla\phi_{11}^-)]+2(\nabla\phi_{20}+\nabla\phi_{11}^++\nabla\phi_{11}^-$$

$$+\nabla\phi_{02})\cdot[\sigma_0^{(1)}\frac{\partial}{\partial t_1}(\nabla\phi_{10})+\sigma_0^{(2)}\frac{\partial}{\partial t_2}(\nabla\phi_{01})]+2(\eta_{10}+\eta_{01})\frac{\partial}{\partial z}\{(\nabla\phi_{10}$$

$$+\nabla\phi_{01})\cdot[\sigma_0^{(1)}\frac{\partial}{\partial t_1}(\nabla\phi_{10})+\sigma_0^{(2)}\frac{\partial}{\partial t_2}(\nabla\phi_{01})]\}+\frac{1}{2}(\nabla\phi_{10}+\nabla\phi_{01})$$

$$\cdot\nabla[(\nabla\phi_{10})^2+2\nabla\phi_{10}\cdot\nabla\phi_{01}+(\nabla\phi_{01})^2]=0,\quad z=0 \tag{4.18c}$$

$$g(\eta_{30}+\eta_{21}^++\eta_{21}^-+\eta_{12}^++\eta_{12}^-+\eta_{03})+\sigma_0^{(1)}\frac{\partial\phi_{30}}{\partial t_1}+(2\sigma_0^{(1)}+\sigma_0^{(2)})\frac{\partial\phi_{21}^+}{\partial t_5}$$

$$+(2\sigma_0^{(1)}-\sigma_0^{(2)})\frac{\partial\phi_{21}^-}{\partial t_6}+(\sigma_0^{(1)}+2\sigma_0^{(2)})\frac{\partial\phi_{12}^+}{\partial t_7}+(\sigma_0^{(1)}-2\sigma_0^{(2)})\frac{\partial\phi_{12}^-}{\partial t_8}$$

$$+\sigma_0^{(2)}\frac{\partial\phi_{03}}{\partial t_2}+\sigma_2^{(1)}\frac{\partial\phi_{10}}{\partial t_1}+\sigma_2^{(2)}\frac{\partial\phi_{01}}{\partial t_2}+\sigma_1^{(1)}\frac{\partial\phi_{20}}{\partial t_1}+(\sigma_1^{(1)}+\sigma_1^{(2)})\frac{\partial\phi_{11}^+}{\partial t_3}$$

$$+(\sigma_1^{(1)}-\sigma_1^{(2)})\frac{\partial\phi_{11}^-}{\partial t_4}+\sigma_1^{(2)}\frac{\partial\phi_{02}}{\partial t_2}+(\eta_{10}+\eta_{01})\frac{\partial}{\partial z}[\sigma_1^{(1)}\frac{\partial\phi_{10}}{\partial t_1}+\sigma_1^{(2)}\frac{\partial\phi_{01}}{\partial t_2}$$

$$+\sigma_0^{(1)}\frac{\partial\phi_{20}}{\partial t_1}+(\sigma_0^{(1)}+\sigma_0^{(2)})\frac{\partial\phi_{11}^+}{\partial t_3}+(\sigma_0^{(1)}-\sigma_0^{(2)})\frac{\partial\phi_{11}^-}{\partial t_4}+\sigma_0^{(2)}\frac{\partial\phi_{02}}{\partial t_2}]$$

$$+(\eta_{20}+\eta_{11}^++\eta_{11}^-+\eta_{02})\frac{\partial}{\partial z}(\sigma_0^{(1)}\frac{\partial\phi_{10}}{\partial t_1}+\sigma_0^{(2)}\frac{\partial\phi_{01}}{\partial t_2})+\frac{1}{2}(\eta_{10}$$

$$+\eta_{01})^2\frac{\partial^2}{\partial z^2}(\sigma_0^{(1)}\frac{\partial\phi_{10}}{\partial t_1}+\sigma_0^{(2)}\frac{\partial\phi_{01}}{\partial t_2})+(\nabla\phi_{10}+\nabla\phi_{01})\cdot(\nabla\phi_{20}+\nabla\phi_{11}^{+}$$

$$+\nabla\phi_{11}^{-}+\nabla\phi_{02})+\frac{1}{2}(\eta_{10}+\eta_{01})\frac{\partial}{\partial z}[(\nabla\phi_{10}+\nabla\phi_{01})^2]=0,\ z=0 \qquad (4.18d)$$

$$\int_0^{2\pi}\int_0^{2\pi}\eta_{30}d(\vec{k}_1\cdot\vec{X})d(\vec{k}_2\cdot\vec{X})=\int_0^{2\pi}\int_0^{2\pi}\eta_{21}^{+}d(\vec{k}_1\cdot\vec{X})d(\vec{k}_2\cdot\vec{X})$$

$$=\int_0^{2\pi}\int_0^{2\pi}\eta_{21}^{-}d(\vec{k}_1\cdot\vec{X})d(\vec{k}_2\cdot\vec{X})=\int_0^{2\pi}\int_0^{2\pi}\eta_{12}^{+}d(\vec{k}_1\cdot\vec{X})d(\vec{k}_2\cdot\vec{X})$$

$$=\int_0^{2\pi}\int_0^{2\pi}\eta_{12}^{-}d(\vec{k}_1\cdot\vec{X})d(\vec{k}_2\cdot\vec{X})=\int_0^{2\pi}\int_0^{2\pi}\eta_{03}d(\vec{k}_1\cdot\vec{X})d(\vec{k}_2\cdot\vec{X})=0$$

$$(4.18e)$$

至此，對照作者前文（Chen & Hsu，1988，1990）之論述，可顯然地得知，對所考慮的兩波列交會系統之波動流場的解析，其所必要滿足的逐階次基本控制條件式，（至第三階量下）本節所列出的（4.16a－e）～（4.18a－e）式是與前文者完全一致；此即表示，對此波動系統其流場內部的脈動機構特性，是直接可由兩來源波列之本質結構間的交錯作用結果給予直覺化明確地掌握與描述，如圖2所示，且進而對其解析建立系統化的流程，而不必再如先前者需經抽絲剝繭般的繁雜冗長迂迴的處理。換言之，本節的陳述，已直接掌握問題的核心，而使其處理方式更直接簡單且系統明朗化，當然地，如此將使問題更易為人所洞察，而又可避免因繁雜的處理致生疏漏與誤解。

五、整個流場解

依上節對任一均匀等深水中，二自由表面規則前進重力波列相交會所構成之波動系統所列出其整體流場結構之逐階次的控制方程式下，則其至第三階次量的各階解可被求之於下。

5-1 第一階(線性)解

由（4.16a-e）式，即刻可解得任一均匀等深d水中之二自由表面規則前進重力波列相交會所構成之波動流場的第一階（或線性）解爲

$$\left.\begin{aligned}
\phi_{10} &= \frac{a_1 g}{\sigma_0^{(1)}} \frac{\cosh k_1(d+z)}{\cosh k_1 d} \sin S_1 \;,\; \eta_{10} = a_1 \cos S_1 \;, \\
&\sigma_0^{(1)2} = g k_1 \tanh k_1 d \;,\; S_1 = \vec{k_1} \cdot \vec{X} - \sigma_1 t + \epsilon_1 \\
\phi_{01} &= \frac{a_2 g}{\sigma_0^{(2)}} \frac{\cosh k_2(d+z)}{\cosh k_2 d} \sin S_2 \;,\; \eta_{01} = a_2 \cos S_2 \\
&\sigma_0^{(2)2} = g k_2 \tanh k_2 d \;,\; S_2 = \vec{k_2} \cdot \vec{X} - \sigma_2 t + \epsilon_2
\end{aligned}\right\} \qquad (5.1)$$

如上節所示，式中a_1、a_2各表示爲正比於相交之二來源成份波列振幅的長度因次量常數，而S_1、S_2各爲其對應的變動相位；其中$\vec{X} = \vec{i}x + \vec{j}y$表示水平面上的位置向量，$\epsilon_1$與$\epsilon_2$各爲此二波列交會時在座標原點處的相位，方便上可適當地調整座標使其中一者（特別情況時爲兩者）爲零。這是顯然地，在線性考慮下，上式所示之波動流場解是完全符合於攝動法解析一般波動場所得之結果。

5-2 第二階解

至於求解所考慮的波動流場之第二階解，即ϕ_{20}、ϕ_{11}^{+}、ϕ_{11}^{-}、ϕ_{02}、η_{20}、η_{11}^{+}、η_{11}^{-}、η_{02}與$\sigma_1^{(1)}$、$\sigma_1^{(2)}$者，則需將上小節所解的線性解代入所對應第二階次控制條件（4.17a-e）式中求解之，今爲清楚解析起見，先算出其中可先求算的部份。此部份可由（5.1）式代入（4.17c）與（4.17d）式中得之，若分別以F與G來表示時，則有

$$F=2\left(\sigma_0^{(1)}\sigma_1^{(1)}\frac{\partial^2\phi_{10}}{\partial t_1^2}+\sigma_0^{(2)}\sigma_1^{(2)}\frac{\partial^2\phi_{01}}{\partial t_2^2}\right)+(\eta_{10}+\eta_{01})$$

$$\times\frac{\partial}{\partial z}\left(\sigma_0^{(1)2}\frac{\partial^2\phi_{10}}{\partial t_1^2}+\sigma_0^{(2)2}\frac{\partial^2\phi_{01}}{\partial t_2^2}+g\frac{\partial\phi_{10}}{\partial z}+g\frac{\partial\phi_{01}}{\partial z}\right)$$

$$+\sigma_0^{(1)}\frac{\partial}{\partial t_1}(\vec{V}_{10}^2)+\sigma_0^{(2)}\frac{\partial}{\partial t_2}(\vec{V}_{01}^2)+2\sigma_0^{(1)}\left(\frac{\partial\vec{V}_{10}}{\partial t_1}\cdot\vec{V}_{01}\right)$$

$$+2\sigma_0^{(2)}\left(\vec{V}_{10}\cdot\frac{\partial\vec{V}_{01}}{\partial t_2}\right),\quad z=0 \tag{5.2}$$

$$G=\sigma_1^{(1)}\frac{\partial\phi_{10}}{\partial t_1}+\sigma_1^{(2)}\frac{\partial\phi_{01}}{\partial t_2}+(\eta_{10}+\eta_{01})\frac{\partial}{\partial z}\left(\sigma_0^{(1)}\frac{\partial\phi_{10}}{\partial t_1}+\sigma_0^{(2)}\frac{\partial\phi_{01}}{\partial t_2}\right)$$

$$+\frac{1}{2}\left(\vec{V}_{10}^2+2\vec{V}_{10}\cdot\vec{V}_{01}+\vec{V}_{01}^2\right),\quad z=0 \tag{5.3}$$

求解 ϕ_{20} 、 ϕ_{11}^{+} 、 ϕ_{11}^{-} 、 ϕ_{02} 等，則需計算（5.2）式中之右邊各項，依（5.1）式之應用其可被列出各爲

$$2\left(\sigma_0^{(1)}\sigma_1^{(1)}\frac{\partial^2\phi_{10}}{\partial t_1^2}+\sigma_0^{(2)}\sigma_1^{(2)}\frac{\partial^2\phi_{01}}{\partial t_2^2}\right)_{z=0}$$

$$=-2\left(a_1g\sigma_1^{(1)}\sin S_1+a_2g\sigma_1^{(2)}\sin S_2\right)$$

$$(\eta_{10}+\eta_{01})\left(\sigma_0^{(1)2}\frac{\partial^3\phi_{10}}{\partial z\partial t_1^2}+g\frac{\partial^2\phi_{10}}{\partial z^2}\right)_{z=0}$$

$$=\frac{1}{2}a_1^2\left(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3}\right)\sin 2S_1$$

$$+\frac{1}{2}a_1a_2\left(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3}\right)\left[\sin(S_1+S_2)+\sin(S_1-S_2)\right]$$

$$(\eta_{10}+\eta_{01})\left(\sigma_0^{(2)2}\frac{\partial^3\phi_{01}}{\partial z\partial t_2^2}+g\frac{\partial^2\phi_{01}}{\partial z^2}\right)_{z=0}$$

$$=\frac{1}{2}a_2^2\left(\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3}\right)\sin 2S_2$$

$$+\frac{1}{2}a_1a_2\left(\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3}\right)\left[\sin(S_1+S_2)+\sin(S_2-S_1)\right]$$

$$
\left.
\begin{aligned}
&\sigma_0^{(1)}\left(\frac{\partial}{\partial t_1}\vec{V}_{10}^2\right)_{z=0}=\sigma_0^{(1)}\left[\frac{\partial}{\partial t_1}(\nabla\phi_{10})^2\right]_{z=0}\\
&\qquad=\frac{a_1^2g^2k_1^2}{\sigma_0^{(1)}}\frac{\sin 2S_1}{\cosh^2k_1d}\\
&\sigma_0^{(2)}\left(\frac{\partial}{\partial t_2}\vec{V}_{01}^2\right)_{z=0}=\sigma_0^{(2)}\left[\frac{\partial}{\partial t_2}(\nabla\phi_{01})^2\right]_{z=0}\\
&\qquad=\frac{a_2^2g^2k_2^2}{\sigma_0^{(2)}}\frac{\sin 2S_2}{\cosh^2k_2d}\\
&2\sigma_0^{(1)}\left(\frac{\partial\vec{V}_{10}}{\partial t_1}\cdot\vec{V}_{01}\right)_{z=0}=2\sigma_0^{(1)}\left[\left(\frac{\partial}{\partial t_1}\nabla\phi_{10}\right)\cdot\nabla\phi_{01}\right]_{z=0}\\
&=\frac{a_1a_2g^2}{\sigma_0^{(2)}}k_1k_2\left\{\left(\cos\theta-\frac{\sigma_0^{(1)2}\sigma_0^{(2)2}}{g^2k_1k_2}\right)\sin(S_1+S_2)\right.\\
&\qquad\left.+\left(\cos\theta+\frac{\sigma_0^{(1)2}\sigma_0^{(2)2}}{g^2k_1k_2}\right)\sin(S_1-S_2)\right\}\\
&2\sigma_0^{(2)}\left(\vec{V}_{10}\cdot\frac{\partial\vec{V}_{01}}{\partial t_2}\right)_{z=0}=2\sigma_0^{(2)}\left[\nabla\phi_{10}\cdot\left(\frac{\partial}{\partial t_2}\nabla\phi_{01}\right)\right]_{z=0}\\
&=\frac{a_1a_2g^2}{\sigma_0^{(1)}}k_1k_2\left\{\left(\cos\theta-\frac{\sigma_0^{(1)2}\sigma_0^{(2)2}}{g^2k_1k_2}\right)\sin(S_1+S_2)\right.\\
&\qquad\left.-\left(\cos\theta+\frac{\sigma_0^{(1)2}\sigma_0^{(2)2}}{g^2k_1k_2}\right)\sin(S_1-S_2)\right\}
\end{aligned}
\right\}
\tag{5.4}
$$

因此，得F爲

$$
\begin{aligned}
F=&-2g\left(a_1\sigma_1^{(1)}\sin S_1+a_2\sigma_1^{(2)}\sin S_2\right)+\frac{3}{2}\frac{a_1^2g^2k_1^2}{\sigma_0^{(1)}\cosh^2k_1d}\sin 2S_1\\
&+\frac{3}{2}\frac{a_2^2g^2k_2^2}{\sigma_0^{(2)}\cosh^2k_2d}\sin 2S_2+a_1a_2\left\{\frac{1}{2}\left(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3}+\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3}\right)\right.\\
&\left.+\left(g^2k_1k_2\cos\theta-\sigma_0^{(1)2}\sigma_0^{(2)2}\right)\left(\frac{1}{\sigma_0^{(1)}}+\frac{1}{\sigma_0^{(2)}}\right)\right\}\sin(S_1+S_2)\\
&+a_1a_2\left\{\frac{1}{2}\left(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3}-\frac{g^2k_2^2}{\sigma_0^{(2)}}+\sigma_0^{(2)3}\right)\right.
\end{aligned}
$$

$$+(g^2k_1k_2\cos\theta+\sigma_0^{(1)2}\sigma_0^{(2)2})(\frac{1}{\sigma_0^{(2)}}-\frac{1}{\sigma_0^{(1)}})\}\sin(S_1-S_2) \quad (5.5)$$

這是明顯地，依攝動法解析下的相位比較，如圖２或（4.2）式所示，可知在此二波列交會之相互作用結果裡，ϕ_{20} 與 ϕ_{02} 是分別對應於 $\sin nS_1$ 與 $\sin nS_2$ 項，$n=1，2$，而 $\phi_{11}=\phi_{11}^{+}+\phi_{11}^{-}$ 是因其間之相互作用所衍生出者應對應於 $\sin(S_1\pm S_2)$ 項；如（4.2）式所示，ϕ_{11}^{+} 與 ϕ_{11}^{-} 分別和 $\sin(S_1+S_2)$ 與 $\sin(S_1-S_2)$ 項對應之。

因此，由（5.2）、（5.4）與（5.5）式之應用，得第二階下的控制式（4.17c）式爲

$$(\sigma_0^{(1)2}\frac{\partial^2\phi_{20}}{\partial t_1^2}+g\frac{\partial\phi_{20}}{\partial z})+(\sigma_0^{(2)2}\frac{\partial^2\phi_{02}}{\partial t_2^2}+g\frac{\partial\phi_{02}}{\partial z})$$

$$+[(\sigma_0^{(1)}+\sigma_0^{(2)})^2\frac{\partial^2\phi_{11}^{+}}{\partial t_3^2}+g\frac{\partial\phi_{11}^{+}}{\partial z}]$$

$$+[(\sigma_0^{(1)}-\sigma_0^{(2)})^2\frac{\partial^2\phi_{11}^{-}}{\partial t_4^2}+g\frac{\partial\phi_{11}^{-}}{\partial z}]=-F$$

$$=2g(a_1\sigma_1^{(1)}\sin S_1+a_2\sigma_1^{(2)}\sin S_2)-\frac{3}{2}\frac{a_1^2g^2k_1^2}{\sigma_0^{(1)}\cosh^2k_1d}\sin 2S_1$$

$$-\frac{3}{2}\frac{a_2^2g^2k_2^2}{\sigma_0^{(2)}\cosh^2k_2d}\sin 2S_2-a_1a_2\{\frac{1}{2}(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3}$$

$$+\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3})+(g^2k_1k_2\cos\theta-\sigma_0^{(1)2}\sigma_0^{(2)2})(\frac{1}{\sigma_0^{(1)}}+\frac{1}{\sigma_0^{(2)}})\}$$

$$\times\sin(S_1+S_2)-a_1a_2\{\frac{1}{2}(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3}-\frac{g^2k_2^2}{\sigma_0^{(2)}}+\sigma_0^{(2)3})$$

$$+(g^2k_1k_2\cos\theta+\sigma_0^{(1)2}\sigma_0^{(2)2})(\frac{1}{\sigma_0^{(2)}}-\frac{1}{\sigma_0^{(1)}})\}$$

$$\times\sin(S_1-S_2)\ ,\ z=0 \quad (5.6)$$

則依上述所言的相位對應關係，ϕ_{20}、ϕ_{02} 與 ϕ_{11}^{+}、ϕ_{11}^{-} 分別滿足

$$\nabla^2\phi_{20}=0\ ,\ \nabla^2\phi_{02}=0\ ,\ \nabla^2\phi_{11}^{+}=0\ ,\ \nabla^2\phi_{11}^{-}=0$$

$$
\left.
\begin{aligned}
&\frac{\partial \phi_{20}}{\partial z}=0\,,\ \frac{\partial \phi_{02}}{\partial z}=0\,,\ \frac{\partial \phi_{11}^{+}}{\partial z}=0\,,\ \frac{\partial \phi_{11}^{-}}{\partial z}=0\,,\ z=-d\\
&\left(\sigma_0^{(1)2}\frac{\partial^2 \phi_{20}}{\partial t_1^2}+g\frac{\partial \phi_{20}}{\partial z}\right)\\
&=2ga_1\sigma_1^{(1)}\sin S_1-\frac{3}{2}\frac{a_1^2g^2k_1^2}{\sigma_0^{(1)}\cosh^2 k_1 d}\sin 2S_1\,,\ z=0\\
&\left(\sigma_0^{(2)2}\frac{\partial^2 \phi_{02}}{\partial t_2^2}+g\frac{\partial \phi_{02}}{\partial z}\right)\\
&=2ga_2\sigma_1^{(2)}\sin S_2-\frac{3}{2}\frac{a_2^2g^2k_2^2}{\sigma_0^{(2)}\cosh^2 k_2 d}\sin 2S_2\,,\ z=0\\
&\left(\sigma_0^{(1)}+\sigma_0^{(2)}\right)^2\frac{\partial^2 \phi_{11}^{+}}{\partial t_3^2}+g\frac{\partial \phi_{11}^{+}}{\partial z}\\
&=-a_1a_2\left\{\frac{1}{2}\left(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3}+\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3}\right)\right.\\
&\quad\left.+\left(g^2k_1k_2\cos\theta-\sigma_0^{(1)2}\sigma_0^{(2)2}\right)\left(\frac{1}{\sigma_0^{(1)}}+\frac{1}{\sigma_0^{(2)}}\right)\right\}\\
&\quad\times\sin\left(S_1+S_2\right)\,,\ z=0\\
&\left(\sigma_0^{(1)}-\sigma_0^{(2)}\right)^2\frac{\partial^2 \phi_{11}^{-}}{\partial t_4^2}+g\frac{\partial \phi_{11}^{-}}{\partial z}\\
&=-a_1a_2\left\{\frac{1}{2}\left(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3}-\frac{g^2k_2^2}{\sigma_0^{(2)}}+\sigma_0^{(2)3}\right)\right.\\
&\quad\left.+\left(g^2k_1k_2\cos\theta+\sigma_0^{(1)2}\sigma_0^{(2)2}\right)\left(\frac{1}{\sigma_0^{(2)}}-\frac{1}{\sigma_0^{(1)}}\right)\right\}\\
&\quad\times\sin\left(S_1-S_2\right)\,,\ z=0
\end{aligned}
\right\}\quad(5.7)
$$

解(5.7)式，避免所謂的庸凡項(secular term)的發生，因此可得$\sigma_1^{(1)}$、$\sigma_1^{(2)}$、ϕ_{20}、ϕ_{02}、ϕ_{11}^{+}與ϕ_{11}^{-}之解各為

$$
\begin{aligned}
&\sigma_1^{(1)}=\sigma_1^{(2)}=0\\
&\phi_{20}=\frac{3}{8}a_1^2\sigma_0^{(1)}\frac{\cosh 2k_1(d+z)}{\sinh^4 k_1 d}\sin 2S_1+B_{20}t
\end{aligned}
$$

$$
\left.
\begin{aligned}
\phi_{02} &= \frac{3}{8} a_2{}^2 \sigma_0{}^{(2)} \frac{\cosh 2k_2(d+z)}{\sinh^4 k_2 d} \sin 2S_2 + B_{02} t \\
\phi_{11}{}^{+} &= A_{11}{}^{+} \frac{\cosh|\vec{k_1}+\vec{k_2}|(d+z)}{\cosh|\vec{k_1}+\vec{k_2}|d} \sin(S_1+S_2) + B_{11}{}^{+} t \\
\phi_{11}{}^{-} &= A_{11}{}^{-} \frac{\cosh|\vec{k_1}-\vec{k_2}|(d+z)}{\cosh|\vec{k_1}-\vec{k_2}|d} \sin(S_1-S_2) + B_{11}{}^{-} t \\
A_{11}{}^{+} &= a_1 a_2 \Big[\frac{1}{2} \Big(\frac{g^2 k_1^2}{\sigma_0{}^{(1)}} - \sigma_0{}^{(1)3} + \frac{g^2 k_2^2}{\sigma_0{}^{(2)}} - \sigma_0{}^{(2)3} \Big) \\
&\quad + (g^2 k_1 k_2 \cos\theta - \sigma_0{}^{(1)2} \sigma_0{}^{(2)2}) \Big(\frac{1}{\sigma_0{}^{(1)}} + \frac{1}{\sigma_0{}^{(2)}} \Big) \Big] \\
&\quad / \Big[(\sigma_0{}^{(1)} + \sigma_0{}^{(2)})^2 - g|\vec{k_1}+\vec{k_2}| \tanh|\vec{k_1}+\vec{k_2}|d \Big] \\
A_{11}{}^{-} &= a_1 a_2 \Big[\frac{1}{2} \Big(\frac{g^2 k_1^2}{\sigma_0{}^{(1)}} - \sigma_0{}^{(1)3} - \frac{g^2 k_2^2}{\sigma_0{}^{(2)}} + \sigma_0{}^{(2)3} \Big) \\
&\quad + (g^2 k_1 k_2 \cos\theta + \sigma_0{}^{(1)2} \sigma_0{}^{(2)2}) \Big(\frac{1}{\sigma_0{}^{(2)}} - \frac{1}{\sigma_0{}^{(1)}} \Big) \Big] \\
&\quad / \Big[(\sigma_0{}^{(1)} - \sigma_0{}^{(2)})^2 - g|\vec{k_1}-\vec{k_2}| \tanh|\vec{k_1}-\vec{k_2}|d \Big]
\end{aligned}
\right\} \quad (5.8)
$$

此處B_{20}、B_{02}、$B_{11}{}^{+}$與$B_{11}{}^{-}$爲待求的常數，其可被決定於求解對應的水位η_{20}、η_{02}與$\eta_{11}{}^{+}$、$\eta_{11}{}^{-}$之解析中，將緊接地陳述於下。

至於求解第二階水位η_{20}、η_{02}與$\eta_{11}{}^{+}$、$\eta_{11}{}^{-}$，則需由對應的自由表面動力邊界條件（4.17d）式爲之。這是類似於求解ϕ_{20}、ϕ_{02}與$\phi_{11}{}^{+}$、$\phi_{11}{}^{-}$之過程，首先需先計算出可先算出的部份，即此部份G可依（5.1）式代入（5.3）式之右邊而逐項求得，其被列出如下：

$$
\sigma_1{}^{(1)} \frac{\partial \phi_{10}}{\partial t_1} = \sigma_1{}^{(2)} \frac{\partial \phi_{01}}{\partial t_2} = 0 \quad (\text{因}\ \sigma_1{}^{(1)} = \sigma_1{}^{(2)} = 0)
$$

$$
\sigma_0{}^{(1)} (\eta_{10} + \eta_{01}) \Big(\frac{\partial^2 \phi_{10}}{\partial z \partial t_1} \Big)_{z=0} = -a_1 \sigma_0{}^{(1)2} \Big\{ \frac{a_1}{2} (1 + \cos 2S_1)
$$

$$
\left.\begin{aligned}
&+\frac{a_2}{2}\left[\cos(S_1-S_2)+\cos(S_1+S_2)\right]\Big\}\\
&\sigma_0^{(2)}(\eta_{10}+\eta_{01})\left(\frac{\partial^2\phi_{01}}{\partial z\partial t_2}\right)_{z=0}=-a_2\sigma_0^{(2)2}\Big\{\frac{a_1}{2}\left[\cos(S_1-S_2)\right.\\
&\quad\left.+\cos(S_1+S_2)\right]+\frac{a_2}{2}(1+\cos 2S_2)\Big\}\\
&\frac{1}{2}(\vec{V}_{10}^2)_{z=0}=\frac{1}{2}(\nabla\phi_{10})^2_{z=0}=\frac{1}{2}a_1^2\sigma_0^{(1)2}\left(1+\frac{1}{2}\frac{1+\cos 2S_1}{\sinh^2 k_1 d}\right)\\
&(\vec{V}_{10}\cdot\vec{V}_{01})_{z=0}=(\nabla\phi_{10}\cdot\nabla\phi_{01})_{z=0}\\
&=\frac{1}{2}a_1a_2\Big\{\left(\frac{g^2k_1k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}\cos\theta+\sigma_0^{(1)}\sigma_0^{(2)}\right)\cos(S_1-S_2)\\
&\quad+\left(\frac{g^2k_1k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}\cos\theta-\sigma_0^{(1)}\sigma_0^{(2)}\right)\cos(S_1+S_2)\Big\}\\
&\frac{1}{2}(\vec{V}_{01}^2)_{z=0}=\frac{1}{2}(\nabla\phi_{01})^2_{z=0}=\frac{1}{2}a_2^2\sigma_0^{(2)2}\left(1+\frac{1}{2}\frac{1+\cos 2S_2}{\sinh^2 k_2 d}\right)
\end{aligned}\right\}\quad(5.9)
$$

故得G

$$
\begin{aligned}
G=&\frac{1}{4}\frac{a_1^2\sigma_0^{(1)2}}{\sinh^2 k_1 d}+\frac{1}{4}\frac{a_2^2\sigma_0^{(2)2}}{\sinh^2 k_2 d}+a_1^2\sigma_0^{(1)2}\left(\frac{1}{4}\frac{1}{\sinh^2 k_1 d}-\frac{1}{2}\right)\cos 2S_1\\
&+a_2^2\sigma_0^{(2)2}\left(\frac{1}{4}\frac{1}{\sinh^2 k_2 d}-\frac{1}{2}\right)\cos 2S_2\\
&+\frac{1}{2}a_1a_2\left[\left(\frac{g^2k_1k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}\cos\theta-\sigma_0^{(1)}\sigma_0^{(2)}\right)-\sigma_0^{(1)2}-\sigma_0^{(2)2}\right]\cos(S_1+S_2)\\
&+\frac{1}{2}a_1a_2\left[\left(\frac{g^2k_1k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}\cos\theta+\sigma_0^{(1)}\sigma_0^{(2)}\right)-\sigma_0^{(1)2}-\sigma_0^{(2)2}\right]\\
&\times\cos(S_1-S_2)
\end{aligned}\qquad(5.10)
$$

依 (5.3) 式之應用，則 (4.17d) 式之第二階次量形式可被完整地寫出爲

$$
g(\eta_{20}+\eta_{11}^{+}+\eta_{11}^{-}+\eta_{02})+\sigma_0^{(1)}\frac{\partial\phi_{20}}{\partial t_1}+\sigma_0^{(2)}\frac{\partial\phi_{02}}{\partial t_2}+(\sigma_0^{(1)}+\sigma_0^{(2)})\frac{\partial\phi_{11}^{+}}{\partial t_3}
$$

$$+(\sigma_0^{(1)}-\sigma_0^{(2)})\frac{\partial\phi_{11}^-}{\partial t_4}+G=0\ ,\ z=0 \tag{5.11}$$

將(5.8)式所得之$\sigma_1^{(1)}$、$\sigma_1^{(2)}$、ϕ_{20}、ϕ_{02}、ϕ_{11}^+、ϕ_{11}^-解與(5.10)式之G代入上式(且令$\eta_{11}=\eta_{11}^++\eta_{11}^-$),則(5.11)式變爲

$$\begin{aligned}
&g(\eta_{20}+\eta_{11}^++\eta_{11}^-+\eta_{02})+B_{20}+B_{11}^++B_{11}^-+B_{02}+\frac{1}{4}\frac{a_1^2\sigma_0^{(1)2}}{\sinh^2k_1d}\\
&+\frac{1}{4}\frac{a_2^2\sigma_0^{(2)2}}{\sinh^2k_2d}-\frac{1}{4}\frac{a_1^2\sigma_0^{(1)2}}{\sinh^4k_1d}(2\sinh^2k_1d+3)\cosh^2k_1d\cdot\cos 2S_1\\
&-\frac{1}{4}\frac{a_2^2\sigma_0^{(2)2}}{\sinh^2k_2d}(2\sinh^2k_2d+3)\cosh^2k_2d\cdot\cos 2S_2\\
&+a_1a_2\{\frac{1}{2}(\frac{g^2k_1k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}\cos\theta-\sigma_0^{(1)2}-\sigma_0^{(1)}\sigma_0^{(2)}-\sigma_0^{(2)2})\\
&-(\sigma_0^{(1)}+\sigma_0^{(2)})[\frac{1}{2}(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3}+\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3})\\
&+(g^2k_1k_2\cos\theta-\sigma_0^{(1)2}\sigma_0^{(2)2})(\frac{1}{\sigma_0^{(1)}}+\frac{1}{\sigma_0^{(2)}})]\\
&/[(\sigma_0^{(1)}+\sigma_0^{(2)})^2-g|\vec{k}_1+\vec{k}_2|\tanh|\vec{k}_1+\vec{k}_2|d]\}\cos(S_1+S_2)\\
&+a_1a_2\{\frac{1}{2}(\frac{g^2k_1k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}\cos\theta-\sigma_0^{(1)2}+\sigma_0^{(1)}\sigma_0^{(2)}-\sigma_0^{(2)2})\\
&-(\sigma_0^{(1)}-\sigma_0^{(2)})[\frac{1}{2}(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3}-\frac{g^2k_2^2}{\sigma_0^{(2)}}+\sigma_0^{(2)3})\\
&+(g^2k_1k_2\cos\theta+\sigma_0^{(1)2}\sigma_0^{(2)2})(\frac{1}{\sigma_0^{(2)}}-\frac{1}{\sigma_0^{(1)}})]\\
&/[(\sigma_0^{(1)}-\sigma_0^{(2)})^2-g|\vec{k}_1-\vec{k}_2|\tanh|\vec{k}_1-\vec{k}_2|d]\}\cos(S_1-S_2)\\
&=0
\end{aligned}\tag{5.12}$$

最後,依平均靜止水平面之條件(4.17e)式的應用,可得

$$B_{20}=-\frac{1}{4}\frac{a_1^2\sigma_0^{(1)2}}{\sinh^2k_1d},\ B_{02}=-\frac{1}{4}\frac{a_2^2\sigma_0^{(2)2}}{\sinh^2k_2d}\ ,\ B_{11}^+=0\ ,\ B_{11}^-=0 \tag{5.13}$$

因此，所欲求解的波動流場之第二階次量解可被完全地寫出，如下：

$$
\left.
\begin{aligned}
&\sigma_1^{(1)}=0\ ,\ \sigma_1^{(2)}=0\\
&\phi_{20}=\frac{3}{8}a_1^2\sigma_0^{(1)}\frac{\cosh 2k_1(d+z)}{\sinh^4 k_1 d}\sin 2S_1-\frac{1}{4}\frac{a_1^2\sigma_0^{(1)2}}{\sinh^2 k_1 d}t\ ,\\
&\eta_{20}=\frac{1}{4}a_1^2k_1\frac{\cosh k_1 d}{\sinh^3 k_1 d}(2\sinh^2 k_1 d+3)\cos 2S_1\\
&\phi_{02}=\frac{3}{8}a_2^2\sigma_0^{(2)}\frac{\cosh 2k_2(d+z)}{\sinh^4 k_2 d}\sin 2S_2-\frac{1}{4}\frac{a_2^2\sigma_0^{(2)2}}{\sinh^2 k_2 d}t\ ,\\
&\eta_{02}=\frac{1}{4}a_2^2k_2\frac{\cosh k_2 d}{\sinh^3 k_2 d}(2\sinh^2 k_2 d+3)\cos 2S_2\\
&\phi_{11}=\phi_{11}^{+}+\phi_{11}^{-}\ ,\\
&\phi_{11}^{+}=A_{11}^{+}\frac{\cosh|\vec{k}_1+\vec{k}_2|(d+z)}{\cosh|\vec{k}_1+\vec{k}_2|d}\sin(S_1+S_2)\ ,\\
&\phi_{11}^{-}=A_{11}^{-}\frac{\cosh|\vec{k}_1-\vec{k}_2|(d+z)}{\cosh|\vec{k}_1-\vec{k}_2|d}\sin(S_1-S_2)\\
&\eta_{11}=\eta_{11}^{+}+\eta_{11}^{-}\ ,\\
&\eta_{11}^{+}=\frac{1}{g}\{\frac{1}{2}a_1a_2(\sigma_0^{(1)2}+\sigma_0^{(1)}\sigma_0^{(2)}+\sigma_0^{(2)2}\\
&\qquad-\frac{g^2k_1k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}\cos\theta)+(\sigma_0^{(1)}+\sigma_0^{(2)})A_{11}^{+}\}\cos(S_1+S_2)\\
&\eta_{11}^{-}=\frac{1}{g}\{\frac{1}{2}a_1a_2(\sigma_0^{(1)2}-\sigma_0^{(1)}\sigma_0^{(2)}+\sigma_0^{(2)2}\\
&\qquad-\frac{g^2k_1k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}\cos\theta)+(\sigma_0^{(1)}-\sigma_0^{(2)})A_{11}^{-}\}\cos(S_1-S_2)
\end{aligned}
\right\}\quad(5.14)
$$

此處 A_{11}^{+} 與 A_{11}^{-} 為 (5.8) 式中所示者。

當 $d\rightarrow\infty$ 時，則 (5.14) 式退化成深海情況之解，且與往昔者 (Longuet - Higgins，1962) 及作者前文 (Chen，1988) 完全一致之。

5-3 第三階解

這是已被陳述在作者前文（Chen，1988）之對深海情況的解析中的，由於往昔對所考慮的波動流場之整體結構的描述尚有不足，即到第三階的求解就無法獲得全盤的融通性。因此，爲適足地解決往昔所尚遺留的此種瓶頸而妥當地給予全面性的打通，且更可完全地推廣展開至任一階次量的求解，於此將必要陳述依本文之解析對所考慮的波動系統，進行其整體流場結構之第三階解的解析，如下。

如同上小節所示之第二階次量的求解手續般，第三階的求解，亦需先列出其各控制條件式中之可先求算的部份。對自由表面邊界條件（4.18c）與（4.18d）式而言，其可先求算的第三階次量部份各以M、N表示，則有

$$
\begin{aligned}
M=&\left(\sigma_1^{(1)2}+2\sigma_0^{(1)}\sigma_2^{(1)}\right)\frac{\partial^2\phi_{10}}{\partial t_1^2}+\left(\sigma_1^{(2)2}+2\sigma_0^{(2)}\sigma_2^{(2)}\right)\frac{\partial^2\phi_{01}}{\partial t_2^2}\\
&+2\sigma_0^{(1)}\sigma_1^{(1)}\frac{\partial^2\phi_{20}}{\partial t_1^2}+2\sigma_0^{(2)}\sigma_1^{(2)}\frac{\partial^2\phi_{02}}{\partial t_2^2}\\
&+2\left(\sigma_0^{(1)}+\sigma_0^{(2)}\right)\left(\sigma_1^{(1)}+\sigma_1^{(2)}\right)\frac{\partial^2\phi_{11}^+}{\partial t_3^2}\\
&+2\left(\sigma_0^{(1)}-\sigma_0^{(2)}\right)\left(\sigma_1^{(1)}-\sigma_1^{(2)}\right)\frac{\partial^2\phi_{11}^-}{\partial t_4^2}\\
&+2\sigma_0^{(1)}\sigma_1^{(1)}\left(\eta_{10}+\eta_{01}\right)\frac{\partial^3\phi_{10}}{\partial z\partial t_1^2}+2\sigma_0^{(2)}\sigma_1^{(2)}\left(\eta_{10}+\eta_{01}\right)\frac{\partial^3\phi_{01}}{\partial z\partial t_2^2}\\
&+\left(\eta_{20}+\eta_{11}^++\eta_{11}^-+\eta_{02}\right)\frac{\partial}{\partial z}\left(\sigma_0^{(1)2}\frac{\partial^2\phi_{10}}{\partial t_1^2}+g\frac{\partial\phi_{10}}{\partial z}+\sigma_0^{(2)2}\frac{\partial^2\phi_{01}}{\partial t_2^2}\right.\\
&\left.+g\frac{\partial\phi_{01}}{\partial z}\right)+\left(\eta_{10}+\eta_{01}\right)\frac{\partial}{\partial z}\left[\sigma_0^{(1)2}\frac{\partial^2\phi_{20}}{\partial t_1^2}+g\frac{\partial\phi_{20}}{\partial z}\right.\\
&+\left(\sigma_0^{(1)}+\sigma_0^{(2)}\right)^2\frac{\partial^2\phi_{11}^+}{\partial t_3^2}+g\frac{\partial\phi_{11}^+}{\partial z}+\left(\sigma_0^{(1)}-\sigma_0^{(2)}\right)^2\frac{\partial^2\phi_{11}^-}{\partial t_4^2}+g\frac{\partial\phi_{11}^-}{\partial z}\\
&\left.+\sigma_0^{(2)2}\frac{\partial^2\phi_{02}}{\partial t_2^2}+g\frac{\partial\phi_{02}}{\partial z}\right]+\frac{1}{2}\left(\eta_{10}+\eta_{01}\right)^2\frac{\partial^2}{\partial z^2}\left(\sigma_0^{(1)2}\frac{\partial^2\phi_{10}}{\partial t_1^2}\right.\\
&\left.+g\frac{\partial\phi_{10}}{\partial z}+\sigma_0^{(2)2}\frac{\partial^2\phi_{01}}{\partial t_2^2}+g\frac{\partial\phi_{01}}{\partial z}\right)+\sigma_1^{(1)}\frac{\partial}{\partial t_1}\left(\vec{V}_{10}^2\right)+\sigma_1^{(2)}\frac{\partial}{\partial t_2}\left(\vec{V}_{01}^2\right)
\end{aligned}
$$

$$+2\sigma_1^{(1)}\left(\frac{\partial \vec{V}_{10}}{\partial t_1}\cdot\vec{V}_{01}\right)+2\sigma_1^{(2)}\left(\vec{V}_{10}\cdot\frac{\partial \vec{V}_{01}}{\partial t_2}\right)$$

$$+2\left(\sigma_0^{(1)}\frac{\partial \vec{V}_{10}}{\partial t_1}+\sigma_0^{(2)}\frac{\partial \vec{V}_{01}}{\partial t_2}\right)\cdot\left(\vec{V}_{20}+\vec{V}_{02}+\vec{V}_{11}^{+}+\vec{V}_{11}^{-}\right)$$

$$+2\left(\vec{V}_{10}+\vec{V}_{01}\right)\cdot\left[\sigma_0^{(1)}\frac{\partial \vec{V}_{20}}{\partial t_1}+\sigma_0^{(2)}\frac{\partial \vec{V}_{02}}{\partial t_2}+\left(\sigma_0^{(1)}+\sigma_0^{(2)}\right)\frac{\partial \vec{V}_{11}^{+}}{\partial t_3}\right.$$

$$\left.+\left(\sigma_0^{(1)}-\sigma_0^{(2)}\right)\frac{\partial \vec{V}_{11}^{-}}{\partial t_4}\right)\Big]+\left(\eta_{10}+\eta_{01}\right)\frac{\partial}{\partial z}\Big[\sigma_0^{(1)}\frac{\partial}{\partial t_1}\left(\vec{V}_{10}^{2}\right)$$

$$+\sigma_0^{(2)}\frac{\partial}{\partial t_2}\left(\vec{V}_{01}^{2}\right)+2\sigma_0^{(1)}\left(\frac{\partial \vec{V}_{10}}{\partial t_1}\cdot\vec{V}_{01}\right)+2\sigma_0^{(2)}\left(\vec{V}_{10}\cdot\frac{\partial \vec{V}_{01}}{\partial t_2}\right)\Big]$$

$$+\left(\vec{V}_{10}+\vec{V}_{01}\right)\cdot\nabla\left[\frac{1}{2}\left(\vec{V}_{10}^{2}+2\vec{V}_{10}\cdot\vec{V}_{01}+\vec{V}_{01}^{2}\right)\right],\quad z=0 \qquad (5.15)$$

與

$$N=\sigma_2^{(1)}\frac{\partial\phi_{10}}{\partial t_1}+\sigma_2^{(2)}\frac{\partial\phi_{01}}{\partial t_2}+\sigma_1^{(1)}\frac{\partial\phi_{20}}{\partial t_1}+\sigma_1^{(2)}\frac{\partial\phi_{02}}{\partial t_2}+\left(\sigma_1^{(1)}+\sigma_1^{(2)}\right)\frac{\partial\phi_{11}^{+}}{\partial t_3}$$

$$+\left(\sigma_1^{(1)}-\sigma_1^{(2)}\right)\frac{\partial\phi_{11}^{-}}{\partial t_4}+\left(\eta_{10}+\eta_{01}\right)\frac{\partial}{\partial z}\left(\sigma_1^{(1)}\frac{\partial\phi_{10}}{\partial t_1}+\sigma_1^{(2)}\frac{\partial\phi_{01}}{\partial t_2}\right)$$

$$+\left(\eta_{20}+\eta_{11}^{+}+\eta_{11}^{-}+\eta_{02}\right)\frac{\partial}{\partial z}\left(\sigma_0^{(1)}\frac{\partial\phi_{10}}{\partial t_1}+\sigma_0^{(2)}\frac{\partial\phi_{01}}{\partial t_2}\right)$$

$$+\left(\eta_{10}+\eta_{01}\right)\frac{\partial}{\partial z}\Big[\sigma_0^{(1)}\frac{\partial\phi_{20}}{\partial t_1}+\left(\sigma_0^{(1)}+\sigma_0^{(2)}\right)\frac{\partial\phi_{11}^{+}}{\partial t_3}$$

$$+\left(\sigma_0^{(1)}-\sigma_0^{(2)}\right)\frac{\partial\phi_{11}^{-}}{\partial t_4}+\sigma_0^{(2)}\frac{\partial\phi_{02}}{\partial t_2}\Big]+\frac{1}{2}\left(\eta_{10}+\eta_{01}\right)^2$$

$$\cdot\frac{\partial^2}{\partial z^2}\left(\sigma_0^{(1)}\frac{\partial\phi_{10}}{\partial t_1}+\sigma_0^{(2)}\frac{\partial\phi_{01}}{\partial t_2}\right)+\left(\vec{V}_{10}+\vec{V}_{01}\right)\cdot\left(\vec{V}_{20}+\vec{V}_{11}^{+}+\vec{V}_{11}^{-}\right.$$

$$\left.+\vec{V}_{02}\right)+\frac{1}{2}\left(\eta_{10}+\eta_{01}\right)\frac{\partial}{\partial z}\left(\vec{V}_{10}^{2}+2\vec{V}_{10}\cdot\vec{V}_{01}+\vec{V}_{01}^{2}\right),\quad z=0 \qquad (5.16)$$

(5.15)與(5.16)兩式之求算，可由已解出的第一與第二階次量之解(5.1)與(5.14)式之代入得之。爲清楚解析起見，於此將逐項計算結果列出，以資往後便於

對其檢核，如下。

$$\left.\begin{array}{l}(\sigma_1^{(1)2}+2\sigma_0^{(1)}\sigma_2^{(1)})(\frac{\partial^2\phi_{10}}{\partial t_1^2})_{z=0}=-2a_1g\sigma_2^{(1)}\sin S_1\\ (\sigma_1^{(2)2}+2\sigma_0^{(2)}\sigma_2^{(2)})(\frac{\partial^2\phi_{01}}{\partial t_2^2})_{z=0}=-2a_2g\sigma_2^{(2)}\sin S_2\end{array}\right\} \tag{5.17a}$$

$$2\sigma_0^{(1)}\sigma_1^{(1)}(\frac{\partial^2\phi_{20}}{\partial t_1^2})_{z=0}=2\sigma_0^{(2)}\sigma_1^{(2)}(\frac{\partial^2\phi_{02}}{\partial t_2^2})_{z=0}$$

$$=2(\sigma_0^{(1)}+\sigma_0^{(2)})(\sigma_1^{(1)}+\sigma_1^{(2)})(\frac{\partial^2\phi_{11}^+}{\partial t_3^2})_{z=0}$$

$$=2(\sigma_0^{(1)}-\sigma_0^{(2)})(\sigma_1^{(1)}-\sigma_1^{(2)})(\frac{\partial^2\phi_{11}^-}{\partial t_4^2})_{z=0}=0 \tag{5.17b}$$

$$2\sigma_0^{(1)}\sigma_1^{(1)}(\eta_{10}+\eta_{01})(\frac{\partial^3\phi_{10}}{\partial z\partial t_1^2})_{z=0}=2\sigma_0^{(2)}\sigma_1^{(2)}(\eta_{10}+\eta_{01})(\frac{\partial^3\phi_{01}}{\partial z\partial t_2^2})_{z=0}$$

$$=0 \tag{5.17c}$$

$$(\eta_{20}+\eta_{11}^++\eta_{11}^-+\eta_{02})[\frac{\partial}{\partial z}(\sigma_0^{(1)2}\frac{\partial^2\phi_{10}}{\partial t_1^2}+g\frac{\partial\phi_{10}}{\partial z}+\sigma_0^{(2)2}\frac{\partial^2\phi_{01}}{\partial t_2^2}$$

$$+g\frac{\partial\phi_{01}}{\partial z})]_{z=0}$$

$$=\frac{1}{2}\{\frac{a_2}{g}(\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3})[-a_1a_2\sigma_0^{(1)}\sigma_0^{(2)}+(\sigma_0^{(1)}-\sigma_0^{(2)})A_{11}^-$$

$$-(\sigma_0^{(1)}+\sigma_0^{(2)})A_{11}^+]-\frac{1}{4}a_1^3k_1(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3})$$

$$\times\frac{\cosh k_1d}{\sinh^3k_1d}(2\sinh^2k_1d+3)\}\sin S_1+\frac{1}{8}a_1^3k_1(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3})$$

$$\times\frac{\cosh k_1d}{\sinh^3k_1d}(2\sinh^2k_1d+3)\sin 3S_1+\frac{1}{2}\{\frac{1}{4}a_1^2a_2k_1(\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3})$$

$$\times\frac{\cosh k_1d}{\sinh^3k_1d}(2\sinh^2k_1d+3)+\frac{a_1}{g}(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3})$$

$$\times\Big[\frac{1}{2}a_1a_2\,(\sigma_0^{(1)2}+\sigma_0^{(1)}\sigma_0^{(2)}+\sigma_0^{(2)2}-\frac{g^2k_1k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}\cos\theta)$$

$$+(\sigma_0^{(1)}+\sigma_0^{(2)})A_{11}^{+}\Big]\Big\}\sin(2S_1+S_2)+\frac{1}{2}\Big\{\frac{1}{4}a_1^2a_2k_1\,(\sigma_0^{(2)3}-\frac{g^2k_2^2}{\sigma_0^{(2)}})$$

$$\times\frac{\cosh k_1d}{\sinh^3k_1d}(2\sinh^2k_1d+3)+\frac{a_1}{g}(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3})$$

$$\times\Big[\frac{1}{2}a_1a_2\,(\sigma_0^{(1)2}-\sigma_0^{(1)}\sigma_0^{(2)}+\sigma_0^{(2)2}-\frac{g^2k_1k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}\cos\theta)$$

$$+(\sigma_0^{(1)}-\sigma_0^{(2)})A_{11}^{-}\Big]\Big\}\sin(2S_1-S_2)+\frac{1}{2}\Big\{\frac{1}{4}a_1a_2^2k_2\,(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3})$$

$$\times\frac{\cosh k_2d}{\sinh^3k_2d}(2\sinh^2k_2d+3)+\frac{a_2}{g}(\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3})$$

$$\times\Big[\frac{1}{2}a_1a_2\,(\sigma_0^{(1)2}+\sigma_0^{(1)}\sigma_0^{(2)}+\sigma_0^{(2)2}-\frac{g^2k_1k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}\cos\theta)$$

$$+(\sigma_0^{(1)}+\sigma_0^{(2)})A_{11}^{+}\Big]\Big\}\sin(S_1+2S_2)+\frac{1}{2}\Big\{\frac{1}{4}a_1a_2^2k_2\,(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3})$$

$$\times\frac{\cosh k_2d}{\sinh^3k_2d}(2\sinh^2k_2d+3)-\frac{a_2}{g}(\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3})$$

$$\times\Big[\frac{1}{2}a_1a_2\,(\sigma_0^{(1)2}-\sigma_0^{(1)}\sigma_0^{(2)}+\sigma_0^{(2)2}-\frac{g^2k_1k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}\cos\theta)$$

$$+(\sigma_0^{(1)}-\sigma_0^{(2)})A_{11}^{-}\Big]\Big\}\sin(S_1-2S_2)+\frac{1}{8}a_2^3k_2\,(\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3})$$

$$\times\frac{\cosh k_2d}{\sinh^3k_2d}(2\sinh^2k_2d+3)\sin 3S_2+\frac{1}{2}\Big\{\frac{1}{4}a_2^3k_2\,(\sigma_0^{(2)3}-\frac{g^2k_2^2}{\sigma_0^{(2)}})$$

$$\times\frac{\cosh k_2d}{\sinh^3k_2d}(2\sinh^2k_2d+3)+\frac{a_1}{g}(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3})$$

$$\times\Big[-a_1a_2\sigma_0^{(1)}\sigma_0^{(2)}+(\sigma_0^{(1)}-\sigma_0^{(2)})A_{11}^{-}-(\sigma_0^{(1)}+\sigma_0^{(2)})A_{11}^{+}\Big]\Big\}\sin S_2 \tag{5.17d}$$

$$(\eta_{10}+\eta_{01})\{\frac{\partial}{\partial z}[\sigma_0^{(1)2}\frac{\partial^2\phi_{20}}{\partial t_1^2}+g\frac{\partial\phi_{20}}{\partial z}+(\sigma_0^{(1)}+\sigma_0^{(2)})^2\frac{\partial^2\phi_{11}^+}{\partial t_3^2}+g\frac{\partial\phi_{11}^+}{\partial z}$$

$$+(\sigma_0^{(1)}-\sigma_0^{(2)})^2\frac{\partial^2\phi_{11}^-}{\partial t_4^2}+g\frac{\partial\phi_{11}^-}{\partial z}+\sigma_0^{(2)2}\frac{\partial^2\phi_{02}}{\partial t_2^2}+g\frac{\partial\phi_{02}}{\partial z}]\}_{z=0}$$

$$=\frac{1}{2}\{\frac{3}{8}a_1^3\sigma_0^{(1)}\frac{\cosh 2k_1d}{\sinh^4k_1d}(-8\sigma_0^{(1)2}k_1\tanh 2k_1d+4gk_1^2)$$

$$+[-(\sigma_0^{(1)}+\sigma_0^{(2)})^2|\vec{k}_1+\vec{k}_2|\tanh|\vec{k}_1+\vec{k}_2|d+g|\vec{k}_1+\vec{k}_2|^2]a_2A_{11}^+$$

$$+[-(\sigma_0^{(1)}-\sigma_0^{(2)})^2|\vec{k}_1-\vec{k}_2|\tanh|\vec{k}_1-\vec{k}_2|d+g|\vec{k}_1-\vec{k}_2|^2]a_2A_{11}^-\}$$

$$\times\sin S_1+\frac{3}{16}a_1^3\sigma_0^{(1)}\frac{\cosh 2k_1d}{\sinh^4k_1d}(-8\sigma_0^{(1)2}k_1\tanh 2k_1d+4gk_1^2)\sin 3S_1$$

$$+\frac{1}{2}\{\frac{3}{8}a_1^2a_2\sigma_0^{(1)}\frac{\cosh 2k_1d}{\sinh^4k_1d}(-8\sigma_0^{(1)2}k_1\tanh 2k_1d+4gk_1^2)$$

$$+[-(\sigma_0^{(1)}+\sigma_0^{(2)})^2|\vec{k}_1+\vec{k}_2|\tanh|\vec{k}_1+\vec{k}_2|d+g|\vec{k}_1+\vec{k}_2|^2]a_1A_{11}^+\}$$

$$\times\sin(2S_1+S_2)+\frac{1}{2}\{\frac{3}{8}a_1^2a_2\sigma_0^{(1)}\frac{\cosh 2k_1d}{\sinh^4k_1d}(-8\sigma_0^{(1)2}k_1\tanh 2k_1d$$

$$+4gk_1^2)+[-(\sigma_0^{(1)}-\sigma_0^{(2)})^2|\vec{k}_1-\vec{k}_2|\tanh|\vec{k}_1-\vec{k}_2|d$$

$$+g|\vec{k}_1-\vec{k}_2|^2]a_1A_{11}^-\}\sin(2S_1-S_2)+\frac{1}{2}\{\frac{3}{8}a_1a_2^2\sigma_0^{(2)}\frac{\cosh 2k_2d}{\sinh^4k_2d}$$

$$\times(-8\sigma_0^{(2)2}k_2\tanh 2k_2d+4gk_2^2)+[-(\sigma_0^{(1)}+\sigma_0^{(2)})^2|\vec{k}_1+\vec{k}_2|$$

$$\times\tanh|\vec{k}_1+\vec{k}_2|d+g|\vec{k}_1+\vec{k}_2|^2]a_2A_{11}^+\}$$

$$\times\sin(S_1+2S_2)+\frac{1}{2}\{-\frac{3}{8}a_1a_2^2\sigma_0^{(2)}\frac{\cosh 2k_2d}{\sinh^4k_2d}$$

$$\times(-8\sigma_0^{(2)2}k_2\tanh 2k_2d+4gk_2^2)+[-(\sigma_0^{(1)}-\sigma_0^{(2)})^2|\vec{k}_1-\vec{k}_2|$$

$$\times \tanh|\vec{k}_1-\vec{k}_2|d+g|\vec{k}_1-\vec{k}_2|^2\rbrack a_2A_{11}^-\}\sin(S_1-2S_2)$$

$$+\frac{3}{16}a_2{}^3\sigma_0{}^{(2)}\frac{\cosh 2k_2d}{\sinh^4k_2d}(-8\sigma_0{}^{(2)2}k_2\tanh 2k_2d+4gk_2^2)\sin 3S_2$$

$$+\frac{1}{2}\{\frac{3}{8}a_2{}^3\sigma_0{}^{(2)}\frac{\cosh 2k_2d}{\sinh^4k_2d}(-8\sigma_0{}^{(2)2}k_2\tanh 2k_2d+4gk_2^2)$$

$$+\lbrack-(\sigma_0{}^{(1)}+\sigma_0{}^{(2)})^2|\vec{k}_1+\vec{k}_2|\tanh|\vec{k}_1+\vec{k}_2|d+g|\vec{k}_1+\vec{k}_2|^2\rbrack a_1A_{11}^+$$

$$-\lbrack-(\sigma_0{}^{(1)}-\sigma_0{}^{(2)})^2|\vec{k}_1-\vec{k}_2|\tanh|\vec{k}_1-\vec{k}_2|d+g|\vec{k}_1-\vec{k}_2|^2\rbrack a_1A_{11}^-\}$$

$$\times\sin S_2 \tag{5.17e}$$

$$\frac{1}{2}(\eta_{10}+\eta_{01})^2\lbrack\frac{\partial^2}{\partial z^2}(\sigma_0{}^{(1)2}\frac{\partial^2\phi_{10}}{\partial t_1{}^2}+g\frac{\partial\phi_{10}}{\partial z}+\sigma_0{}^{(2)2}\frac{\partial^2\phi_{01}}{\partial t_2{}^2}+g\frac{\partial\phi_{01}}{\partial z})\rbrack_{z=0}$$

$$=0 \tag{5.17f}$$

$$\sigma_1{}^{(1)}\lbrack\frac{\partial}{\partial t_1}(\vec{V}_{10}^2)\rbrack_{z=0}=\sigma_1{}^{(2)}\lbrack\frac{\partial}{\partial t_2}(\vec{V}_{01}^2)\rbrack_{z=0}=2\sigma_1{}^{(1)}(\frac{\partial\vec{V}_{10}}{\partial t_1}\cdot\vec{V}_{01})_{z=0}$$

$$=2\sigma_1{}^{(2)}(\vec{V}_{10}\cdot\frac{\partial\vec{V}_{01}}{\partial t_2})_{z=0}=0 \tag{5.17g}$$

$$2\{(\sigma_0{}^{(1)}\frac{\partial\vec{V}_{10}}{\partial t_1}+\sigma_0{}^{(2)}\frac{\partial\vec{V}_{01}}{\partial t_2})\cdot(\vec{V}_{20}+\vec{V}_{11}^++\vec{V}_{11}^-+\vec{V}_{02})+(\vec{V}_{10}+\vec{V}_{01})$$

$$\cdot\lbrack\sigma_0{}^{(1)}\frac{\partial\vec{V}_{20}}{\partial t_1}+(\sigma_0{}^{(1)}+\sigma_0{}^{(2)})\frac{\partial\vec{V}_{11}^+}{\partial t_3}+(\sigma_0{}^{(1)}-\sigma_0{}^{(2)})\frac{\partial\vec{V}_{11}^-}{\partial t_4}$$

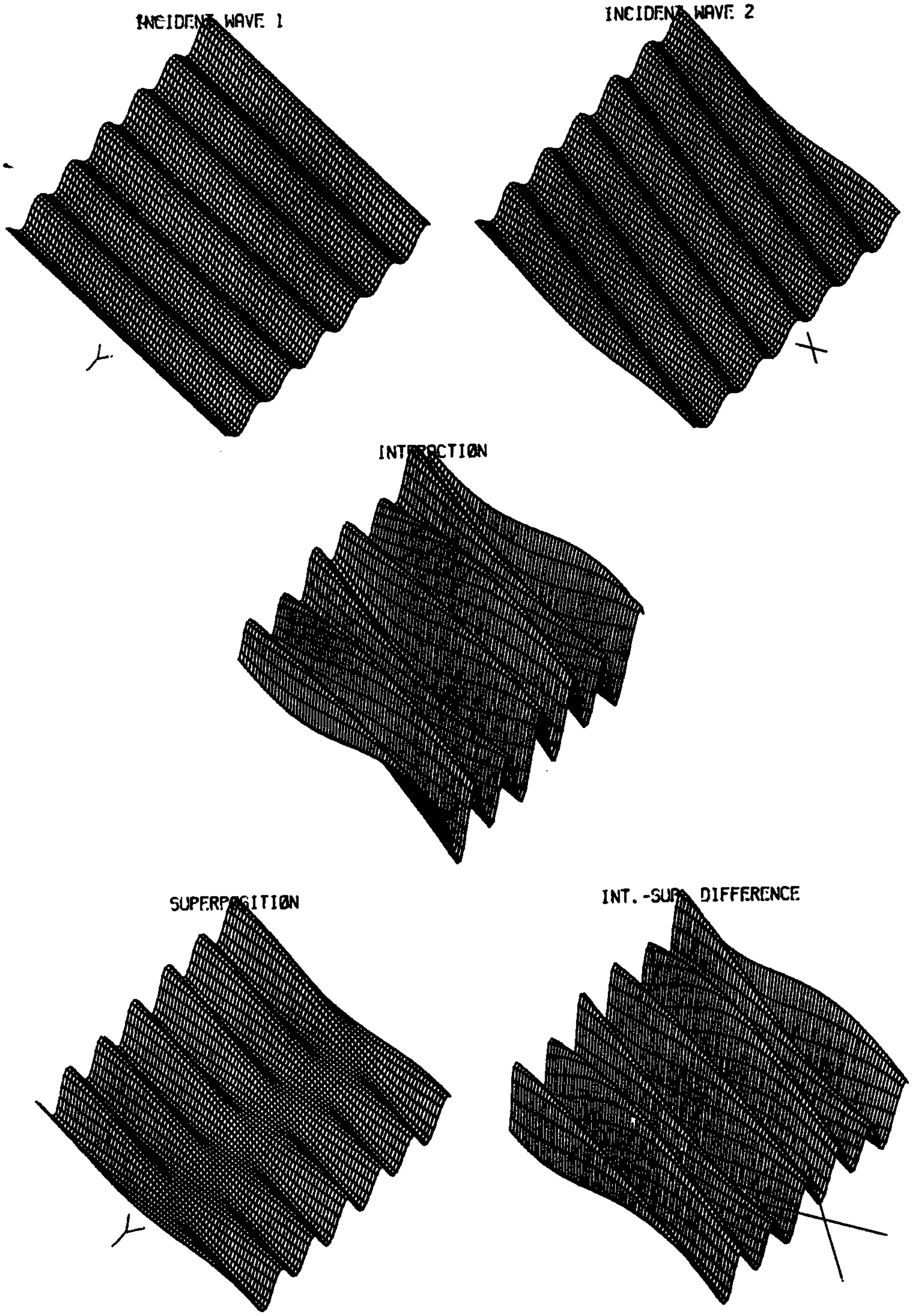
$$+\sigma_0{}^{(2)}\frac{\partial\vec{V}_{02}}{\partial t_2}\rbrack\}_{z=0}$$

$$=\sigma_0{}^{(1)}\lbrack\frac{3}{4}a_1{}^3\frac{\cosh 2k_1d}{\sinh^4k_1d}(gk_1^2+\sigma_0{}^{(1)2}k_1\tanh 2k_1d)$$

$$+\frac{a_2g}{\sigma_0{}^{(2)}}A_{11}^+(k_1k_2\cos\theta+k_2^2)+a_2\sigma_0{}^{(2)}A_{11}^+|\vec{k}_1+\vec{k}_2|\tanh|\vec{k}_1+\vec{k}_2|d$$

$$+\frac{a_2g}{\sigma_0{}^{(2)}}A_{11}^-(k_1k_2\cos\theta-k_2^2)-a_2\sigma_0{}^{(2)}A_{11}^-|\vec{k}_1-\vec{k}_2|\tanh|\vec{k}_1-\vec{k}_2|d\rbrack$$

$$\times\sin S_1+\frac{9}{4}a_1{}^3\sigma_0{}^{(1)}\frac{\tanh 2k_1d}{\sinh^4k_1d}(gk_1^2-\sigma_0{}^{(1)2}k_1\tanh 2k_1d)\sin 3S_1$$



第一列來源波列(incident waves 1)
k_1d = 0.196 (相對水深)
k_1a_1= 0.076 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.196 (相對水深)
k_2a_2= 0.076 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 10°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-7a
Fig. 5-7a

$$\times \sin S_2 \tag{5.17h}$$

$$(\eta_{10}+\eta_{01})\{\frac{\partial}{\partial z}[\sigma_0^{(1)}\frac{\partial}{\partial t_1}(\vec{V}_{10}^2)+\sigma_0^{(2)}\frac{\partial}{\partial t_2}(\vec{V}_{01}^2)$$

$$+2\sigma_0^{(1)}(\frac{\partial \vec{V}_{10}}{\partial t_1}\cdot\vec{V}_{01})+2\sigma_0^{(2)}(\vec{V}_{10}\cdot\frac{\partial \vec{V}_{01}}{\partial t_2})]\}_{z=0}$$

$$=a_1a_2^2[\frac{gk_1k_2}{\sigma_0^{(2)}}(\sigma_0^{(1)2}+\sigma_0^{(2)2})\cos\theta-\frac{g}{\sigma_0^{(1)}}(k_1^2\sigma_0^{(2)2}+k_2^2\sigma_0^{(1)2})]\sin S_1$$

$$+\frac{1}{2}a_1^2a_2(\frac{1}{\sigma_0^{(2)}}+\frac{1}{\sigma_0^{(1)}})[gk_1k_2(\sigma_0^{(1)2}+\sigma_0^{(2)2})\cos\theta$$

$$-g(k_1^2\sigma_0^{(2)2}+k_2^2\sigma_0^{(1)2})]\sin(2S_1+S_2)+\frac{1}{2}a_1^2a_2(\frac{1}{\sigma_0^{(2)}}-\frac{1}{\sigma_0^{(1)}})$$

$$\times[gk_1k_2(\sigma_0^{(1)2}+\sigma_0^{(2)2})\cos\theta+g(k_1^2\sigma_0^{(2)2}+k_2^2\sigma_0^{(1)2})]\sin(2S_1-S_2)$$

$$+\frac{1}{2}a_1a_2^2(\frac{1}{\sigma_0^{(2)}}+\frac{1}{\sigma_0^{(1)}})[gk_1k_2(\sigma_0^{(1)2}+\sigma_0^{(2)2})\cos\theta$$

$$-g(k_1^2\sigma_0^{(2)2}+k_2^2\sigma_0^{(1)2})]\sin(S_1+2S_2)+\frac{1}{2}a_1a_2^2(\frac{1}{\sigma_0^{(2)}}-\frac{1}{\sigma_0^{(1)}})$$

$$\times[gk_1k_2(\sigma_0^{(1)2}+\sigma_0^{(2)2})\cos\theta+g(k_1^2\sigma_0^{(2)2}+k_2^2\sigma_0^{(1)2})]\sin(S_1-2S_2)$$

$$+a_1^2a^2[\frac{gk_1k_2}{\sigma_0^{(1)}}(\sigma_0^{(1)2}+\sigma_0^{(2)2})\cos\theta-\frac{g}{\sigma_0^{(2)}}(k_1^2\sigma_0^{(2)2}+k_2^2\sigma_0^{(1)2})]\sin S_2 \tag{5.17i}$$

$$\{(\vec{V}_{10}+\vec{V}_{01})\cdot\nabla[\frac{1}{2}(\vec{V}_{10}^2+2\vec{V}_{10}\cdot\vec{V}_{01}+\vec{V}_{01}^2)]\}_{z=0}$$

$$=\frac{a_1g}{2}\{-\frac{1}{2}\frac{a_1^2g^2k_1^4}{\sigma_0^{(1)3}\cosh^2k_1d}-\frac{a_2^2g^2k_1^2k_2^2}{\sigma_0^{(1)}\sigma_0^{(2)2}}\cos^2\theta+4a_2^2k_2^2\sigma_0^{(1)}$$

$$+2a_1^2k_1^2\sigma_0^{(1)}+\frac{a_2^2}{\sigma_0^{(1)}}k_1^2\sigma_0^{(2)2}\}\sin S_1-\frac{1}{4}\frac{a_1^3g^3k_1^4}{\sigma_0^{(1)3}\cosh^2k_1d}\sin 3S_1$$

$$+\frac{a_1^2a_2g}{4}\{-[\frac{g^2k_1^2k_2}{\sigma_0^{(1)2}\sigma_0^{(2)}}\cos\theta-k_1\sigma_0^{(2)}](k_1+k_2\cos\theta)$$

$$-\frac{g^2k_1^3k_2\cos\theta}{\sigma_0^{(1)2}\sigma_0^{(2)}\cosh^2k_1d}+\frac{1}{\sigma_0^{(2)}}[k_1k_2\cos\theta\cdot(\sigma_0^{(1)2}+\sigma_0^{(2)2})$$

$$-(k_1^2\ \sigma_0^{(2)2}+k_2^2\ \sigma_0^{(1)2})]\}\sin(2S_1+S_2)+\frac{a_1{}^2a_2g}{4}\{-[\frac{g^2k_1^2\ k_2}{\sigma_0{}^{(1)2}\sigma_0{}^{(2)}}\cos\theta$$

$$+k_1\sigma_0{}^{(2)}](k_1-k_2\cos\theta)-\frac{g^2k_1^3\ k_2\cos\theta}{\sigma_0{}^{(1)2}\sigma_0{}^{(2)}\cosh^2k_1d}$$

$$+\frac{1}{\sigma_0{}^{(2)}}[\mathbf{k_1k_2\cos\theta}\cdot(\sigma_0{}^{(1)2}+\sigma_0{}^{(2)2})+(k_1^2\ \sigma_0{}^{(2)2}+k_2^2\ \sigma_0{}^{(1)2})]\}$$

$$\times\sin(2S_1-S_2)+\frac{a_1a_2{}^2g}{4}\{-[\frac{g^2k_1k_2^2}{\sigma_0{}^{(1)}\sigma_0{}^{(2)2}}\cos\theta-k_2\sigma_0{}^{(1)}](k_1\cos\theta+k_2)$$

$$-\frac{g^2k_1k_2^3\cos\theta}{\sigma_0{}^{(1)}\sigma_0{}^{(2)2}\cosh^2k_2d}+\frac{1}{\sigma_0{}^{(1)}}[k_1k_2\cos\theta\cdot(\sigma_0{}^{(1)2}+\sigma_0{}^{(2)2})-(k_1^2\ \sigma_0{}^{(2)2}+k_2{}^2\sigma_0{}^{(1)2})]\}$$

$$\times\sin(S_1+2S_2)+\frac{a_1a_2{}^2g}{4}\{-[\frac{g^2k_1k_2^2}{\sigma_0{}^{(1)}\sigma_0{}^{(2)2}}\cos\theta+k_2\sigma_0{}^{(1)}]$$

$$\times(k_1\cos\theta-k_2)+\frac{g^2k_1k_2^3\cos\theta}{\sigma_0{}^{(1)}\sigma_0{}^{(2)2}\cosh^2k_2d}$$

$$-\frac{1}{\sigma_0{}^{(1)}}[\mathbf{k_1k_2\cos\theta}\cdot(\sigma_0{}^{(1)2}+\sigma_0{}^{(2)2})+(k_1^2\ \sigma_0{}^{(2)2}+k_2^2\ \sigma_0{}^{(1)2})]\}$$

$$\times\sin(S_1-2S_2)-\frac{1}{4}\frac{a_2{}^3g^3k_2^4}{\sigma_0{}^{(2)3}\cosh^2k_2d}\sin3S_2$$

$$+\frac{a_2g}{2}\{-\frac{1}{2}\frac{a_2{}^2g^2k_2^4}{\sigma_0{}^{(2)3}\cosh^2k_2d}-\frac{a_1{}^2g^2k_1^2\ k_2^2}{\sigma_0{}^{(1)2}\sigma_0{}^{(2)}}\cos^2\theta+4a_1{}^2k_1^2\ \sigma_0{}^{(2)}$$

$$+2a_2{}^2k_2^2\ \sigma_0{}^{(2)}+\frac{a_1{}^2}{\sigma_0{}^{(2)}}k_2^2\ \sigma_0{}^{(1)2}\}\sin S_2 \qquad (5.17j)$$

與

$$\left.\begin{aligned}\sigma_2{}^{(1)}(\frac{\partial\phi_{10}}{\partial t_1})_{z=0}&=-\sigma_2{}^{(1)}\frac{a_1g}{\sigma_0{}^{(1)}}\cos S_1\\ \sigma_2{}^{(2)}(\frac{\partial\phi_{01}}{\partial t_2})_{z=0}&=-\sigma_2{}^{(2)}\frac{a_2g}{\sigma_0{}^{(2)}}\cos S_2\end{aligned}\right\} \qquad (5.18a)$$

$$\sigma_1{}^{(1)}(\frac{\partial\phi_{20}}{\partial t_1})_{z=0}=\sigma_1{}^{(2)}(\frac{\partial\phi_{02}}{\partial t_2})_{z=0}=(\sigma_1{}^{(1)}+\sigma_1{}^{(2)})(\frac{\partial\phi_{11}{}^{+}}{\partial t_3})_{z=0}$$

$$=(\sigma_1{}^{(1)}-\sigma_1{}^{(2)})(\frac{\partial\phi_{11}{}^{-}}{\partial t_4})_{z=0}=(\eta_{10}+\eta_{01})[\frac{\partial}{\partial z}(\sigma_1{}^{(1)}\frac{\partial\phi_{10}}{\partial t_1}$$

$$+\sigma_1^{(2)}\frac{\partial\phi_{01}}{\partial t_2})]_{z=0}=0 \tag{5.18b}$$

$$(\eta_{20}+\eta_{11}^{+}+\eta_{11}^{-}+\eta_{02})[\frac{\partial}{\partial z}(\sigma_0^{(1)}\frac{\partial\phi_{10}}{\partial t_1}+\sigma_0^{(2)}\frac{\partial\phi_{01}}{\partial t_2})]_{z=0}$$

$$=-\{\frac{1}{8}gk_1^2a_1^3\ (3\tanh^{-2}k_1d-1)$$

$$+\frac{1}{2}(\sigma_0^{(1)2}+\sigma_0^{(2)2}-\frac{g^2k_1k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}\cos\theta)k_2a_1a_2^2\tanh k_2d$$

$$+\frac{1}{2}[(\sigma_0^{(1)}+\sigma_0^{(2)})A_{11}^{+}+(\sigma_0^{(1)}-\sigma_0^{(2)})A_{11}^{-}]k_2a_2\tanh k_2d\}\ \cos S_1$$

$$-[\frac{1}{8}gk_1k_2a_1^2a_2\ (3\tanh^{-3}k_1d-\tanh^{-1}k_1d)\tanh k_2d$$

$$+\frac{1}{4}k_1a_1^2a_2\tanh k_1d\cdot(\sigma_0^{(1)2}+\sigma_0^{(1)}\sigma_0^{(2)}+\sigma_0^{(2)2}-\frac{g^2k_1k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}\cos\theta)$$

$$+\frac{1}{2}(\sigma_0^{(1)}+\sigma_0^{(2)})k_1a_1A_{11}^{+}\tanh k_1d]\cos\ (2S_1+S_2)$$

$$-[\frac{1}{8}gk_1k_2a_1^2a_2\ (3\tanh^{-3}k_1d-\tanh^{-1}k_1d)\tanh k_2d$$

$$+\frac{1}{4}k_1a_1^2a_2\tanh k_1d\ (\sigma_0^{(1)2}-\sigma_0^{(1)}\sigma_0^{(2)}+\sigma_0^{(2)2}-\frac{g^2k_1k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}\cos\theta)$$

$$+\frac{1}{2}(\sigma_0^{(1)}-\sigma_0^{(2)})k_1a_1A_{11}^{-}\tanh k_1d]\cos\ (2S_1-S_2)$$

$$-[\frac{1}{8}gk_1k_2a_1a_2^2\tanh k_1d\cdot(3\tanh^{-3}k_2d-\tanh^{-1}k_2d)$$

$$+\frac{1}{4}k_2a_1a_2^2\tanh k_2d\cdot(\sigma_0^{(1)2}+\sigma_0^{(1)}\sigma_0^{(2)}+\sigma_0^{(2)2}-\frac{g^2k_1k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}\cos\theta)$$

$$+\frac{1}{2}(\sigma_0^{(1)}+\sigma_0^{(2)})k_2a_2A_{11}^{+}\tanh k_2d]\cos\ (S_1+2S_2)$$

$$-[\frac{1}{8}gk_1k_2a_1a_2^2\tanh k_1d\cdot(3\tanh^{-3}k_2d-\tanh^{-1}k_2d)$$

$$+\frac{1}{4}k_2 a_1 a_2^2 \tanh k_2 d \cdot \left(\sigma_0^{(1)2} - \sigma_0^{(1)}\sigma_0^{(2)} + \sigma_0^{(2)2} - \frac{g^2 k_1 k_2}{\sigma_0^{(1)}\sigma_0^{(2)}} \cos\theta \right)$$

$$+\frac{1}{2}\left(\sigma_0^{(1)} - \sigma_0^{(2)} \right) k_2 a_2 A_{11}^- \tanh k_2 d \Big] \cos\left(S_1 - 2S_2 \right)$$

$$-\Big\{ \frac{1}{8} g\, k_2^2 a_2^3 \; \left(3 \tanh^{-2} k_2 d - 1 \right)$$

$$+\frac{1}{2}\left(\sigma_0^{(1)2} + \sigma_0^{(2)2} - \frac{g^2 k_1 k_2}{\sigma_0^{(1)}\sigma_0^{(2)}} \cos\theta \right) k_1 a_1^2 a_2 \tanh k_1 d$$

$$+\frac{1}{2}\Big[\left(\sigma_0^{(1)} + \sigma_0^{(2)} \right) A_{11}^+ + \left(\sigma_0^{(1)} - \sigma_0^{(2)} \right) A_{11}^- \Big] k_1 a_1 \tanh k_1 d \Big\} \cos S_2$$

$$-\frac{1}{8} g\, k_1^2 a_1^3 \; \left(3 \tanh^{-2} k_1 d - 1 \right) \cos 3S_1$$

$$-\frac{1}{8} g k_2^2\, a_2^3 \; \left(3 \tanh^{-2} k_2 d - 1 \right) \cos 3S_2 \qquad (5.18c)$$

$$\left(\eta_{10} + \eta_{01} \right) \Big\{ \frac{\partial}{\partial z} \Big[\sigma_0^{(1)} \frac{\partial \phi_{20}}{\partial t_1} + \left(\sigma_0^{(1)} + \sigma_0^{(2)} \right) \frac{\partial \phi_{11}^+}{\partial t_3}$$

$$+ \left(\sigma_0^{(1)} - \sigma_0^{(2)} \right) \frac{\partial \phi_{11}^-}{\partial t_4} + \sigma_0^{(2)} \frac{\partial \phi_{02}}{\partial t_2} \Big] \Big\}_{z=0}$$

$$= -\Big[\frac{3}{2} g\, k_1^2 a_1^3 \; \left(\tanh^{-2} k_1 d - 1 \right)$$

$$+\frac{1}{2}\left(\sigma_0^{(1)} + \sigma_0^{(2)} \right) \left| \vec{k}_1 + \vec{k}_2 \right| a_2 A_{11}^+ \tanh \left| \vec{k}_1 + \vec{k}_2 \right| d$$

$$+\frac{1}{2}\left(\sigma_0^{(1)} - \sigma_0^{(2)} \right) \left| \vec{k}_1 - \vec{k}_2 \right| a_2 A_{11}^- \tanh \left| \vec{k}_1 - \vec{k}_2 \right| d \Big] \cos S_1$$

$$-\frac{3}{2} g\, k_1^2 a_1^3 \; \left(\tanh^{-2} k_1 d - 1 \right) \cos 3S_1$$

$$-\Big[\frac{3}{2} g\, k_1^2 a_1^2 a_2 \left(\tanh^{-2} k_1 d - 1 \right)$$

$$+\frac{1}{2}\left(\sigma_0^{(1)} + \sigma_0^{(2)} \right) \left| \vec{k}_1 + \vec{k}_2 \right| a_1 A_{11}^+ \tanh \left| \vec{k}_1 + \vec{k}_2 \right| d \Big] \cos\left(2S_1 + S_2 \right)$$

$$-\Big[\frac{3}{2}gk_1^2a_1^2a_2\,(\tanh^{-2}k_1d-1)$$

$$+\frac{1}{2}(\sigma_0^{(1)}-\sigma_0^{(2)})\,|\vec{k_1}-\vec{k_2}|\,a_1A_{11}^-\tanh|\vec{k_1}-\vec{k_2}|\,d\Big]\cos(2S_1-S_2)$$

$$-\Big[\frac{3}{2}gk_2^2a_1a_2^2\,(\tanh^{-2}k_2d-1)$$

$$+\frac{1}{2}(\sigma_0^{(1)}+\sigma_0^{(2)})\,|\vec{k_1}+\vec{k_2}|\,a_2A_{11}^+\tanh|\vec{k_1}+\vec{k_2}|\,d\Big]\cos(S_1+2S_2)$$

$$-\Big[\frac{3}{2}gk_2^2a_1a_2^2\,(\tanh^{-2}k_2d-1)$$

$$+\frac{1}{2}(\sigma_0^{(1)}-\sigma_0^{(2)})\,|\vec{k_1}-\vec{k_2}|\,a_2A_{11}^-\tanh|\vec{k_1}-\vec{k_2}|\,d\Big]\cos(S_1-2S_2)$$

$$-\frac{3}{2}gk_2^2a_2^3\,(\tanh^{-2}k_2d-1)\cos 3S_2-\Big[\frac{3}{2}gk_2^2a_2^3\,(\tanh^{-2}k_2d-1)$$

$$+\frac{1}{2}(\sigma_0^{(1)}+\sigma_0^{(2)})\,|\vec{k_1}+\vec{k_2}|\,a_1A_{11}^+\tanh|\vec{k_1}+\vec{k_2}|\,d$$

$$+\frac{1}{2}(\sigma_0^{(1)}-\sigma_0^{(2)})\,|\vec{k_1}-\vec{k_2}|\,a_1A_{11}^-\tanh|\vec{k_1}-\vec{k_2}|\,d\Big]\cos S_2 \quad (5.18d)$$

$$\frac{1}{2}(\eta_{10}+\eta_{01})^2\Big[\frac{\partial^2}{\partial z^2}\Big(\sigma_0^{(1)}\frac{\partial\phi_{10}}{\partial t_1}+\sigma_0^{(2)}\frac{\partial\phi_{01}}{\partial t_2}\Big)\Big]_{z=0}$$

$$=-\Big[\frac{3}{8}gk_1^2a_1^3+\frac{1}{4}gk_1^2a_1a_2^2+\frac{1}{2}gk_2^2a_1a_2^2\Big]\cos S_1-\frac{1}{8}gk_1^2a_1^3\cos 3S_1$$

$$-\Big[\frac{1}{8}gk_2^2a_1^2a_2+\frac{1}{4}gk_1^2a_1^2a_2\Big]\cos(2S_1+S_2)$$

$$-\Big[\frac{1}{8}gk_2^2a_1^2a_2+\frac{1}{4}gk_1^2a_1^2a_2\Big]\cos(2S_1-S_2)$$

$$-\Big[\frac{1}{8}gk_1^2a_1a_2^2+\frac{1}{4}gk_2^2a_1a_2^2\Big]\cos(S_1-2S_2)$$

$$-\Big[\frac{1}{8}gk_1^2a_1a_2^2+\frac{1}{4}gk_2^2a_1a_2^2\Big]\cos(S_1+2S_2)$$

$$-\frac{1}{8}g k_2^2 a_2^3 \cos 3S_2 - \left[\frac{3}{8}g k_2^2 a_2^3 + \frac{1}{4}g k_2^2 a_1^2 a_2 + \frac{1}{2}g k_1^2 a_1^2 a_2\right]\cos S_2 \tag{5.18e}$$

$$\{(\vec{V}_{10}+\vec{V}_{01})\cdot(\vec{V}_{20}+\vec{V}_{11}^{+}+\vec{V}_{11}^{-}+\vec{V}_{02})\}_{z=0}$$

$$=\left[\frac{3}{8}g k_1^2 a_1^3(\tanh^{-4}k_1d+2\tanh^{-2}k_1d-3)\right.$$

$$+\frac{1}{2}\frac{a_2 g}{\sigma_0^{(2)}}A_{11}^{+}(k_1k_2\cos\theta+k_2^2)+\frac{1}{2}a_2\sigma_0^{(2)}A_{11}^{+}|\vec{k}_1+\vec{k}_2|\tanh|\vec{k}_1+\vec{k}_2|d$$

$$+\frac{1}{2}\frac{a_2 g}{\sigma_0^{(2)}}A_{11}^{-}(k_1k_2\cos\theta-k_2^2)$$

$$\left.-\frac{1}{2}a_2\sigma_0^{(2)}A_{11}^{-}|\vec{k}_1-\vec{k}_2|\tanh|\vec{k}_1-\vec{k}_2|d\right]\cos S_1$$

$$+\frac{3}{8}g k_1^2 a_1^3(\tanh^{-4}k_1d-2\tanh^{-2}k_1d+1)\cos 3S_1$$

$$+\left[\frac{3}{8}gk_1k_2a_1^2a_2\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}\cos\theta\cdot(\tanh^{-4}k_1d-1)\right.$$

$$-\frac{3}{4}g k_1^2 a_1^2 a_2\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}(\tanh^{-2}k_1d-1)+\frac{1}{2}\frac{a_1 g}{\sigma_0^{(1)}}A_{11}^{+}(k_1^2+k_1k_2\cos\theta)$$

$$\left.-\frac{1}{2}a_1\sigma_0^{(1)}A_{11}^{+}|\vec{k}_1+\vec{k}_2|\tanh|\vec{k}_1+\vec{k}_2|d\right]\cos(2S_1+S_2)$$

$$+\left[\frac{3}{8}gk_1k_2a_1^2a_2\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}\cos\theta\cdot(\tanh^{-4}k_1d-1)\right.$$

$$+\frac{3}{4}gk_1^2 a_1^2 a_2\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}(\tanh^{-2}k_1d-1)+\frac{1}{2}\frac{a_1 g}{\sigma_0^{(1)}}A_{11}^{-}(k_1^2-k_1k_2\cos\theta)$$

$$\left.-\frac{1}{2}a_1\sigma_0^{(1)}A_{11}^{-}|\vec{k}_1-\vec{k}_2|\tanh|\vec{k}_1-\vec{k}_2|d\right]\cos(2S_1-S_2)$$

$$+\left[\frac{3}{8}gk_1k_2a_1a_2^2\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}\cos\theta\cdot(\tanh^{-4}k_2d-1)\right.$$

$$-\frac{3}{4}g k_2^2 a_1 a_2^2\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}(\tanh^{-2}k_2d-1)+\frac{1}{2}\frac{a_2 g}{\sigma_0^{(2)}}A_{11}^{+}(k_1k_2\cos\theta+k_2^2)$$

$$-\frac{1}{2}a_2\sigma_0^{(2)}A_{11}^{+}\,|\vec{k}_1+\vec{k}_2|\tanh|\vec{k}_1+\vec{k}_2|d\,]\cos(S_1+2S_2)$$

$$+[\frac{3}{8}gk_1k_2a_1a_2^2\,\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}\cos\theta\cdot(\tanh^{-4}k_2d-1)$$

$$+\frac{3}{4}g\,k_2^2a_1a_2^2\,\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}(\tanh^{-2}k_2d-1)+\frac{1}{2}\frac{a_2g}{\sigma_0^{(2)}}A_{11}^{-}(k_1k_2\cos\theta-k_2^2)$$

$$+\frac{1}{2}a_2\sigma_0^{(2)}A_{11}^{-}\,|\vec{k}_1-\vec{k}_2|\tanh|\vec{k}_1-\vec{k}_2|d\,]\cos(S_1-2S_2)$$

$$+\frac{3}{8}g\,k_2^2a_2^3\,(\tanh^{-4}k_2d-2\tanh^{-2}k_2d+1)\cos 3S_2$$

$$+[\frac{3}{8}g\,k_2^2a_2^3\,(\tanh^{-4}k_2d+2\tanh^{-2}k_2d-3)$$

$$+\frac{1}{2}\frac{a_1g}{\sigma_0^{(1)}}A_{11}^{+}\,(k_1^2+k_1k_2\cos\theta)$$

$$+\frac{1}{2}a_1\sigma_0^{(1)}A_{11}^{+}\,|\vec{k}_1+\vec{k}_2|\tanh|\vec{k}_1+\vec{k}_2|d$$

$$+\frac{1}{2}\frac{a_1g}{\sigma_0^{(1)}}A_{11}^{-}\,(k_1^2-k_1k_2\cos\theta)$$

$$+\frac{1}{2}a_1\sigma_0^{(1)}A_{11}^{-}\,|\vec{k}_1-\vec{k}_2|\tanh|\vec{k}_1-\vec{k}_2|d\,]\cos S_2 \qquad (5.18f)$$

$$\frac{1}{2}(\eta_{10}+\eta_{01})[\frac{\partial}{\partial z}(\vec{V}_{10}^2+2\vec{V}_{10}\cdot\vec{V}_{01}+\vec{V}_{01}^2)]_{z=0}$$

$$=ga_1[k_1^2a_1^2+k_2^2a_2^2+\frac{1}{2}k_1k_2a_2^2\,(\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}+\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}})\cos\theta\,]\cos S_1$$

$$+\frac{1}{4}ga_1^2a_2[k_1k_2(\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}+\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}})\cos\theta$$

$$-(k_1^2\,\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}+k_2^2\,\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}})]\cos(2S_1+S_2)$$

$$+\frac{1}{4}ga_1^2a_2[k_1k_2(\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}+\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}})\cos\theta$$

$$+\left(k_1^2\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}+k_2^2\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}\right)\Big]\cos(2S_1-S_2)+\frac{1}{4}ga_1a_2^2\Big[k_1k_2\left(\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}\right.$$

$$\left.+\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}\right)\cos\theta-\left(k_1^2\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}+k_2^2\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}\right)\Big]\cos(S_1+2S_2)$$

$$+\frac{1}{4}ga_1a_2^2\Big[k_1k_2\left(\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}+\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}\right)\cos\theta$$

$$+\left(k_1^2\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}+k_2^2\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}\right)\Big]\cos(S_1-2S_2)$$

$$+ga_2\Big[k_1^2a_1^2+k_2^2a_2^2+\frac{1}{2}k_1k_2a_1^2\left(\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}+\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}\right)\cos\theta\Big]\cos S_2$$

(5.18g)

於求解ϕ_{30}、ϕ_{21}^+、ϕ_{21}^-、ϕ_{12}^+、ϕ_{12}^-與ϕ_{03}時，便於其內各相位變化之比較起見，則M由(5.8)式之A_{11}^+與A_{11}^-代入下，經(5.15)與(5.17a-j)式之運算整理，可被寫成爲

$$M=\mu_{30}^{(1)}\sin S_1+\mu_{30}^{(3)}\sin 3S_1+\mu_{21}^+\sin(2S_1+S_2)+\mu_{21}^-\sin(2S_1-S_2)$$
$$+\mu_{12}^+\sin(S_1+2S_2)+\mu_{12}^-\sin(S_1-2S_2)+\mu_{03}^{(3)}\sin 3S_2+$$
$$+\mu_{03}^{(1)}\sin S_2 \qquad (5.19)$$

此處

$$\mu_{30}^{(1)}=ga_1\Big\{-2\sigma_2^{(1)}+\frac{1}{8}k_1^2a_1^2\sigma_0^{(1)}(9-10\tanh^{-2}k_1d+9\tanh^{-4}k_1d)$$

$$-\frac{1}{2}\frac{k_2^2a_2^2\sigma_0^{(1)}}{\cosh^2k_2d}+\Big[\frac{1}{2}\frac{k_2^2a_2^2}{\cosh^2k_2d}\left(\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}-1\right)$$

$$-\frac{1}{2}k_2|\vec{k}_1-\vec{k}_2|a_2^2\left(1+\frac{\sigma_0^{(1)2}}{\sigma_0^{(2)2}}\right)\tanh k_2d\cdot\tanh|\vec{k}_1-\vec{k}_2|d$$

$$+\frac{1}{2}|\vec{k}_1-\vec{k}_2|^2a_2^2+(k_1k_2\cos\theta-k_2^2)\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}a_2^2\Big]$$

$$\times\Big[\frac{1}{2}\left(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3}-\frac{g^2k_2^2}{\sigma_0^{(2)}}+\sigma_0^{(2)3}\right)$$

$$+(g^2k_1k_2\cos\theta+\sigma_0^{(1)2}\sigma_0^{(2)2})\left(\frac{1}{\sigma_0^{(2)}}-\frac{1}{\sigma_0^{(1)}}\right)\Big]$$

$$/\left[(\sigma_0^{(1)}-\sigma_0^{(2)})^2-g\,|\vec{k}_1-\vec{k}_2|\tanh|\vec{k}_1-\vec{k}_2|d\right]$$

$$+\left[-\frac{1}{2}\frac{k_2^2\,a_2^2}{\cosh^2 k_2 d}\left(\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}+1\right)\right.$$

$$-\frac{1}{2}k_2\,|\vec{k}_2+\vec{k}_2|\,a_2^2\left(\frac{\sigma_0^{(1)2}}{\sigma_0^{(2)2}}+1\right)\tanh k_2 d\cdot\tanh|\vec{k}_1+\vec{k}_2|d$$

$$\left.+\frac{1}{2}|\vec{k}_1+\vec{k}_2|^2 a_2^2+(k_1 k_2\cos\theta+k_2^2)\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}a_2^2\right]$$

$$\times\left[\frac{1}{2}\left(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3}+\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3}\right)\right.$$

$$\left.+(g^2k_1k_2\cos\theta-\sigma_0^{(1)2}-\sigma_0^{(2)2})\left(\frac{1}{\sigma_0^{(1)}}+\frac{1}{\sigma_0^{(2)}}\right)\right]$$

$$/\left[(\sigma_0^{(1)}+\sigma_0^{(2)})^2-g\,|\vec{k}_1+\vec{k}_2|\tanh|\vec{k}_1+\vec{k}_2|d\right]$$

$$+k_2^2\,a_2^2\sigma_0^{(1)}-\frac{1}{2}k_1^2\,a_2^2\,\frac{\sigma_0^{(2)2}}{\sigma_0^{(1)}}+k_1k_2a_2^2\left(\frac{\sigma_0^{(1)2}}{\sigma_0^{(2)}}+\sigma_0^{(2)}\right)\cos\theta$$

$$\left.-\frac{1}{2}\frac{g^2k_1^2\,k_2^2\,a_2^2}{\sigma_0^{(1)}\sigma_0^{(2)2}}\cos^2\theta\right\}$$

$$=2ga_1\left\{-\sigma_2^{(1)}+\frac{1}{16}k_1^2\,a_1^2\sigma_0^{(1)}(9-10\tanh^{-2}k_1d+9\tanh^{-4}k_1d)\right.$$

$$\left.+f_1(k_1,k_2,d,\theta)\,k_2^2\,a_2^2\sigma_0^{(1)}\right\} \tag{5.20a}$$

$$\mu_{30}^{(3)}=\frac{1}{8}gk_1^2\,a_1^3\sigma_0^{(1)}(27\tanh^{-4}k_1d-66\tanh^{-2}k_1d+39) \tag{5.20b}$$

$$\mu_{21}^{+}=ga_1^2a_2\left\{\frac{1}{4}k_1^2\,\sigma_0^{(1)}(3\tanh^{-4}k_1d-12\tanh^{-2}k_1d+10-\tanh^2k_1d)\right.$$

$$+\frac{1}{4}k_1^2\,\sigma_0^{(2)}(-12\tanh^{-2}k_1d+11-\tanh^2k_1d)$$

$$-\frac{1}{4}k_1^2\sigma_0^{(2)}\cos\theta\cdot\tanh^{-1}k_2d\cdot(2\tanh^{-1}k_1d-\tanh k_1d)$$

$$+\frac{1}{4}k_1k_2\sigma_0^{(1)}\tanh k^2d\cdot(-6\tanh^{-3}k_1d+7\tanh^{-1}k_1d-\tanh k_1d)$$

$$+\frac{1}{4}k_1k_2\sigma_0^{(1)}\cos\theta\cdot(3\tanh^{-4}k_1d-1)$$

$$+\frac{1}{8}k_1k_2\sigma_0^{(2)}(\tanh^{-1}k_2d-\tanh k_2d)(3\tanh^{-3}k_1d-\tanh^{-1}k_1d)$$

$$+k_1k_2\sigma_0^{(2)}\cos\theta-\frac{1}{4}k_1k_2\sigma_0^{(2)}\cos^2\theta\cdot\tanh^{-1}k_1d\cdot\tanh^{-1}k_2d$$

$$+\frac{1}{4}k_1k_2\frac{\sigma_0^{(1)2}}{\sigma_0^{(2)}}\cos\theta\cdot(6\tanh^{-4}k_1d-\tanh^{-2}k_1d-2)$$

$$+\frac{1}{2}k_1k_2\frac{\sigma_0^{(2)2}}{\sigma_0^{(1)}}\cos\theta-\frac{1}{2}k_2^2\ \sigma_0^{(1)}-\frac{3}{4}k_2^2\ \frac{\sigma_0^{(1)2}}{\sigma_0^{(2)}}-\frac{1}{2}k_1^2\ \frac{\sigma_0^{(2)2}}{\sigma_0^{(1)}}$$

$$+\left[\frac{1}{2}k_1^2\ (6-\tanh^2k_1d)+\frac{1}{2}k_1^2\ \frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}(3-\tanh^2k_1d)\right.$$

$$+3k_1k_2\cos\theta+k_1k_2\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}\cos\theta+\frac{1}{2}k_2^2$$

$$\left.-\frac{1}{2}\left(5+4\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}+\frac{\sigma_0^{(2)2}}{\sigma_0^{(1)2}}\right)k_1\ |\vec{k}_1+\vec{k}_2|\tanh k_1d\cdot\tanh|\vec{k}_1+\vec{k}_2|\,d\right]$$

$$\times\left[\frac{1}{2}\left(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3}+\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3}\right)\right.$$

$$\left.+\left(g^2k_1k_2\cos\theta-\sigma_0^{(1)2}\sigma_0^{(2)2}\right)\left(\frac{1}{\sigma_0^{(1)}}+\frac{1}{\sigma_0^{(2)}}\right)\right]$$

$$/\left[(\sigma_0^{(1)}+\sigma_0^{(2)})^2-g\,|\vec{k}_1+\vec{k}_2|\tanh|\vec{k}_1+\vec{k}_2|\,d\right]\}\qquad(5.20c)$$

$$\mu_{21}^-=ga_1^2a_2\ \{\frac{1}{4}k_1^2\ \sigma_0^{(1)}(3\tanh^{-4}k_1d-12\tanh^{-2}k_1d+10-\tanh^2k_1d)$$

$$+\frac{1}{4}k_1^2\ \sigma_0^{(2)}(12\tanh^{-2}k_1d-11+\tanh^2k_1d)$$

$$-\frac{1}{4}\ k_1^2\sigma_0^{(2)}\cos\theta\cdot\tanh^{-1}k_2d\cdot(2\tanh^{-1}k_1d-\tanh k_1d)$$

$$+\frac{1}{4}k_1k_2\sigma_0^{(1)}\tanh k_2d(-6\tanh^{-3}k_1d+7\tanh^{-1}k_1d-\tanh k_1d)$$

$$-\frac{1}{4}k_1k_2\sigma_0^{(1)}\cos\theta\cdot(3\tanh^{-4}k_1d-1)$$

$$-\frac{1}{8}k_1k_2\sigma_0^{(2)}(\tanh^{-1}k_2d-\tanh k_2d)(3\tanh^{-3}k_1d-\tanh^{-1}k_1d)$$

$$+k_1k_2\sigma_0^{(2)}\cos\theta+\frac{1}{4}k_1k_2\sigma_0^{(2)}\cos^2\theta\cdot\tanh^{-1}k_1d\cdot\tanh^{-1}k_2d$$

$$+\frac{1}{4}k_1k_2\frac{\sigma_0^{(1)2}}{\sigma_0^{(2)}}\cos\theta\cdot(6\tanh^{-4}k_1d-\tanh^{-2}k_1d-2)$$

$$-\frac{1}{2}k_1k_2\frac{\sigma_0^{(2)2}}{\sigma_0^{(1)}}\cos\theta-\frac{1}{2}k_2^2\ \sigma_0^{(1)}+\frac{3}{4}k_2^2\ \frac{\sigma_0^{(1)2}}{\sigma_0^{(2)}}-\frac{1}{2}k_1^2\ \frac{\sigma_0^{(2)2}}{\sigma_0^{(1)}}$$

$$+[\frac{1}{2}k_1^2\ (6-\tanh^2k_1d)-\frac{1}{2}k_1^2\ \frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}(3-\tanh^2k_1d)$$

$$-3k_1k_2\cos\theta+k_1k_2\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}\cos\theta+\frac{1}{2}k_2^2$$

$$-\frac{1}{2}(5-4\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}+\frac{\sigma_0^{(2)2}}{\sigma_0^{(1)2}})k_1|\vec{k_1}-\vec{k_2}|\tanh k_1d\cdot\tanh|\vec{k_1}-\vec{k_2}|d]$$

$$\times[\frac{1}{2}(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3}-\frac{g^2k_2^2}{\sigma_0^{(2)}}+\sigma_0^{(2)3})$$

$$+(g^2k_1k_2\cos\theta+\sigma_0^{(1)2}\sigma_0^{(2)2})(\frac{1}{\sigma_0^{(2)}}-\frac{1}{\sigma_0^{(1)}})]$$

$$/[(\sigma_0^{(1)}-\sigma_0^{(2)})^2-g|\vec{k_1}-\vec{k_2}|\tanh|\vec{k_1}-\vec{k_2}|d]\}\qquad(5.20d)$$

$$\mu_{12}^+=ga_1a_2^2\ \{\frac{1}{4}k_2^2\ \sigma_0^{(2)}(3\tanh^{-4}k_2d-12\tanh^{-2}k_2d+10-\tanh^2k_2d)$$

$$+\frac{1}{4}k_2^2\ \sigma_0^{(1)}(-12\tanh^{-2}k_2d+11-\tanh^2k_2d)$$

$$-\frac{1}{4}k_2^2\ \sigma_0^{(1)}\cos\theta\cdot\tanh^{-1}k_1d\cdot(2\tanh^{-1}k_2d-\tanh k_2d)$$

$$+\frac{1}{4}k_1k_2\sigma_0^{(2)}\tanh k_1d\cdot(-6\tanh^{-3}k_2d+7\tanh^{-1}k_2d-\tanh k_2d)$$

$$+\frac{1}{4}k_1k_2\sigma_0^{(2)}\cos\theta\cdot(3\tanh^{-4}k_2d-1)$$

$$+\frac{1}{8}k_1k_2\sigma_0^{(1)}(\tanh^{-1}k_1d-\tanh k_1d)(3\tanh^{-3}k_2d-\tanh^{-1}k_2d)$$

$$+k_1k_2\sigma_0^{(1)}\cos\theta-\frac{1}{4}k_1k_2\sigma_0^{(1)}\cos^2\theta\cdot\tanh^{-1}k_1d\cdot\tanh^{-1}k_2d$$

$$+\frac{1}{4}k_1k_2\frac{\sigma_0^{(2)2}}{\sigma_0^{(1)}}\cos\theta\cdot(6\tanh^{-4}k_2d-\tanh^{-2}k_2d-2)$$

$$+\frac{1}{2}k_1k_2\frac{\sigma_0^{(1)2}}{\sigma_0^{(2)}}\cos\theta-\frac{1}{2}k_1^2\sigma_0^{(2)}-\frac{3}{4}k_1^2\frac{\sigma_0^{(2)2}}{\sigma_0^{(1)}}-\frac{1}{2}k_2^2\frac{\sigma_0^{(1)2}}{\sigma_0^{(2)}}$$

$$+[\frac{1}{2}k_2^2(6-\tanh^2k_2d)+\frac{1}{2}k_2^2\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}(3-\tanh^2k_2d)$$

$$+3k_1k_2\cos\theta+k_1k_2\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}\cos\theta+\frac{1}{2}k_1^2$$

$$-\frac{1}{2}(5+4\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}+\frac{\sigma_0^{(1)2}}{\sigma_0^{(2)2}})k_2|\vec{k}_1+\vec{k}_2|\tanh k_2d\cdot\tanh|\vec{k}_1+\vec{k}_2|d]$$

$$\times[\frac{1}{2}(\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3}+\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3})$$

$$+(g^2k_1k_2\cos\theta-\sigma_9^{(1)2}\sigma_0^{(2)2})(\frac{1}{\sigma_0^{(2)}}+\frac{1}{\sigma_0^{(1)}})]$$

$$/[(\sigma_0^{(1)}+\sigma_0^{(2)})^2-g|\vec{k}_1+\vec{k}_2|\tanh|\vec{k}_1+\vec{k}_2|d]\} \qquad (5.20e)$$

$$\mu_{12}^{-}=ga_1a_2^2\{-\frac{1}{4}k_2^2\sigma_0^{(2)}(3\tanh^{-4}k_2d-12\tanh^{-2}k_2d+10-\tanh^2k_2d)$$

$$-\frac{1}{4}k_2^2\sigma_0^{(1)}(12\tanh^{-2}k_2d-11+\tanh^2k_2d)$$

$$+\frac{1}{4}k_2^2\sigma_0^{(1)}\cos\theta\cdot\tanh^{-1}k_1d\cdot(2\tanh^{-1}k_2d-\tanh k_2d)$$

$$-\frac{1}{4}k_1k_2\sigma_0^{(2)}\tanh k_1d\cdot(-6\tanh^{-3}k_2d+7\tanh^{-1}k_2d-\tanh k_2d)$$

$$+\frac{1}{4}k_1k_2\sigma_0^{(2)}\cos\theta\cdot(3\tanh^{-4}k_2d-1)$$

$$+\frac{1}{8}k_1k_2\sigma_0^{(1)}(\tanh^{-1}k_1d-\tanh k_1d)(3\tanh^{-3}k_2d-\tanh^{-1}k_2d)$$

$$-k_1k_2\sigma_0^{(1)}\cos\theta-\frac{1}{4}k_1k_2\sigma_0^{(1)}\cos^2\theta\cdot\tanh^{-1}k_1d\cdot\tanh^{-1}k_2d$$

$$-\frac{1}{4}k_1k_2\frac{\sigma_0^{(2)2}}{\sigma_0^{(1)}}\cos\theta\cdot(6\tanh^{-4}k_2d-\tanh^{-2}k_2d-2)$$

$$+\frac{1}{2}k_1k_2\frac{\sigma_0^{(1)2}}{\sigma_0^{(2)}}\cos\theta+\frac{1}{2}k_1^2\sigma_0^{(2)}-\frac{3}{4}k_1^2\frac{\sigma_0^{(2)2}}{\sigma_0^{(1)}}+\frac{1}{2}k_2^2\frac{\sigma_0^{(1)2}}{\sigma_0^{(2)}}$$

$$-[\frac{1}{2}k_2^2(6-\tanh^2k_2d)-\frac{1}{2}k_2^2\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}(3-\tanh^2k_2d)$$

$$-3k_1k_2\cos\theta+k_1k_2\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}\cos\theta+\frac{1}{2}k_1^2$$

$$-\frac{1}{2}(5-4\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}+\frac{\sigma_0^{(1)2}}{\sigma_0^{(2)2}})k_2|\vec{k}_1-\vec{k}_2|\tanh k_2d\cdot\tanh|\vec{k}_1-\vec{k}_2|d]$$

$$\times[\frac{1}{2}(\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3}-\frac{g^2k_1^2}{\sigma_0^{(1)}}+\sigma_0^{(1)3})$$

$$+(g^2k_1k_2\cos\theta+\sigma_0^{(1)2}\sigma_0^{(2)2})(\frac{1}{\sigma_0^{(1)}}-\frac{1}{\sigma_0^{(2)}})]$$

$$/[(\sigma_0^{(1)}-\sigma_0^{(2)})^2-g|\vec{k}_1-\vec{k}_2|\tanh|\vec{k}_1-\vec{k}_2|d]\} \qquad (5.20f)$$

$$\mu_{03}^{(3)}=\frac{1}{8}gk_2^2a_2^3\sigma_0^{(2)}(27\tanh^{-4}k_2d-66\tanh^{-2}k_2d+39) \qquad (5.20g)$$

$$\mu_{03}^{(1)}=ga_2\{-2\sigma_2^{(2)}+\frac{1}{8}k_2^2a_2^2\sigma_0^{(2)}(9-10\tanh^{-2}k_2d+9\tanh^{-4}k_2d)$$

$$-\frac{1}{2}\frac{k_1^2a_1^2\sigma_0^{(2)}}{\cosh^2k_1d}+[\frac{1}{2}\frac{k_1^2a_1^2}{\cosh^2k_1d}(\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}-1)$$

$$-\frac{1}{2}k_1|\vec{k}_1-\vec{k}_2|a_1^2(1+\frac{\sigma_0^{(2)2}}{\sigma_0^{(1)2}})\tanh k_1d\cdot\tanh|\vec{k}_1-\vec{k}_2|d$$

$$+\frac{1}{2}|\vec{k}_1-\vec{k}_2|^2a_1^2+(k_1k_2\cos\theta-k_1^2)\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}a_1^2]$$

$$\times[\frac{1}{2}(\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3}-\frac{g^2k_1^2}{\sigma_0^{(1)}}+\sigma_0^{(1)3})$$

$$+(g^2k_1k_2\cos\theta+\sigma_0^{(1)2}\sigma_0^{(2)2})(\frac{1}{\sigma_0^{(1)}}-\frac{1}{\sigma_0^{(2)}}))]$$

$$/[(\sigma_0^{(1)}-\sigma_0^{(2)})^2-g|\vec{k}_1-\vec{k}_2|\tanh|\vec{k}_1-\vec{k}_2|d]$$

$$+[-\frac{1}{2}\frac{k_1^2\ a_1^2}{\cosh^2k_1d}(\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}+1)$$

$$-\frac{1}{2}k_1|\vec{k}_1+\vec{k}_2|a_1^2\ (\frac{\sigma_0^{(2)2}}{\sigma_0^{(1)2}}+1)\tanh k_1d\cdot\tanh|\vec{k}_1+\vec{k}_2|d$$

$$+\frac{1}{2}|\vec{k}_1+\vec{k}_2|^2a_1^2+(k_1k_2\cos\theta+k_1^2)\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}a_1^2]$$

$$\times[\frac{1}{2}(\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3}+\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3})$$

$$+(g^2k_1k_2\cos\theta-\sigma_0^{(1)2}\sigma_0^{(2)2})(\frac{1}{\sigma_0^{(1)}}+\frac{1}{\sigma_0^{(2)}}))]$$

$$/[(\sigma_0^{(1)}+\sigma_0^{(2)})^2-g|\vec{k}_1+\vec{k}_2|\tanh|\vec{k}_1+\vec{k}_2|d]+k_1^2\ a_1{}^2\sigma_0^{(2)}$$

$$-\frac{1}{2}k_2^2\ a_1^2\ \frac{\sigma_0^{(1)2}}{\sigma_0^{(2)}}+k_1k_2a_1^2\ (\sigma_0^{(1)}+\frac{\sigma_0^{(2)2}}{\sigma_0^{(1)}})\cos\theta$$

$$-\frac{1}{2}\frac{g^2k_1^2\ k_2^2\ a_1^2}{\sigma_0^{(1)2}\sigma_0^{(2)}}\cos^2\theta\}$$

$$=2ga_2\{-\sigma_2^{(2)}+\frac{1}{16}k_2^2\ a_2^2\ \sigma_0^{(2)}(9-10\tanh^{-2}k_2d+9\tanh^{-4}k_2d)$$

$$+f_2(k_1,k_2,d,\theta)k_1^2\ a_1{}^2\sigma_0^{(2)}\}\qquad(5.20\text{h})$$

至此，依上述M的計算結果可明顯地得知，所欲求解的整體波動流場結構之第三階次量解，即流速勢函數（ velocity potential function ）者，其所關連到的變動相位有S_1、$3S_1$、（$2S_1 \pm S_2$）、（$S_1 \pm 2S_2$）、$3S_2$與S_2。於此，便於清楚解析起見，將以ϕ_{30}對應於S_1與$3S_1$、$\phi_{21} = \phi_{21}^+ + \phi_{21}^-$對應於（$2S_1 \pm S_2$），$\phi_{12} = \phi_{12}^+ + \phi_{12}^-$對應於（$S_1 \pm 2S_2$）及$\phi_{03}$對應於$S_2$與$3S_2$，如(4.2)式所示。因此，依上述的逐次推演，這已是很明朗化的，對所欲求解的整個波動流場結構之第三階次解ϕ_{30}、$\phi_{21} = \phi_{21}^+ + \phi_{21}^-$、$\phi_{12} = \phi_{12}^+ + \phi_{12}^-$與$\phi_{03}$等量，則可由第四節中所述之控制條件式(4.18a)、(4.18b)、(4.18c)與(5.15)、(5.17a-j)及(5.19)、(5.20a-h)等式之應用爲之，於第三階次下，它們有

$$\left.\begin{aligned}
&\nabla^2\phi_{30} = 0\ ,\ \nabla^2\phi_{21}^+ = 0\ ,\ \nabla^2\phi_{21}^- = 0\ ,\ \nabla^2\phi_{12}^+ = 0\ ,\\
&\nabla^2\phi_{12}^- = 0\ ,\ \nabla^2\phi_{03} = 0\ ,\\
&\frac{\partial\phi_{30}}{\partial z} = 0\ ,\ \frac{\partial\phi_{21}^+}{\partial z} = 0\ ,\ \frac{\partial\phi_{21}^-}{\partial z} = 0\ ,\ \frac{\partial\phi_{12}^+}{\partial z} = 0\ ,\\
&\frac{\partial\phi_{12}^-}{\partial z} = 0\ ,\ \frac{\partial\phi_{03}}{\partial z} = 0\ ,\ z = -d\\
&\sigma_0^{(1)2}\frac{\partial^2\phi_{30}}{\partial t_1^2} + g\frac{\partial\phi_{30}}{\partial z} = -\mu_{30}^{(1)}\sin S_1 - \mu_{30}^{(3)}\sin 3S_1\ ,\ z = 0\\
&\sigma_0^{(2)2}\frac{\partial^2\phi_{03}}{\partial t_2^2} + g\frac{\partial\phi_{03}}{\partial z} = -\mu_{03}^{(1)}\sin S_2 - \mu_{03}^{(3)}\sin 3S_2\ ,\ z = 0\\
&(2\sigma_0^{(1)} + \sigma_0^{(2)})^2\frac{\partial^2\phi_{21}^+}{\partial t_5^2} + g\frac{\partial\phi_{21}^+}{\partial z} = -\mu_{21}^+\sin(2S_1 + S_2)\ ,\\
&\qquad z = 0\\
&(2\sigma_0^{(1)} - \sigma_0^{(2)})^2\frac{\partial^2\phi_{21}^-}{\partial t_6^2} + g\frac{\partial\phi_{21}^-}{\partial z} = -\mu_{21}^-\sin(2S_1 - S_2)\ ,\\
&\qquad z = 0\\
&(\sigma_0^{(1)} + 2\sigma_0^{(2)})^2\frac{\partial^2\phi_{12}^+}{\partial t_7^2} + g\frac{\partial\phi_{12}^+}{\partial z} = -\mu_{12}^+\sin(S_1 + 2S_2)\ ,\\
&\qquad z = 0
\end{aligned}\right\}\quad(5.21)$$

$$(\sigma_0^{(1)}-2\sigma_0^{(2)})^2\frac{\partial^2\phi_{12}^-}{\partial t_8^2}+g\frac{\partial\phi_{12}^-}{\partial z}=-\mu_{12}^-\sin(S_1-2S_2)\ ,\ z=0\ \rfloor$$

因此，於避免產生庸凡項（ secular terms ）下，所欲求的第三階次量之解可容易地得爲

因 $\mu_{30}^{(1)}=0$，

$$\sigma_2^{(1)}=\frac{1}{16}(9-10\tanh^{-2}k_1d+9\tanh^{-4}k_1d)k_1^2a_1^2\sigma_0^{(1)}+f_1(k_1,k_2,d,\theta)k_2^2a_2^2\sigma_0^{(1)} \tag{5.22a}$$

因 $\mu_{03}^{(1)}=0$，

$$\sigma_2^{(2)}=\frac{1}{16}(9-10\tanh^{-2}k_2d+9\tanh^{-4}k_2d)k_2^2a_2^2\sigma_0^{(2)}+f_2(k_1,k_2,d,\theta)k_1^2a_1^2\sigma_0^{(2)} \tag{5.22b}$$

$$\phi_{30}=\frac{1}{64}(9\tanh^{-7}k_1d+5\tanh^{-5}k_1d-53\tanh^{-3}k_1d+39\tanh^{-1}k_1d)k_1a_1^3\sigma_0^{(1)}\frac{\cosh 3k_1(d+z)}{\cosh 3k_1d}\sin 3S_1+B_{30}t \tag{5.22c}$$

$$\phi_{03}=\frac{1}{64}(9\tanh^{-7}k_2d+5\tanh^{-5}k_2d-53\tanh^{-3}k_2d+39\tanh^{-1}k_2d)k_2a_2^3\sigma_0^{(2)}\frac{\cosh 3k_2(d+z)}{\cosh 3k_1d}\sin 3S_2+B_{03}t \tag{5.22d}$$

$$\phi_{21}^+=\frac{\mu_{21}^+}{(2\sigma_0^{(1)}+\sigma_0^{(2)})^2-g|2\vec{k}_1+\vec{k}_2|\tanh|2\vec{k}_1+\vec{k}_2|d}\times\frac{\cosh|2\vec{k}_1+\vec{k}_2|(d+z)}{\cosh|2\vec{k}_1+\vec{k}_2|d}\sin(2S_1+S_2)+B_{21}^+t \tag{5.22e}$$

$$\phi_{21}^-=\frac{\mu_{21}^-}{(2\sigma_0^{(1)}-\sigma_0^{(2)})^2-g|2\vec{k}_1-\vec{k}_2|\tanh|2\vec{k}_1-\vec{k}_2|d}$$

$$\times\frac{\cosh|2\vec{k}_1-\vec{k}_2|(d+z)}{\cosh|2\vec{k}_1-\vec{k}_2|d}\sin(2S_1-S_2)+B_{21}^{-}t \quad (5.22f)$$

$$\phi_{12}^{+}=\frac{\mu_{12}^{+}}{(\sigma_0^{(1)}+2\sigma_0^{(2)})^2-g|\vec{k}_1+2\vec{k}_2|\tanh|\vec{k}_1+2\vec{k}_2|d}$$

$$\times\frac{\cosh|\vec{k}_1+2\vec{k}_2|(d+z)}{\cosh|\vec{k}_1+2\vec{k}_2|d}\sin(S_1+2S_2)+B_{12}^{+}t \quad (5.22g)$$

$$\phi_{12}^{-}=\frac{\mu_{12}^{-}}{(\sigma_0^{(1)}-2\sigma_0^{(2)})^2-g|\vec{k}_1-2\vec{k}_2|\tanh|\vec{k}_1-2\vec{k}_2|d}$$

$$\times\frac{\cosh|\vec{k}_1-2\vec{k}_2|(d+z)}{\cosh|\vec{k}_1-2\vec{k}_2|d}\sin(S_1-2S_2)+B_{12}^{-}t \quad (5.22h)$$

上式中 f_1 、f_2 各被示之於(5.20a)與(5.20h)式中，而 $\mu_{21}^{\pm}$ 與 $\mu_{12}^{\pm}$ 則分別爲(5.20c-f)式所示；至於 $B_{21}^{\pm}$ 與 $B_{12}^{\pm}$ 皆爲未定的常數，其可由對應的第三階次水位量的求算中決定之，如下。

求解水位 η_{30} 、$\eta_{21}=\eta_{21}^{+}+\eta_{21}^{-}$ 、$\eta_{12}=\eta_{12}^{+}+\eta_{12}^{-}$ 與 η_{30} 等量，則需由仍尙未被應用的控制條件式(4.18d)與(4.18e)式進行之。以(5.16)式之應用，則於第三階次量的考慮下，(4.18d)式可被寫成

$$g(\eta_{30}+\eta_{21}^{+}+\eta_{21}^{-}+\eta_{12}^{+}+\eta_{12}^{-}+\eta_{03})+\sigma_0^{(1)}\frac{\partial\phi_{30}}{\partial t_1}$$

$$+(2\sigma_0^{(1)}+\sigma_0^{(2)})\frac{\partial\phi_{21}^{+}}{\partial\phi_5}+(2\sigma_0^{(1)}-\sigma_0^{(2)})\frac{\partial\phi_{21}^{-}}{\partial t_6}$$

$$+(\sigma_0^{(1)}+2\sigma_0^{(2)})\frac{\partial\phi_{12}^{+}}{\partial t_7}+(\sigma_0^{(1)}-2\sigma_0^{(2)})\frac{\partial\phi_{12}^{-}}{\partial t_8}+\sigma_0^{(2)}\frac{\partial\phi_{03}}{\partial t_2}+N$$

$$=0 \quad , \quad z=0 \quad (5.23)$$

因此，由(5.16)、(5.18a-g)與(5.22a-h)式之代入(5.23)式中且於(4.18e)式之應用下，可求得

$$B_{30}=B_{21}^{+}=B_{21}^{-}=B_{12}^{+}=B_{12}^{-}=B_{03}=0 \quad (5.24)$$

與

$$
\left.\begin{aligned}
&\eta_{30}=\zeta_{30}^{(1)}\cos S_1+\zeta_{30}^{(3)}\cos 3S_1,\\
&\eta_{03}=\zeta_{03}^{(1)}\cos S_2+\zeta_{03}^{(3)}\cos 3S_2\\
&\eta_{21}^{+}=\zeta_{21}^{+}\cos(2S_1+S_2),\quad \eta_{21}^{-}=\zeta_{21}^{-}\cos(2S_1-S_2)\\
&\eta_{12}^{+}=\zeta_{12}^{-}\cos(S_1+2S_2),\quad \eta_{12}^{-}=\zeta_{12}^{-}\cos(S_1-2S_2)
\end{aligned}\right\}\qquad(5.25)
$$

此處

$$
\begin{aligned}
\zeta_{30}^{(1)}=&\frac{1}{16}k_1^2a_1^3\,(3\tanh^{-4}k_1d+8\tanh^{-2}k_1d-9)\\
&+a_1a_2^2\Big[\frac{1}{4}k_1^2\,(1-\frac{\sigma_0^{(2)2}}{\sigma_0^{(1)2}})-\frac{1}{4}k_2^2\,(1-3\tanh^2k_2d)\\
&-\frac{1}{2}k_1k_2\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}\cos\theta-\frac{1}{4}k_1k_2\cos^2\theta\cdot\tanh^{-1}k_1d\tan^{-1}k_2d\\
&+\frac{1}{2}k_1k_2\tanh k_1d\cdot\tanh k_2d\Big]-\frac{1}{2}\frac{a_1a_2^2}{g}\Big[-(\sigma_0^{(1)}+\sigma_0^{(2)})\\
&\times k_2\tanh k_2d+\frac{1}{2}(\frac{1}{\sigma_0^{(2)}}+\frac{1}{\sigma_0^{(1)}})gk_2^2\,(1-\tanh^2k_2d)\\
&+\frac{1}{2}(\frac{\sigma_0^{(2)2}}{\sigma_0^{(1)}}-\sigma_0^{(1)})\,|\vec{k}_1+\vec{k}_2|\,\tanh|\vec{k}_1+\vec{k}_2|\,d\\
&-\frac{1}{2}\frac{g}{\sigma_0^{(1)}}|\vec{k}_1+\vec{k}_2|^2\Big]\Big[\frac{1}{2}(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3}+\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3})\\
&+(g^2k_1k_2\cos\theta-\sigma_0^{(1)2}\sigma_0^{(2)2})(\frac{1}{\sigma_0^{(1)}}+\frac{1}{\sigma_0^{(2)}})\Big]\\
&/\Big[(\sigma_0^{(1)}+\sigma_0^{(2)})^2-g\,|\vec{k}_1+\vec{k}_2|\,\tanh|\vec{k}_1+\vec{k}_2|\,d\Big]\\
&-\frac{1}{2}\frac{a_1a_2^2}{g}\Big[-(\sigma_0^{(1)}-\sigma_0^{(2)})\,k_2\tanh k_2d-\frac{1}{2}(\frac{1}{\sigma_0^{(2)}}-\frac{1}{\sigma_0^{(1)}})\\
&\times gk_2^2\,(1-\tanh^2k_2d)+\frac{1}{2}(\frac{\sigma_0^{(2)2}}{\sigma_0^{(1)}}-\sigma_0^{(1)})\,|\vec{k}_1-\vec{k}_2|\\
&\times\tanh|\vec{k}_1-\vec{k}_2|\,d-\frac{1}{2}\frac{g}{\sigma_0^{(1)}}|\vec{k}_1-\vec{k}_2|^2\Big]\Big[\frac{1}{2}(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3}
\end{aligned}
$$

$$-\frac{g^2k_2^2}{\sigma_0^{(2)}}+\sigma_0^{(2)3}\,)+(\,g^2k_1k_2\cos\theta+\sigma_0^{(1)2}\sigma_0^{(2)2}\,)$$

$$\times(\frac{1}{\sigma_0^{(2)}}-\frac{1}{\sigma_0^{(1)}}\,)\,]\,/\,[\,(\,\sigma_0^{(1)}-\sigma_0^{(2)}\,)^2-g\,|\,\vec{k}_1-\vec{k}_2\,|$$

$$\times\tanh|\,\vec{k}_1-\vec{k}_2\,|\,d\,] \qquad (5.26a)$$

$$\zeta_{30}^{(3)}=\frac{1}{64}k_1^2a_1^3\,(\,27\tanh^{-6}k_1d-9\tanh^{-4}k_1d+9\tanh^{-2}k_1d$$

$$-3\,) \qquad (5.26b)$$

$$\zeta_{21}^{+}=a_1^2a_2\,[\,\frac{1}{8}k_1k_2\tanh k_2d\cdot(\,3\tanh^{-3}k_1d-\tanh^{-1}k_1d\,)$$

$$+\frac{1}{4}k_1^2\,(\,6\tanh^{-2}k_1d-5+\tanh^2k_1d\,)+\frac{1}{8}k_2^2+\frac{1}{4}k_1^2\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}$$

$$\times(\,3\tanh^{-2}k_1d-2+\tanh^2k_1d\,)+\frac{1}{4}k_2^2\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}$$

$$+\frac{1}{4}k_1k_2\tanh k_1d\cdot\tanh k_2d-\frac{1}{8}k_1k_2\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}\cos\theta\cdot(\,3\tanh^{-4}k_1d+1)$$

$$-\frac{1}{4}k_1k_2\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}\cos\theta\,]-\frac{1}{2}\frac{a_1^2a_2}{g}\,[\frac{g}{\sigma_0^{(1)}}(\,k_1^2+k_1k_2\cos\theta\,)$$

$$-(\,\sigma_0^{(1)}+\sigma_0^{(2)}\,)k_1\tanh k_1d-(\,2\sigma_0^{(1)}+\sigma_0^{(2)}\,)\,|\,\vec{k}_1+\vec{k}_2\,|$$

$$\times\tanh|\,\vec{k}_1+\vec{k}_2\,|\,d\,]\,[\,\frac{1}{2}(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3}+\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3}\,)$$

$$+(\,g^2k_1k_2\cos\theta-\sigma_0^{(1)2}\sigma_0^{(2)2}\,)(\frac{1}{\sigma_0^{(1)}}+\frac{1}{\sigma_0^{(2)}}\,)\,]$$

$$/\,[\,(\,\sigma_0^{(1)}+\sigma_0^{(2)}\,)^2-g\,|\,\vec{k}_1+\vec{k}_2\,|\,\tanh|\,\vec{k}_1+\vec{k}_2\,|\,d\,]$$

$$+(\,2\sigma_0^{(1)}+\sigma_0^{(2)}\,)\frac{\mu_{21}^{+}}{g}/\,[\,(\,2\sigma_0^{(1)}+\sigma_0^{(2)}\,)^2$$

$$-g\,|\,2\vec{k}_1+\vec{k}_2\,|\,\tanh|\,2\vec{k}_1+\vec{k}_2\,|\,d\,] \qquad (5.26c)$$

$$\zeta_{21}^{-}=a_1^2a_2\,[\,\frac{1}{8}k_1k_2\tanh k_2d\cdot(\,3\tanh^{-3}k_1d-\tanh^{-1}k_1d\,)$$

$$+\frac{1}{4}k_1^2\ (6\tanh^{-2}k_1d-5+\tanh^2k_1d)+\frac{1}{8}k_2^2-\frac{1}{4}k_1^2\ \frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}$$

$$\times(3\tanh^{-2}k_1d-2+\tanh^2k_1d)-\frac{1}{4}k_2^2\ \frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}$$

$$+\frac{1}{4}\ k_1k_2\tanh k_1d\cdot\tanh k_2d\ -\frac{1}{8}k_1k_2\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}\cos\theta\cdot(3\tanh^{-4}k_1d+1)$$

$$-\frac{1}{4}k_1k_2\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}\cos\theta\,]-\frac{1}{2}\frac{a_1{}^2a_2}{g}\,[\frac{g}{\sigma_0^{(1)}}(k_1^2\ -k_1k_2\cos\theta)$$

$$-(\sigma_0^{(1)}-\sigma_0^{(2)})k_1\tanh k_1d-(2\sigma_0^{(1)}-\sigma_0^{(2)})\,|\,\vec{k}_1-\vec{k}_2\,|$$

$$\times\tanh|\,\vec{k}_1-\vec{k}_2\,|\,d\,]\,[\,\frac{1}{2}(\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3}-\frac{g^2k_2^2}{\sigma_0^{(2)}}+\sigma_0^{(2)3})$$

$$+(g^2k_1k_2\cos\theta+\sigma_0^{(1)2}\sigma_0^{(2)2})(\frac{1}{\sigma_0^{(2)}}-\frac{1}{\sigma_0^{(1)}})\,]$$

$$/\,[\,(\sigma_0^{(1)}-\sigma_0^{(2)})^2-g\,|\,\vec{k}_1-\vec{k}_2\,|\,\tanh|\,\vec{k}_1-\vec{k}_2\,|\,d\,]$$

$$+(2\sigma_0^{(1)}-\sigma_0^{(2)})\frac{\mu_{21}{}^-}{g}/\,[\,(2\sigma_0^{(1)}-\sigma_0^{(2)})^2$$

$$-g\,|\,2\vec{k}_1-\vec{k}_2\,|\,\tanh|\,2\vec{k}_1-\vec{k}_2\,|\,d\,)\,] \qquad (5.26d)$$

$$\zeta_{12}{}^+=a_1a_2^2\ [\,\frac{1}{8}k_1k_2\tanh k_1d(3\tanh^{-3}k_2d-\tanh^{-1}k_2d)$$

$$+\frac{1}{4}k_2^2\ (6\tanh^{-2}k_2d-5+\tanh^2k_2d)+\frac{1}{8}k_1^2+\frac{1}{4}k_2^2\ \frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}$$

$$\times(3\tanh^{-2}k_2d-2+\tanh^2k_2d)+\frac{1}{4}k_1^2\ \frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}$$

$$+\frac{1}{4}\ k_1k_2\tanh k_1d\cdot\tanh k_2d\ -\frac{1}{8}k_1k_2\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}\cos\theta\cdot(3\tanh^{-4}k_2d+1)$$

$$-\frac{1}{4}k_1k_2\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}\cos\theta\,]-\frac{1}{2}\frac{a_1a_2^2}{g}\,[\frac{g}{\sigma_0^{(2)}}(k_1k_2\cos\theta+k_2^2\ )$$

$$-(\sigma_0^{(1)}+\sigma_0^{(2)})k_2\tanh k_2d-(\sigma_0^{(1)}+2\sigma_0^{(2)})\,|\,\vec{k}_1+\vec{k}_2\,|$$

$$\times\tanh|\,\vec{k}_1+\vec{k}_2\,|\,d\,]\,[\,\frac{1}{2}(\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3}+\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3})$$

$$+\left(g^2k_1k_2\cos\theta-\sigma_0^{(1)2}\sigma_0^{(2)2}\right)\left(\frac{1}{\sigma_0^{(2)}}+\frac{1}{\sigma_0^{(1)}}\right)\Big]$$

$$/\left[\left(\sigma_0^{(1)}+\sigma_0^{(2)}\right)^2-g\,|\vec{k}_1+\vec{k}_2|\tanh|\vec{k}_1+\vec{k}_2|d\right]$$

$$+\left(\sigma_0^{(1)}+2\sigma_0^{(2)}\right)\frac{\mu_{12}^{+}}{g}/\left[\left(\sigma_0^{(1)}+2\sigma_0^{(2)}\right)^2\right.$$

$$\left.-g\,|\vec{k}_1+2\vec{k}_2|\tanh|\vec{k}_1+2\vec{k}_2|d\right] \tag{5.26e}$$

$$\zeta_{12}^{-}=a_1a_2^2\Big[\frac{1}{8}k_1k_2\tanh k_1d\cdot\left(3\tanh^{-3}k_2d-\tanh^{-1}k_2d\right)$$

$$+\frac{1}{4}k_2^2\left(6\tanh^{-2}k_2d-5+\tanh^2k_2d\right)+\frac{1}{8}k_1^2-\frac{1}{4}k_2^2\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}$$

$$\times\left(3\tanh^{-2}k_2d-2+\tanh^2k_2d\right)-\frac{1}{4}k_1^2\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}$$

$$+\frac{1}{4}k_1k_2\tanh k_1d\cdot\tanh k_2d-\frac{1}{8}k_1k_2\frac{\sigma_0^{(2)}}{\sigma_0^{(1)}}\cos\theta\cdot\left(3\tanh^{-4}k_2d+1\right)$$

$$-\frac{1}{4}k_1k_2\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}\cos\theta\Big]-\frac{1}{2}\frac{a_1a_2^2}{g}\Big[\frac{g}{\sigma_0^{(2)}}\left(k_2^2-k_1k_2\cos\theta\right)$$

$$-\left(\sigma_0^{(2)}-\sigma_0^{(1)}\right)k_2\tanh k_2d-\left(2\sigma^{(2)}-\sigma_0^{(1)}\right)|\vec{k}_1-\vec{k}_2|$$

$$\times\tanh|\vec{k}_1-\vec{k}_2|d\Big]\Big[\frac{1}{2}\left(\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3}-\frac{g^2k_1^2}{\sigma_0^{(1)}}+\sigma_0^{(1)3}\right)$$

$$+\left(g^2k_1k_2\cos\theta+\sigma_0^{(1)2}\sigma_0^{(2)2}\right)\left(\frac{1}{\sigma_0^{(1)}}-\frac{1}{\sigma_0^{(2)}}\right]$$

$$/\left[\left(\sigma_0^{(1)}-\sigma_0^{(2)}\right)^2-g\,|\vec{k}_1-\vec{k}_2|\tanh|\vec{k}_1-\vec{k}_2|d\right]$$

$$+\left(\sigma_0^{(1)}-2\sigma_0^{(2)}\right)\frac{\mu_{12}^{-}}{g}/\left[\left(\sigma_0^{(1)}-2\sigma_0^{(2)}\right)^2\right.$$

$$\left.-g\,|\vec{k}_1-2\vec{k}_2|\tanh|\vec{k}_1-2\vec{k}_2|d\right] \tag{5.26f}$$

$$\zeta_{03}^{(1)}=\frac{1}{16}k_2^2a_2^3\left(3\tanh^{-4}k_2d+8\tanh^{-2}k_2d-9\right)$$

$$+a_1^2a_2\Big[\frac{1}{4}k_2^2\left(1-\frac{\sigma_0^{(1)2}}{\sigma_0^{(2)2}}\right)-\frac{1}{4}k_1^2\left(1-3\tanh^2k_1d\right)$$

$$-\frac{1}{2}k_1k_2\frac{\sigma_0^{(1)}}{\sigma_0^{(2)}}\cos\theta-\frac{1}{4}k_1k_2\cos^2\theta\cdot\tanh^{-1}k_1d\cdot\tanh^{-1}k_2d$$

$$+\frac{1}{2}k_1k_2\tanh k_1d\cdot\tanh k_2d\,\Big]-\frac{1}{2}\frac{a_1^2a_2}{g}\Big[-(\sigma_0^{(1)}+\sigma_0^{(2)})$$

$$\times k_1\tanh k_1d+\frac{1}{2}\left(\frac{1}{\sigma_0^{(1)}}+\frac{1}{\sigma_0^{(2)}}\right)gk_1^2\,(1-\tanh^2k_1d)$$

$$+\frac{1}{2}\left(\frac{\sigma_0^{(1)2}}{\sigma_0^{(2)}}-\sigma_0^{(2)}\right)|\vec{k}_1+\vec{k}_2|\tanh|\vec{k}_1+\vec{k}_2|d$$

$$-\frac{1}{2}\frac{g}{\sigma_0^{(2)}}|\vec{k}_1+\vec{k}_2|^2\Big]\Big[\frac{1}{2}\left(\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3}+\frac{g^2k_1^2}{\sigma_0^{(1)}}-\sigma_0^{(1)3}\right)$$

$$+(g^2k_1k_2\cos\theta-\sigma_0^{(1)2}\sigma_0^{(2)2})\left(\frac{1}{\sigma_0^{(2)}}+\frac{1}{\sigma_0^{(1)}}\right)\Big]$$

$$/\Big[(\sigma_0^{(1)}+\sigma_0^{(2)})^2-g|\vec{k}_1+\vec{k}_2|\tanh|\vec{k}_1+\vec{k}_2|d\Big]$$

$$-\frac{1}{2}\frac{a_1^2a_2}{g}\Big[-(\sigma_0^{(2)}-\sigma_0^{(1)})k_1\tanh k_1d-\frac{1}{2}\left(\frac{1}{\sigma_0^{(1)}}-\frac{1}{\sigma_0^{(2)}}\right)$$

$$\times gk_1^2\,(1-\tanh^2k_1d)+\frac{1}{2}\left(\frac{\sigma_0^{(1)2}}{\sigma_0^{(2)}}-\sigma_0^{(2)}\right)|\vec{k}_1-\vec{k}_2|$$

$$\times\tanh|\vec{k}_1-\vec{k}_2|d-\frac{1}{2}\frac{g}{\sigma_0^{(2)}}|\vec{k}_1-\vec{k}_2|^2\Big]$$

$$\times\Big[\frac{1}{2}\left(\frac{g^2k_2^2}{\sigma_0^{(2)}}-\sigma_0^{(2)3}-\frac{g^2k_1^2}{\sigma_0^{(1)}}+\sigma_0^{(1)3}\right)+(g^2k_1k_2\cos\theta$$

$$+\sigma_0^{(1)2}\sigma_0^{(2)2})\left(\frac{1}{\sigma_0^{(1)}}-\frac{1}{\sigma_0^{(2)}}\right)\Big]/\Big[(\sigma_0^{(1)}-\sigma_0^{(2)})^2$$

$$-g|\vec{k}_1-\vec{k}_2|\tanh|\vec{k}_1-\vec{k}_2|d\Big] \qquad (5.26g)$$

$$\zeta_{03}^{(3)}=\frac{1}{16}k_2^2\,a_2^3\,(27\tanh^{-6}k_2d-9\tanh^{-4}k_2d+9\tanh^{-2}k_2d$$

$$-3) \qquad (5.26h)$$

縱論至此，這是明白的，依(5.1)、(5.14)、(5.22a～h)與(5.24)～

(5.26a～h)等式可確知，所考慮的任一均勻等深水中之任意二自由表面規則前進重力波列相交會所構成的波動系統，其整個流場結構解，於本文所述之適當的解析處理下，至第三階次量，共振情況將被另予詳述外，已被完全地求得。當然，當 $d\to\infty$ 之深海情況時，則本節所得之解恰好退化成深海中者，即爲作者已述者，見Chen (1988)。

爲便於本文此後之論述中，對所解出的波動流場解進行驗證與描述其所具有的相關特性起見，於此將所得之至第三階的整體流場結構解(除共振或近於共振情況將被單獨地論述外)歸納列出之，以資方便利用，爲

$$
\begin{aligned}
\phi =& \frac{a_1 g}{\sigma_0^{(1)}}\frac{\cosh k_1(d+z)}{\cosh k_1 d}\sin S_1+\frac{a_2 g}{\sigma_0^{(2)}}\frac{\cosh k_2(d+z)}{\cosh k_2 d}\sin S_2 \\
&+\frac{3}{8}a_1^2\sigma_0^{(1)}\frac{\cosh 2k_1(d+z)}{\sinh^4 k_1 d}\sin 2S_1-\frac{1}{4}\frac{a_1^2\,\sigma_0^{(1)2}\,t}{\sinh^2 k_1 d}-\frac{1}{4}\frac{a_2^2\sigma_0^{(2)2}\,t}{\sinh^2 k_2 d} \\
&+\frac{3}{8}a_2^2\sigma_0^{(2)}\frac{\cosh 2k_2(d+z)}{\sinh^4 k_2 d}\sin 2S_2 \\
&+A_{11}^{+}\frac{\cosh|\vec{k}_1+\vec{k}_2|(d+z)}{\cosh|\vec{k}_1+\vec{k}_2|d}\sin(S_1+S_2) \\
&+A_{11}^{-}\frac{\cosh|\vec{k}_1-\vec{k}_2|(d+z)}{\cosh|\vec{k}_1-\vec{k}_2|d}\sin(S_1-S_2) \\
&+\frac{1}{64}(9\tanh^{-7}k_1 d+5\tanh^{-5}k_1 d-53\tanh^{-3}k_1 d \\
&+39\tanh^{-1}k_1 d)\,k_1 a_1^3\sigma_0^{(1)}\frac{\cosh 3k_1(d+z)}{\cosh 3k_1 d}\sin 3S_1 \\
&+\frac{\mu_{21}^{+}}{(2\sigma_0^{(1)}+\sigma_0^{(2)})^2-g|2\vec{k}_1+\vec{k}_2|\tanh|2\vec{k}_1+\vec{k}_2|d} \\
&\times\frac{\cosh|2\vec{k}_1+\vec{k}_2|(d+z)}{\cosh|2\vec{k}_1+\vec{k}_2|d}\sin(2S_1+S_2)
\end{aligned}
$$

$$
+\frac{\mu_{21}^{-}}{(2\sigma_0^{(1)}-\sigma_0^{(2)})^2-g\,|2\vec{k}_1-\vec{k}_2|\tanh|2\vec{k}_1-\vec{k}_2|d}
$$

$$
\times\frac{\cosh|2\vec{k}_1-\vec{k}_2|(d+z)}{\cosh|2\vec{k}_1-\vec{k}_2|d}\sin(2S_1-S_2)
$$

$$
+\frac{\mu_{12}^{+}}{(\sigma_0^{(1)}+2\sigma_0^{(2)})^2-g\,|\vec{k}_1+2\vec{k}_2|\tanh|\vec{k}_1+2\vec{k}_2|d}
$$

$$
\times\frac{\cosh|\vec{k}_1+2\vec{k}_2|(d+z)}{\cosh|\vec{k}_1+2\vec{k}_2|d}\sin(S_1+2S_2)
$$

$$
+\frac{\mu_{12}^{-}}{(\sigma_0^{(1)}-2\sigma_0^{(2)})^2-g\,|\vec{k}_1-2\vec{k}_2|\tanh|\vec{k}_1-2\vec{k}_2|d}
$$

$$
\times\frac{\cosh|\vec{k}_1-2\vec{k}_2|(d+z)}{\cosh|\vec{k}_1-2\vec{k}_2|d}\sin(S_1-2S_2)
$$

$$
+\frac{1}{64}(9\tanh^{-7}k_2d+5\tanh^{-5}k_2d-53\tanh^{-3}k_2d
$$

$$
+39\tanh^{-1}k_2d)k_2a_2^3\sigma_0^{(2)}\frac{\cosh 3k_2(d+z)}{\cosh 3k_2d}\sin 3S_2
$$

$$
=\phi_{10}+\phi_{01}+\phi_{20}+\phi_{02}+\phi_{11}^{+}+\phi_{11}^{-}+\phi_{30}+\phi_{21}^{+}+\phi_{21}^{-}+\phi_{12}^{+}+\phi_{12}^{-}+\phi_{03} \tag{5.27}
$$

$$
\eta=(a_1+\zeta_{30}^{(1)})\cos S_1+(a_2+\zeta_{03}^{(1)})\cos S_2
$$

$$
+\frac{1}{4}a_1^2k_1\frac{\cosh k_1d}{\sinh^3 k_1d}(2\sinh^2k_1d+3)\cos 2S_1
$$

$$
+\frac{1}{4}a_2^2k_2\frac{\cosh k_2d}{\sinh^3k_2d}(2\sinh^2k_2d+3)\cos 2S_2
$$

$$
+\frac{1}{g}\{\frac{1}{2}a_1a_2(\sigma_0^{(1)2}+\sigma_0^{(1)}\sigma_0^{(2)}+\sigma_0^{(2)2}-\frac{g^2k_1k_2}{\sigma_0^{(2)}\sigma_0^{(1)}}\cos\theta)
$$

$$
+(\sigma_0^{(2)}+\sigma_0^{(1)})A_{11}^{+}\}\cos(S_1+S_2)
$$

$$+\frac{1}{g}\{\frac{1}{2}a_1a_2(\sigma_0^{(1)2}-\sigma_0^{(1)}\sigma_0^{(2)}+\sigma_0^{(2)2}-\frac{g^2k_1k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}\cos\theta)$$

$$+(\sigma_0^{(1)}-\sigma_0^{(2)})A_{11}^-\}\cos(S_1-S_2)+\zeta_{30}^{(3)}\cos 3S_1$$

$$+\zeta_{21}^+\cos(2S_1+S_2)+\zeta_{21}^-\cos(2S_1-S_2)+\zeta_{12}^+\cos(S_1+2S_2)$$

$$+\zeta_{12}^-\cos(S_1-2S_2)+\zeta_{03}^{(3)}\cos 3S_2$$

$$=\eta_{10}+\eta_{01}+\eta_{20}+\eta_{02}+\eta_{11}^++\eta_{11}^-+\eta_{30}+\eta_{21}^++\eta_{21}^-+\eta_{12}^++\eta_{12}^-+\eta_{03} \tag{5.28}$$

$$\sigma_1=\sigma_0^{(1)}+\frac{1}{16}(9-10\tanh^{-2}k_1d+9\tanh^{-4}k_1d)k_1^2a_1^2\sigma_0^{(1)}$$

$$+\frac{1}{2}\{\sigma_0^{(1)}-\frac{\sigma_0^{(1)}}{2}\operatorname{sech}^2k_2d-\frac{\sigma_0^{(1)}}{2}\frac{k_1}{k_2}\frac{\tanh k_2d}{\tanh k_1d}$$

$$+(\frac{k_1^{3/2}}{k_2^{3/2}}\sqrt{\frac{\tanh k_1d}{\tanh k_2d}}+\sqrt{\frac{k_1}{k_2}\frac{\tanh k_2d}{\tanh k_1d}})\sigma_0^{(1)}\cos\theta$$

$$-\frac{\sigma_0^{(1)}}{2}\frac{k_1}{k_2}\tanh^{-1}k_1d\cdot\tanh^{-1}k_2d\cdot\cos^2\theta$$

$$+[\frac{1}{2}(\sqrt{\frac{k_1}{k_2}\frac{\tanh k_1d}{\tanh k_2d}}-1)\operatorname{sech}^2k_2d-\frac{1}{2}(\frac{k_1^2}{k_2^2}-2\frac{k_1}{k_2}\cos\theta+1)^{1/2}$$

$$\times(\tanh k_2d+\frac{k_1}{k_2}\tanh k_1d)\tanh(k_1^2-2k_1k_2\cos\theta+k_2^2)^{1/2}d$$

$$+\frac{1}{2}(\frac{k_1^2}{k_2^2}-2\frac{k_1}{k_2}\cos\theta+1)+(\frac{k_1}{k_2}\cos\theta-1)\sqrt{\frac{k_1}{k_2}\frac{\tanh k_1d}{\tanh k_2d}})$$

$$\times[\sigma_0^{(1)}\frac{k_1}{k_2}\operatorname{csch}2k_1d+\sigma_0^{(1)}(\sqrt{\frac{k_1}{k_2}}-\sqrt{\frac{\tanh k_2d}{\tanh k_1d}})$$

$$\times(\tanh^{-1}k_1d\cdot\tanh^{-1}k_2d\cdot\cos\theta+1)\sqrt{\tanh k_1d\cdot\tanh k_2d}$$

$$-\sigma_0^{(2)}\operatorname{csch}2k_2d]/[(\sqrt{\frac{k_1}{k_2}\tanh k_1d}-\sqrt{\tanh k_2d})^2$$

$$-(\frac{k_1^2}{k_2^2}-2\frac{k_1}{k_2}\cos\theta+1)^{1/2}\tanh(k_1^2-2k_1k_2\cos\theta+k_2^2)^{1/2}d]$$

$$+[-\frac{1}{2}(\sqrt{\frac{k_1}{k_2}\frac{\tanh k_1d}{\tanh k_2d}}+1)\operatorname{sech}^2k_2d-\frac{1}{2}(\frac{k_1^2}{k_2^2}+2\frac{k_1}{k_2}\cos\theta+1)^{1/2}$$

$$\times(\tanh k_2d+\frac{k_1}{k_2}\tanh k_1d)\tanh(k_1^2+2k_1k_2\cos\theta+k_2^2)^{1/2}d$$

$$+\frac{1}{2}(\frac{k_1^2}{k_2^2}+2\frac{k_1}{k_2}\cos\theta+1)+(\frac{k_1}{k_2}\cos\theta+1)\sqrt{\frac{k_1}{k_2}\frac{\tanh k_1d}{\tanh k_2d}}]$$

$$\times[\sigma_0^{(1)}\frac{k_1}{k_2}\operatorname{csch}2k_1d+\sigma_0^{(1)}(\sqrt{\frac{k_1}{k_2}}+\sqrt{\frac{\tanh k_2d}{\tanh k_1d}})$$

$$\times(\tanh^{-1}k_1d\cdot\tanh^{-1}k_2d\cdot\cos\theta-1)\sqrt{\tanh k_1d\cdot\tanh k_2d}$$

$$+\sigma_0^{(2)}\operatorname{csch}2k_2d]/[(\sqrt{\frac{k_1}{k_2}\tanh k_1d}+\sqrt{\tanh k_2d})^2$$

$$-(\frac{k_1^2}{k_2^2}+2\frac{k_1}{k_2}\cos\theta+1)^{1/2}\tanh(k_1^2+2k_1k_2\cos\theta+k_2^2)^{1/2}d]\}k_2^2a_2^2$$

$$=\sigma_0^{(1)}+\sigma_2^{(1)} \tag{5.29}$$

$$\sigma_2=\sigma_0^{(2)}+\frac{1}{16}(9-10\tanh^{-2}k_2d+9\tanh^{-4}k_2d)k_2^2a_2^2\sigma_0^{(2)}$$

$$+\frac{1}{2}\{\sigma_0^{(2)}-\frac{\sigma_0^{(2)}}{2}\operatorname{sech}^2k_1d-\frac{\sigma_0^{(2)}}{2}\frac{k_2}{k_1}\frac{\tanh k_1d}{\tanh k_2d}$$

$$+(\frac{k_2^{3/2}}{k_1^{3/2}}\sqrt{\frac{\tanh k_2d}{\tanh k_1d}}+\sqrt{\frac{k_2\tanh k_1d}{k_1\tanh k_2d}})\sigma_0^{(2)}\cos\theta$$

$$-\frac{\sigma_0^{(2)}}{2}\frac{k_2}{k_1}\tanh^{-1}k_1d\cdot\tanh^{-1}k_2d\cdot\cos^2\theta$$

$$+[\frac{1}{2}(\sqrt{\frac{k_2\tanh k_2d}{k_1\tanh k_1d}}-1)\operatorname{sech}^2k_1d-\frac{1}{2}(\frac{k_2^2}{k_1^2}-2\frac{k_2}{k_1}\cos\theta+1)^{1/2}$$

$$\times(\tanh k_1d+\frac{k_2}{k_1}\tanh k_2d)\tanh(k_1^2-2k_1k_2\cos\theta+k_2^2)^{1/2}d$$

$$+\frac{1}{2}(\frac{k_2^2}{k_1^2}-2\frac{k_2}{k_1}\cos\theta+1)+(\frac{k_2}{k_1}\cos\theta-1)\sqrt{\frac{k_2\tanh k_2d}{k_1\tanh k_1d}}]$$

$$\times[\sigma_0^{(2)}\frac{k_2}{k_1}\operatorname{csch}2k_2d+\sigma_0^{(2)}(\sqrt{\frac{k_2}{k_1}}-\sqrt{\frac{\tanh k_1d}{\tanh k_2d}})$$

$$\times(\tanh^{-1}k_1d\cdot\tanh^{-1}k_2d\cdot\cos\theta+1)\sqrt{\tanh k_1d\cdot\tanh k_2d}$$

$$-\sigma_0^{(1)}\operatorname{csch}2k_1d]/[(\sqrt{\frac{k_2}{k_1}\tanh k_2d}-\sqrt{\tanh k_1d})^2$$

$$
-\left(\frac{k_2^2}{k_1^2}-2\frac{k_2}{k_1}\cos\theta+1\right)^{1/2}\tanh\left(k_1^2-2k_1k_2\cos\theta+k_2^2\right)^{1/2}d\,\Big]
$$

$$
+\Big[-\frac{1}{2}\left(\sqrt{\frac{k_2\tanh k_2d}{k_1\tanh k_1d}}+1\right)\mathrm{sech}^2k_1d-\frac{1}{2}\left(\frac{k_2^2}{k_2^2}+2\frac{k_2}{k_1}\cos\theta+1\right)^{1/2}
$$

$$
\times\left(\tanh k_1d+\frac{k_2}{k_1}\tanh k_2d\right)\tanh\left(k_1^2+2k_1k_2\cos\theta+k_1^2\right)^{1/2}d
$$

$$
+\frac{1}{2}\left(\frac{k_2^2}{k_1^2}+2\frac{k_2}{k_1}\cos\theta+1\right)+\left(\frac{k_2}{k_1}\cos\theta+1\right)\sqrt{\frac{k_2\tanh k_2d}{k_1\tanh k_1d}}\,\Big]
$$

$$
\times\Big[\sigma_0^{(2)}\frac{k_2}{k_1}\mathrm{csch}\,2k_2d+\sigma_0^{(2)}\left(\sqrt{\frac{k_2}{k_1}}+\sqrt{\frac{\tanh k_1d}{\tanh k_2d}}\right)
$$

$$
\times\left(\tanh^{-1}k_1d\cdot\tanh^{-1}k_2d\cdot\cos\theta-1\right)\sqrt{\tanh k_1d\cdot\tanh k_2d}
$$

$$
+\sigma_0^{(1)}\mathrm{csch}\,2k_1d\,\Big]\Big/\Big[\left(\sqrt{\frac{k_2}{k_1}}\tanh k_2d+\sqrt{\tanh k_1d}\right)^2
$$

$$
-\left(\frac{k_2^2}{k_1^2}+2\frac{k_2}{k_1}\cos\theta+1\right)^{1/2}\tanh\left(k_1^2+2k_1k_2\cos\theta+k_2^2\right)^{1/2}d\Big]\Big\}k_1^2a_1^2
$$

$$
=\sigma_0^{(2)}+\sigma_2^{(2)} \tag{5.30}
$$

上式中有關之係數 $A_{11}^{\pm}$ 、$\mu_{ij}^{\pm}$ 與 $\zeta_{ij}^{\pm}$ 等皆被詳列於上述中。

5-4 波動流場之一些特性

所考慮的二自由表面規則前進重力波列相交會所構成之波動系統，其流場之整體特性，包括各來源成份波列自身原具有的非線性本質與由其間相互作用所衍生出的效應量等，至第三階次量下，雖然可由所求得的波動流場解 (5.27) ～ (5.30) 式給予其全盤詳盡的論述，然此處僅將其中較顯著重要的牢牢三者特加描述之，如下。

(1)週波率影響效應

這是眾所知曉的，由淺至深之所有等深水中，單一自由表面規則前進重力波之週波率皆隨振幅（或波浪尖銳度）增加而增大；然對重力駐波者，其於較深水中是隨其振幅（或波浪尖銳度）之增加反而減少，而在水深對波長之比較小於 0.17 下之較淺水中才變爲隨之增大，謂之重力駐波週波率之逆變特性。此二者之至第三階的完整式

各被示之於第七節之（7.2）與（7.10）式中。由於單一前進波列與駐波爲所慮的二波列相交會所構成的波動系統之二個極端的特例，因此，爲全然清楚地探究出於任一等深 d 水中，一來源成份波列受另一者交會衝擊作用下，其週波率隨交會的情況，即隨此二波列之各有的波長、振幅等基本特性及它們前進方向間的夾角，而增大或變爲減小之逆變現象等之影響因素關係的全貌，於此將所得的二波列交會後所造致之成份波的週波率至第三階完整的通式量（5.29）與（5.30）式列下討論之。如以 σ_i ，$i=1$，2 表示，則經整理後，可被寫之爲

$$\frac{\sigma_i}{\sigma_0^{(i)}}-1-\frac{1}{16}(9\tanh^{-4}k_i d-10\tanh^{-2}k_i d+9)k_i^2 a_i^2$$

$$=f_i(k_i,k_j,d,\theta)k_j^2 a_j^2 \quad , \quad i、j=1,2,\ i\neq j$$

而
$$f_i(k_i,k_j,d,\theta)=\frac{1}{2}\{1-\frac{1}{2}\operatorname{sech}^2 k_j d-\frac{1}{2}\frac{k_i}{k_j}\frac{\tanh k_j d}{\tanh k_i d}$$

$$+(\frac{k_i^{3/2}}{k_j^{3/2}}\sqrt{\frac{\tanh k_i d}{\tanh k_j d}}+\sqrt{\frac{k_i\tanh k_j d}{k_j\tanh k_i d}})\cos\theta$$

$$-\frac{1}{2}\frac{k_i}{k_j}\cos^2\theta(\tanh k_i d\cdot\tanh k_j d)^{-1}$$

$$+[\frac{1}{2}(\sqrt{\frac{k_i\tanh k_i d}{k_j\tanh k_j d}}-1)\operatorname{sech}^2 k_j d$$

$$-\frac{1}{2}(\frac{k_i^2}{k_j^2}-2\frac{k_i}{k_j}\cos\theta+1)^{1/2}(\tanh k_j d+\frac{k_i}{k_j}\tanh k_i d)$$

$$\times\tanh((\frac{k_i^2}{k_j^2}-2\frac{k_j}{k_j}\cos\theta+1)^{1/2}k_j d)$$

$$+\frac{1}{2}(\frac{k_i^2}{k_j^2}-2\frac{k_i}{k_j}\cos\theta+1)+(\frac{k_i}{k_j}\cos\theta-1)$$

$$\times\sqrt{\frac{k_i\tanh k_i d}{k_j\tanh k_j d}}][\frac{k_i}{k_j}\operatorname{csch}2k_i d+(\sqrt{\frac{k_i}{k_j}}-\sqrt{\frac{\tanh k_j d}{\tanh k_i d}})$$

$$\times(1+\frac{\cos\theta}{\tanh k_i d\cdot\tanh k_j d})\sqrt{\tanh k_i d\cdot\tanh k_j d}$$

$$
\begin{aligned}
&-\sqrt{\frac{k_j \tanh k_j d}{k_i \tanh k_i d}}\operatorname{csch} 2k_j d\,]\,/\,[\,\sqrt{\frac{k_i}{k_j}\tanh k_i d}\\
&-\sqrt{\tanh k_j d}\,)^2-(\frac{k_i^2}{k_j^2}-2\frac{k_i}{k_j}\cos\theta+1\,)^{1/2}\\
&\times\tanh\,(\,(\frac{k_i^2}{k_j^2}-2\frac{k_i}{k_j}\cos\theta+1\,)^{1/2}k_j d\,)\,]\\
&+[\,-\frac{1}{2}\,(\,1+\sqrt{\frac{k_i \tanh k_i d}{k_j \tanh k_j d}})\operatorname{sech}^2 k_j d\\
&-\frac{1}{2}(\frac{k_i^2}{k_j^2}+2\frac{k_i}{k_j}\cos\theta+1\,)^{1/2}\,(\,\tanh k_j d+\frac{k_i}{k_j}\tanh k_i d)\\
&\times\tanh(\,(\frac{k_i^2}{k_j^2}+2\frac{k_i}{k_j}\cos\theta+1\,)^{1\,2}k_j d\,)\\
&+\frac{1}{2}(\frac{k_i^2}{k_j^2}+2\frac{k_i}{k_j}\cos\theta+1\,)+(\frac{k_i}{k_j}\cos\theta+1\,)\\
&\times\sqrt{\frac{k_i \tanh k_i d}{k_j \tanh k_j d}}]\,[\frac{k_i}{k_j}\operatorname{csch} 2k_i d+(\,\sqrt{\frac{k_i}{k_j}}+\sqrt{\frac{\tanh k_j d}{\tanh k_i d}})\\
&\times(\frac{\cos\theta}{\tanh k_i d\cdot\tanh k_j d}-1\,)\sqrt{\tanh k_i d\cdot\tanh k_j d}\\
&+\sqrt{\frac{k_j \tanh k_j d}{k_i \tanh k_i d}}\operatorname{csch} 2k_j d\,]\\
&/\,[\,(\,\sqrt{\frac{k_i}{k_j}}\tanh k_i d+\sqrt{\tanh k_j d}\,)^2\\
&-(\frac{k_i^2}{k_j^2}+2\frac{k_i}{k_j}\cos\theta+1\,)^{1/2}\\
&\times\tanh\,(\,(\frac{k_i^2}{k_j^2}+2\frac{k_i}{k_j}\cos\theta+1\,)^{1/2}k_j d\,)\,]\,\}\,,\\
&i\,,\,j=1\,,\,2\,,\,i\neq j
\end{aligned}
\qquad (5.31)
$$

對照於單一前進重力波之週波率的考量下，即由(7.2)式中之最後一式的比較，這顯然可得知，兩來源前進重力波列相交會後，因其間所產生的相互作用，會造致各來源波列的週波率受到衝擊的影響。就如上式，以下標 i 、 j (i ， j = 1 ， 2 ， i ≠ j) 所示的兩來源波列交會相互作用後，第 i 的來源波列其週波率受到第 j 的來源波列之衝擊的影響量爲 $f_i(k_i, k_j, d, \theta)\, k_j^2 a_j^2 \sigma_0^{(i)}$ (至第三階次解) ; 反之，第 j 的來源波列者受到第 i 的來源波列之衝擊影響量爲 $f_j(k_i, k_j, d, \theta)\, k_j^2 a_i^2 \sigma_0^{(j)}$ 。方便上，稱 $f_i(k_i, k_j, d, \theta)$ 爲週波率影響效應函數 (the effect function by interaction on frequency) ，因此，吾人明確可知，只要此效應函數之有關變化特性能被清楚地描述，則兩波交會所造致之各波列的週波率受衝擊變化就可被整體性地掌握。此效應函數 f_i ，於各種水深下，隨相交會的兩波列之波數比與它們前進方向之夾角間的變化特性如下圖 3 所示。

依圖 3a ～ f 可清楚地得知函數 $f_i(k_i, k_j, d, \theta)$ 整體的變化特性，即可對所造致各波列週波率受衝擊的影響做整體性明確的描述，如後。當相對水深 $k_j d$ ($= 2\pi d / L_j$) 由大變小時，即所考慮的波動系統所在的水深由深到淺的情況，則所對應的 f_i 值會隨之增大的趨勢，此現象是符合波動本質特性的；蓋因當水深較淺時，波動非線性的本質較爲顯著，因而，兩波交會間因相互作用所生的非線性效應，相對而言，亦當趨爲強烈。至於 f_i 值因交會的兩波之前進方向夾角 θ 而引起的變化情況，亦可由圖 3 直接明顯地看出；當 θ 低於 90° 內，即它們的傳遞脈動具有同向分量時，則 f_i 爲正值，此時各波列的週波率受衝擊的影響成遞增者，反之，當 θ 略大於 90° 後，即兩波列的傳遞脈動具有逆向分量時，則 f_i 爲負值，此時各波列的週波率受衝擊的影響成遞減者而發生所謂的逆變效應現象。上述所言的各波列週波率受其間交角 θ 的影響結果，從一般作用力學原則的觀點來考察是符合的；即正衝時則效應加強，反衝時則爲倒逆而減低，且以 $\theta = 90°$ 相交時爲其中間而分隔之，如圖 3 所示。再者，從圖 3 中亦顯示， f_i 值受交會的兩波列之波數比 $k_i / k_j = L_j / L_i$ (或言它們波長比的倒數) 影響的變化特性走向，即當 k_i / k_j 增大時，則 f_i 值隨之增強 (除在相當淺水， $\theta < 90°$ ， $k_i / k_j \approx 1$ 之附近處外) ，這是符合短波被長波拖著走的一般眞實現況，見 Longuet - Higgins & Stewart (1960) 。

(2)共振現象

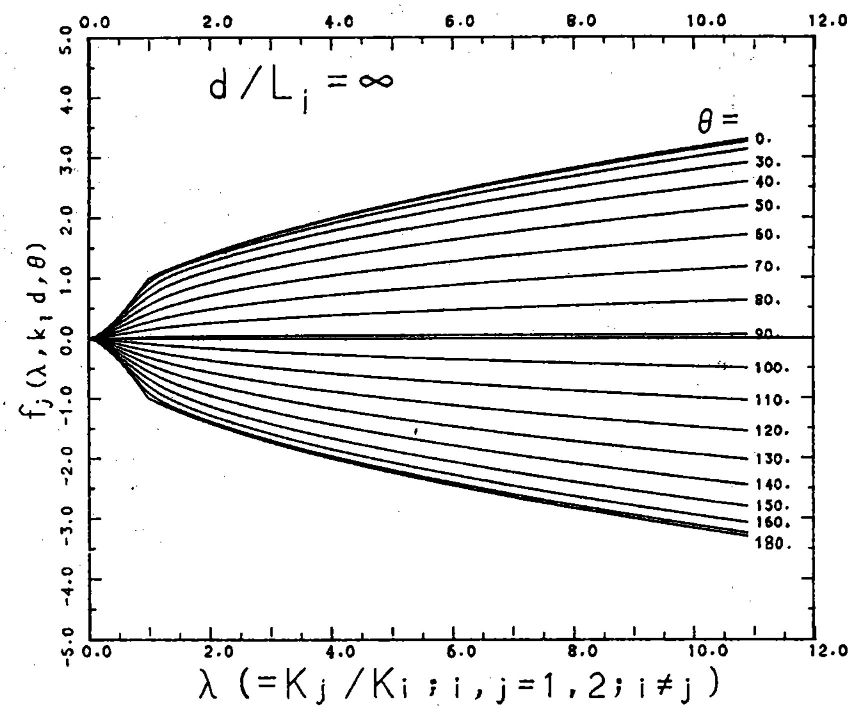


圖 3-a 相對水深$d/L_i=\infty$時，週波率影響效應函數f_j隨交會的兩波間之夾角θ與它們波數比k_j/k_i之變化

Fig. 3-a The effect on frequency by interaction, f_j as function of the intersecting angle θ and the wavelength ratio k_j/k_i of two gravity wave trains in depth $d/L_i=\infty$.

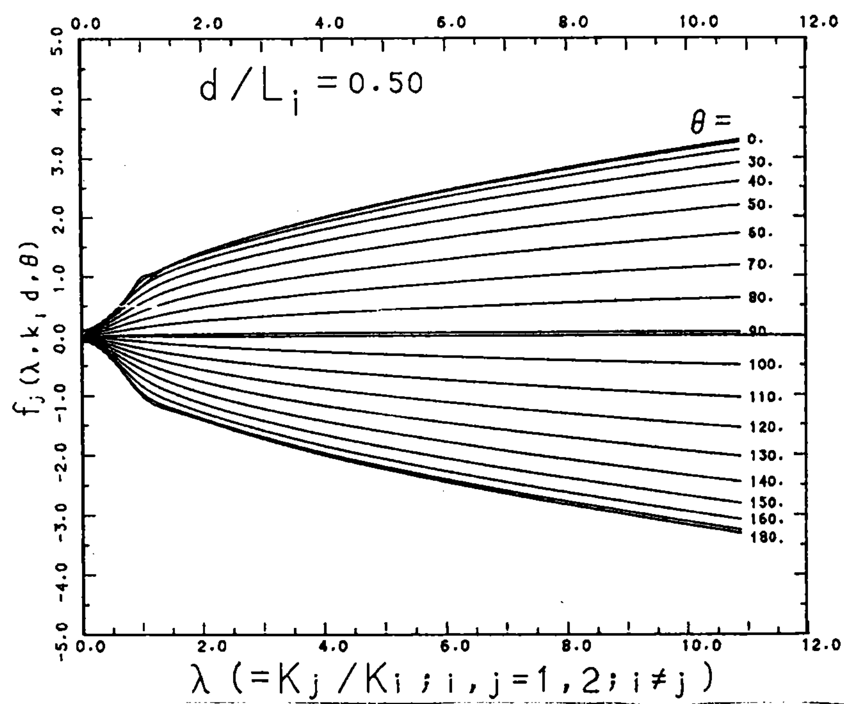


圖 3-b 相對水深$d/L_i=0.50$時，週波率影響效應函數f_j隨交會的兩波間之夾角θ與它們波數比k_j/k_i之變化

Fig. 3-b The effect on frequency by interaction, f_j as function of the intersecting angle θ and the wavelength ratio k_j/k_i of two gravity wave trains in depth $d/L_i=0.50$.

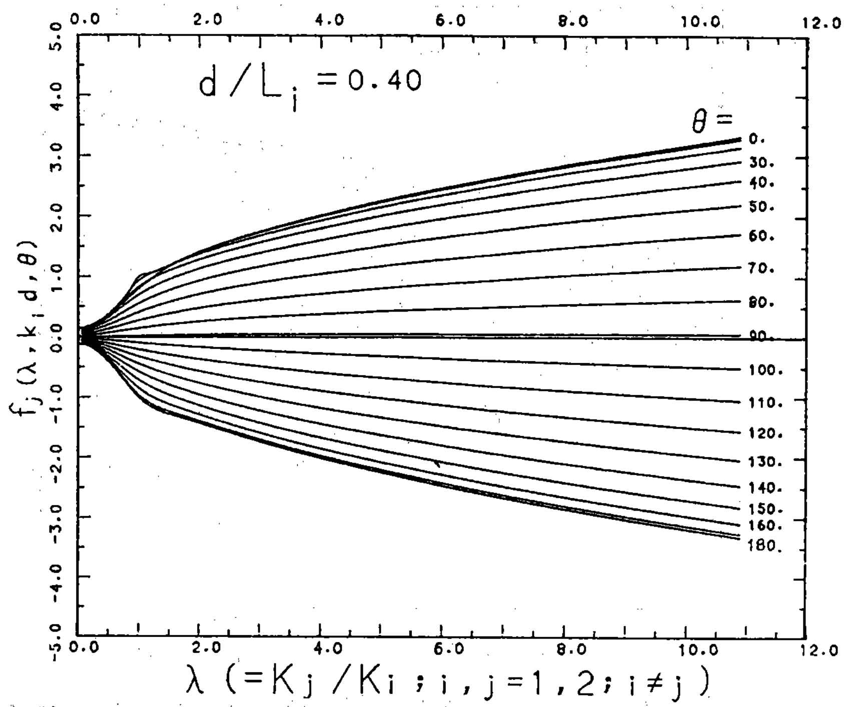


圖 3-c 相對水深d/L_i=0.40時，週波率影響效應函數f_j隨交會的兩波間之夾角θ與它們波數比k_j/k_i之變化

Fig. 3-c The effect on frequency by interaction, f_j as function of the intersecting angle θ and the wavelength ratio k_j/k_i of two gravity wave trains in depth d/L_i=0.40.

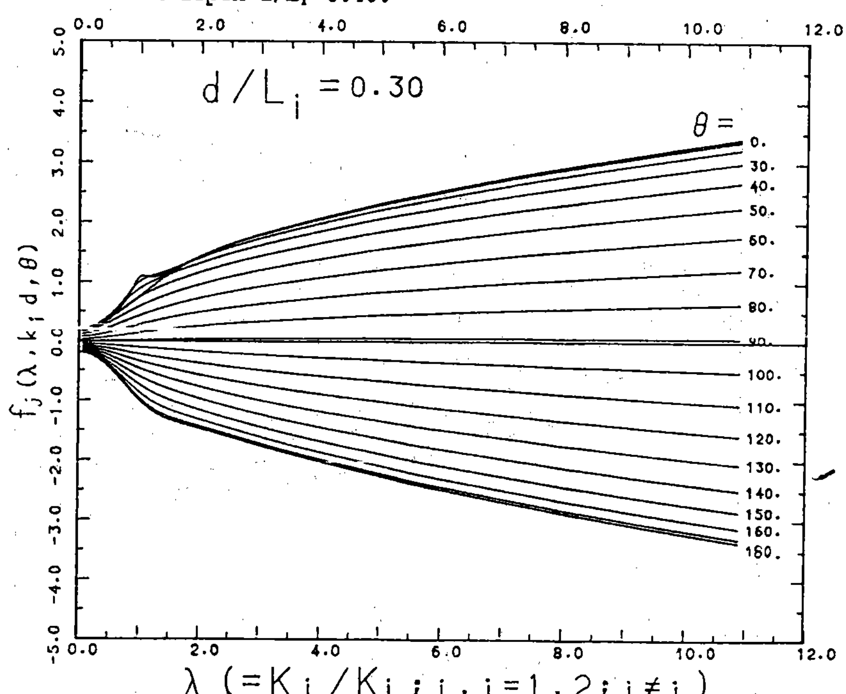


圖 3-d 相對水深d/L_i=0.30時，週波率影響效應函數f_j隨交會的兩波間之夾角θ與它們波數比k_j/k_i之變化

Fig. 3-d The effect on frequency by interaction, f_j as function of the intersecting angle θ and the wavelength ratio k_j/k_i of two gravity wave trains in depth d/L_i=0.30.

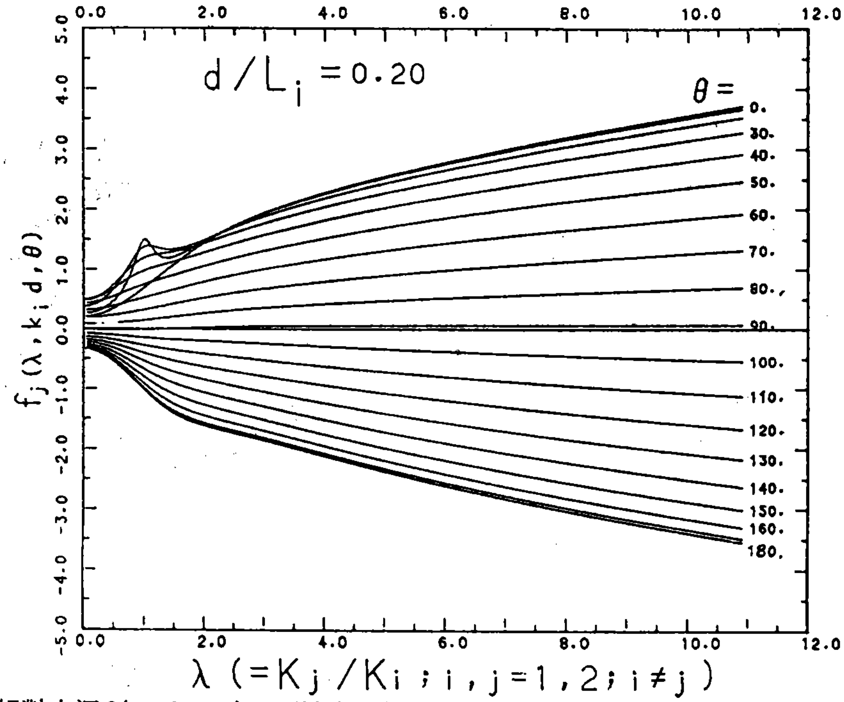


圖 3-e 相對水深d/L_i=0.20時，週波率影響效應函數f_j隨交會的兩波間之夾角θ與它們波數比k_j/k_i之變化

Fig. 3-e The effect on frequency by interaction, f_j as function of the intersecting angle θ and the wavelength ratio k_j/k_i of two gravity wave trains in depth d/L_i=0.20.

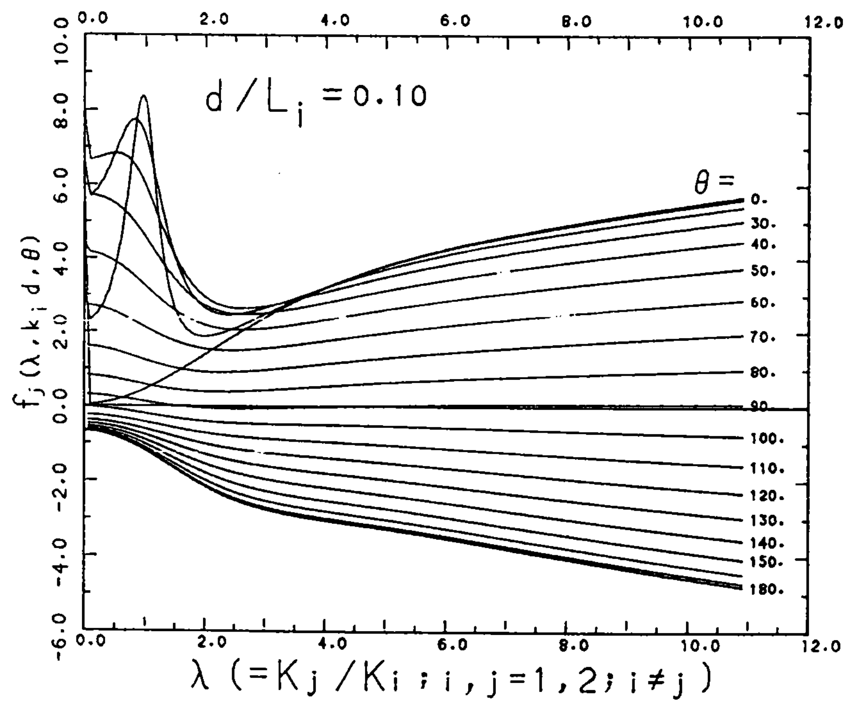


圖 3-f 相對水深d/L_i=0.10時，週波率影響效應函數f_j隨交會的兩波間之夾角θ與它們波數比k_j/k_i之變化

Fig. 3-f The effect on frequency by interaction, f_j as function of the intersecting angle θ and the wavelength ratio k_j/k_i of two gravity wave trains in depth d/L_i=0.10.

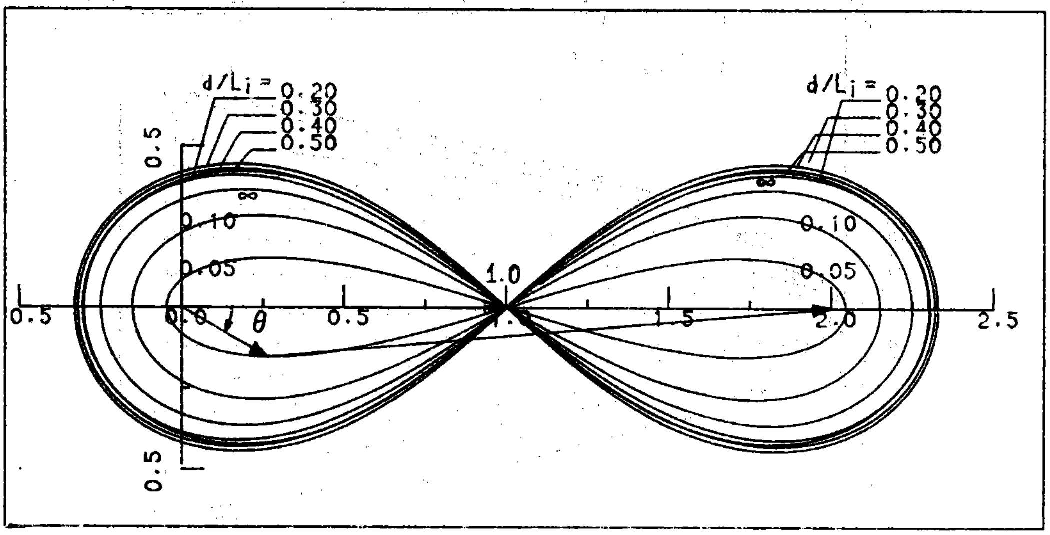


圖 4 於各種相對水深d/L_i中，兩波交會產生共振情況時，各成份波之波數向量關係圖 (三階解下)

Fig.4 The resonance loop of the interaction between two gravity wave trains in various depth d/L_i.

Phillips（1960）從理論解析中，推導出兩波交會時會產生所謂的共振現象，因而，引發了眾多的學者對兩波交會的問題廣泛地關注，並針對此一特別的現象進行一系列深入的研究與探討，如前言中所述。雖然如此，對這種共振現象的清楚闡述似都止於深水情況，鮮少論及於有限等深水中的詳情。因此，基於對兩波交會所形成的流場機構有較全盤性完整描述的本文目的下，於此得將它擴及到有限等深水中情況。由（5.27）式所示之流速勢函數解 ϕ，吾人即刻可知，在第三階解下，於水深 d 中，兩波交會會產生所謂的共振現象（resonance），其條件為

$$
\begin{aligned}
&(2\sigma_0^{(1)}-\sigma_0^{(2)})^2=g\,|2\vec{k}_1-\vec{k}_2|\tanh|2\vec{k}_1-\vec{k}_2|\,d\\
\text{或}\quad&(\sigma_0^{(1)}-2\sigma_0^{(2)})^2=g\,|\vec{k}_1-2\vec{k}_2|\tanh|\vec{k}_1-2\vec{k}_2|\,d
\end{aligned}
\qquad (5.32)
$$

（5.32）式經簡化處理後，可換寫成

$$
\begin{aligned}
&4\tanh k_i d-4(\lambda\tanh k_i d\cdot\tanh\lambda k_i d)^{1/2}+\lambda\tanh\lambda k_i d\\
&-(4-4\lambda\cos\theta+\lambda^2)^{1/2}\tanh[(4-4\lambda\cos\theta+\lambda^2)^{1/2}k_i d]=0\,,\\
&\lambda=k_j/k_i\,,\ i\,,\ j=1\,,\ 2\,,\ i\neq j
\end{aligned}
\qquad (5.33)
$$

故依（5.33）式可得，於水深 d（或 $k_i d$）給定下，所考慮的兩波交會可能出現共振的情況，至第三階時，可被明確地決定，如下圖 4 所示。

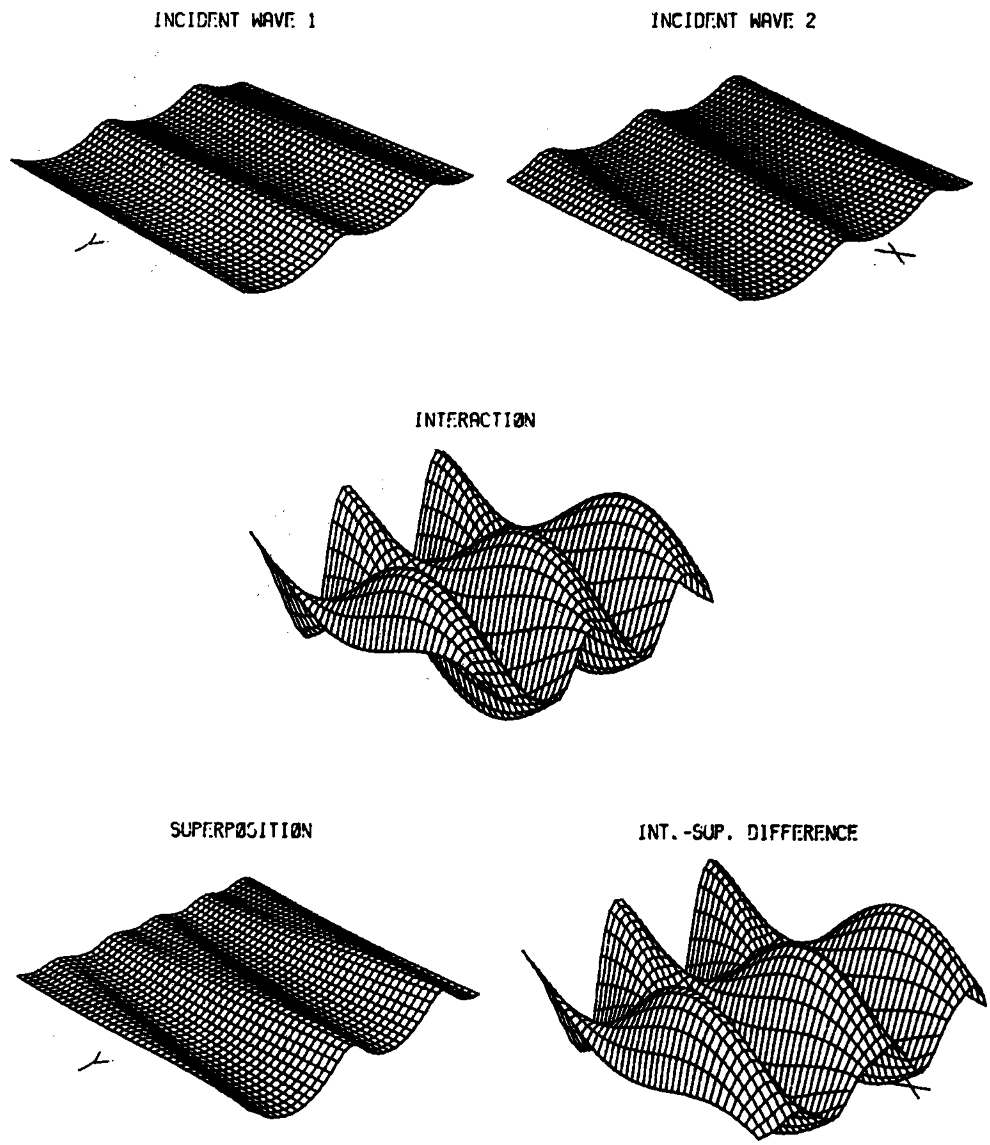
有關所考慮的兩波交會系統發生共振情況時，其波動流場特性的細節，限於篇幅起見，將另闢章節詳論於後。不過，於此處之較清楚完整的本文解析下，再次地給予共振現象之會出現於兩波交會系統中的可能性一個較明確性的佐證。

以上，本文所解析的結果，當波動系統是在深水情況時，即在 $d\to\infty$（或言 $k_i d$、$k_j d\to\infty$）下，則與筆者（1988、1989）所述之深海情況者完全一致之。

(3)水位（或波形）之脈動

為對所考慮的兩波列交會波動系統，於各種情況下，具體地洞識其於時空中所會呈現出之整體的脈動現象，於此將對其之整體實際展現於外的水位脈動或言波形者進行考量伸述之；即對水深由深至淺，各來源波列之波浪尖銳度從小至大，而它們前進方向間的夾角由 0° 至 180°，等之各種不同的組合情形，其所顯現出的直覺外貌之整

體波形的脈動變化特性論究之，如下之圖5所示。於該圖中，為更清楚直接地考察出於各種情況下，兩波列交會所形成之波動系統，其顯現出之整體脈動波形的整體結構及其內部的細節，則分別繪製出兩來源波列直接疊加（即各來源波列之三階解的波形直接線性相加者）與它們含相互作用而合成者及此兩者間之差異等之空間立體圖，俾利給予其很了然明澈地詳述，於下。由圖5所示之各種不同組合情況下的所有分圖，當是顯然可知，所考慮的兩波列交會波動系統，其整體呈現出的脈動波形之整個變化特性為：(1)就水深因素而言，當水深愈淺則對所造致之脈動波形之扭曲及增異量相對明顯地增強，此可由圖5－1a～f與圖5－2a～f至圖5－9a～f等之比較即刻可得知；蓋因此乃由於水深之愈淺，則波動之非線性本質特性相對增加，因而由波浪相交會間所產生出之非線性相互作用亦相對增強所致之故。(2)受來源波列之波浪尖銳度影響而言，當其他條件保持一樣時，則由圖5中之各分圖相對照下可得知，當波浪尖銳度皆大的兩來源波列相交會所形成之波動系統，其所顯現之脈動波形的扭曲與增異量亦為大，反之，當一或兩來源波列之波浪尖銳度為小者，則對所造致之脈動波形的扭曲與增異量亦為小，如見圖5－2a～f，圖5－3a～f，圖5－5a～f，圖5－6a～f，圖5－8a～5與圖5－9a～f等之比照；這種現象其原由亦為很明顯的，因為波浪之非線性本質特性是隨其波浪尖銳度之增大而增強，因此，發生於波浪相交會間的非線性相互作用量亦隨之增烈，反之，則減弱之。(3)就兩來源波列相交會間的夾角因素而言，依圖5所示之結果可發現，當僅此夾角θ有所不同時，如比照圖5－1a～f，或圖5－4a～f，及或5－7a～f等之各分圖，則由兩來源波列相交會所形成之波動系統，其整體呈現出的脈動波形發生的扭曲或增異量，於θ近於直角時為最小，且隨θ之偏離90°而逐漸加遽，至θ等於0°與180°時為兩頭的極端，於此值得注意的一事是，在$\theta=0°$時，所言的兩波列相交會所形成之波動系統會產生所謂的共振奇特狀況；其實，對這裡所述之脈動波形受交會夾角θ之此影響，於力學作用觀點上是即刻可被理解的，即是當兩來源波列在90°夾角相交會下，則其彼此間之波能力通量（Wave ener- getic flux）相垂直通過，因而所會發生之相互作用量近於最小，這對交會後之各來源波列之週波率受交會夾角θ之影響亦為如此，如見上小節中之陳述或圖4所示可知，然當交會夾角θ愈偏離90°時，則其彼此間之波能力通量愈具有同向作用或者是逆向衝擊之分量，因此，造成的非線性相互作用亦愈隨之加強，



第一列來源波列(incident waves 1)
$k_1 d = 4.0$ (相對水深)
$k_1 a_1 = 0.4$ (波浪尖銳度)

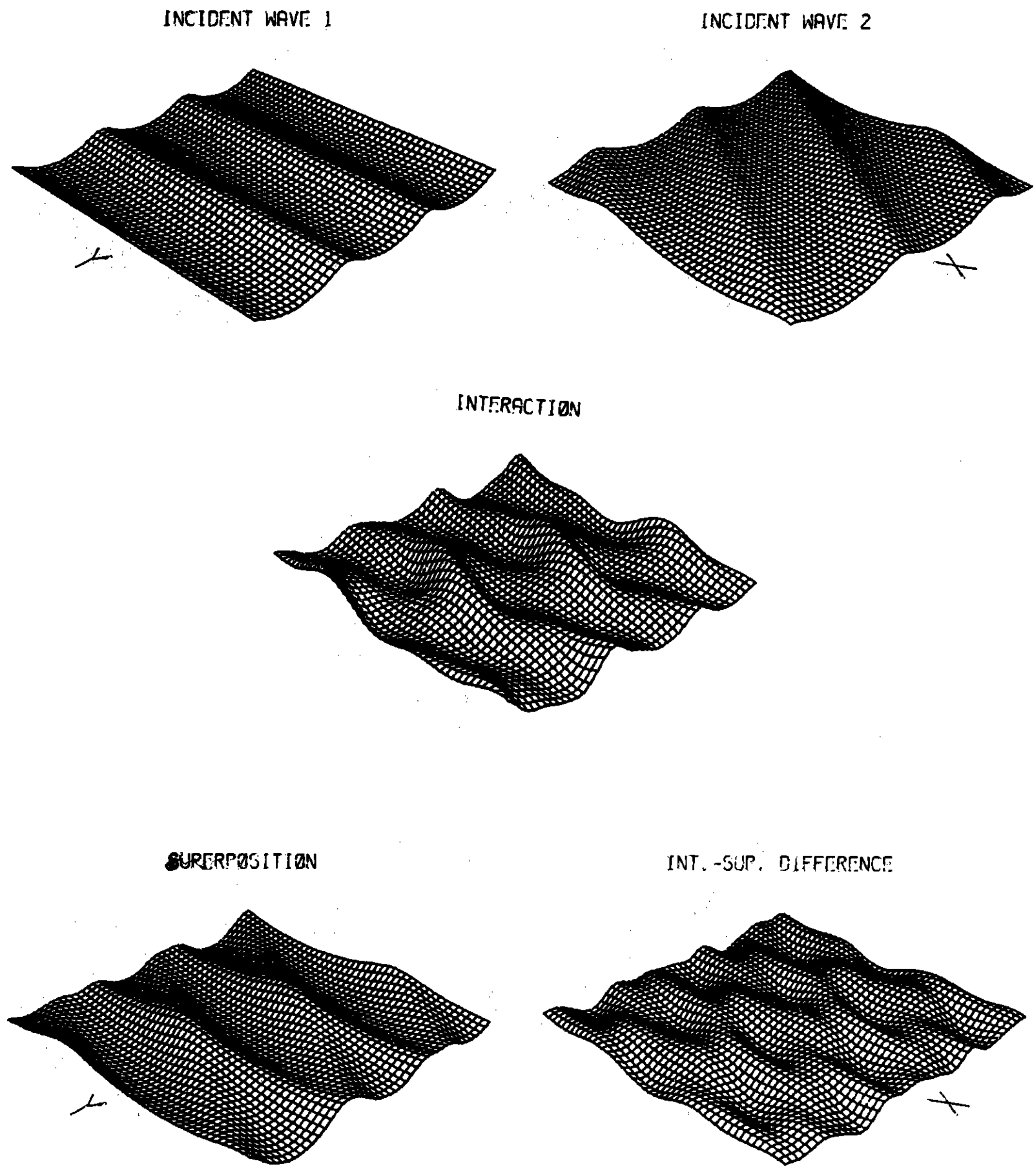
第二列來源波列(incident waves 2)
$k_2 d = 4.0$ (相對水深)
$k_2 a_2 = 0.4$ (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 10°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖 5-1a
Fig. 5-1a

圖 5 兩波交會相互作用所形成之自由表面波形脈動
Fig. 5 The fre-surface evelvation resulted from the interaction between two progressive gravity waves trains.



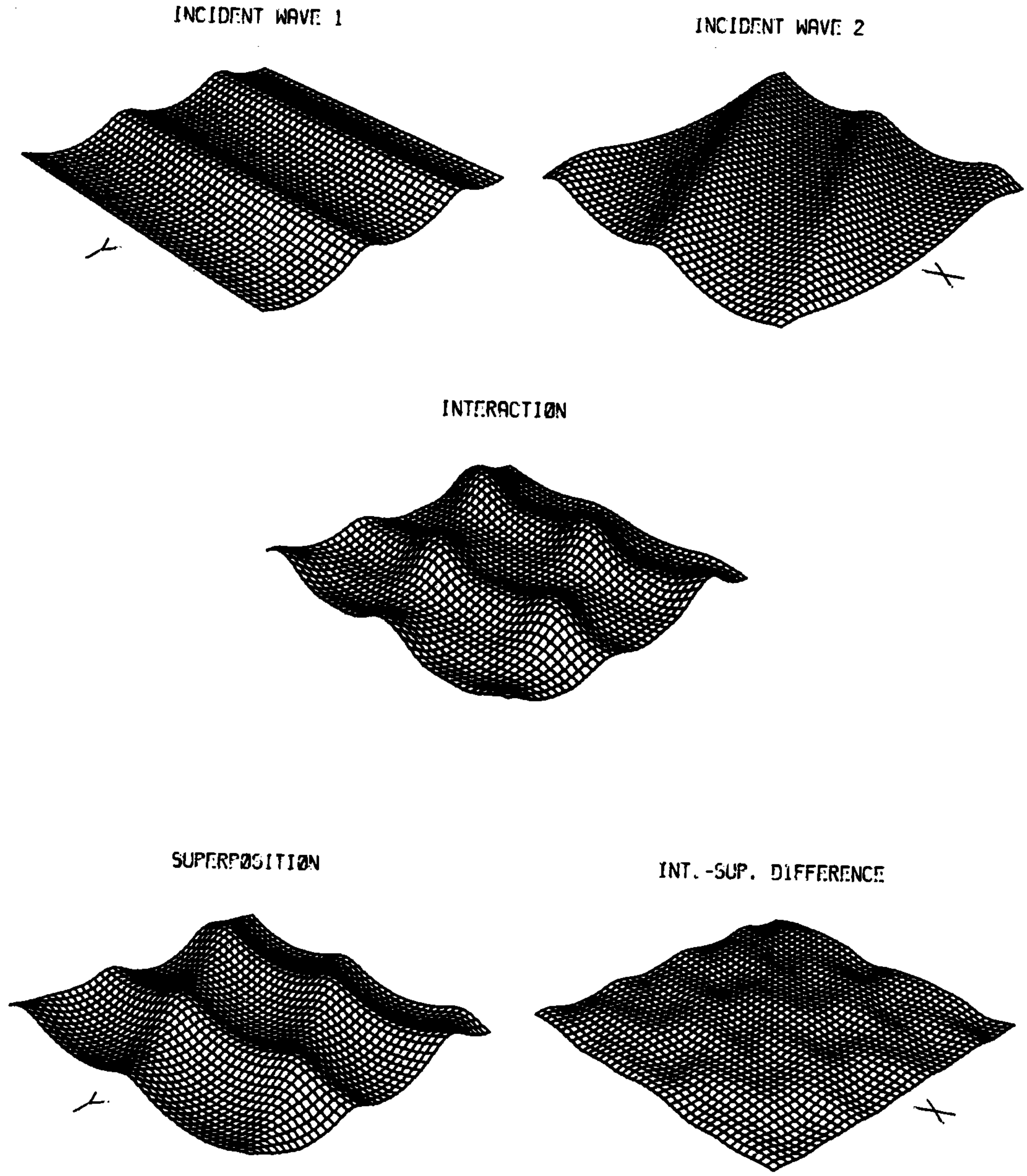
第一列來源波列(incident waves 1)
$k_1 d$ = 4.0 (相對水深)
$k_1 a_1$= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2 d$ = 4.0 (相對水深)
$k_2 a_2$= 0.4 (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 30°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-1b
Fig. 5-1b



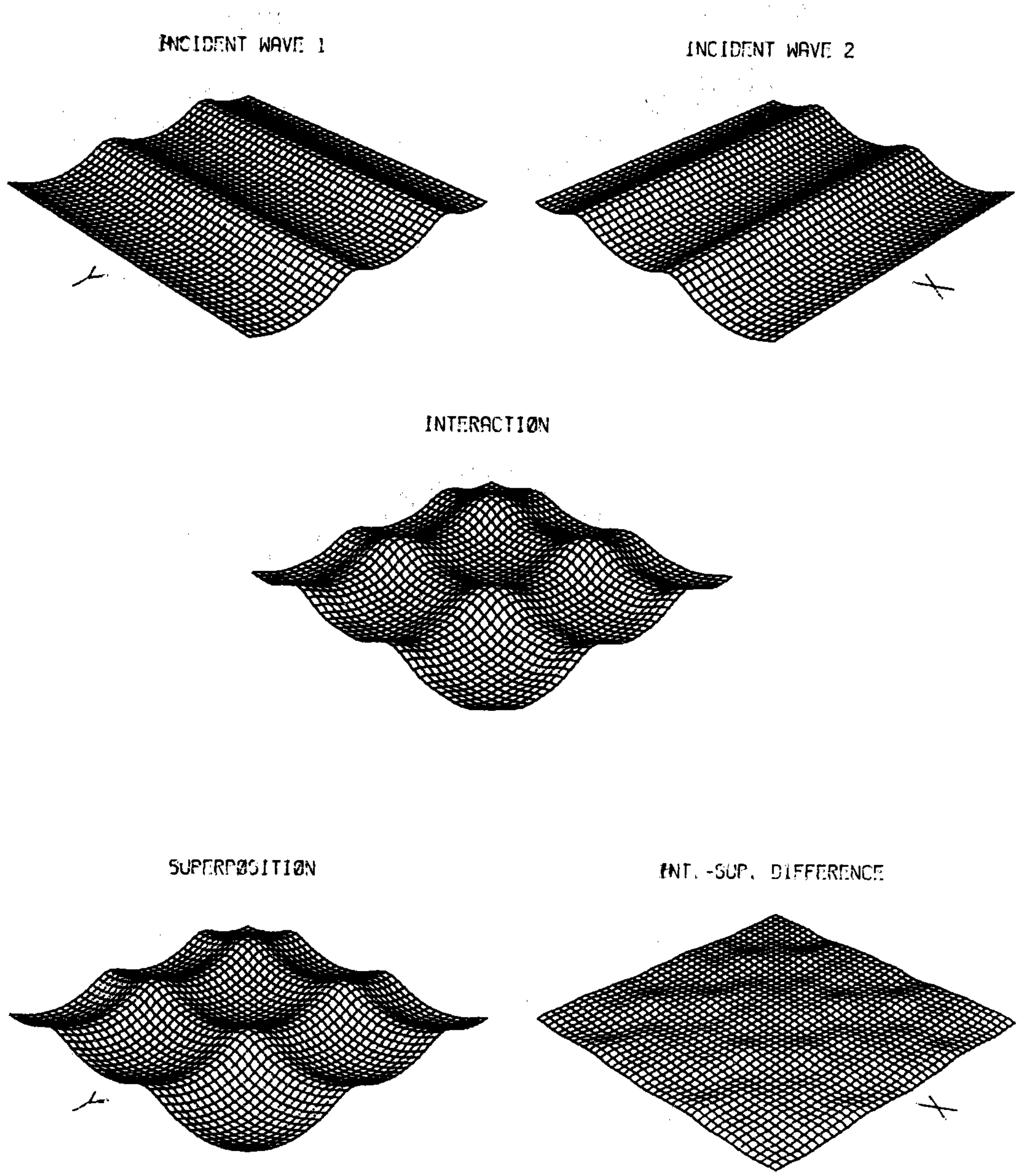
第一列來源波列(incident waves 1)
k_1d = 4.0 (相對水深)
k_1a_1= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 4.0 (相對水深)
k_2a_2= 0.4 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 60^\circ$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-1c
Fig. 5-1c



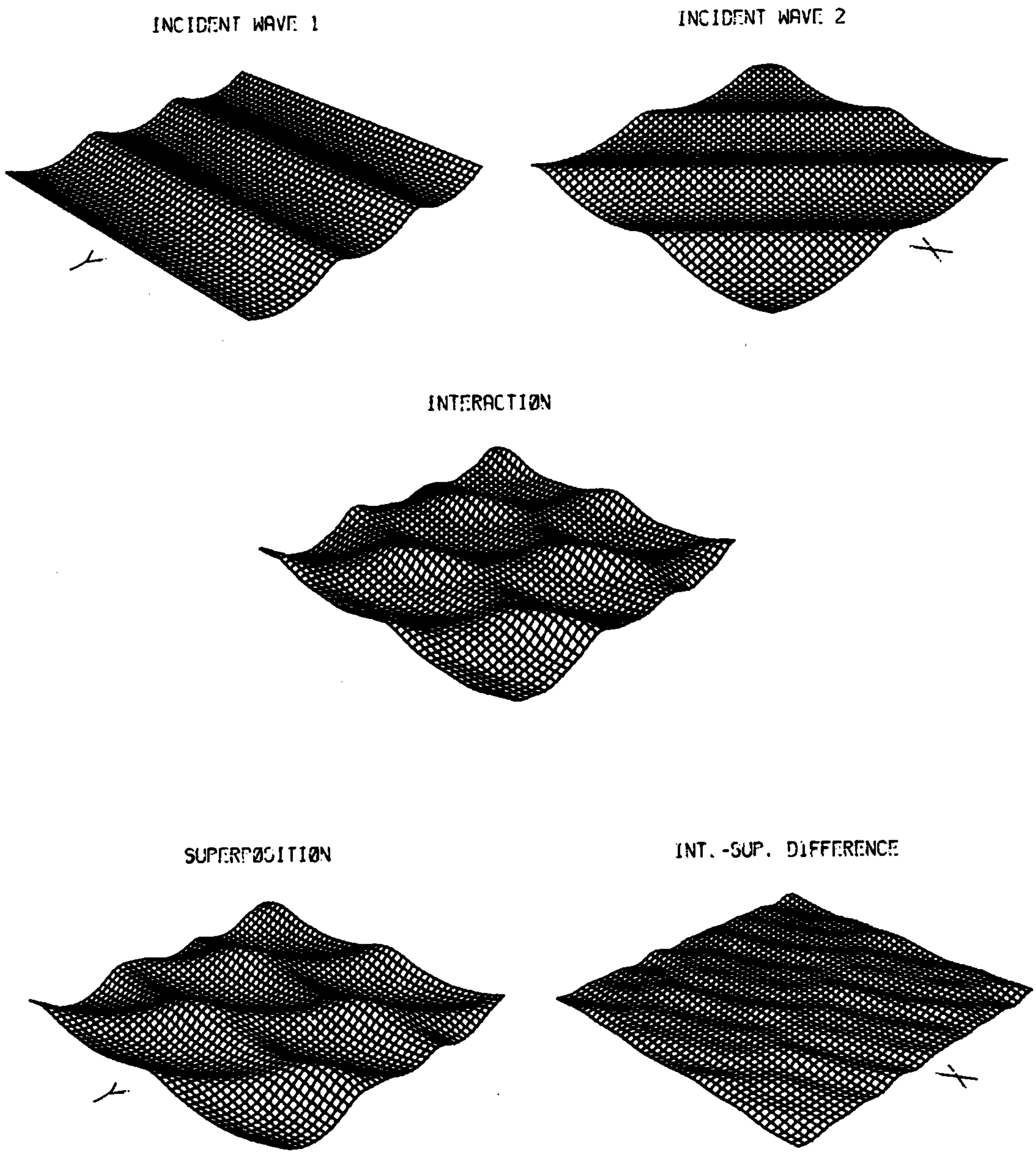
第一列來源波列(incident waves 1)
$k_1d = 4.0$ (相對水深)
$k_1a_1= 0.4$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2d = 4.0$ (相對水深)
$k_2a_2= 0.4$ (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 90°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-1d
Fig. 5-1d



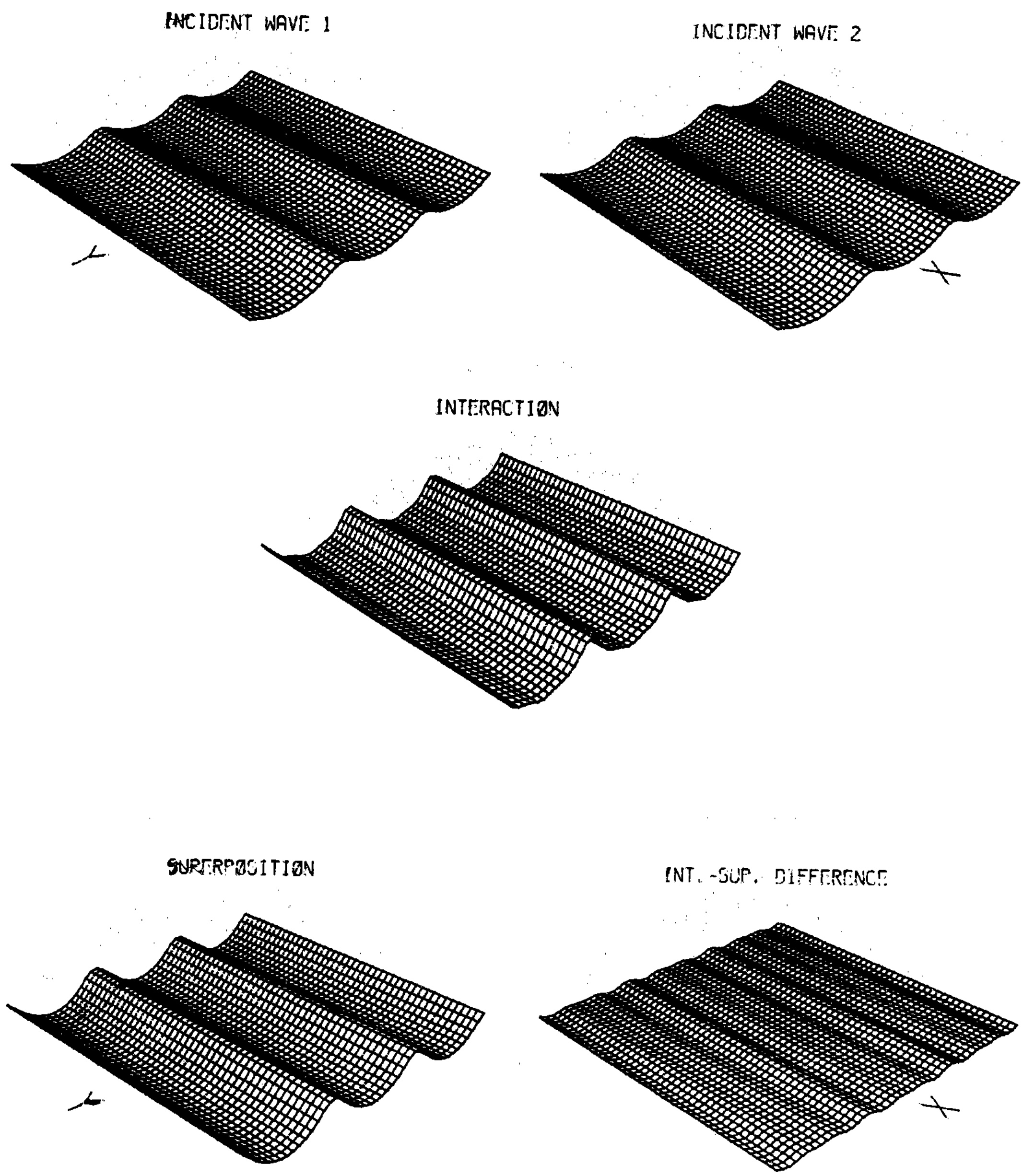
第一列來源波列(incident waves 1)
k_1d = 4.0 (相對水深)
k_1a_1= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 4.0 (相對水深)
k_2a_2= 0.4 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2$= 135°

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-1e
Fig. 5-1e



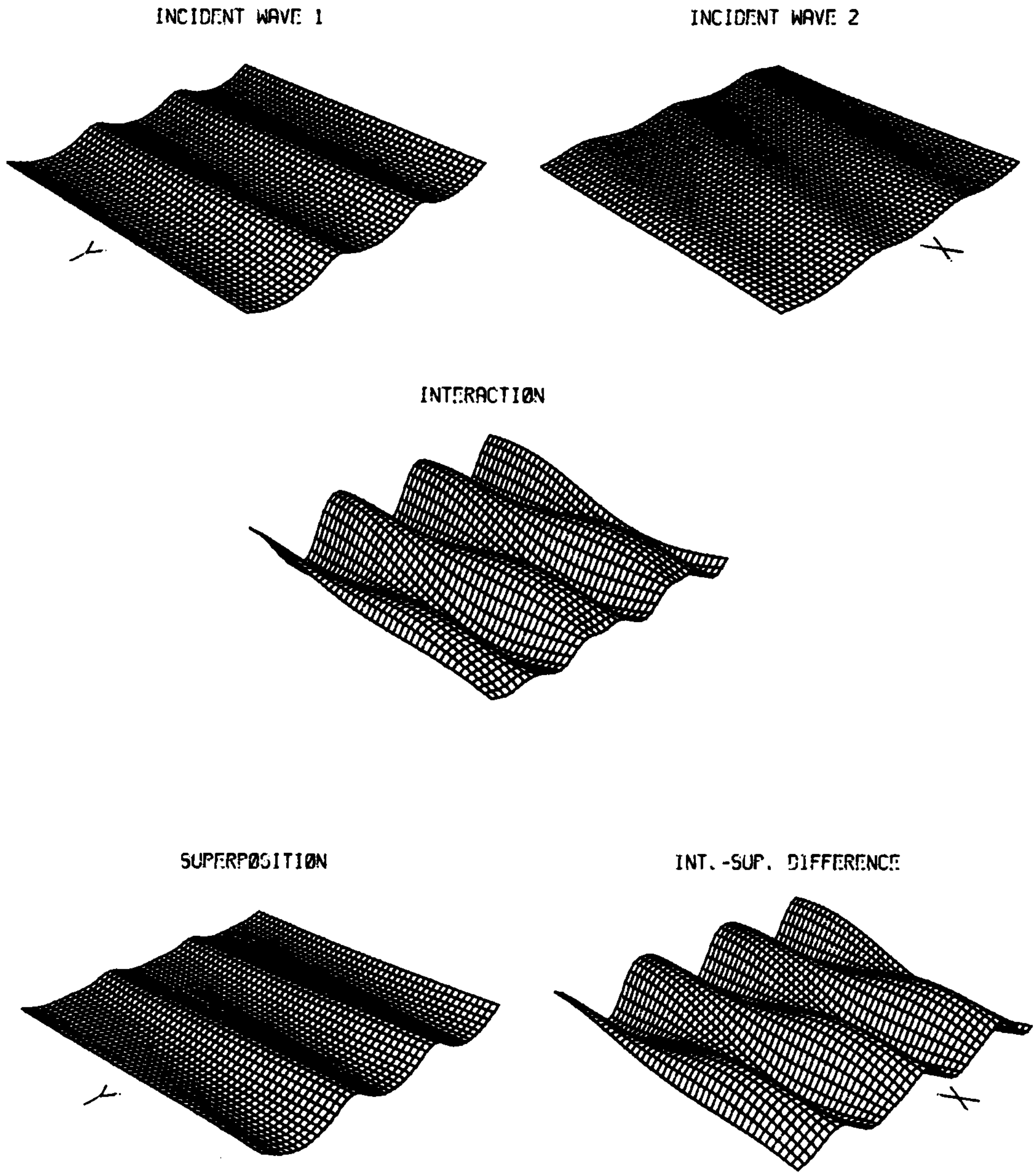
第一列來源波列(incident waves 1)
k_1d = 4.0 (相對水深)
k_1a_1= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 4.0 (相對水深)
k_2a_2= 0.4 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2$= 180°

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-1f
Fig. 5-1f



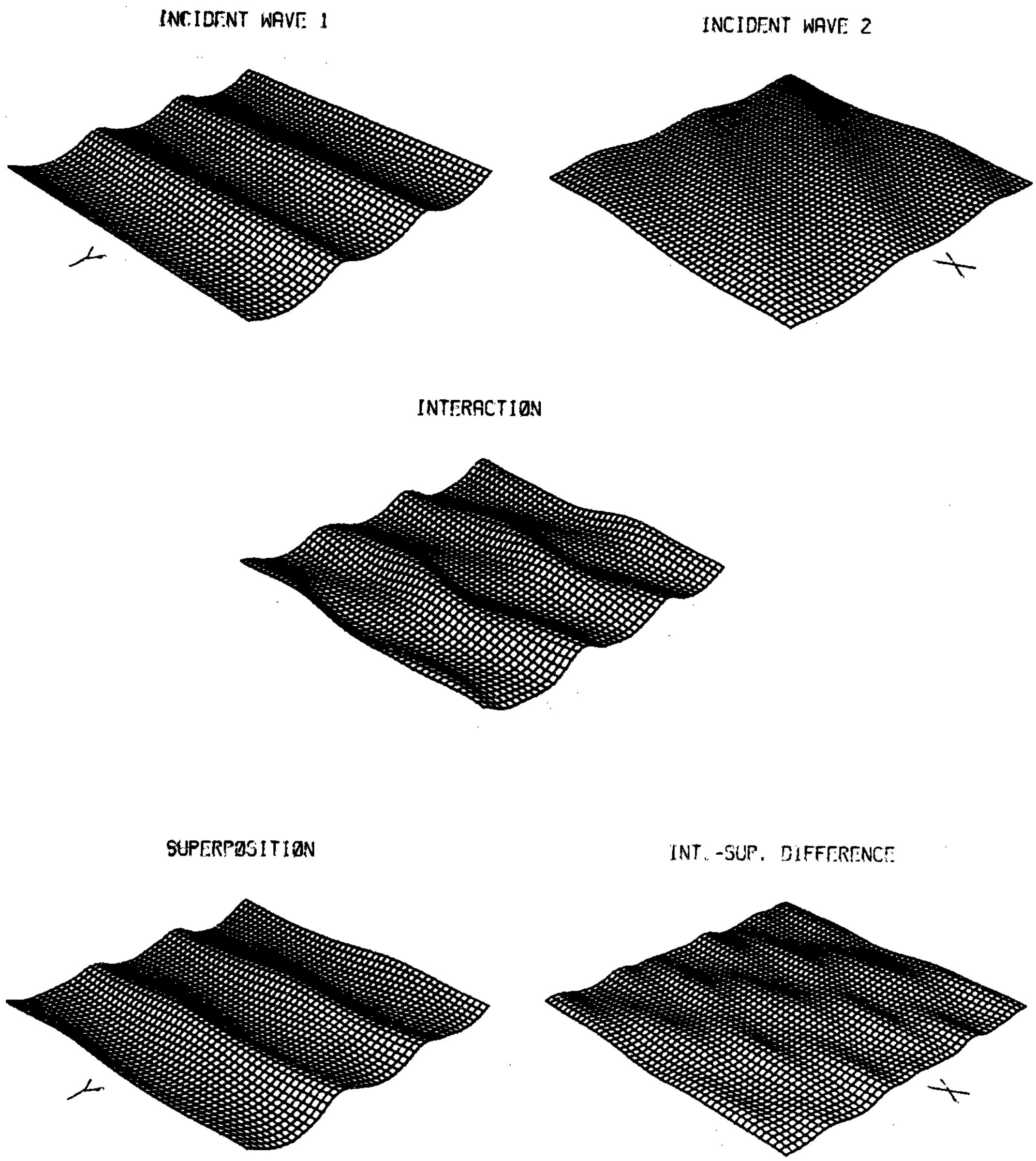
第一列來源波列(incident waves 1)
k_1d = 4.0 (相對水深)
k_1a_1= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 4.0 (相對水深)
k_2a_2= 0.2 (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 10°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-2a
Fig. 5-2a



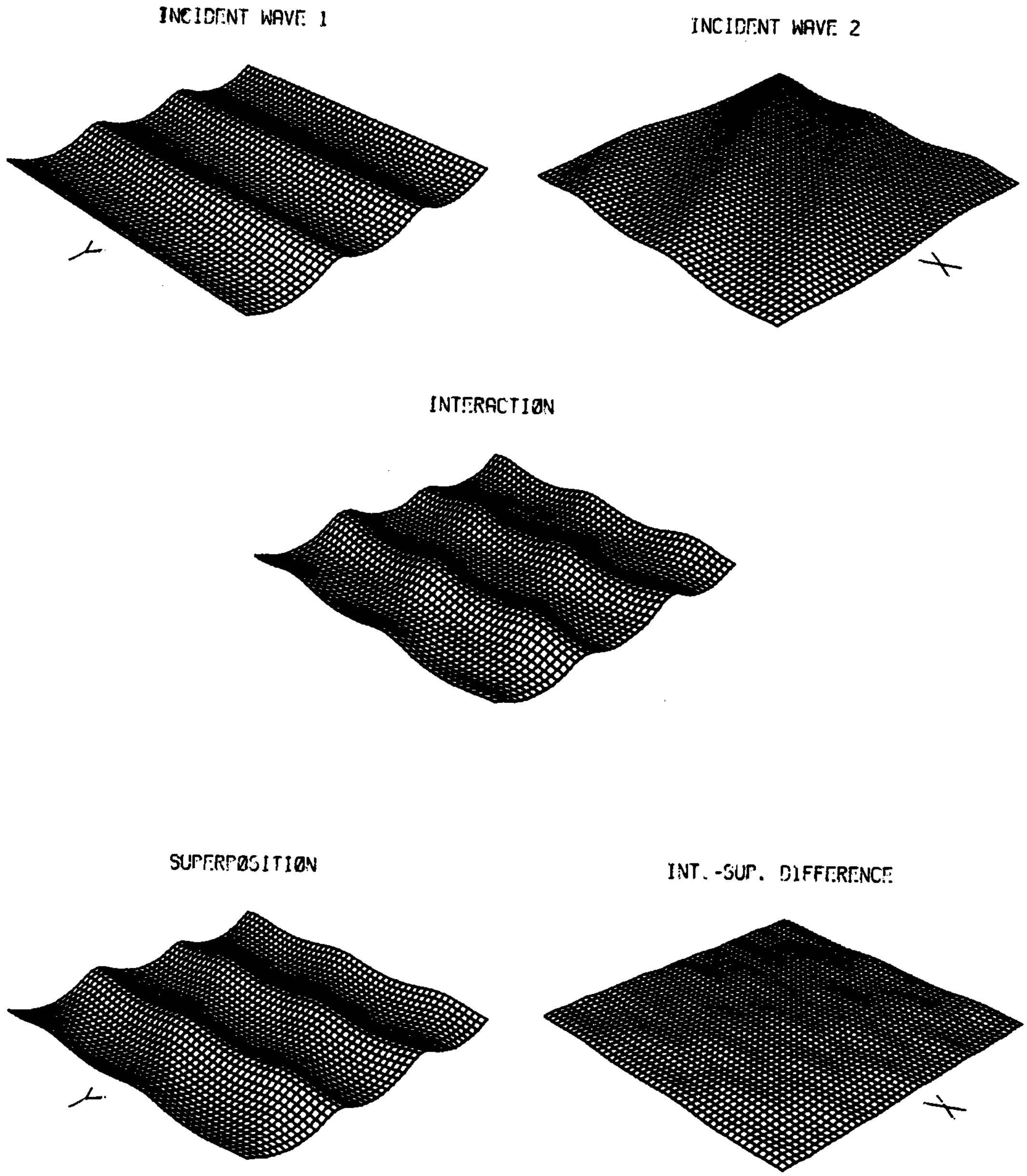
第一列來源波列(incident waves 1)
k_1d = 4.0 (相對水深)
k_1a_1= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 4.0 (相對水深)
k_2a_2= 0.2 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 30^\circ$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-2b
Fig. 5-2b



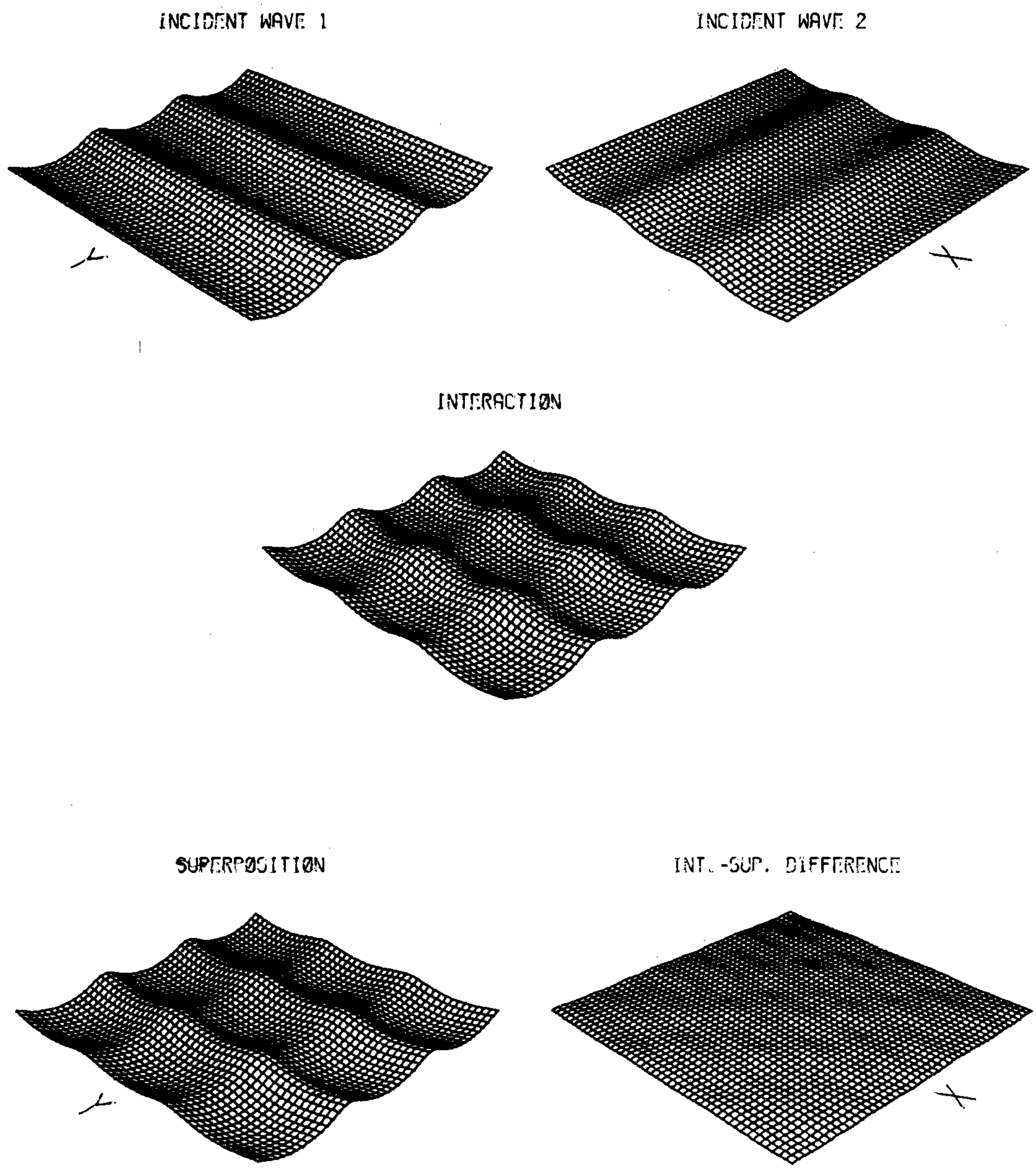
第一列來源波列(incident waves 1)
$k_1d = 4.0$ (相對水深)
$k_1a_1 = 0.4$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2d = 4.0$ (相對水深)
$k_2a_2 = 0.2$ (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 60°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION = 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-2c
Fig. 5-2c



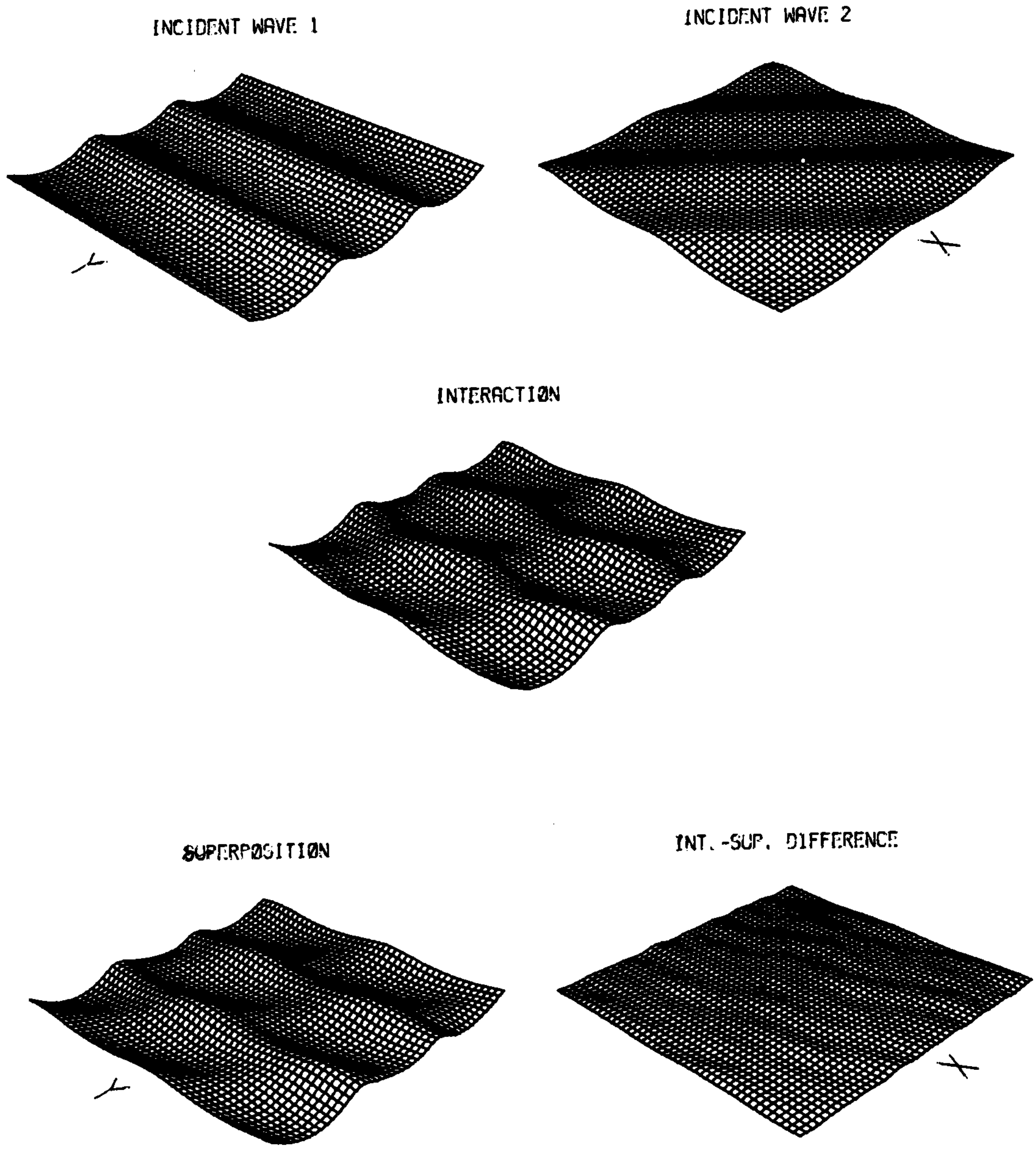
第一列來源波列(incident waves 1)
k_1d = 4.0 (相對水深)
k_1a_1= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 4.0 (相對水深)
k_2a_2= 0.2 (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 90^\circ$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-2d
Fig. 5-2d



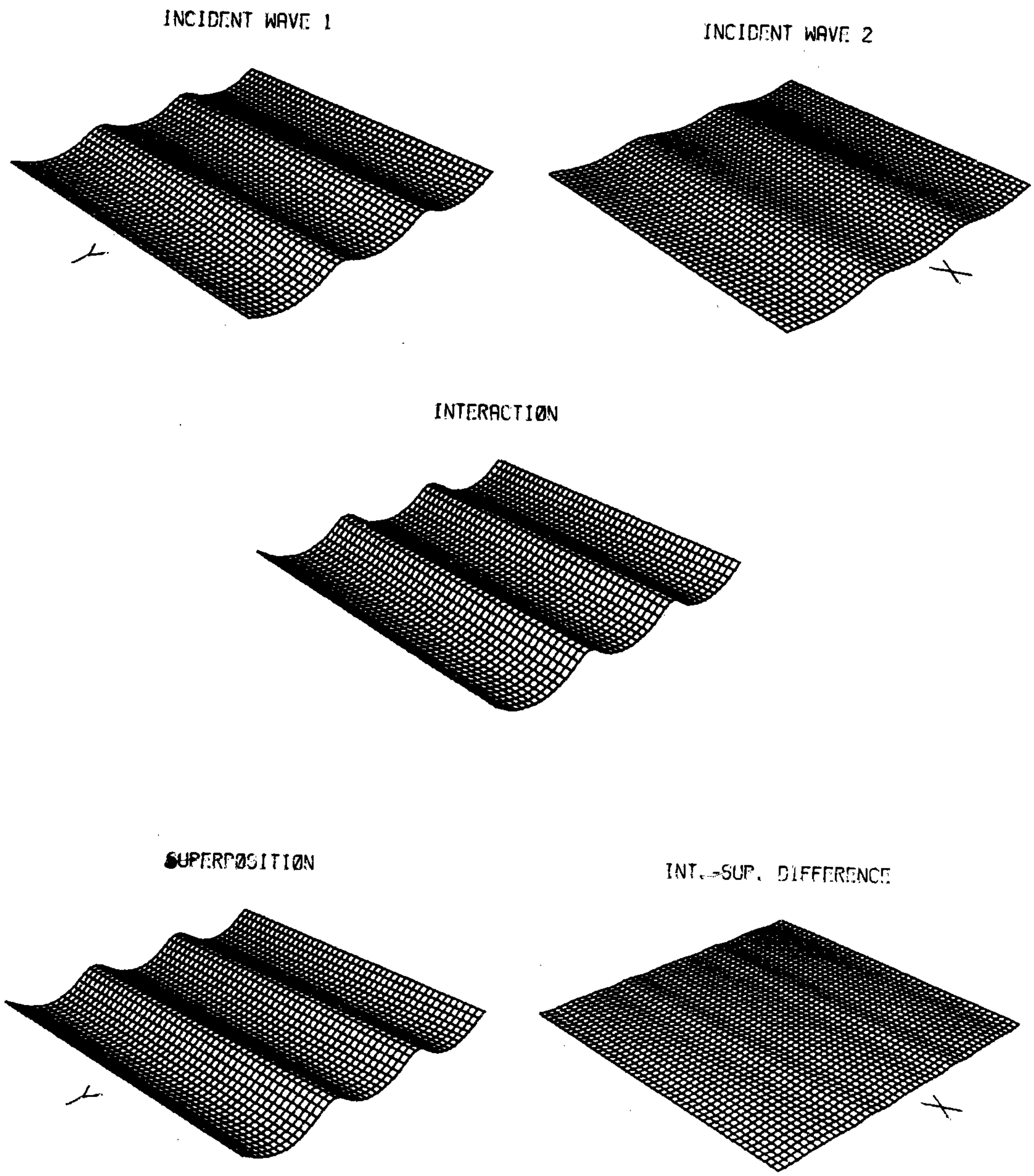
第一列來源波列(incident waves 1)
$k_1d = 4.0$ (相對水深)
$k_1a_1 = 0.4$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2d = 4.0$ (相對水深)
$k_2a_2 = 0.2$ (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 135°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-2e
Fig. 5-2e



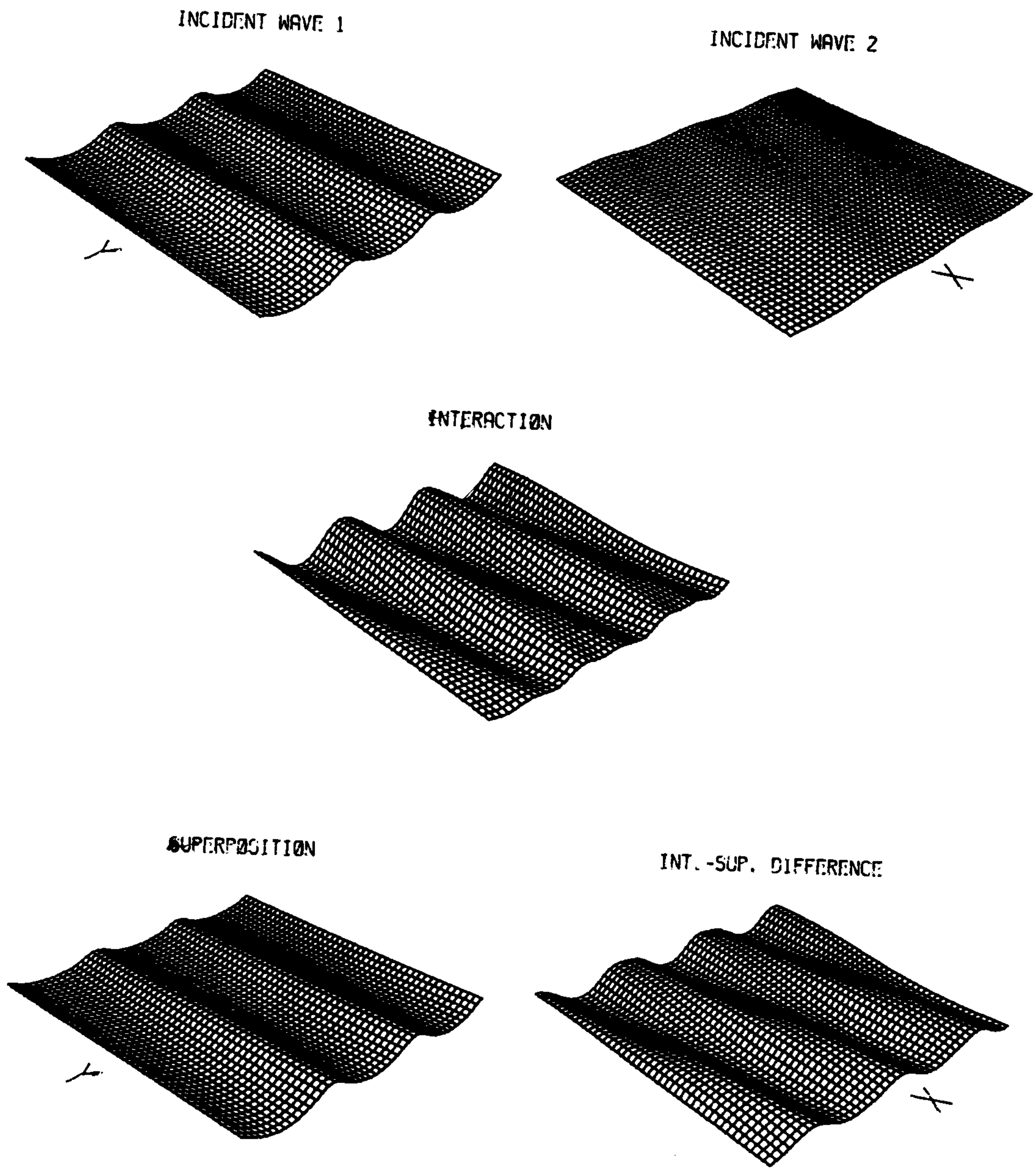
第一列來源波列(incident waves 1)
$k_1 d$ = 4.0 (相對水深)
$k_1 a_1$= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2 d$ = 4.0 (相對水深)
$k_2 a_2$= 0.2 (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 180°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-2f
Fig. 5-2f



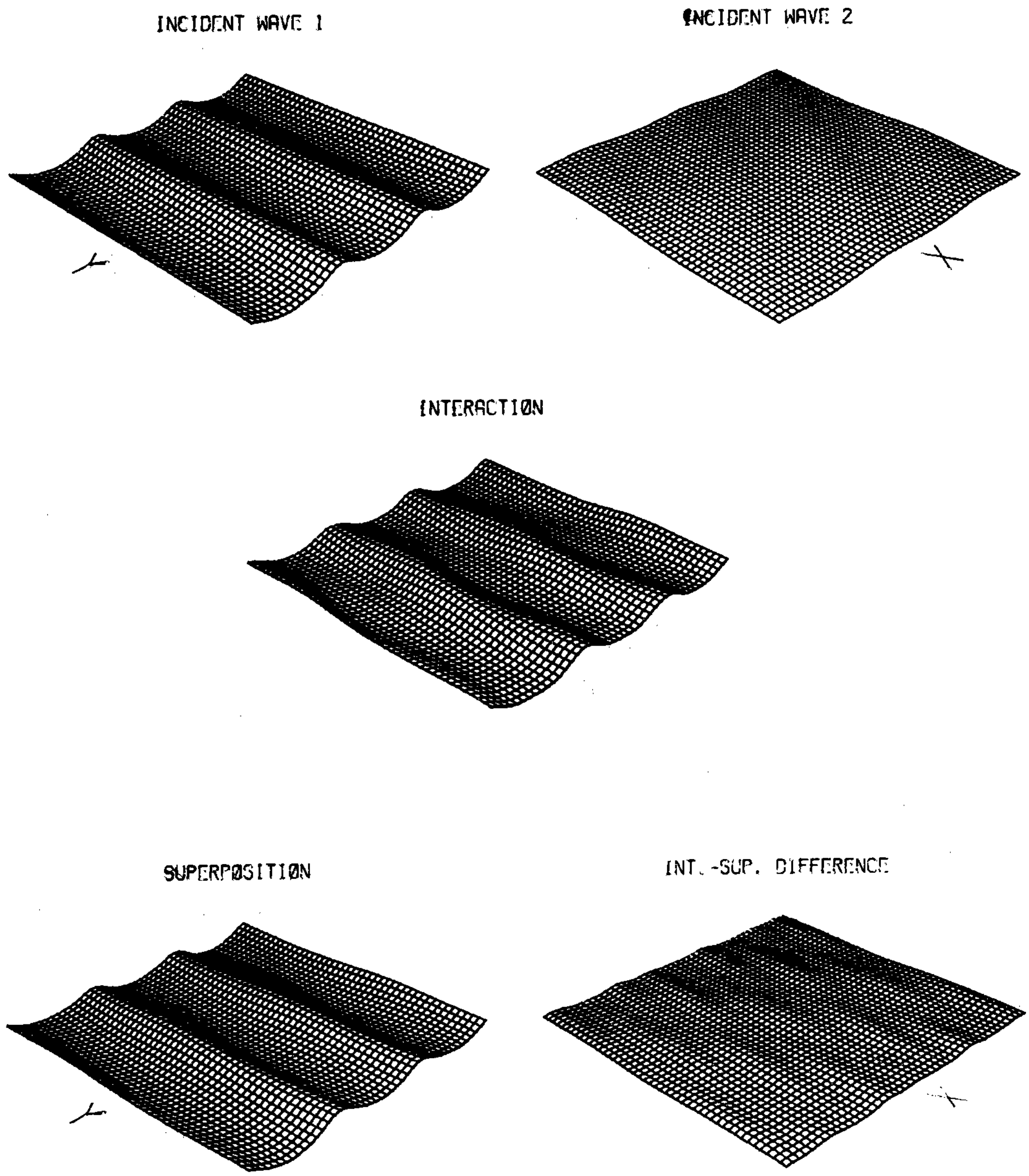
第一列來源波列(incident waves 1)
k_1d = 4.0 (相對水深)
k_1a_1= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 4.0 (相對水深)
k_2a_2= 0.1 (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 10°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-3a
Fig. 5-3a



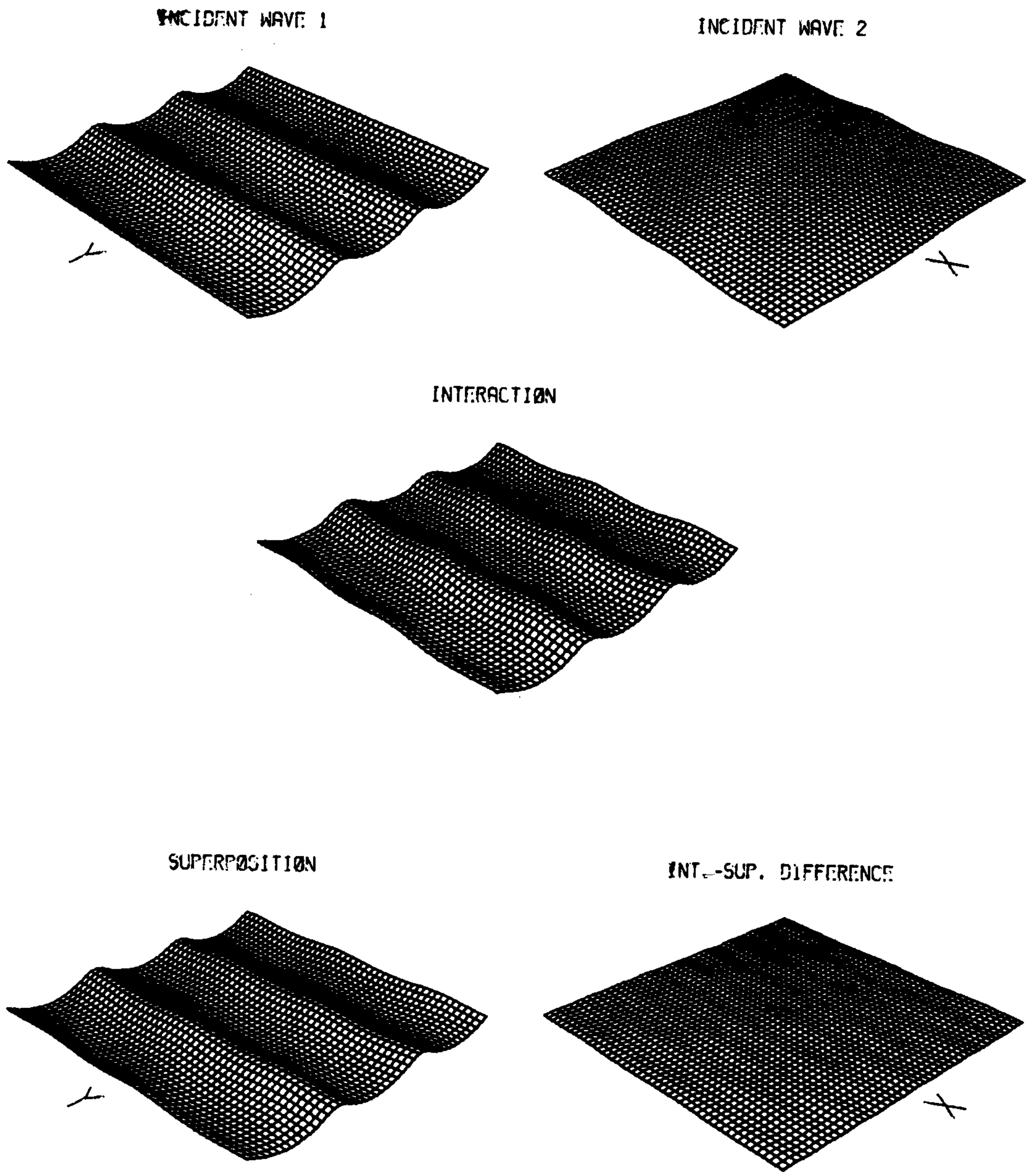
第一列來源波列(incident waves 1)
$k_1d = 4.0$ (相對水深)
$k_1a_1 = 0.4$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2d = 4.0$ (相對水深)
$k_2a_2 = 0.1$ (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 30°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-3b
Fig. 5-3b



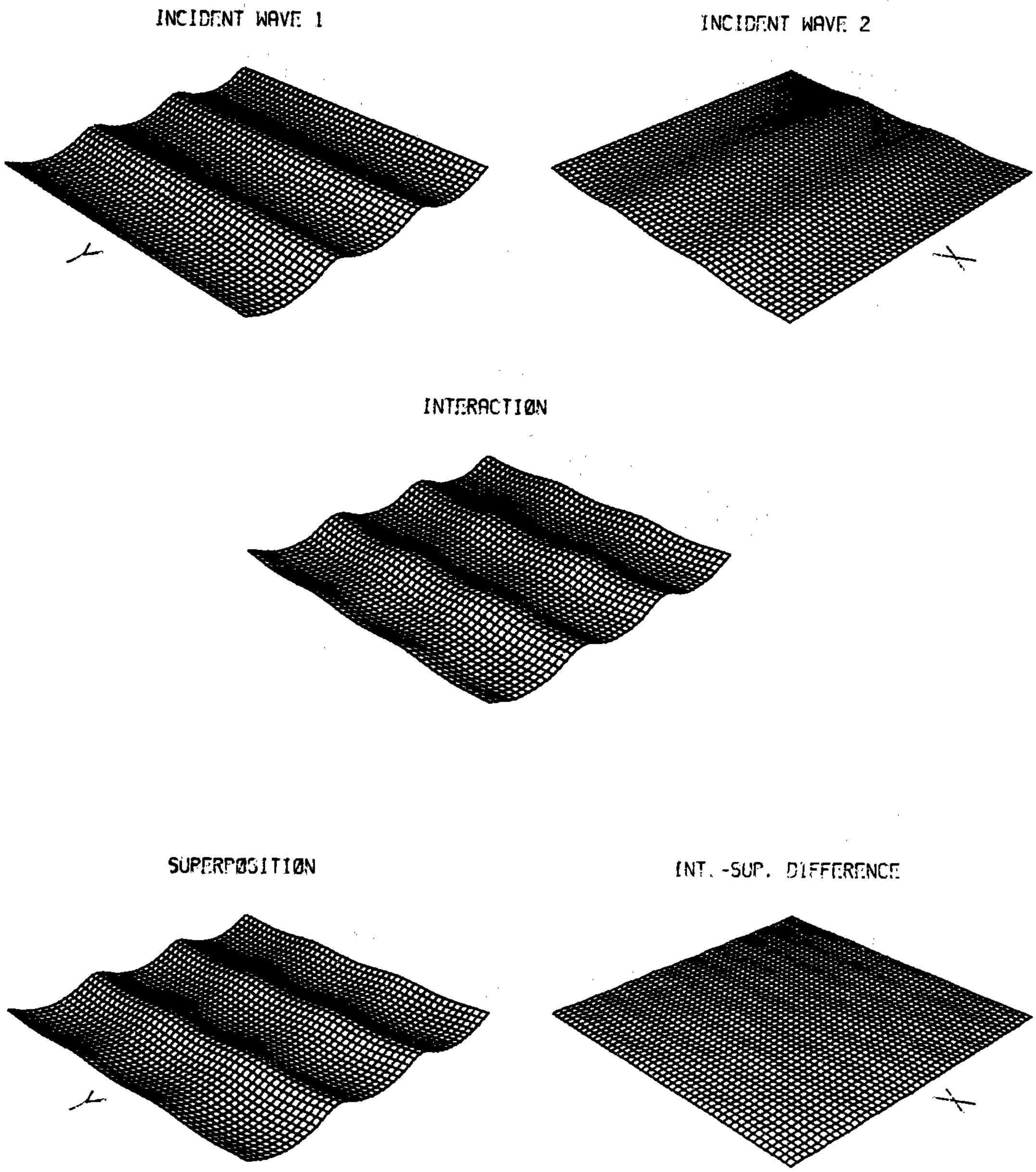
第一列來源波列(incident waves 1)
k_1d = 4.0 (相對水深)
k_1a_1= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 4.0 (相對水深)
k_2a_2= 0.1 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 60^\circ$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-3c
Fig. 5-3c



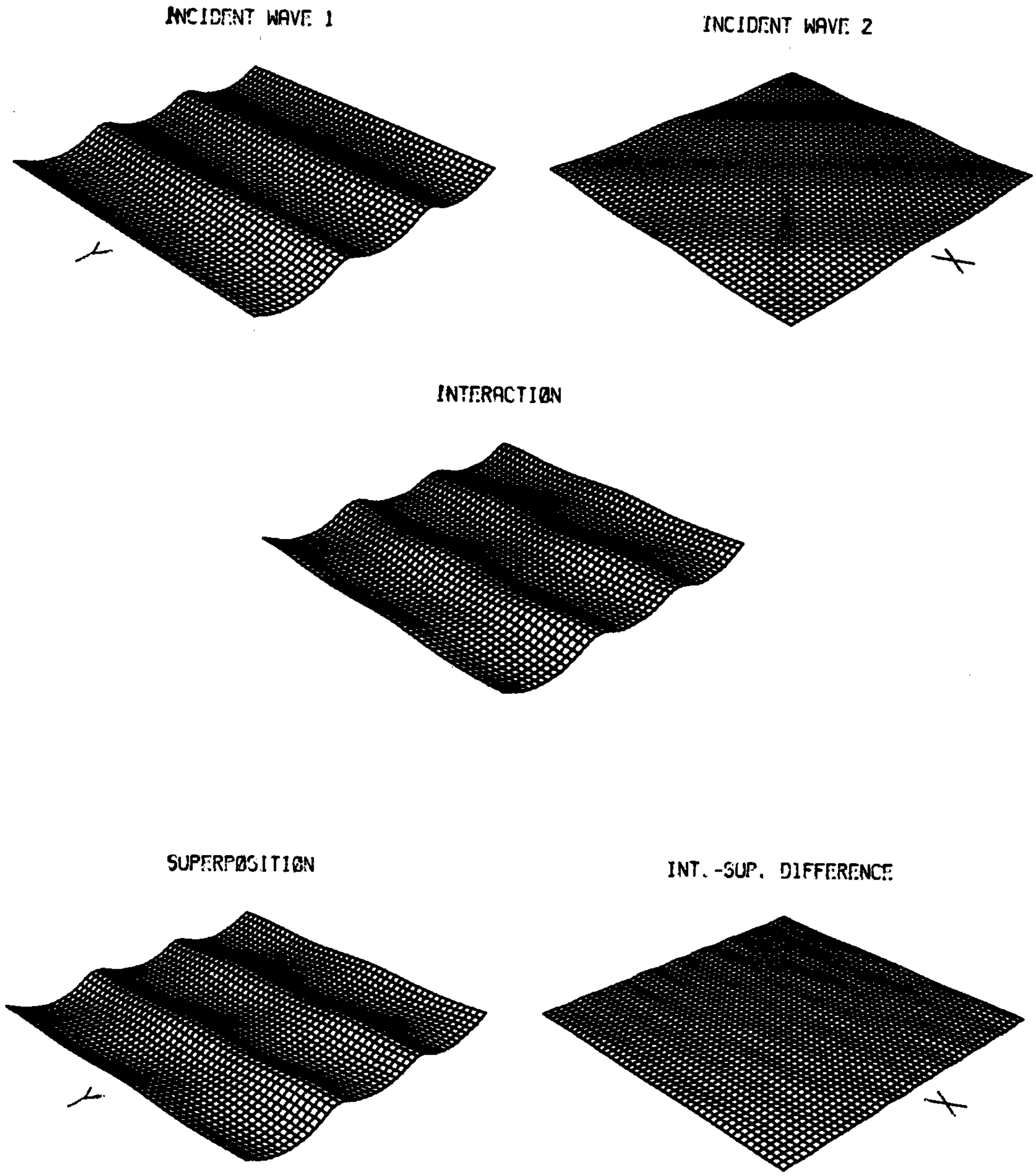
第一列來源波列(incident waves 1)
k_1d = 4.0 (相對水深)
k_1a_1= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 4.0 (相對水深)
k_2a_2= 0.1 (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 90°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-3d
Fig. 5-3d



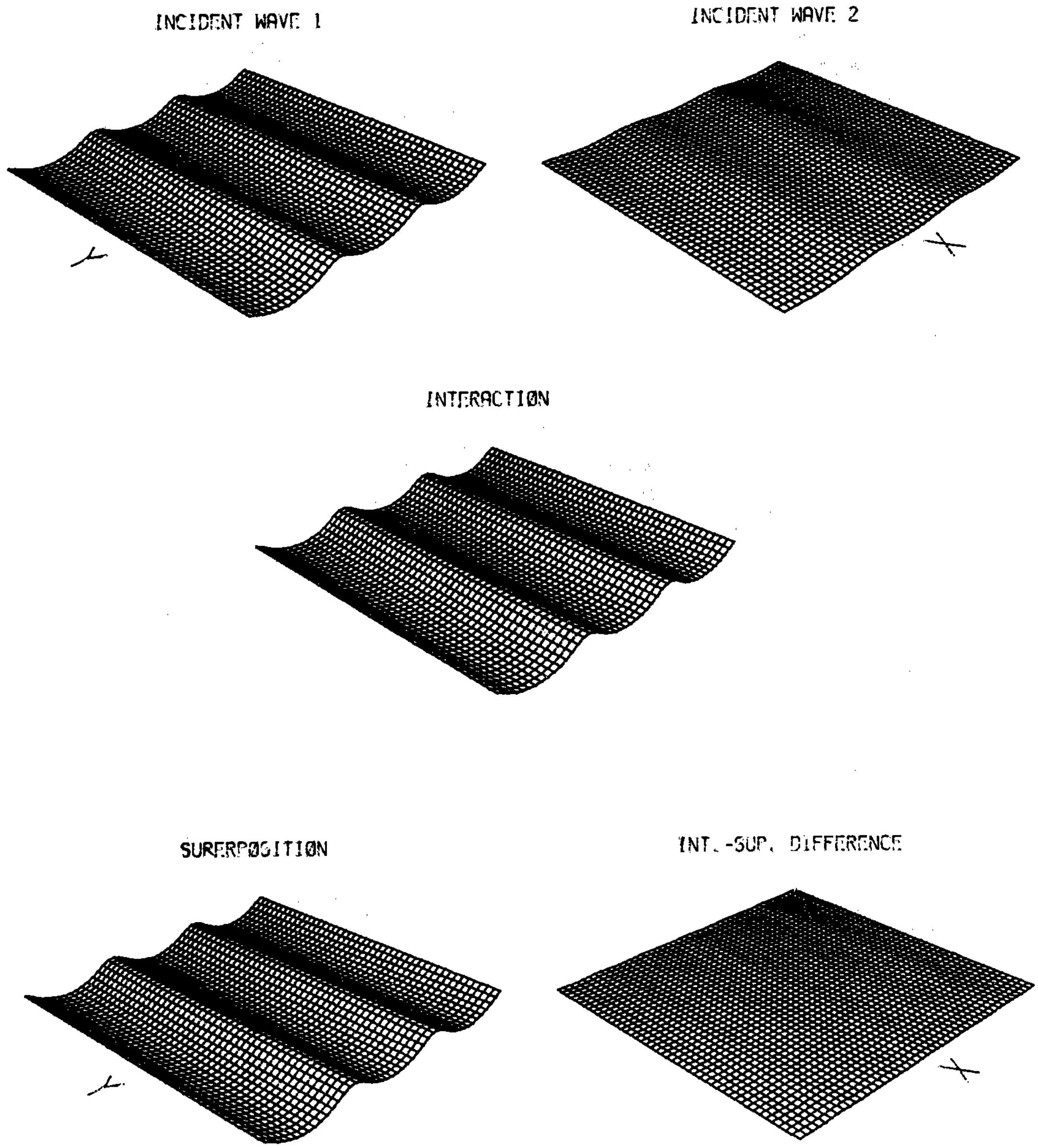
第一列來源波列(incident waves 1)
k_1d = 4.0 (相對水深)
k_1a_1= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 4.0 (相對水深)
k_2a_2= 0.1 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2$= 135°

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-3e
Fig. 5-3e



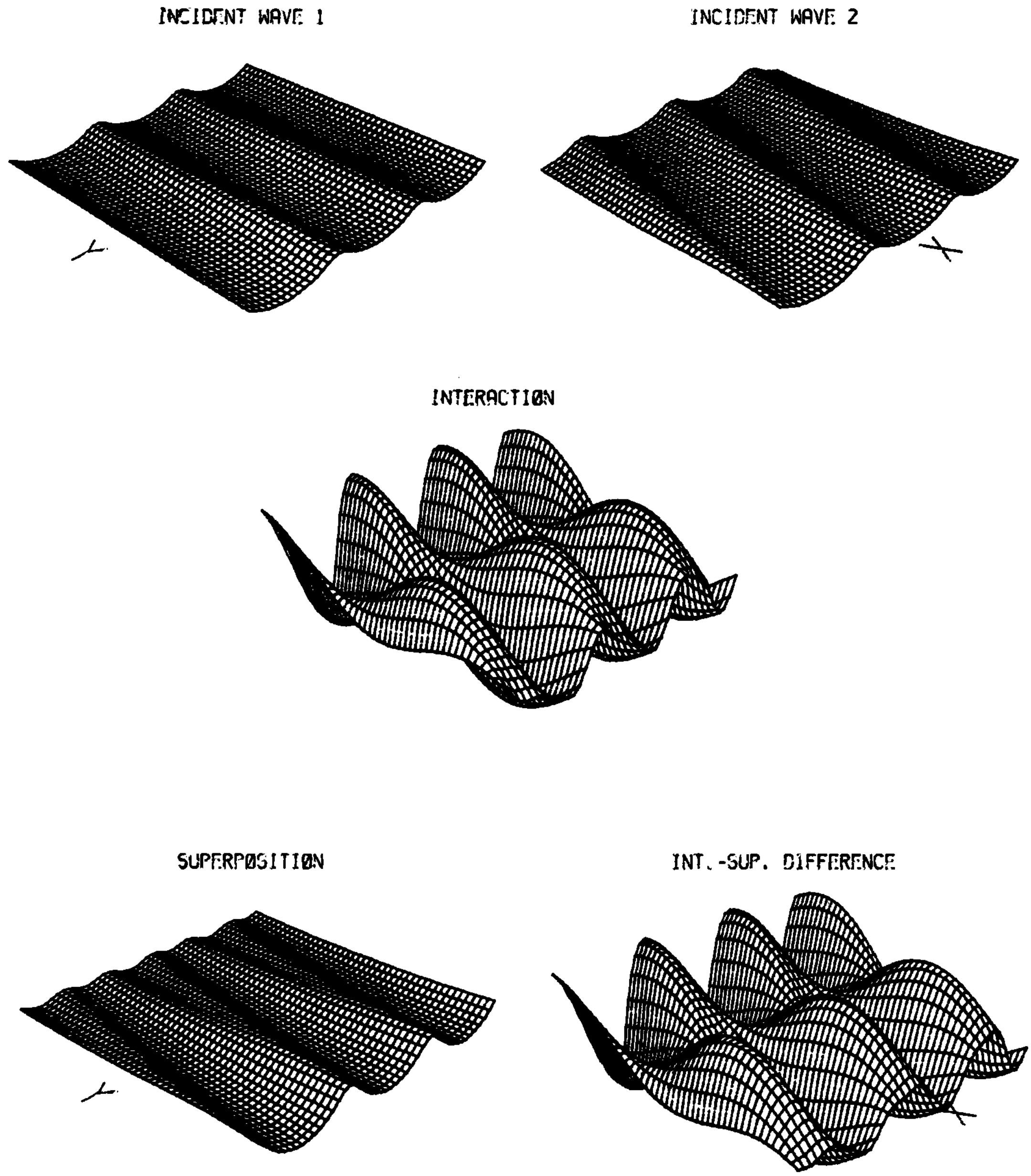
第一列來源波列(incident waves 1)
$k_1d = 4.0$ (相對水深)
$k_1a_1 = 0.4$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2d = 4.0$ (相對水深)
$k_2a_2 = 0.1$ (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 180^\circ$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-3f
Fig. 5-3f



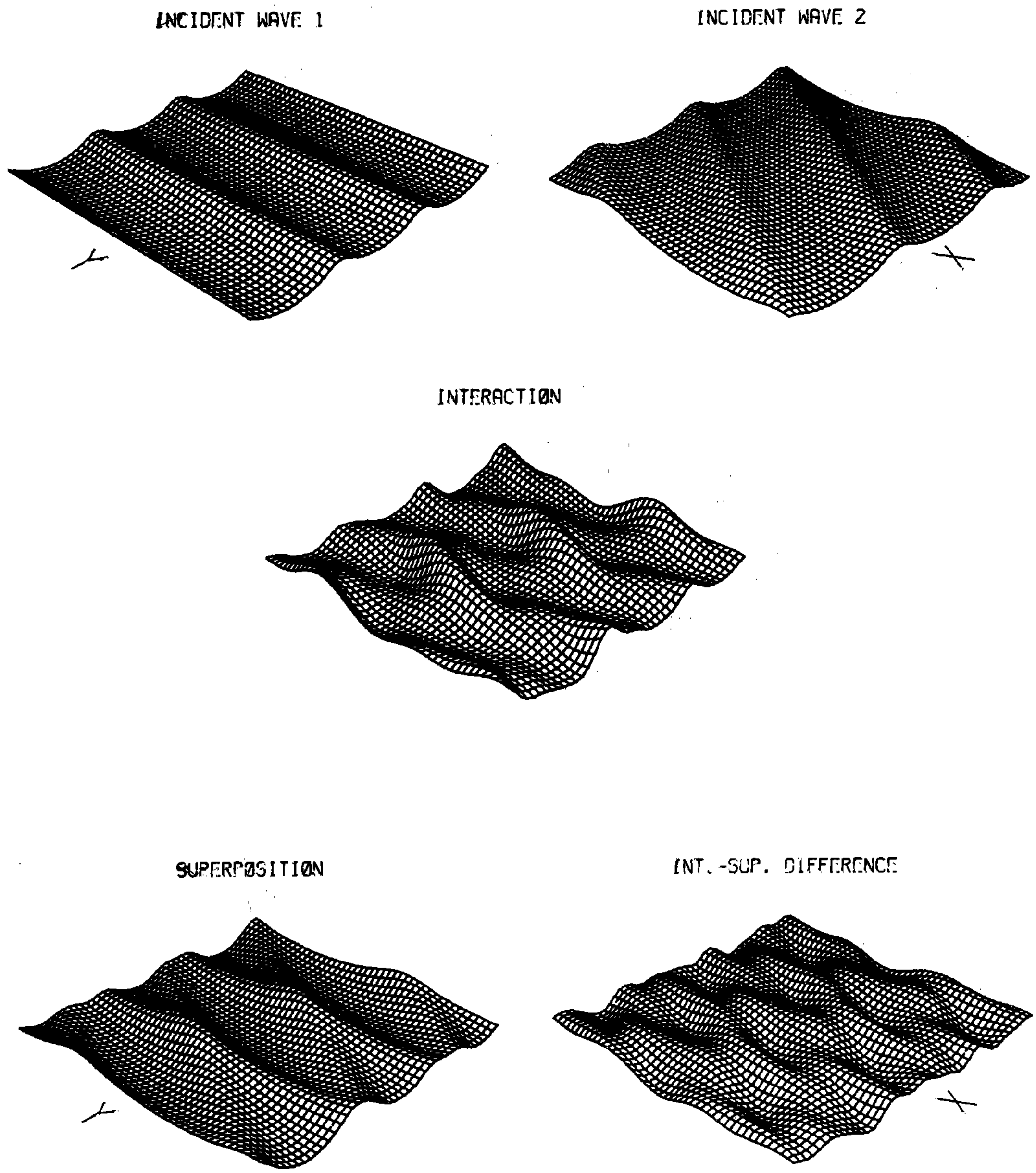
第一列來源波列(incident waves 1)
k_1d = 2.0 (相對水深)
k_1a_1= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 2.0 (相對水深)
k_2a_2= 0.4 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 10^\circ$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-4a
Fig. 5-4a



第一列來源波列(incident waves 1)
k_1d = 2.0 (相對水深)
k_1a_1= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 2.0 (相對水深)
k_2a_2= 0.4 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 30°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-4b
Fig. 5-4b

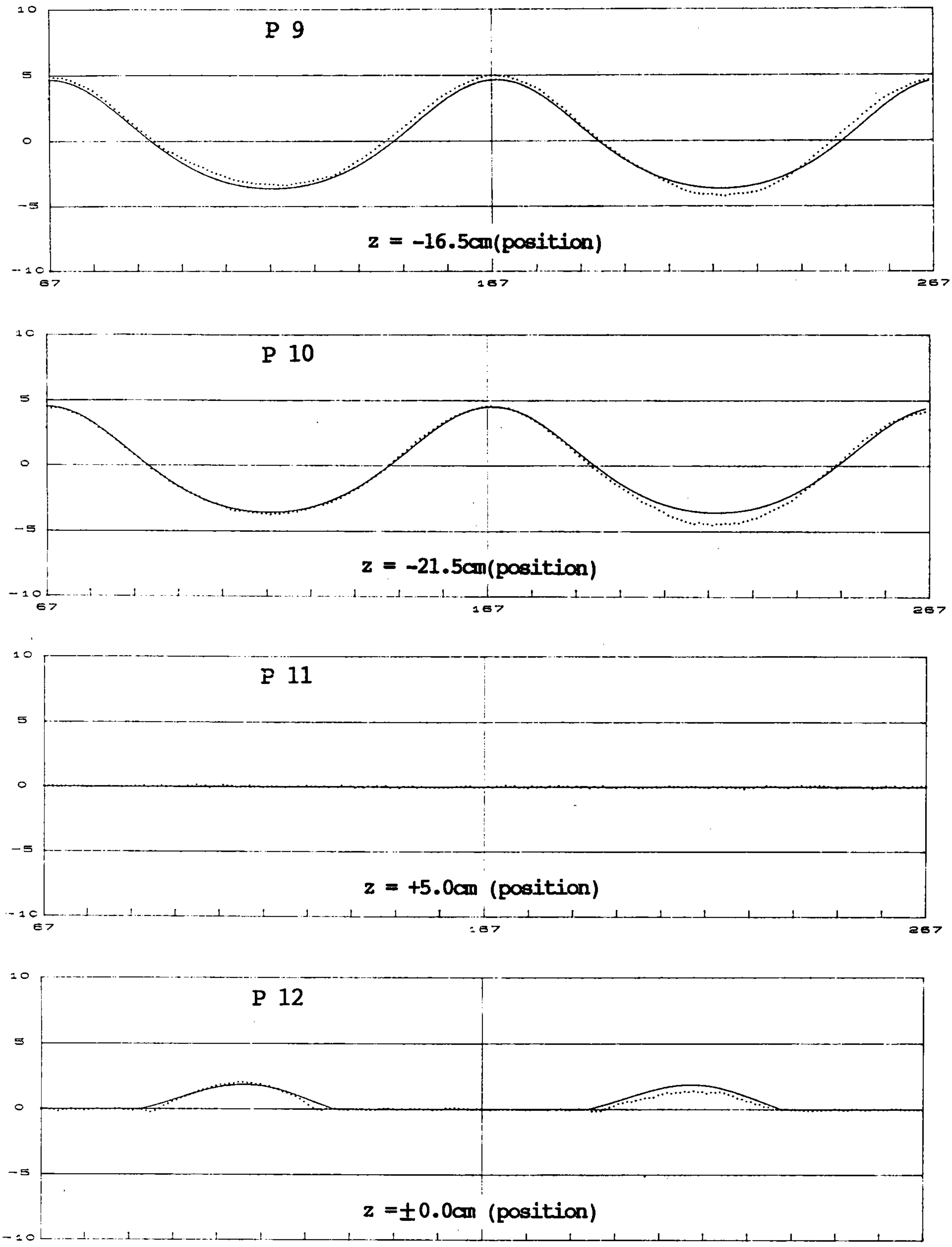
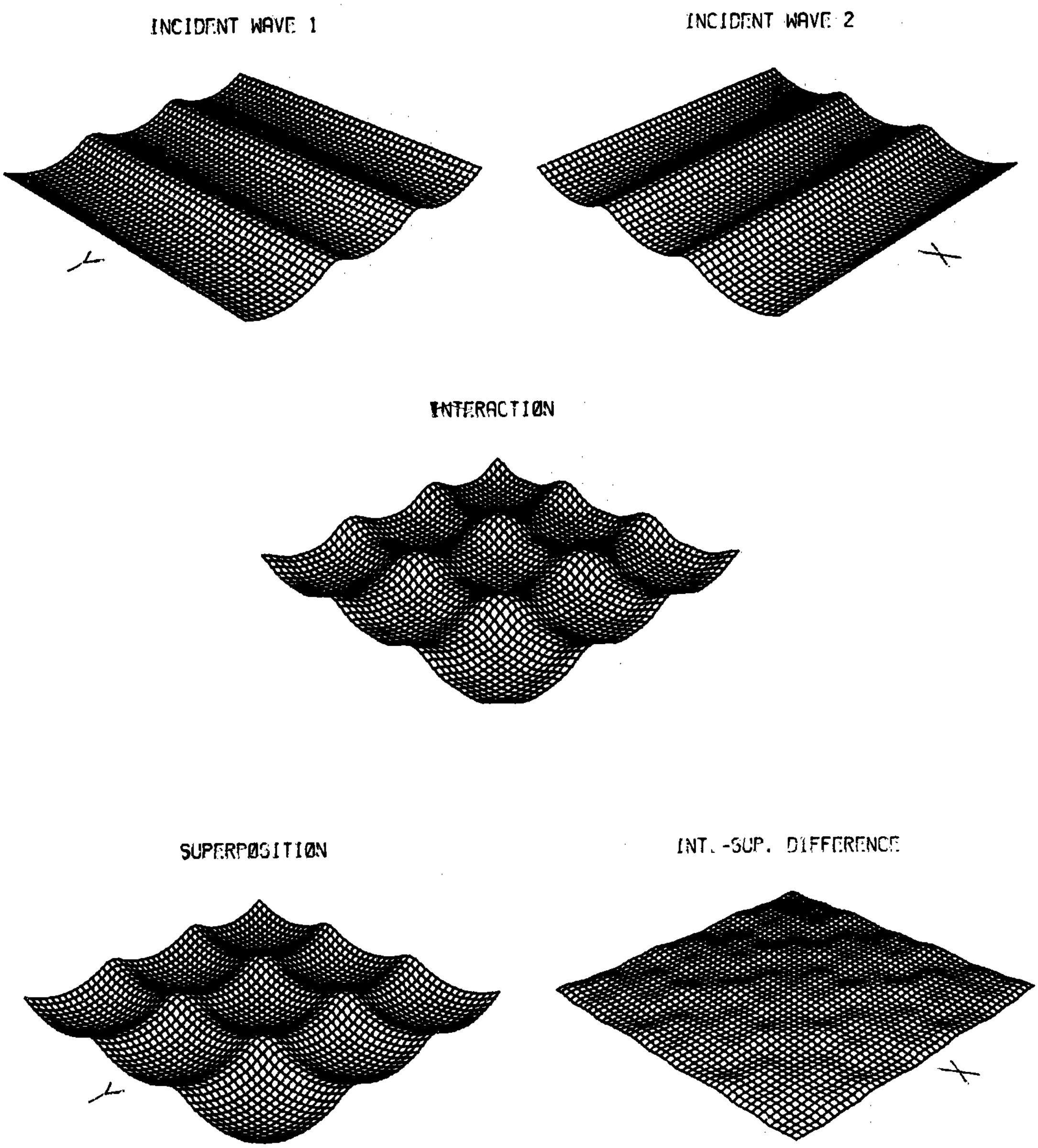

P 9
z = -16.5cm(position)
P 10
z = -21.5cm(position)
P 11
z = +5.0cm (position)
P 12
z =±0.0cm (position)

圖11-23



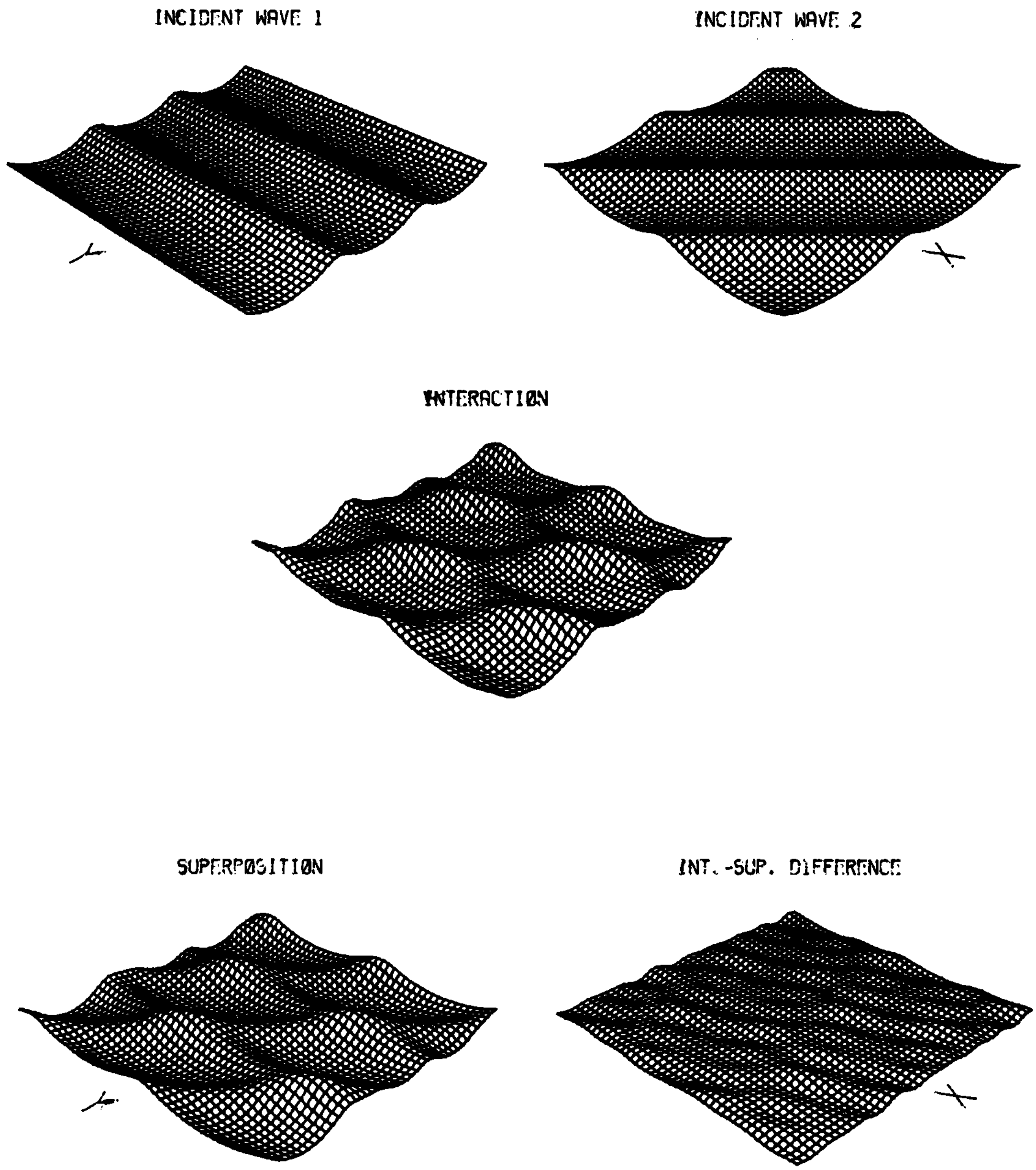
第一列來源波列(incident waves 1)
k_1d = 2.0 (相對水深)
k_1a_1= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 2.0 (相對水深)
k_2a_2= 0.4 (波浪尖銳度)

二波列交會夾角$\theta=\theta_1-\theta_2$= 90°

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-4d
Fig. 5-4d



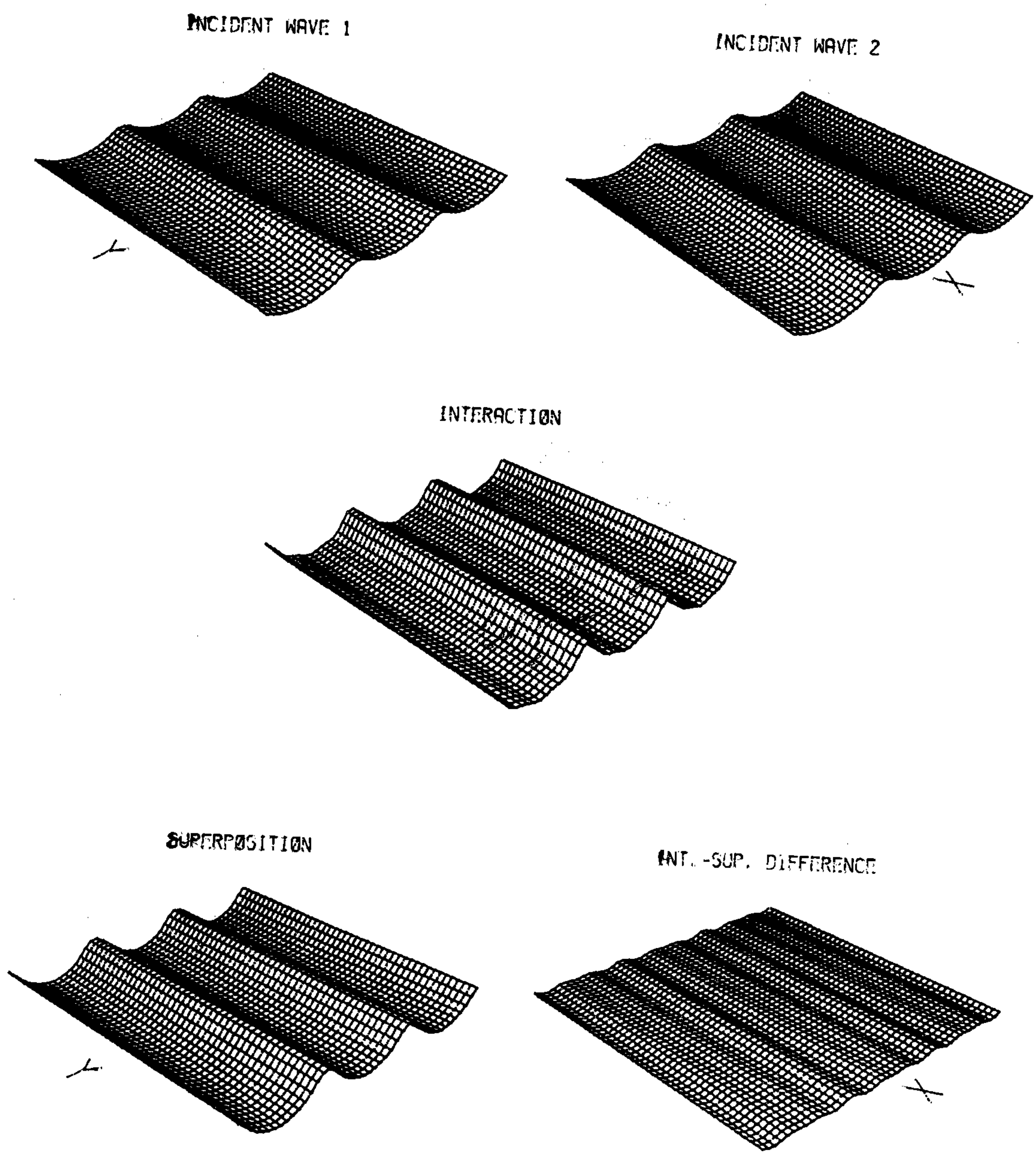
第一列來源波列(incident waves 1)
$k_1d = 2.0$ (相對水深)
$k_1a_1 = 0.4$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2d = 2.0$ (相對水深)
$k_2a_2 = 0.4$ (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 135°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-4e
Fig. 5-4e



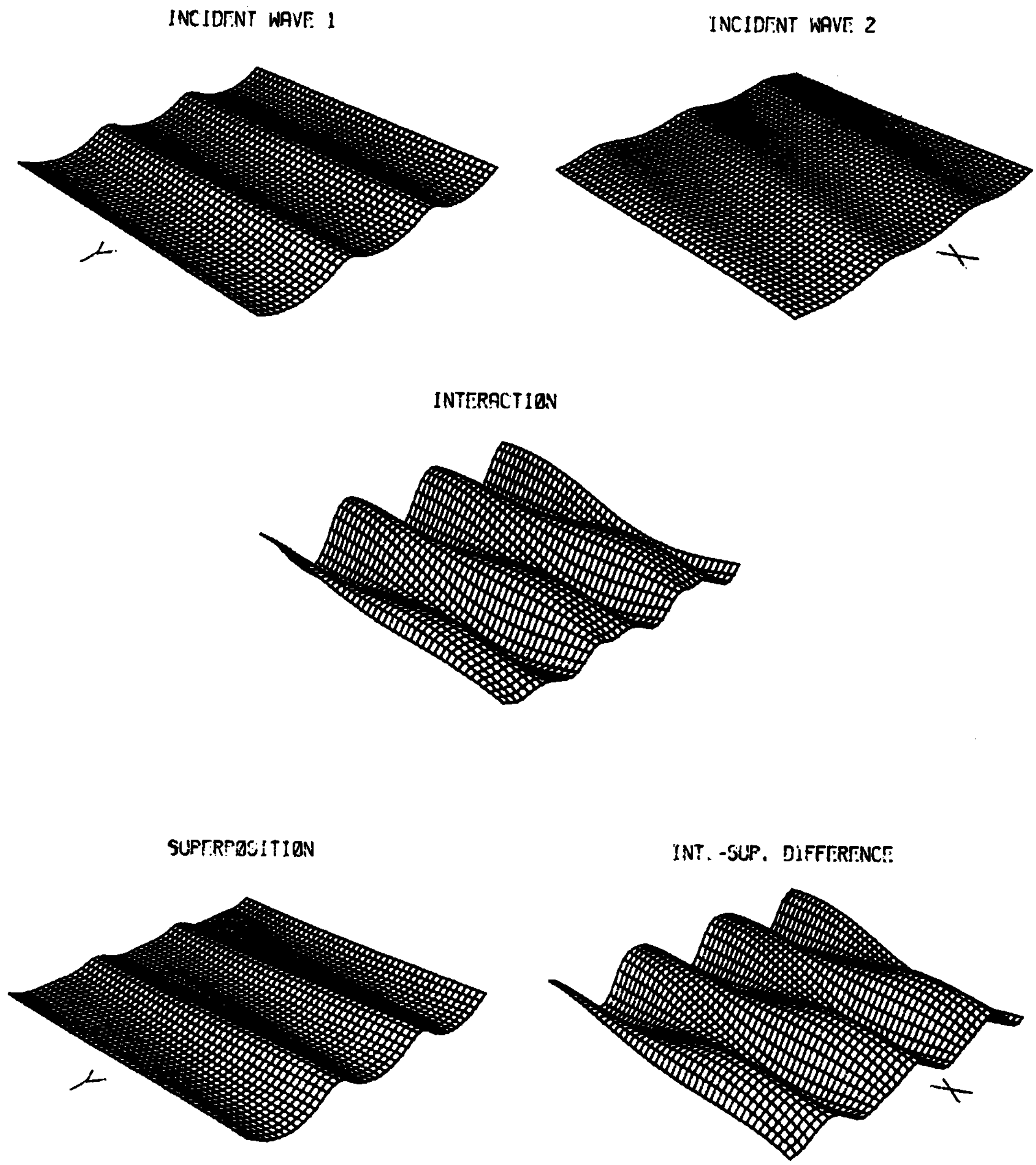
第一列來源波列(incident waves 1)
k_1d = 2.0 (相對水深)
k_1a_1= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 2.0 (相對水深)
k_2a_2= 0.4 (波浪尖銳度)

二波列交會夾角$\theta=\theta_1-\theta_2$= 180°

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-4f
Fig. 5-4f



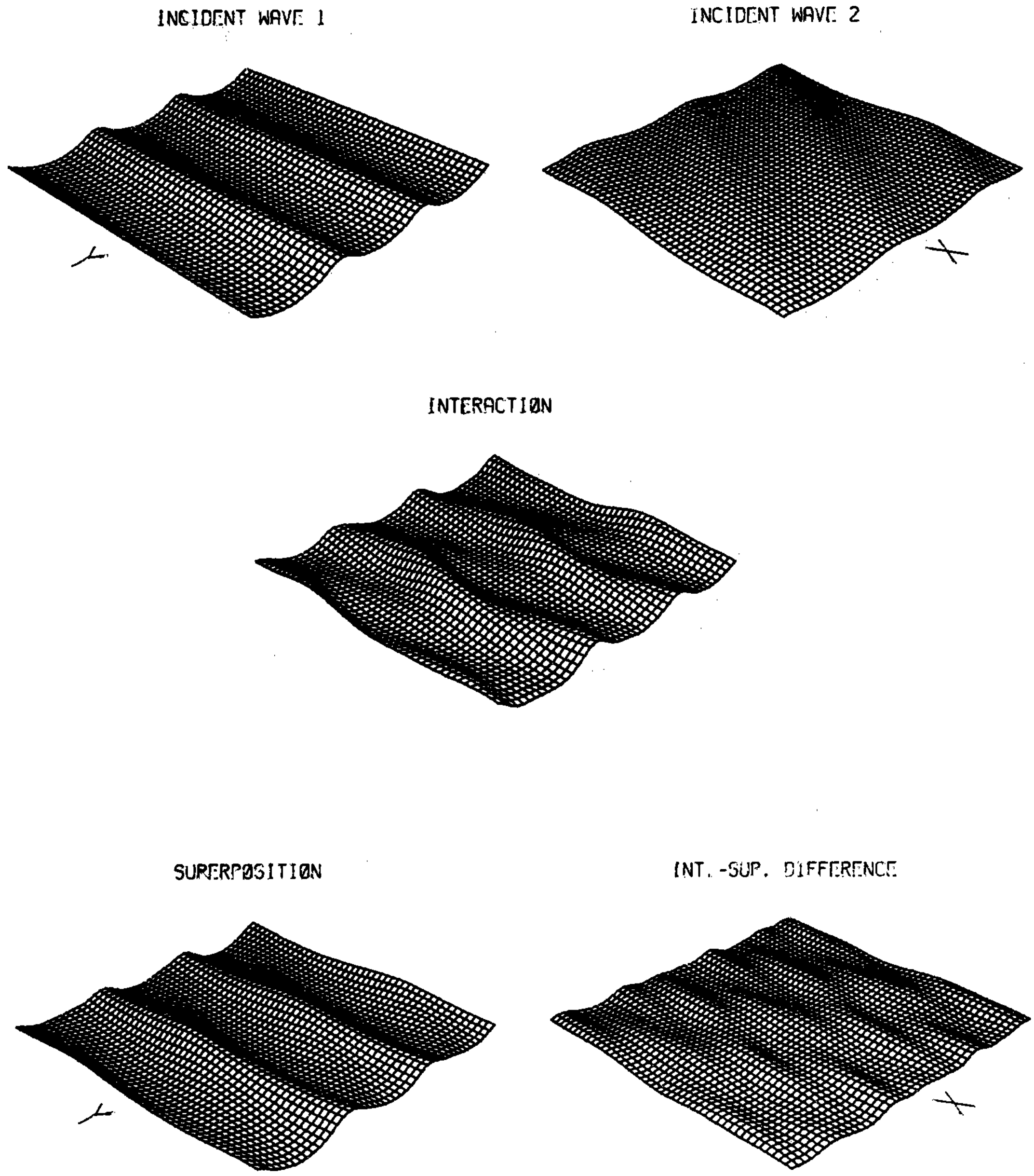
第一列來源波列(incident waves 1)
$k_1d = 2.0$ (相對水深)
$k_1a_1 = 0.4$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2d = 2.0$ (相對水深)
$k_2a_2 = 0.2$ (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 10°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-5a
Fig. 5-5a



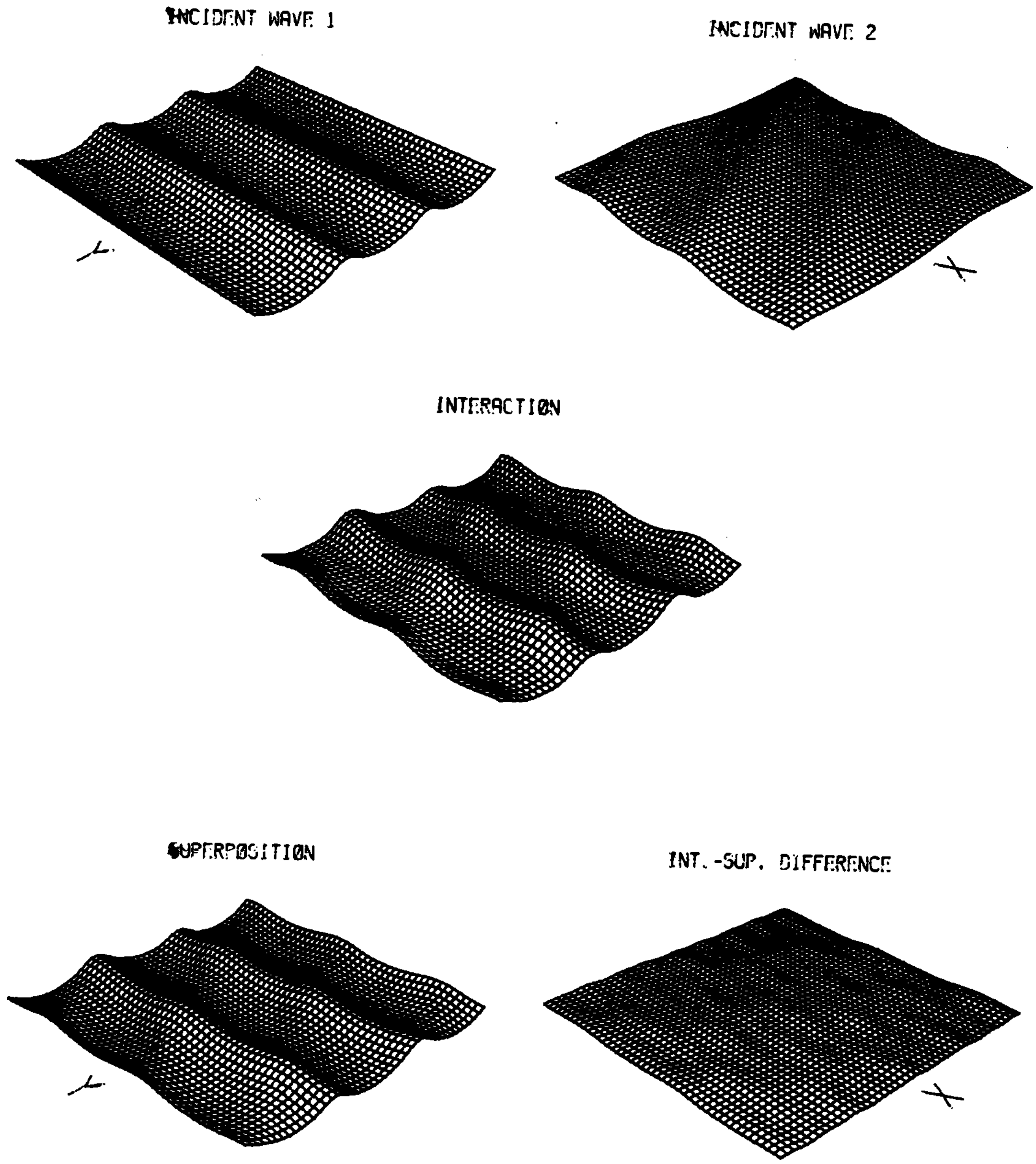
第一列來源波列(incident waves 1)
$k_1d = 2.0$ (相對水深)
$k_1a_1= 0.4$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2d = 2.0$ (相對水深)
$k_2a_2= 0.2$ (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 30°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-5b
Fig. 5-5b



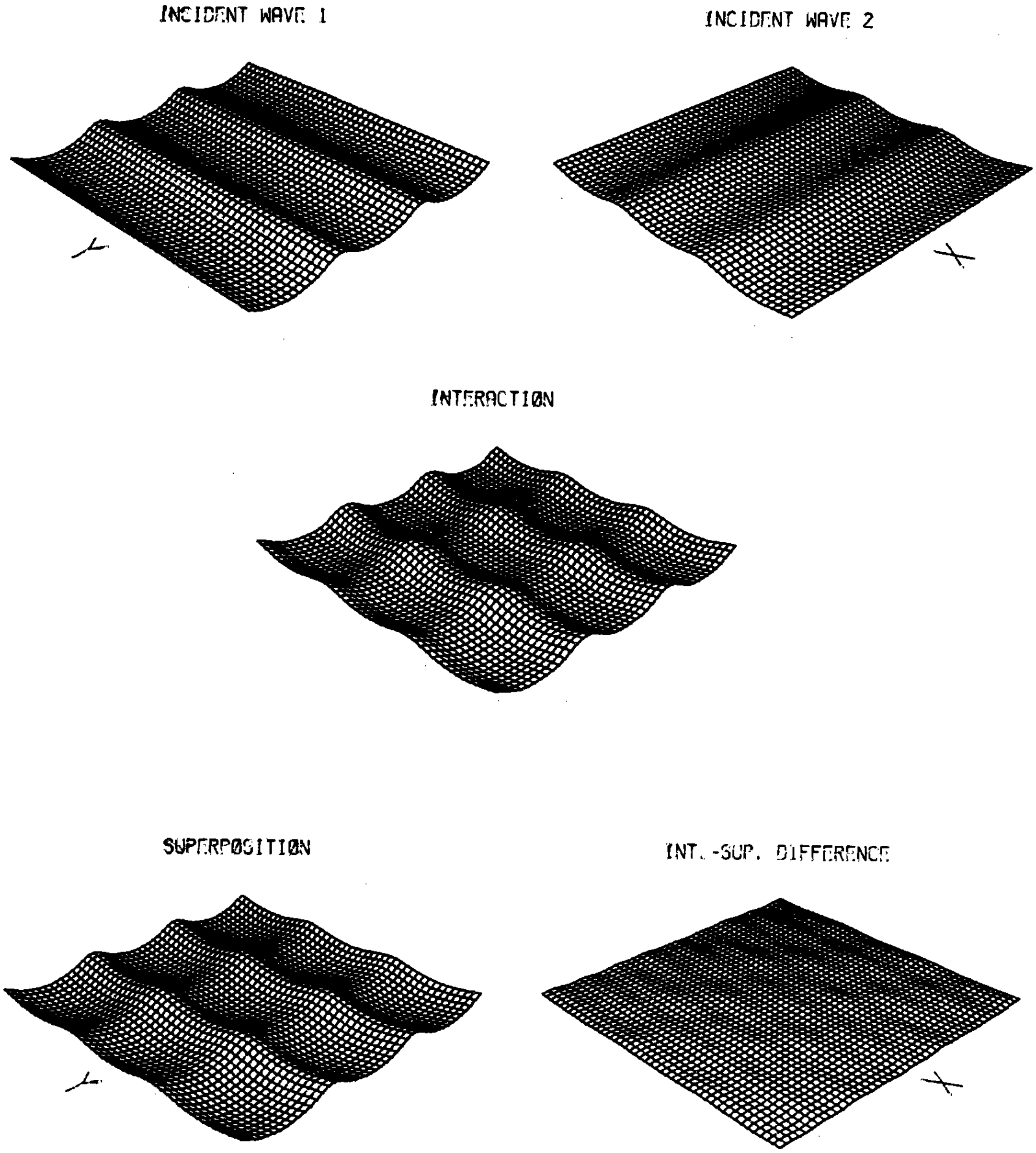
第一列來源波列(incident waves 1)
k_1d = 2.0 (相對水深)
k_1a_1= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 2.0 (相對水深)
k_2a_2= 0.2 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 60°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-5c
Fig. 5-5c



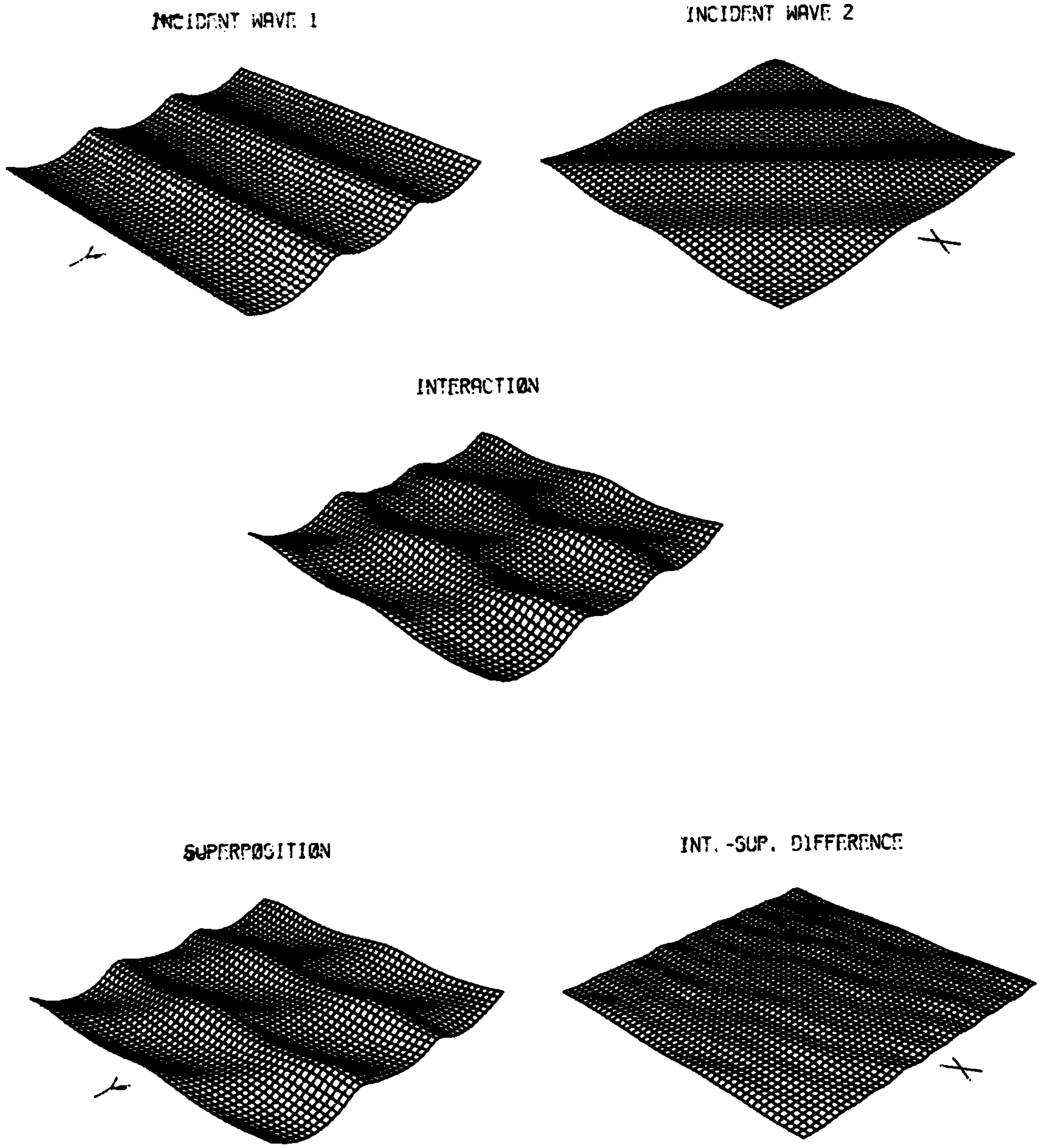
第一列來源波列(incident waves 1)
k_1d = 2.0 (相對水深)
k_1a_1= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 2.0 (相對水深)
k_2a_2= 0.2 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 90°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-5d
Fig. 5-5d



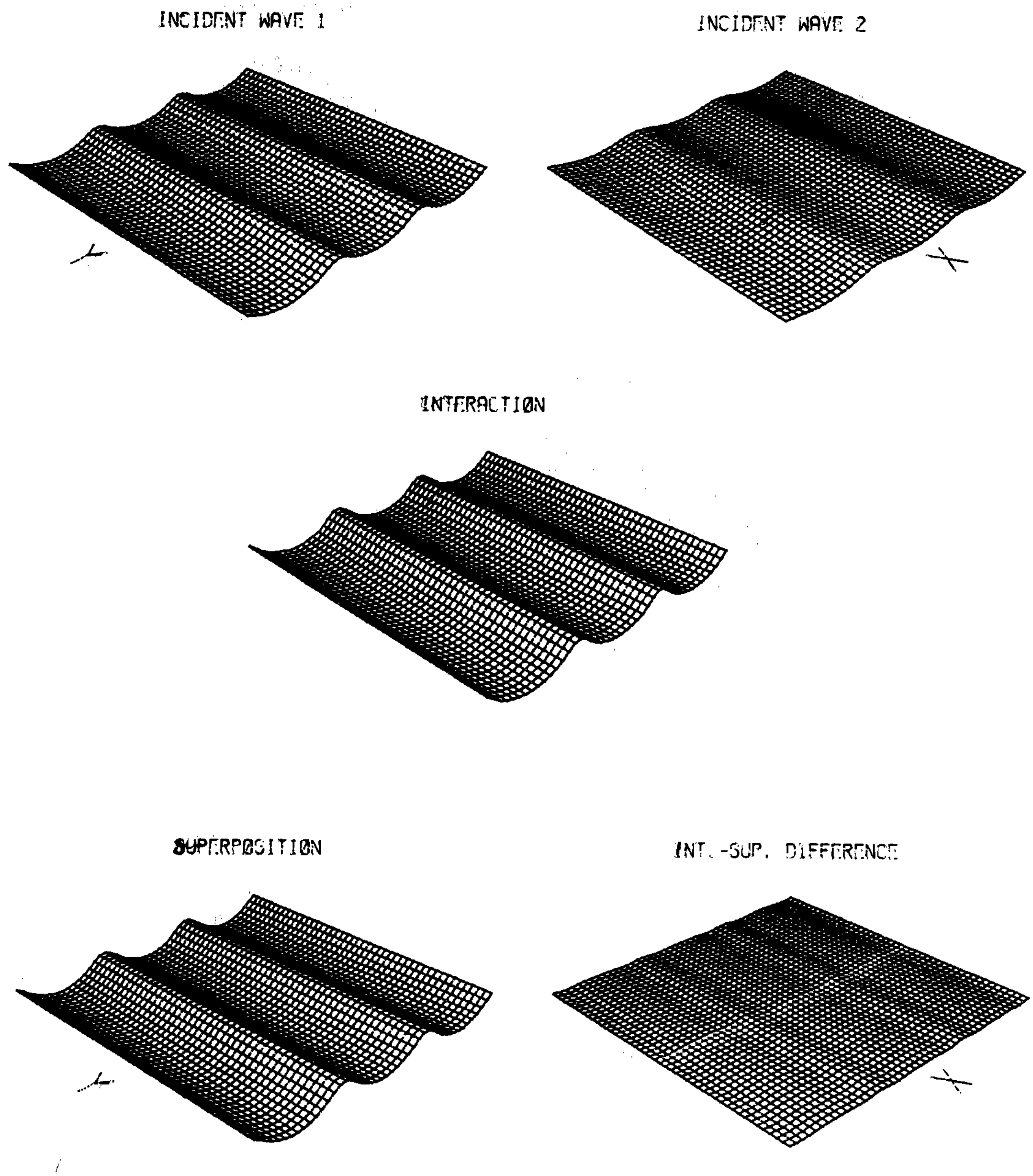
第一列來源波列(incident waves 1)
$k_1d = 2.0$ (相對水深)
$k_1a_1 = 0.4$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2d = 2.0$ (相對水深)
$k_2a_2 = 0.2$ (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 135°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-5e
Fig. 5-5e



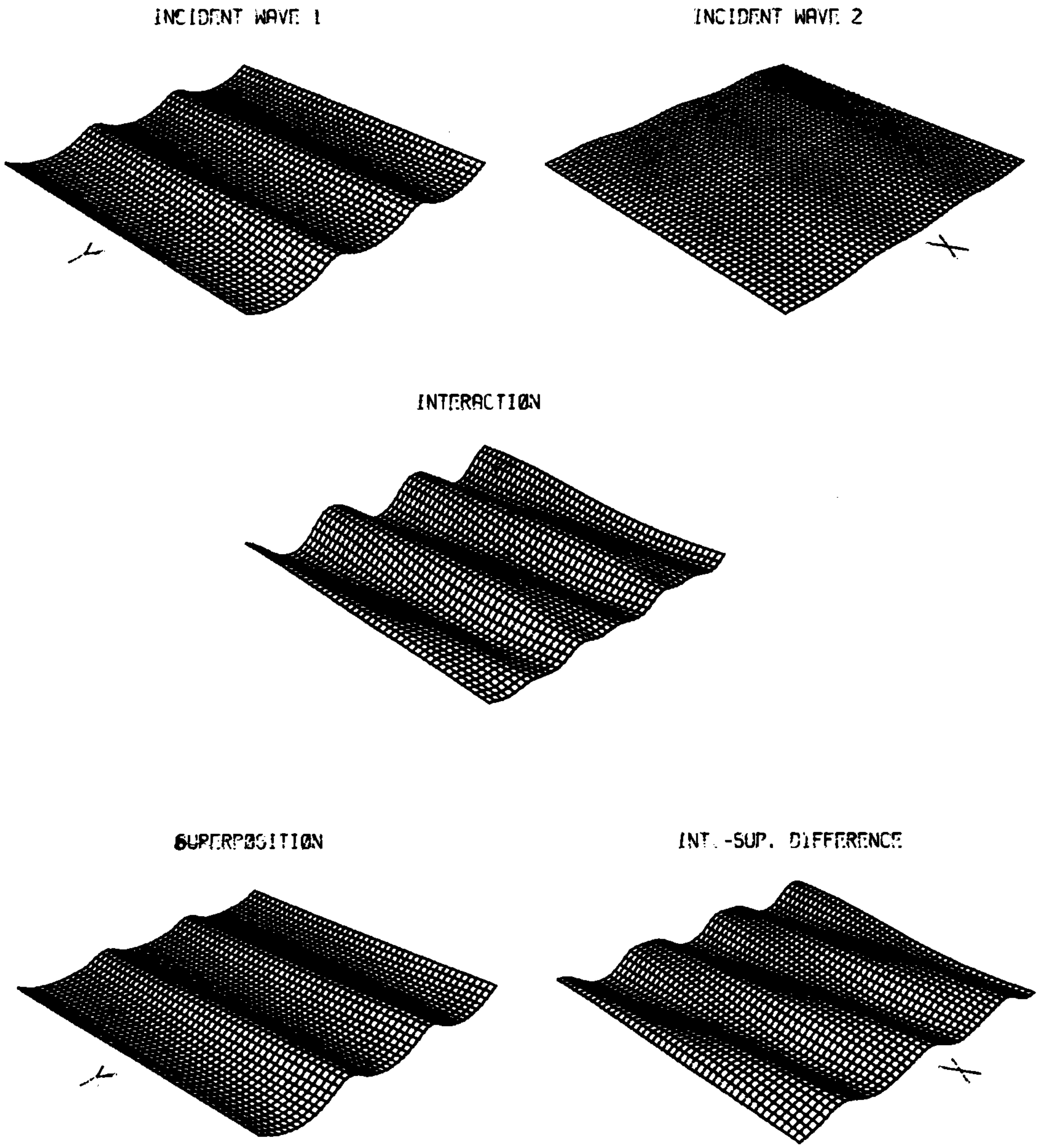
第一列來源波列(incident waves 1)
k_1d = 2.0 (相對水深)
k_1a_1= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 2.0 (相對水深)
k_2a_2= 0.2 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 180°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-5f
Fig. 5-5f



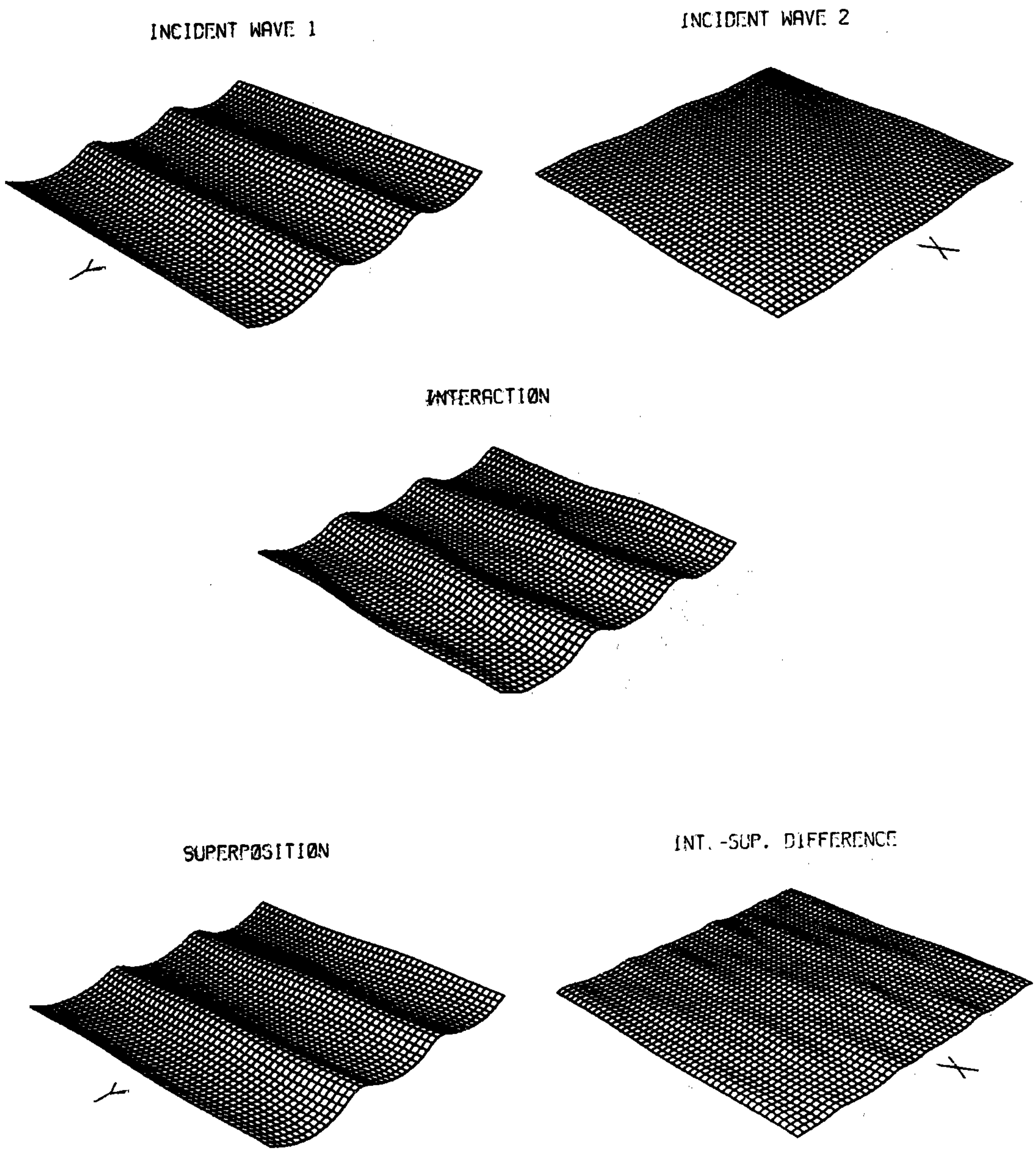
第一列來源波列(incident waves 1)
k_1d = 2.0 (相對水深)
k_1a_1= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 2.0 (相對水深)
k_2a_2= 0.1 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 10°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-6a
Fig. 5-6a



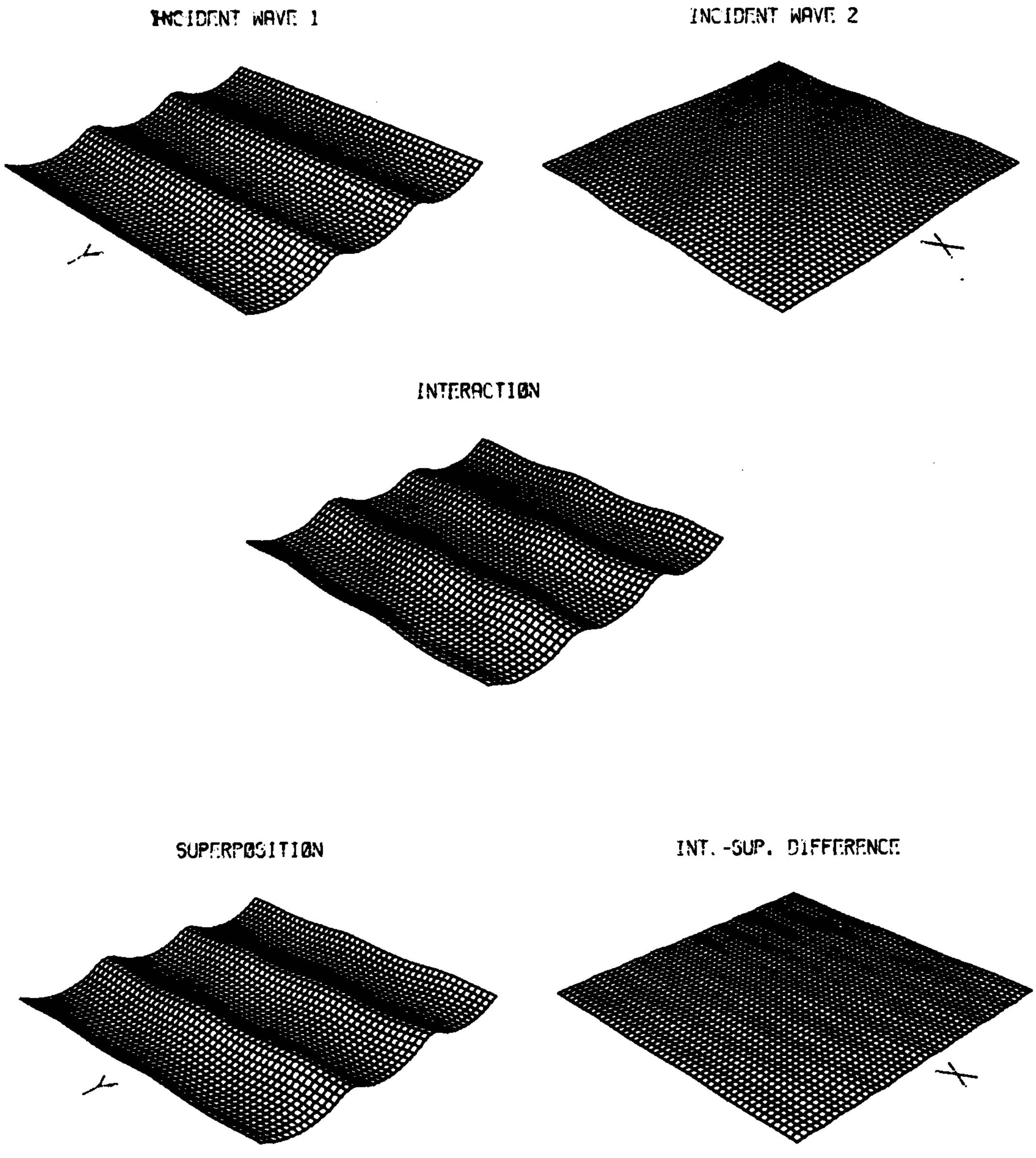
第一列來源波列(incident waves 1)
$k_1d = 2.0$ (相對水深)
$k_1a_1 = 0.4$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2d = 2.0$ (相對水深)
$k_2a_2 = 0.1$ (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 30^\circ$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-6b
Fig. 5-6b



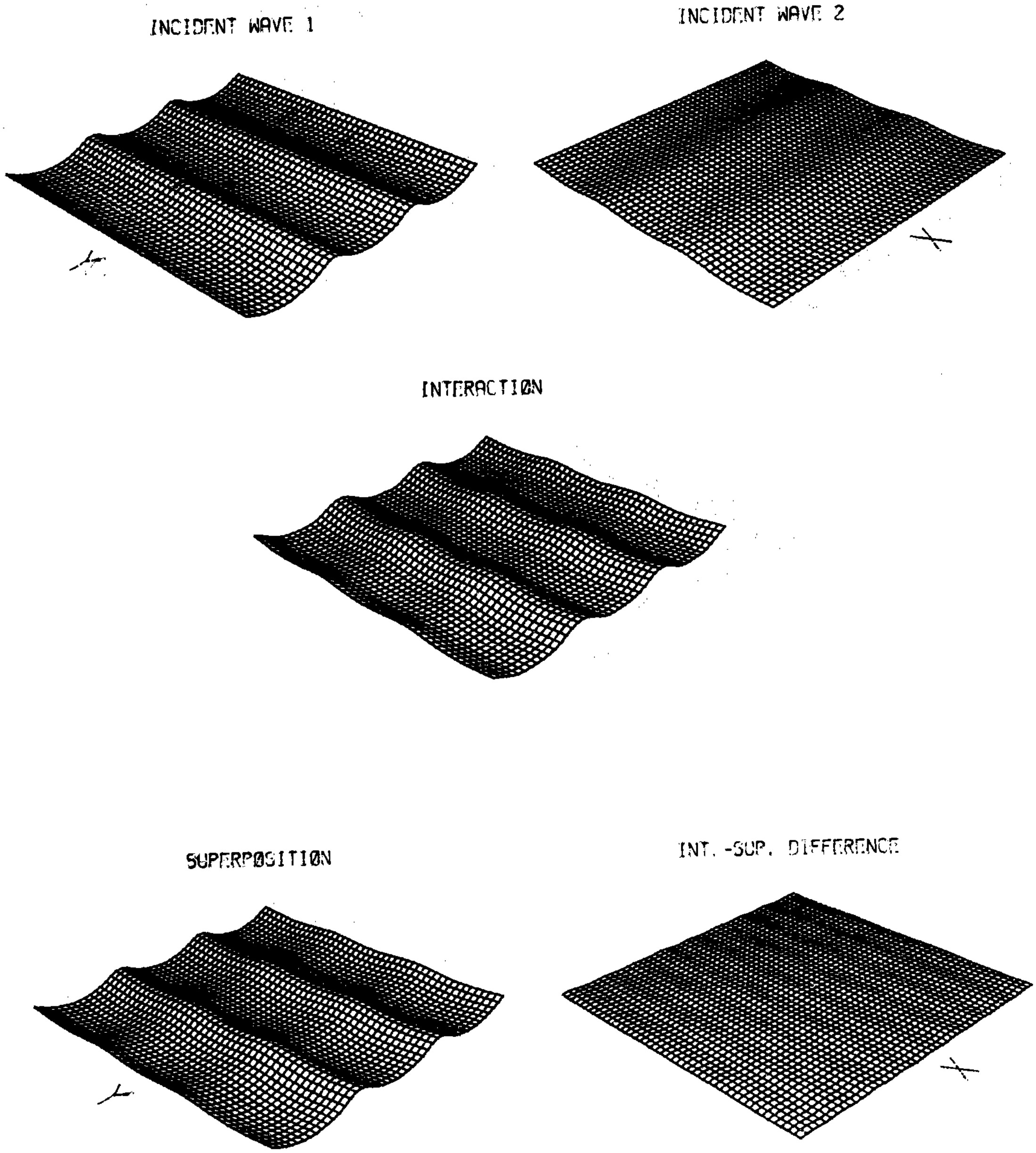
第一列來源波列(incident waves 1)
$k_1 d$ = 2.0 (相對水深)
$k_1 a_1$= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2 d$ = 2.0 (相對水深)
$k_2 a_2$= 0.1 (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 60°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-6c
Fig. 5-6c



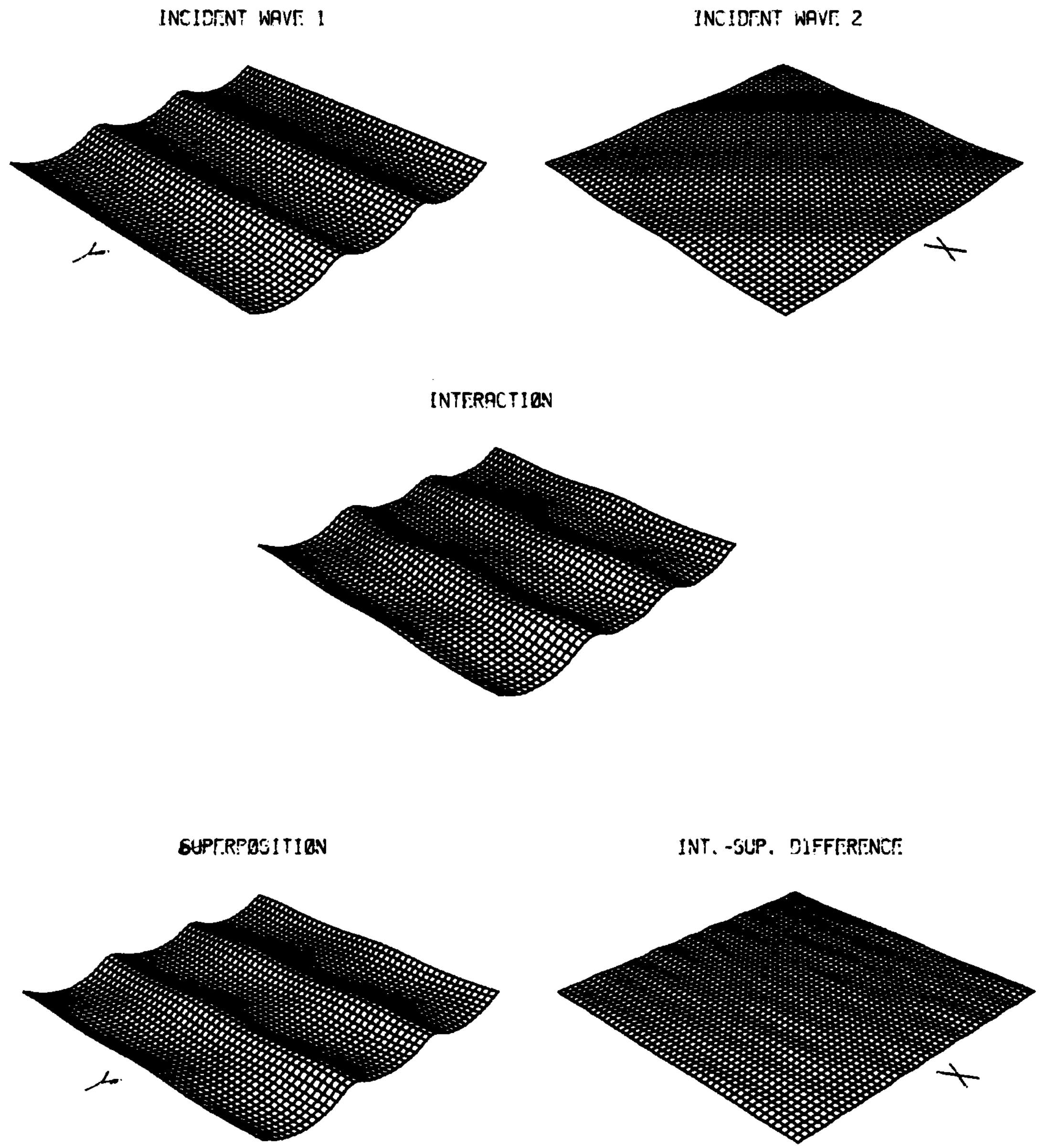
第一列來源波列(incident waves 1)
k_1d = 2.0 (相對水深)
k_1a_1= 0.4 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 2.0 (相對水深)
k_2a_2= 0.1 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 90°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-6d
Fig. 5-6d



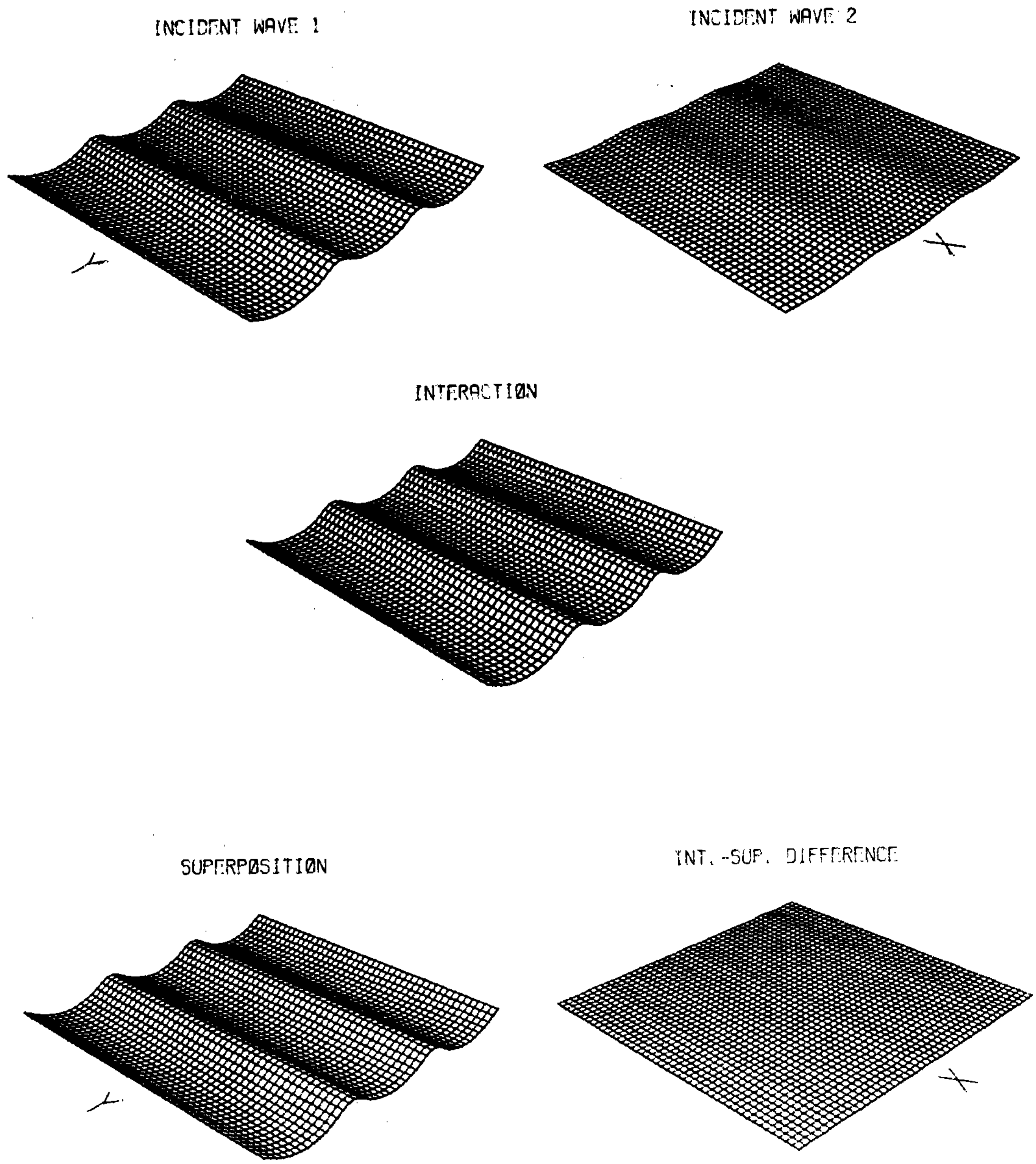
第一列來源波列(incident waves 1)
$k_1d = 2.0$ (相對水深)
$k_1a_1 = 0.4$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2d = 2.0$ (相對水深)
$k_2a_2 = 0.1$ (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 135°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-6e
Fig. 5-6e



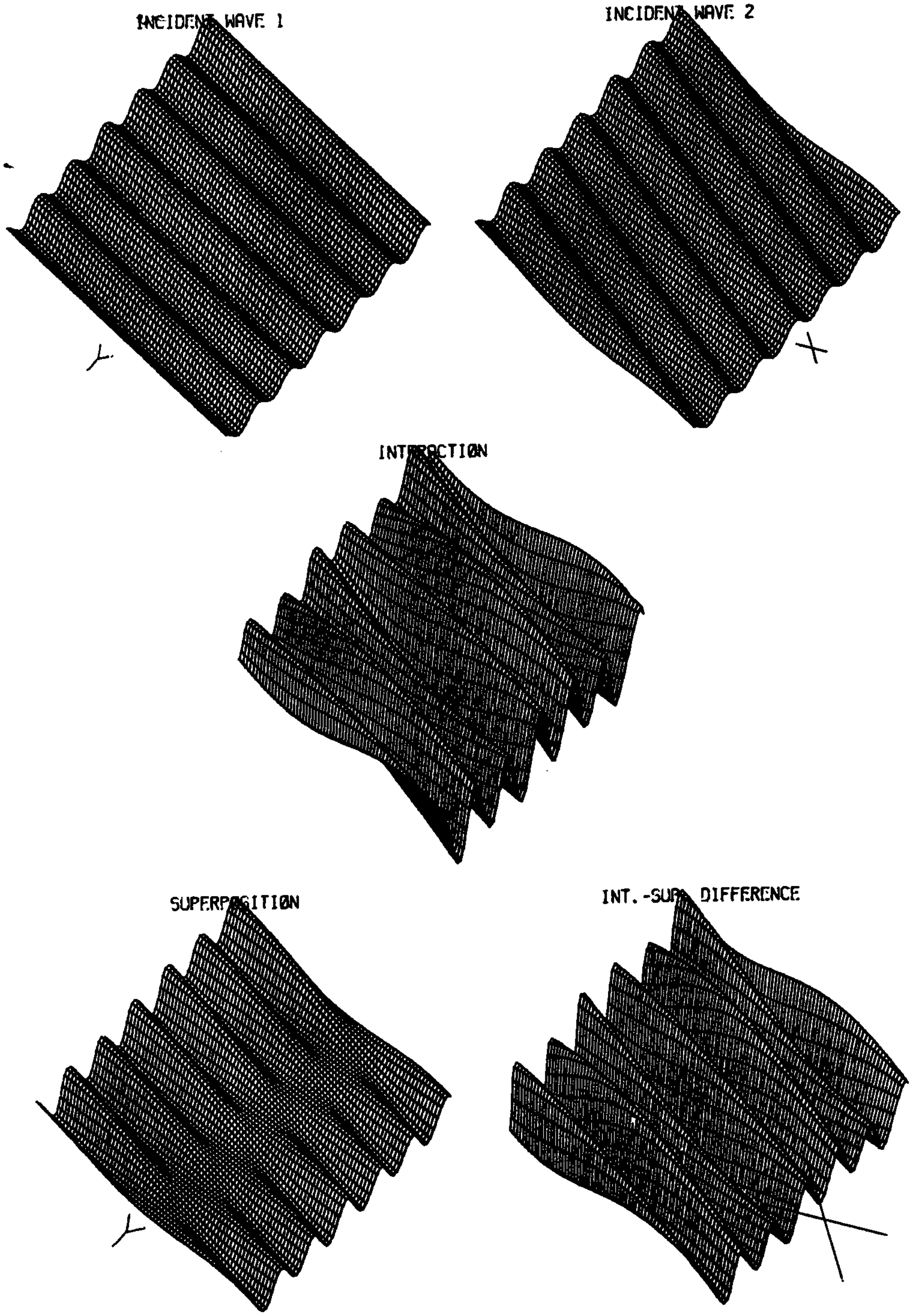
第一列來源波列(incident waves 1)
$k_1d = 2.0$ (相對水深)
$k_1a_1 = 0.4$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2d = 2.0$ (相對水深)
$k_2a_2 = 0.1$ (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 180°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-6f
Fig. 5-6f



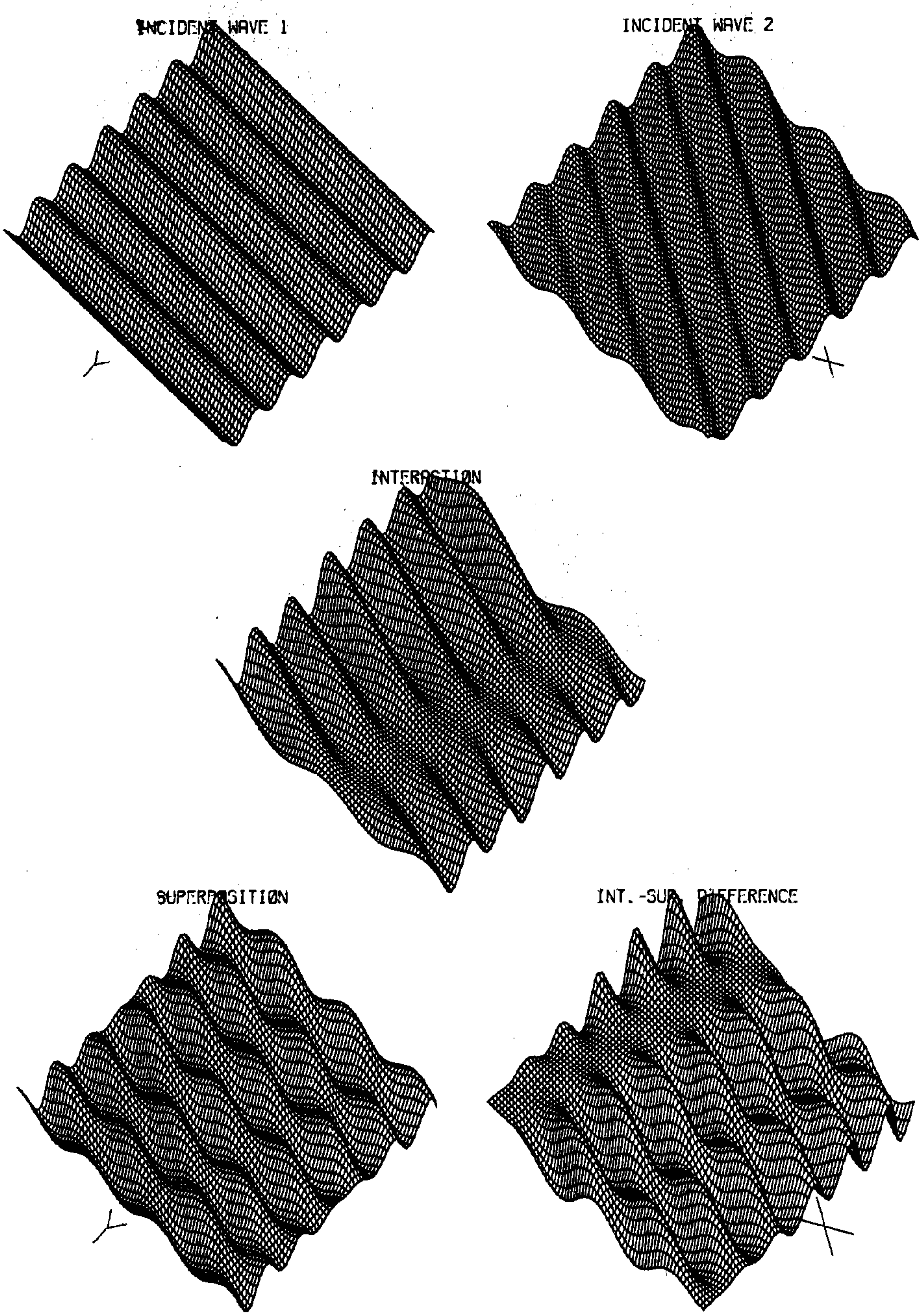
第一列來源波列(incident waves 1)
k_1d = 0.196 (相對水深)
k_1a_1= 0.076 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.196 (相對水深)
k_2a_2= 0.076 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 10^\circ$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-7a
Fig. 5-7a



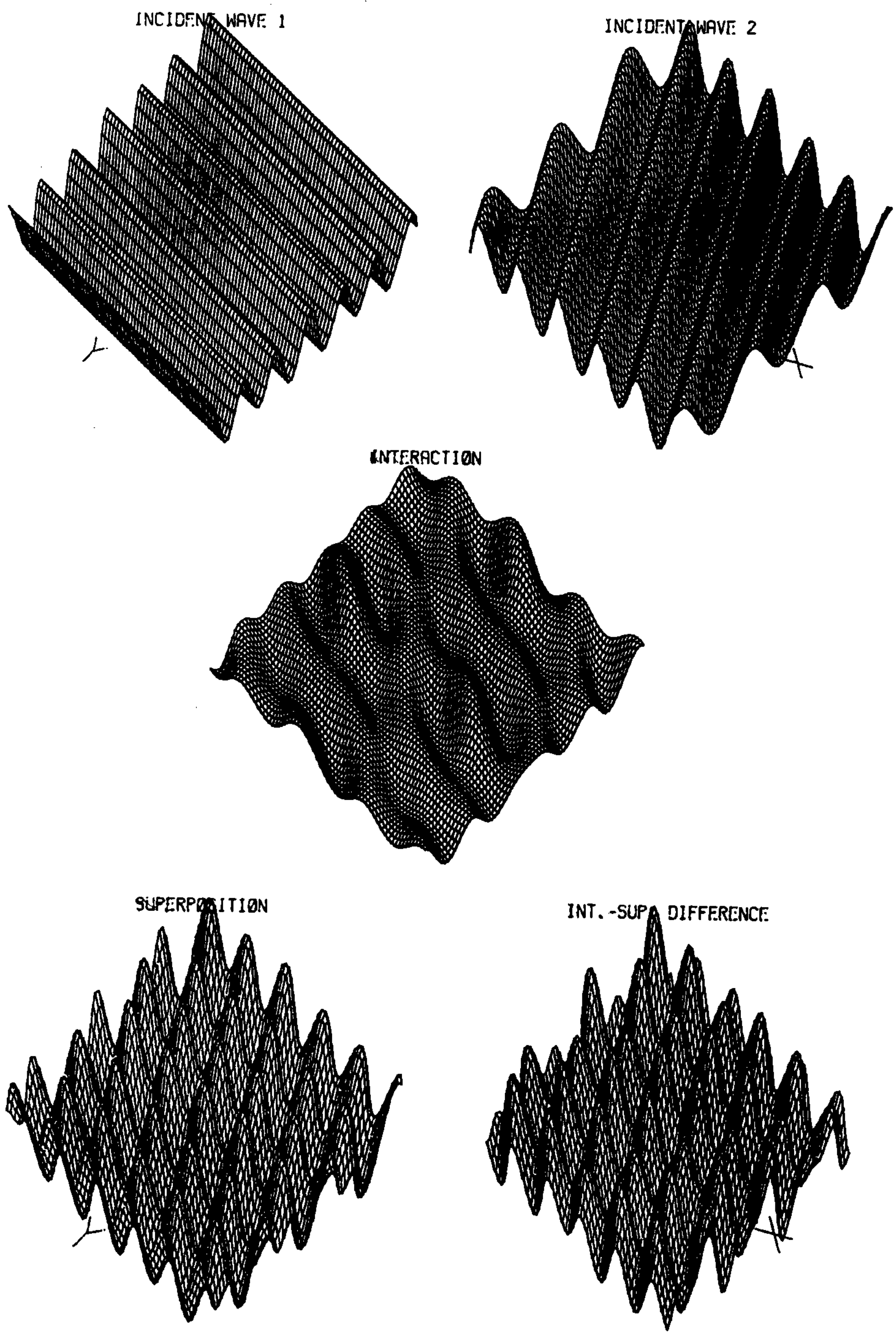
第一列來源波列(incident waves 1)
k_1d = 0.196 (相對水深)
k_1a_1= 0.076 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.196 (相對水深)
k_2a_2= 0.076 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 30^\circ$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-7b
Fig. 5-7b



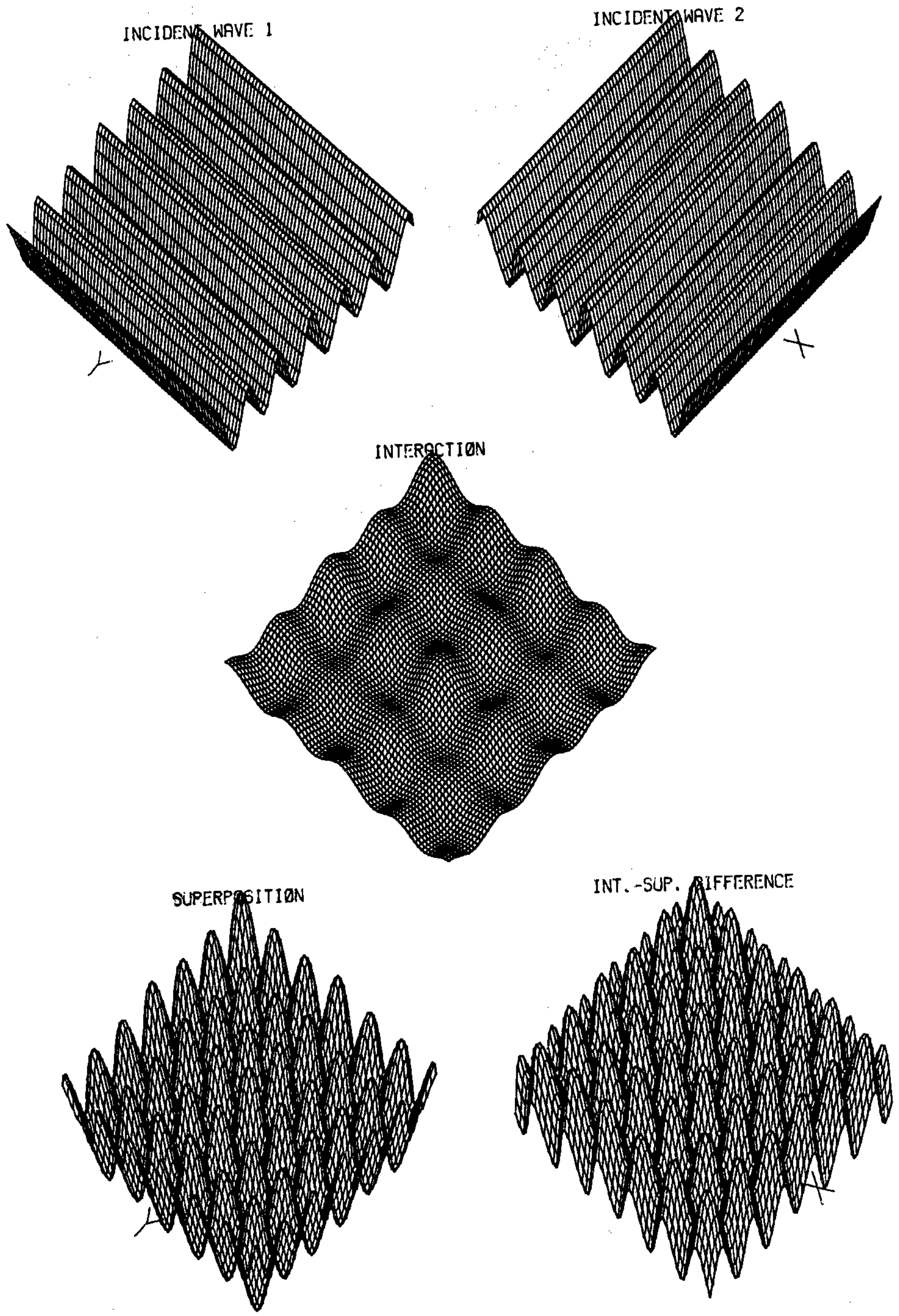
第一列來源波列(incident waves 1)
k_1d = 0.196 (相對水深)
k_1a_1= 0.076 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.196 (相對水深)
k_2a_2= 0.076 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 60°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-7c
Fig. 5-7c



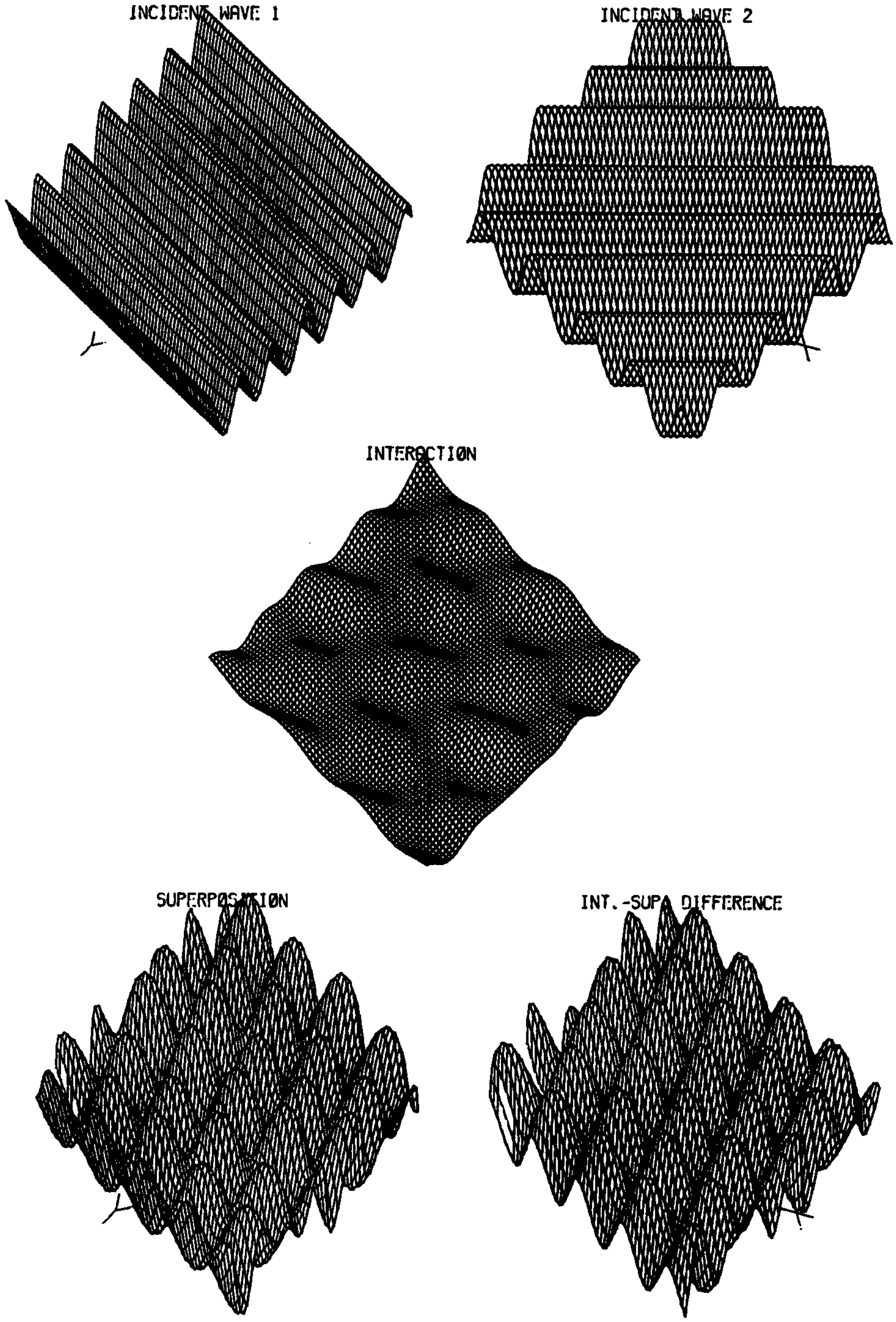
第一列來源波列(incident waves 1)
k_1d = 0.196 (相對水深)
k_1a_1= 0.076 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.196 (相對水深)
k_2a_2= 0.076 (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 90°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-7d
Fig. 5-7d



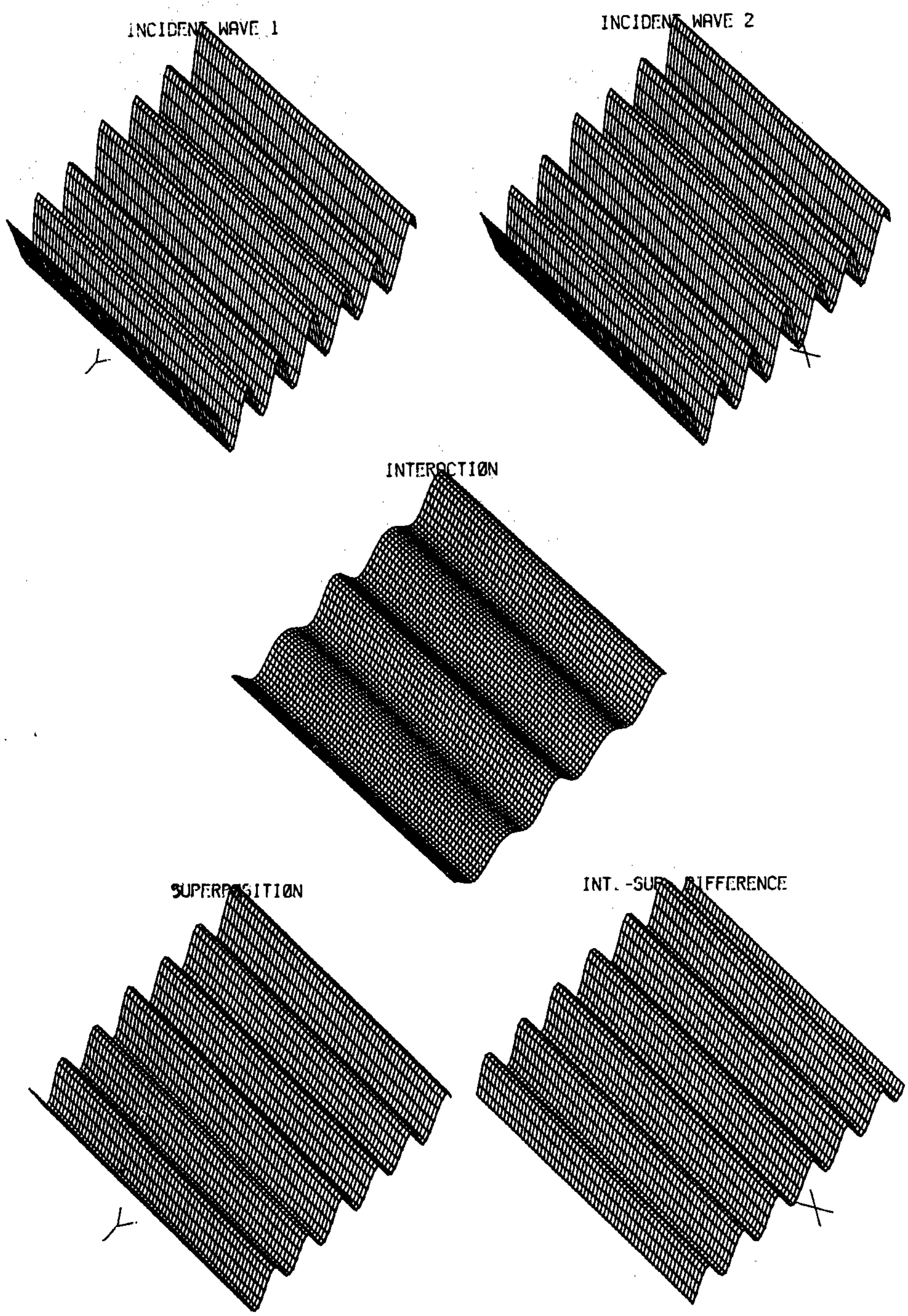
第一列來源波列(incident waves 1)
k_1d = 0.196 (相對水深)
k_1a_1= 0.076 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.196 (相對水深)
k_2a_2= 0.076 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 135°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-7e
Fig. 5-7e



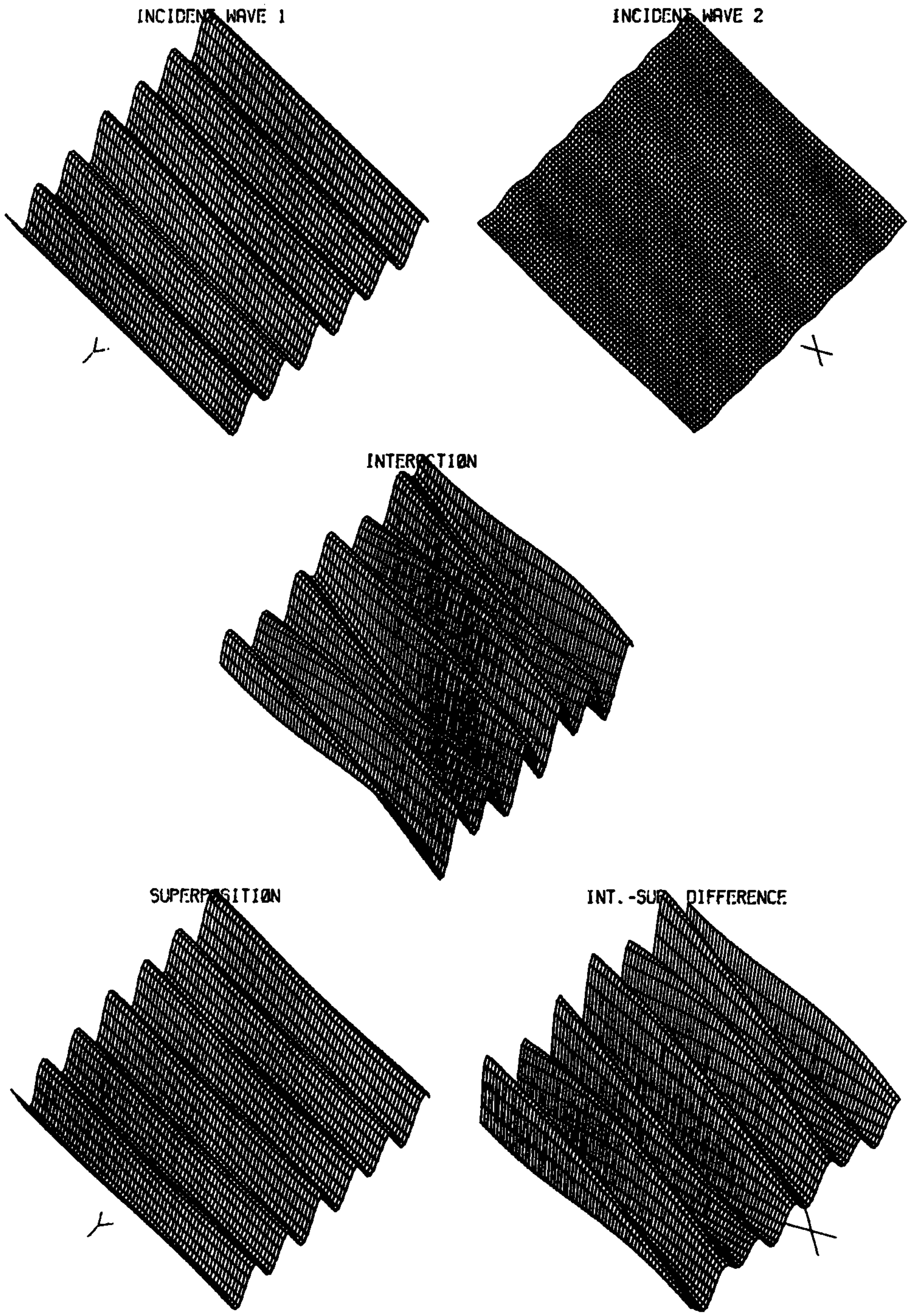
第一列來源波列(incident waves 1)
k_1d = 0.196 (相對水深)
k_1a_1= 0.076 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.196 (相對水深)
k_2a_2= 0.076 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2$= 180°

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-7f
Fig. 5-7f



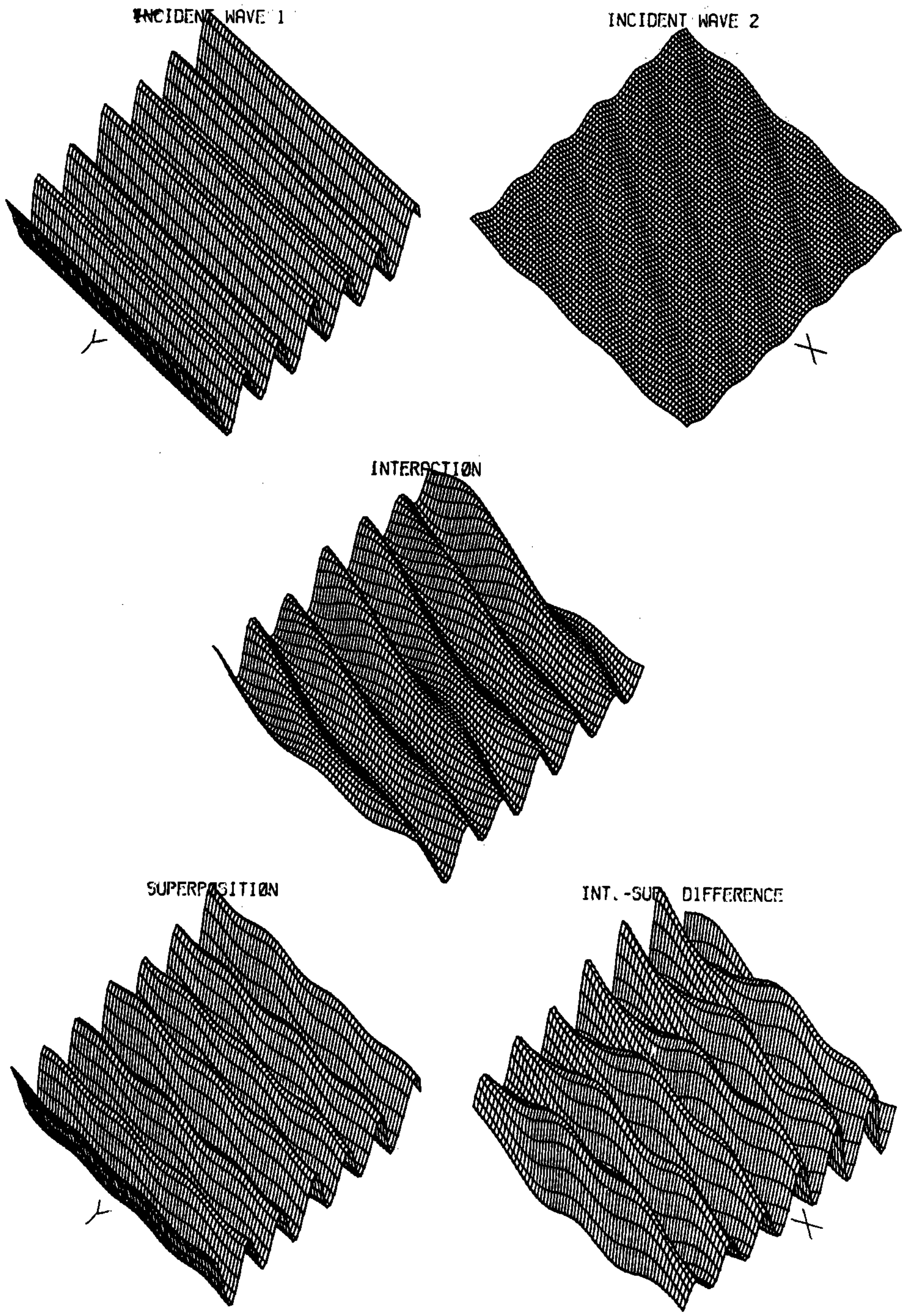
第一列來源波列(incident waves 1)
k_1d = 0.196 (相對水深)
k_1a_1= 0.076 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.196 (相對水深)
k_2a_2= 0.038 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2$= 10°

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-8a
Fig. 5-8a



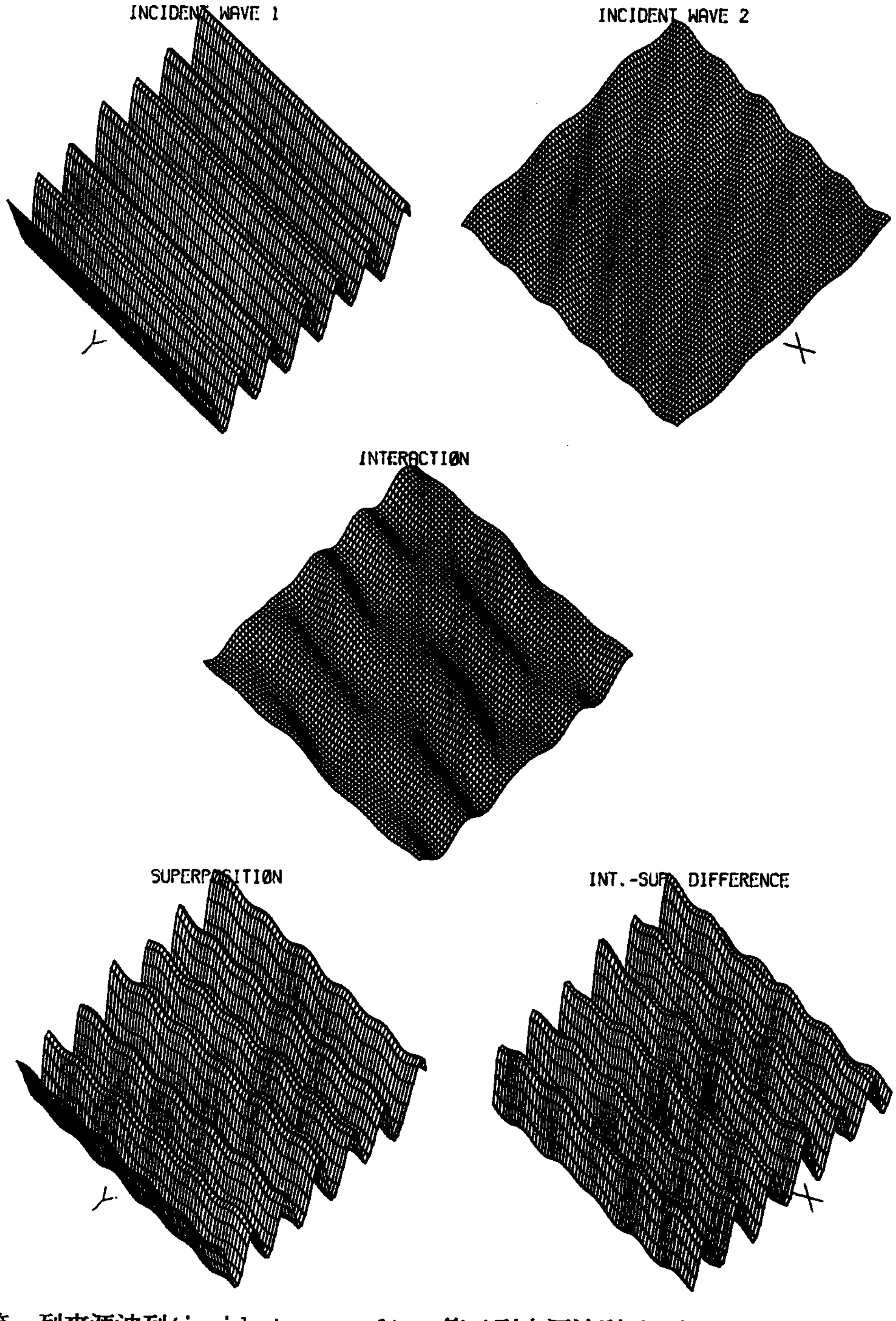
第一列來源波列(incident waves 1)
k_1d = 0.196 (相對水深)
k_1a_1= 0.076 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.196 (相對水深)
k_2a_2= 0.038 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 30^\circ$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-8b
Fig. 5-8b



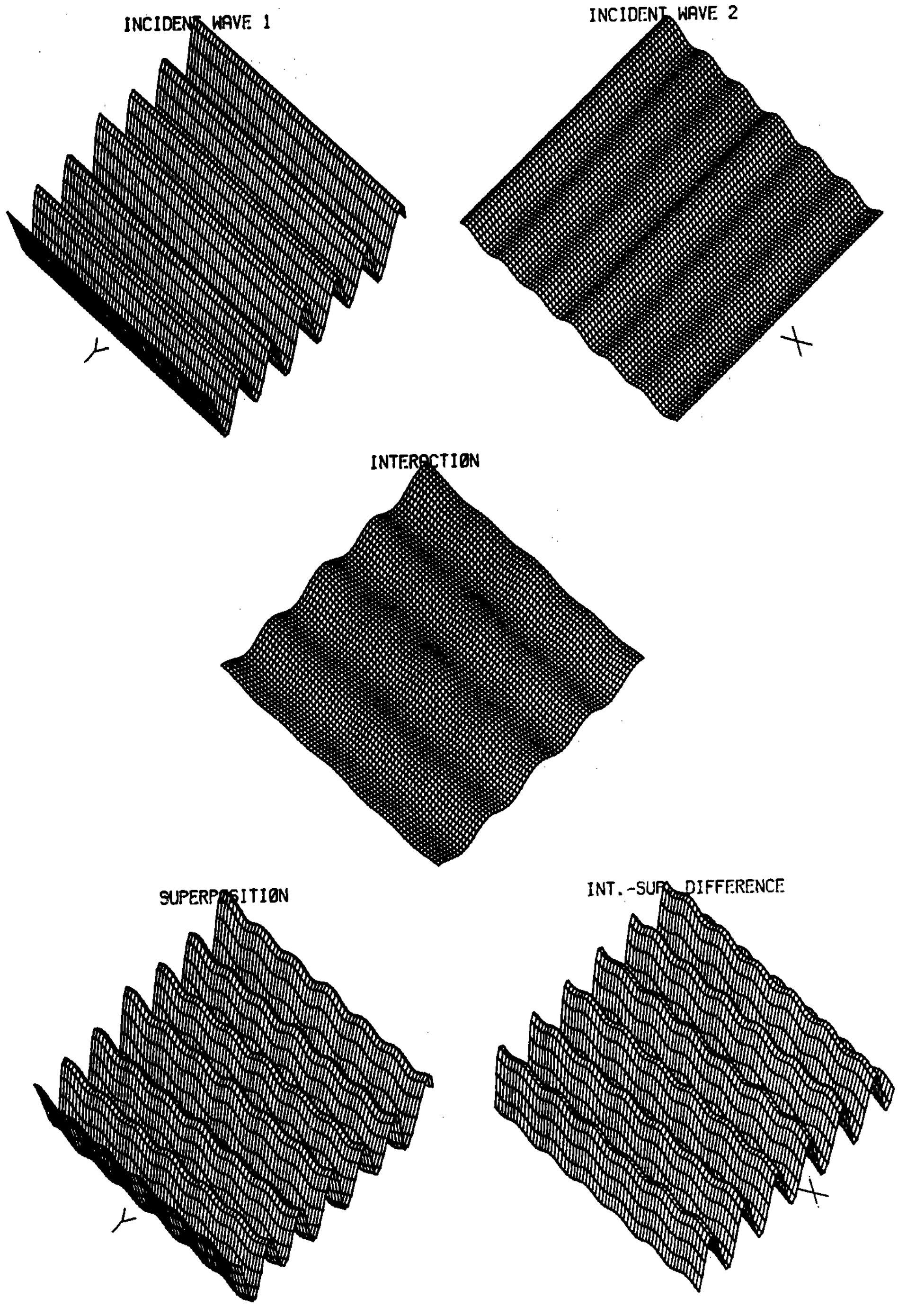
第一列來源波列(incident waves 1)
$k_1 d = 0.196$ (相對水深)
$k_1 a_1 = 0.076$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2 d = 0.196$ (相對水深)
$k_2 a_2 = 0.038$ (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 60°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-8c
Fig. 5-8c



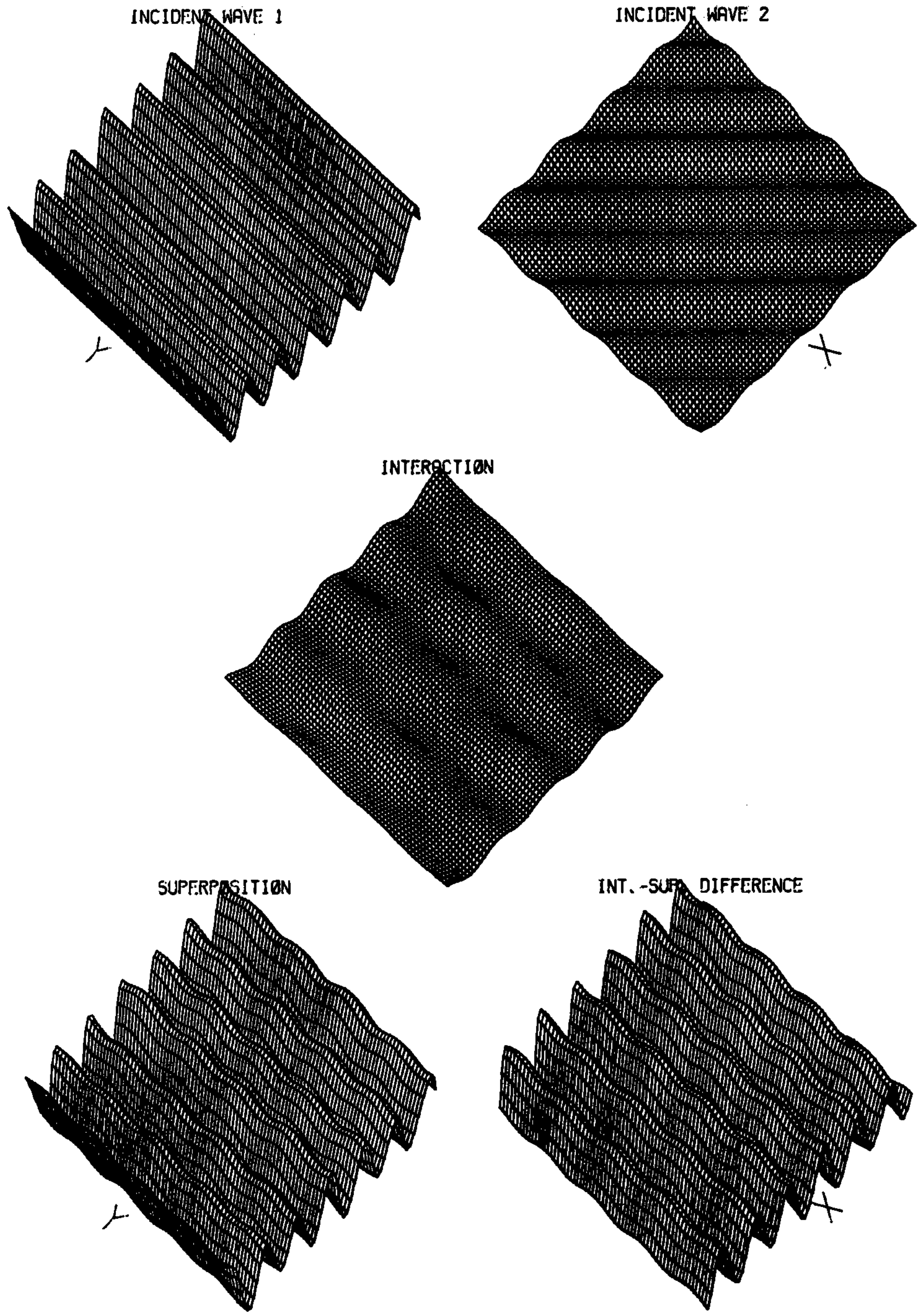
第一列來源波列(incident waves 1)
k_1d = 0.196 (相對水深)
k_1a_1= 0.076 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.196 (相對水深)
k_2a_2= 0.038 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 90°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-8d
Fig. 5-8d



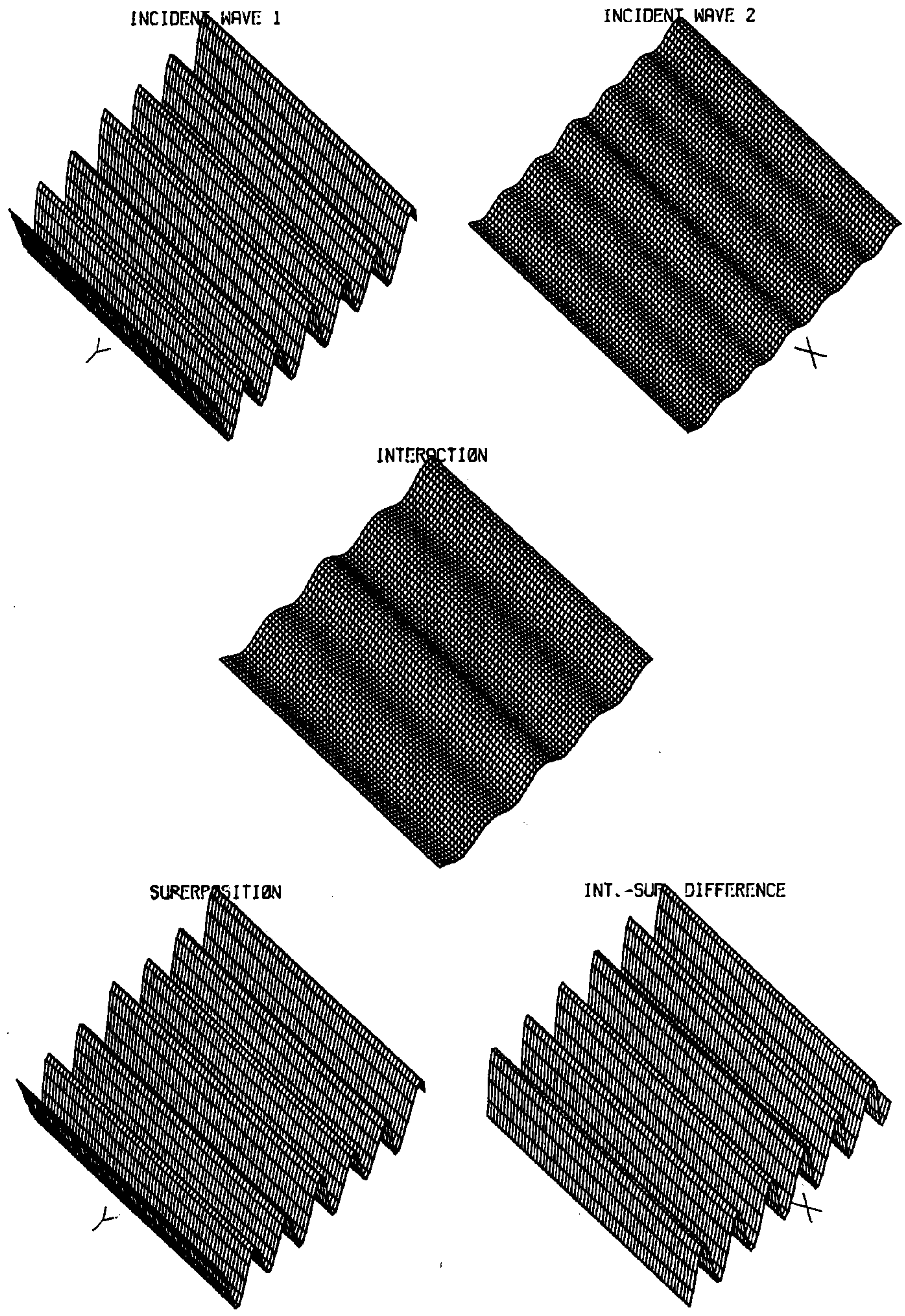
第一列來源波列(incident waves 1)
k_1d = 0.196 (相對水深)
k_1a_1= 0.076 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.196 (相對水深)
k_2a_2= 0.038 (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 135°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-8e
Fig. 5-8e



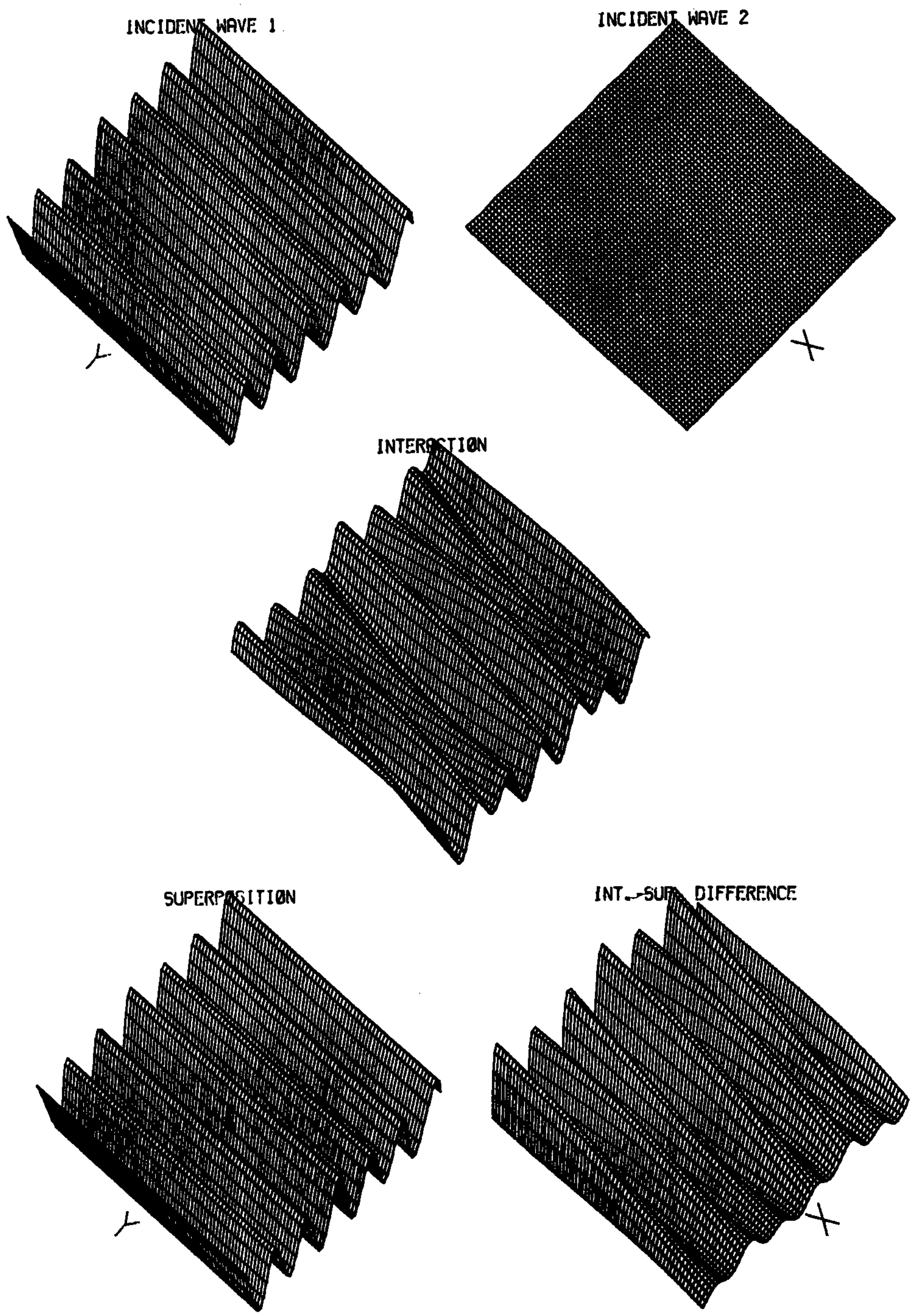
第一列來源波列(incident waves 1)
k_1d = 0.196 (相對水深)
k_1a_1= 0.076 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.196 (相對水深)
k_2a_2= 0.038 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 180°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-8f
Fig. 5-8f



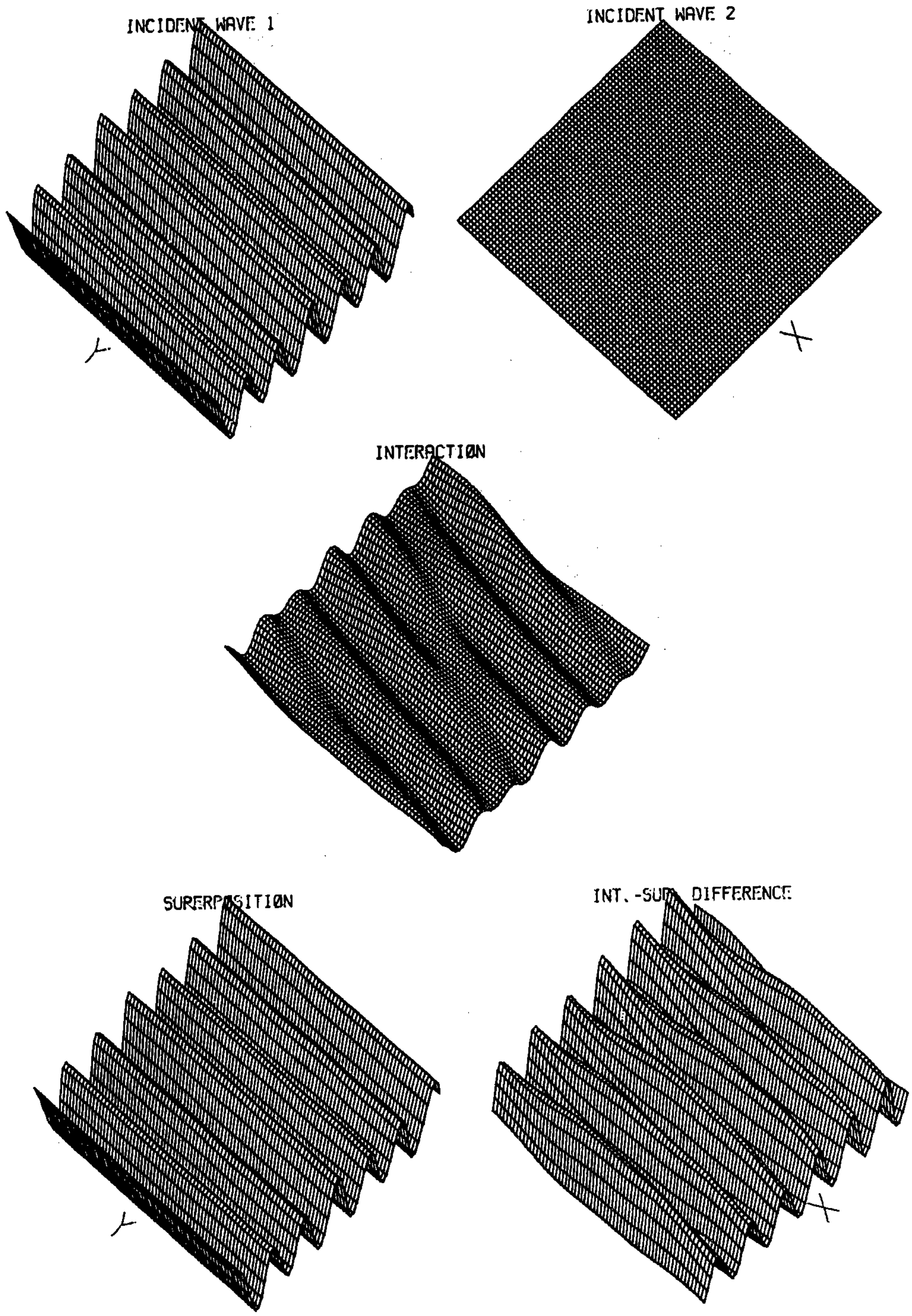
第一列來源波列(incident waves 1)
k_1d = 0.196 (相對水深)
k_1a_1= 0.076 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.196 (相對水深)
k_2a_2= 0.019 (波浪尖銳度)

二波列交會夾角$\theta = \theta_1 - \theta_2 = 10°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-9a
Fig. 5-9a



第一列來源波列(incident waves 1)　第二列來源波列(incident waves 2)

$k_1d = 0.196$ (相對水深)　$k_2d = 0.196$ (相對水深)

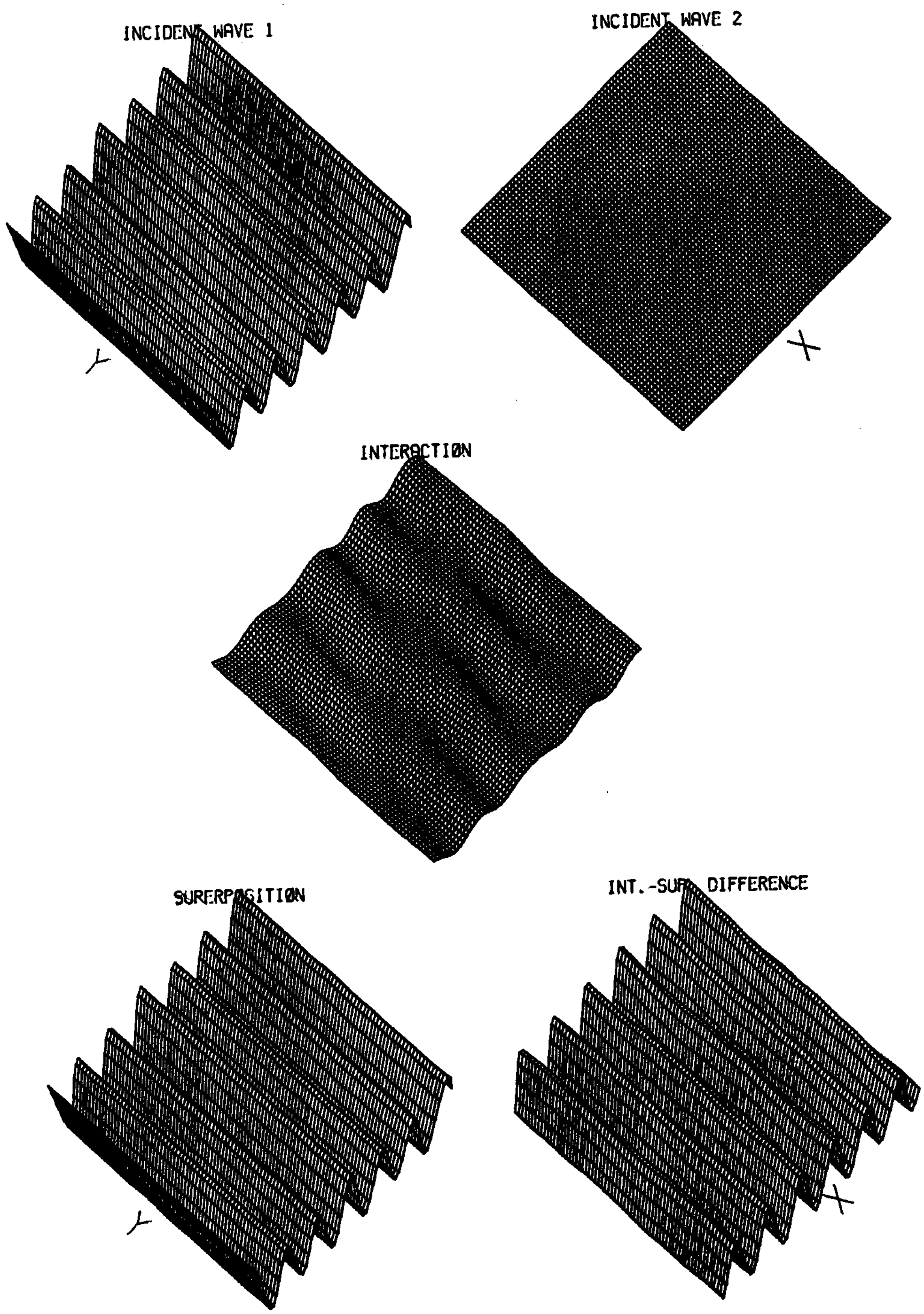
$k_1a_1 = 0.076$ (波浪尖銳度)　$k_2a_2 = 0.019$ (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 30°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-9b

Fig. 5-9b



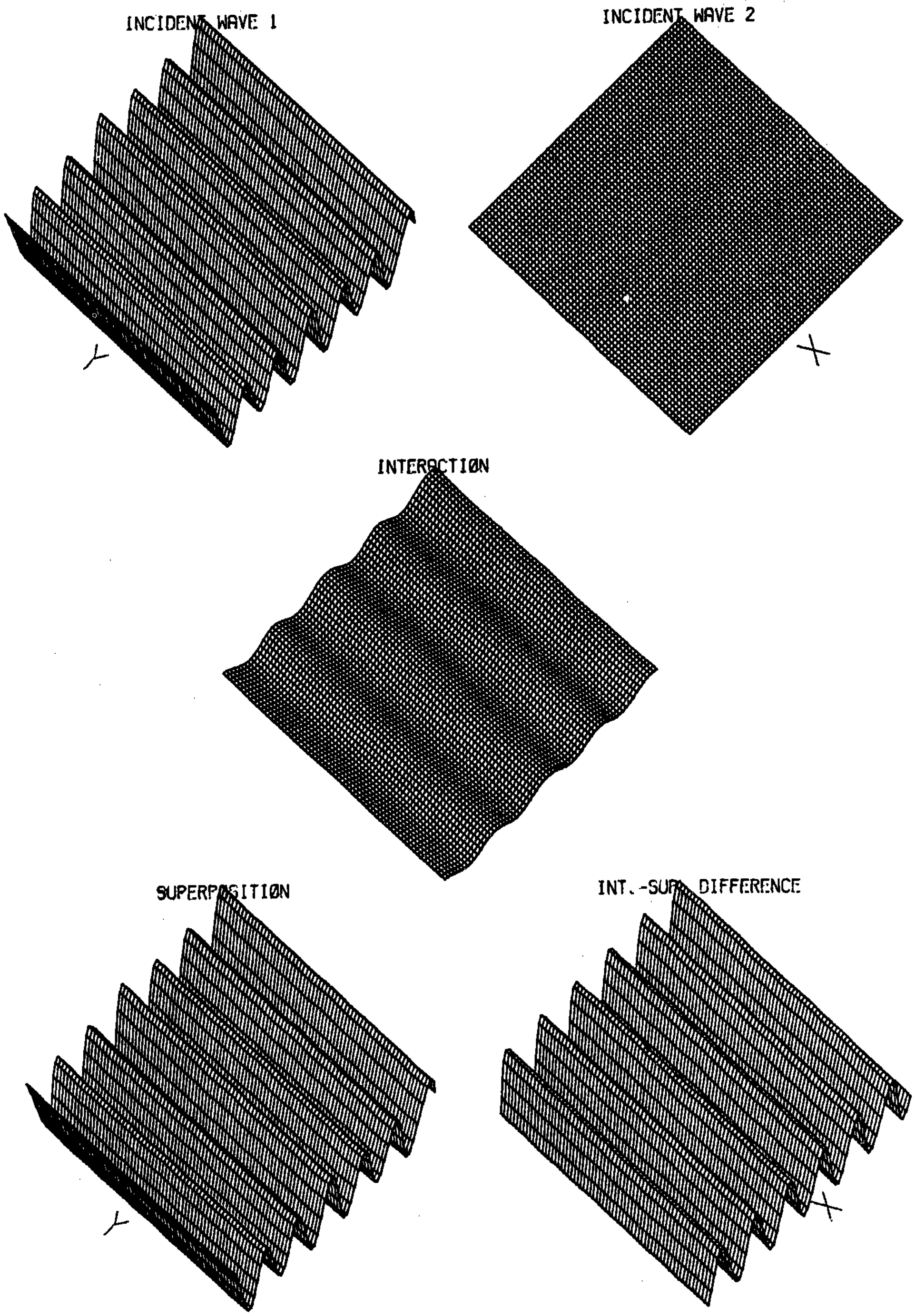
第一列來源波列(incident waves 1)
k_1d = 0.196 (相對水深)
k_1a_1= 0.076 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.196 (相對水深)
k_2a_2= 0.019 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 60°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-9c
Fig. 5-9c



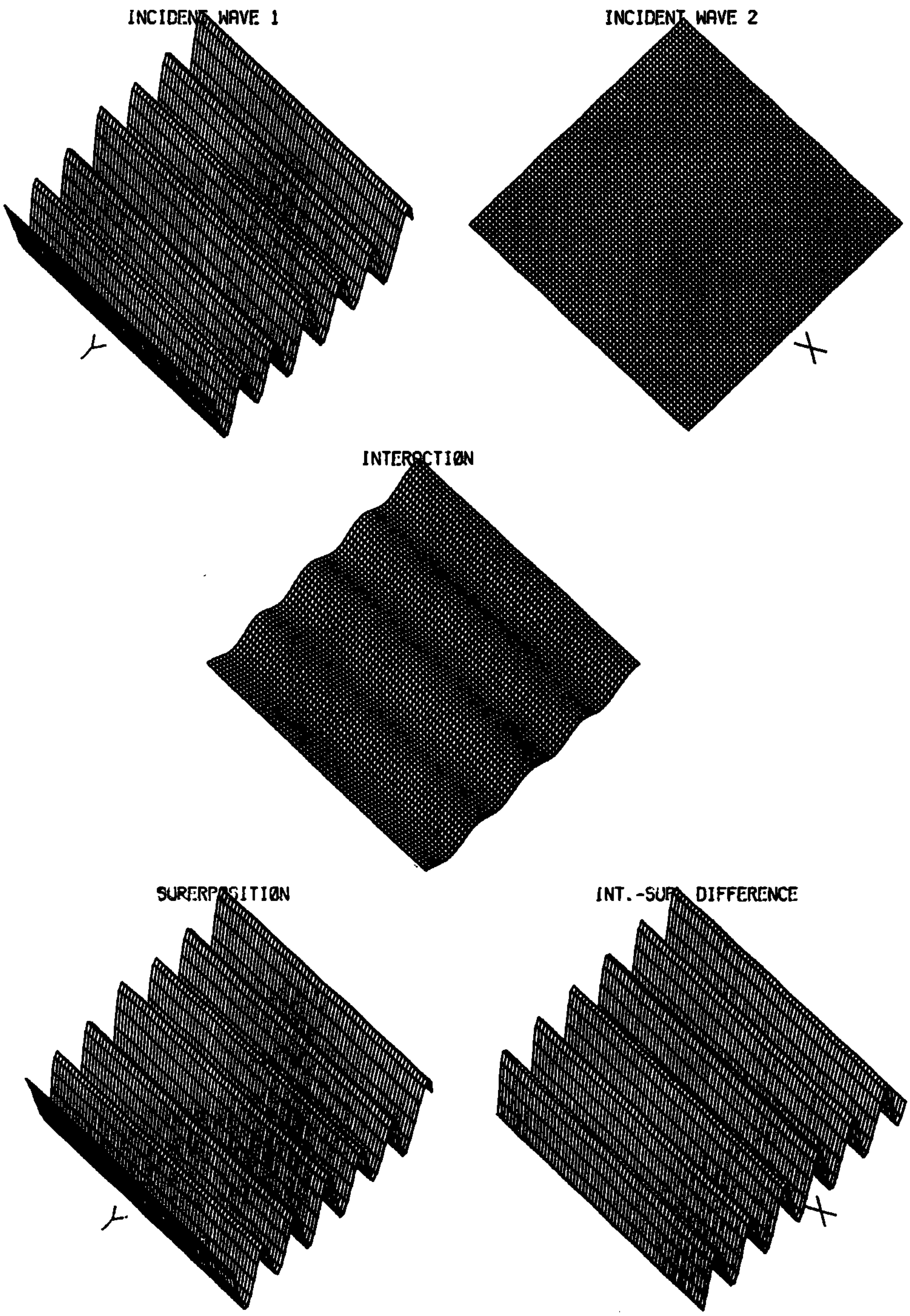
第一列來源波列(incident waves 1)
k_1d = 0.196 (相對水深)
k_1a_1= 0.076 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.196 (相對水深)
k_2a_2= 0.019 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 90^\circ$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-9d
Fig. 5-9d



第一列來源波列(incident waves 1)

k_1d = 0.196 (相對水深)

k_1a_1= 0.076 (波浪尖銳度)

第二列來源波列(incident waves 2)

k_2d = 0.196 (相對水深)

k_2a_2= 0.019 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 135°$

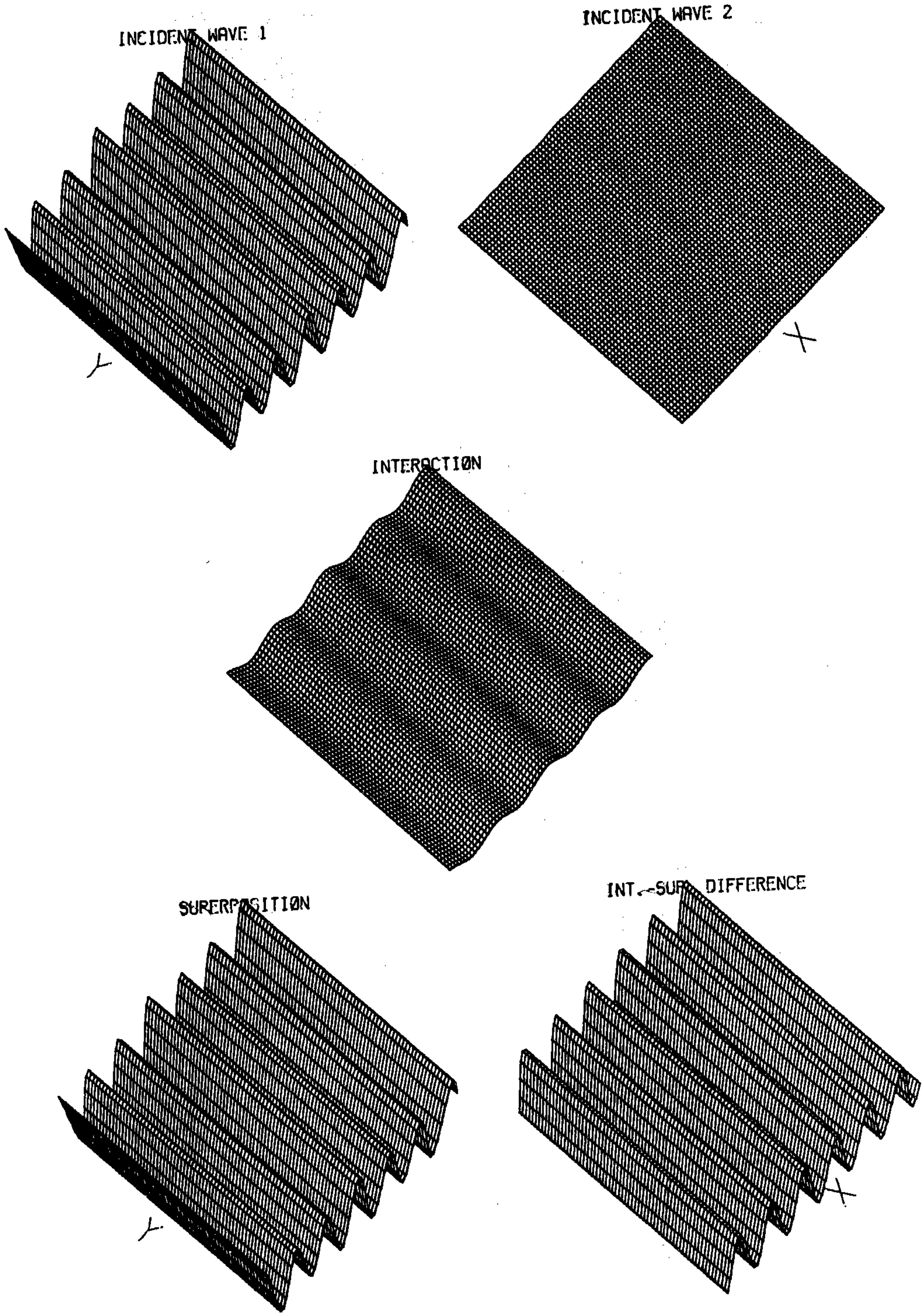
INTERACTION = 二波列交會發生非線性交互作用的結果

SUPERPOSITION= 二波列直接線性疊加

INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-9e

Fig. 5-9e



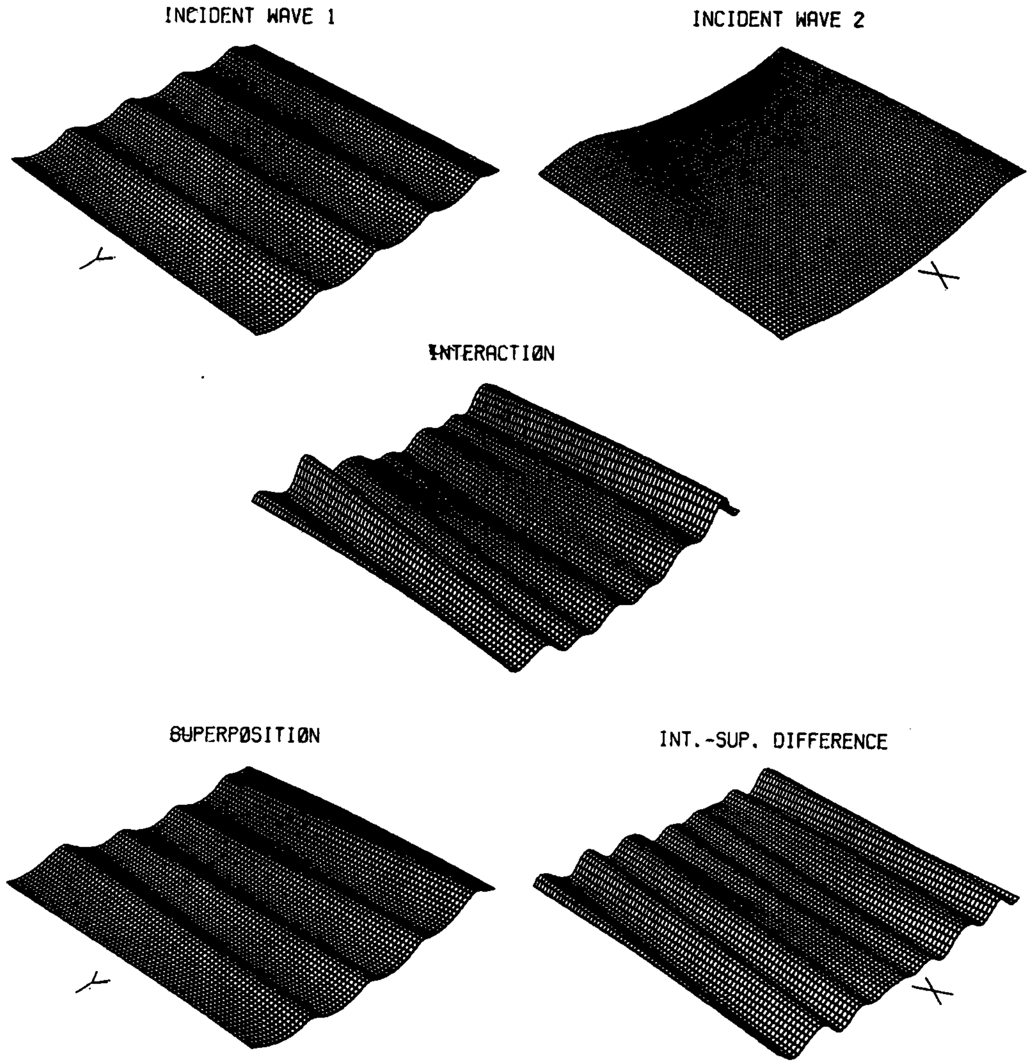
第一列來源波列(incident waves 1)
k_1d = 0.196 (相對水深)
k_1a_1= 0.076 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.196 (相對水深)
k_2a_2= 0.019 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2$= 180°

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-9f
Fig. 5-9f



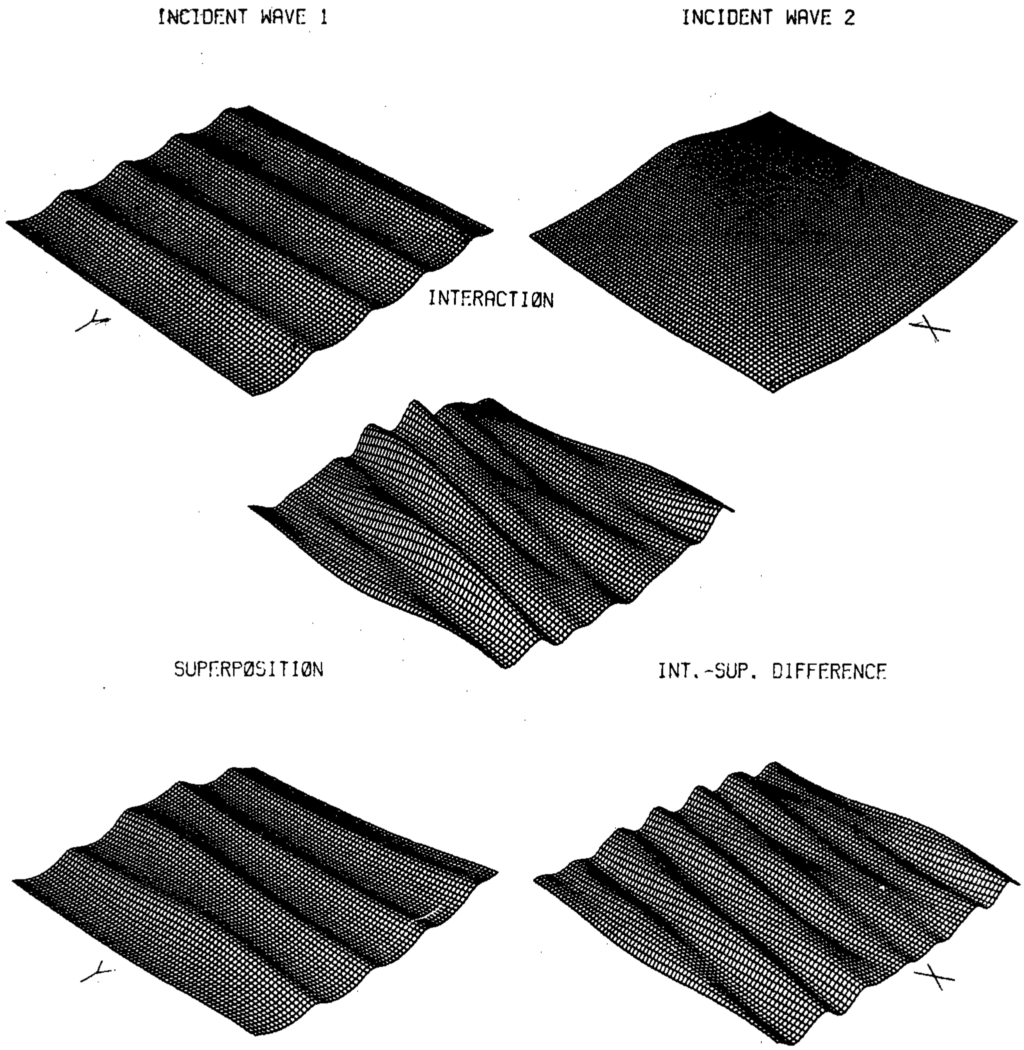
第一列來源波列(incident waves 1)
k_1d = 4.0 (相對水深)
k_1a_1= 0.2 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 1.0 (相對水深)
k_2a_2= 0.2 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 10°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-10a
Fig. 5-10a



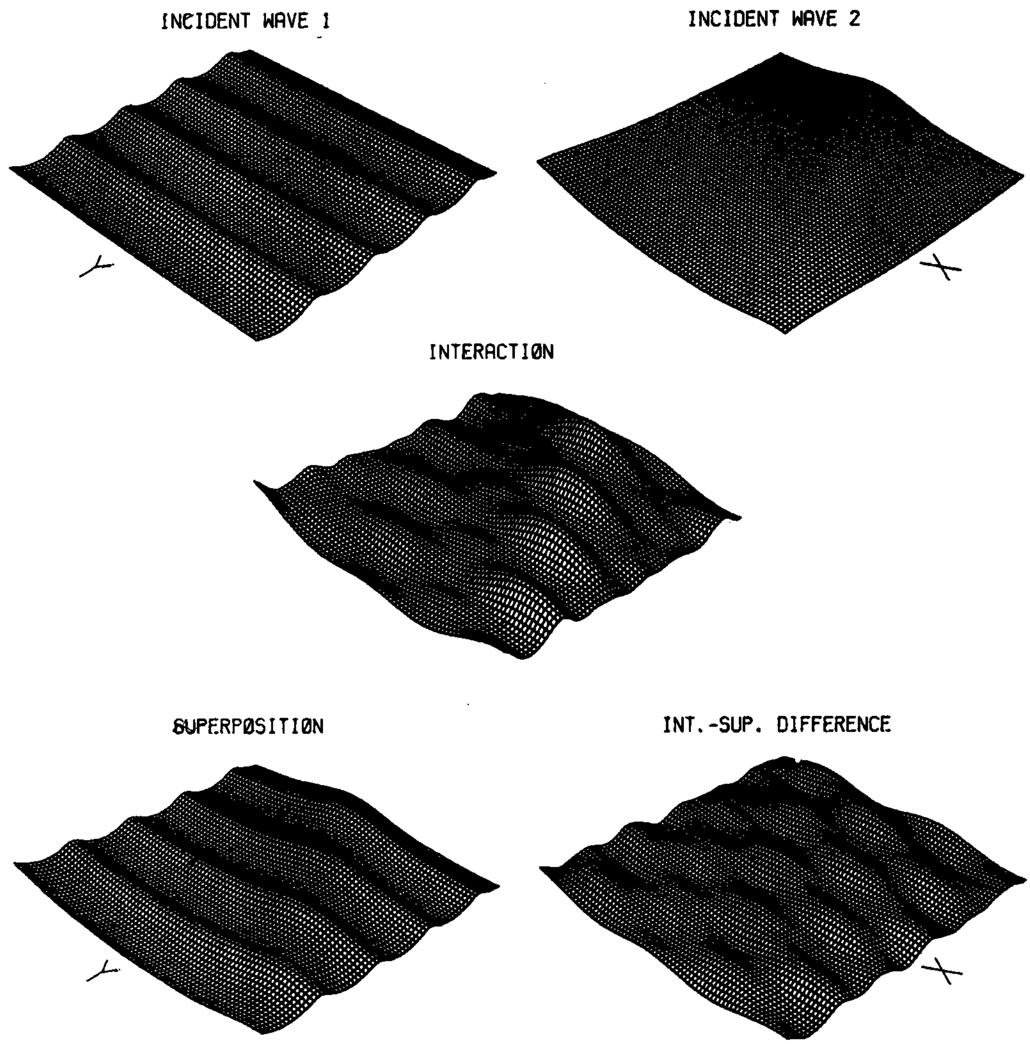
第一列來源波列(incident waves 1)
k_1d = 4.0 (相對水深)
k_1a_1= 0.2 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 1.0 (相對水深)
k_2a_2= 0.2 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 30^\circ$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-10b
Fig. 5-10b



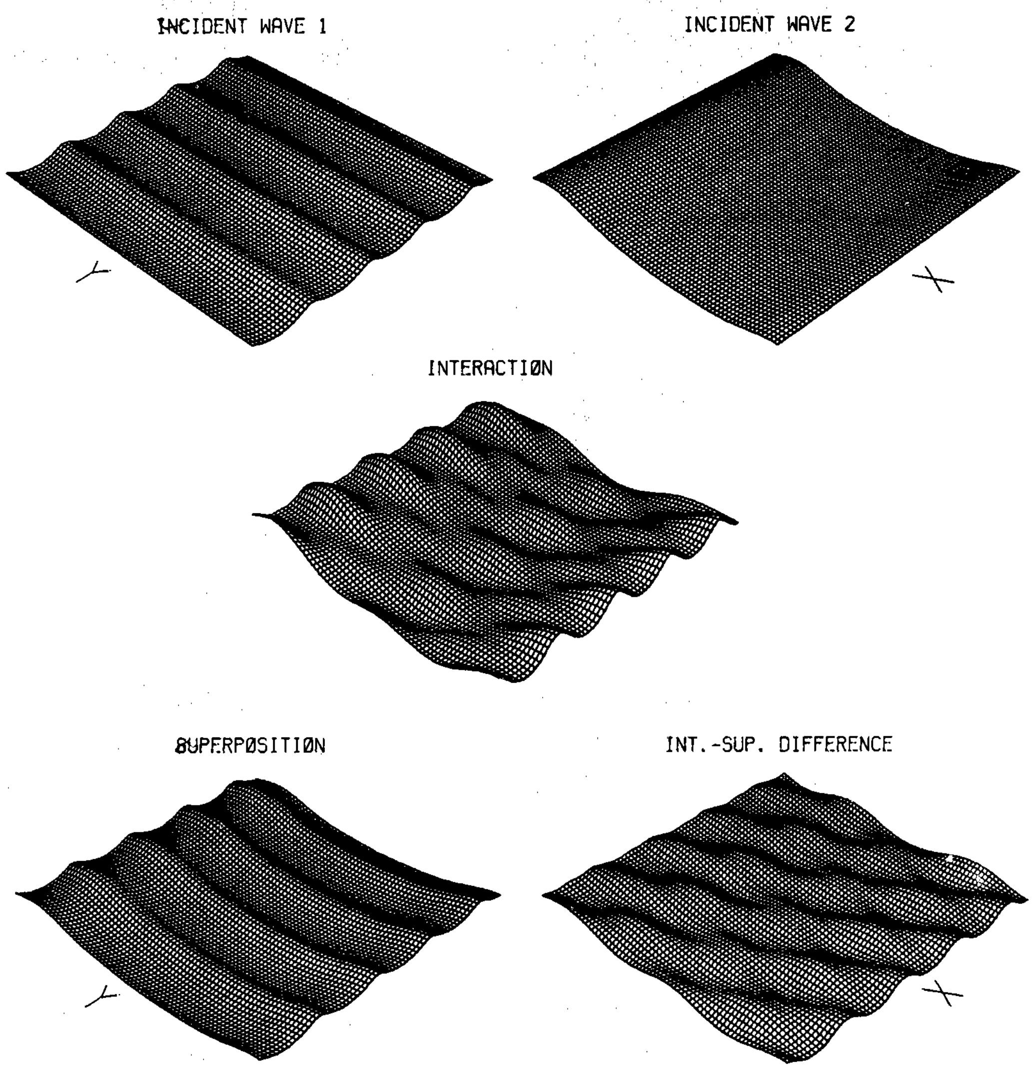
第一列來源波列(incident waves 1)
k_1d = 4.0 (相對水深)
k_1a_1= 0.2 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 1.0 (相對水深)
k_2a_2= 0.2 (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 60°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-10c
Fig. 5-10c



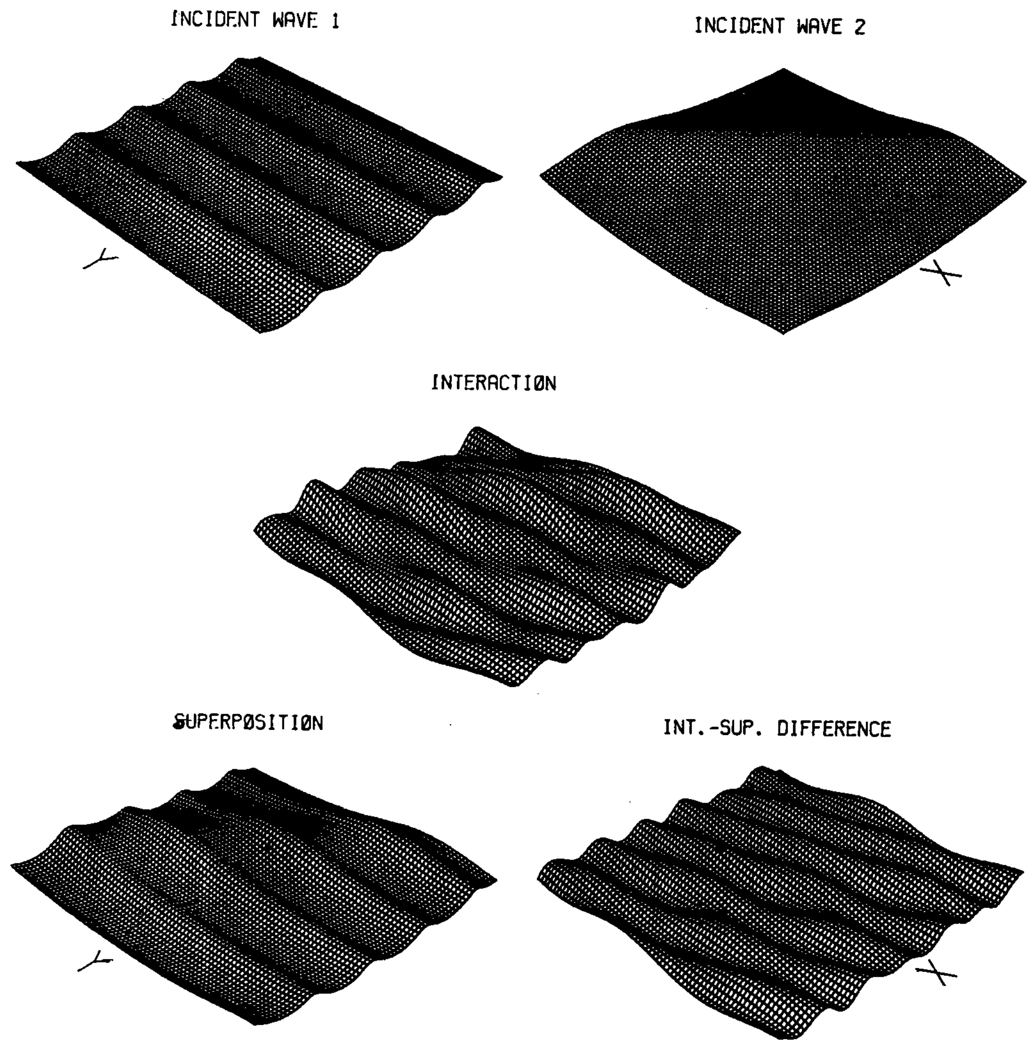
第一列來源波列(incident waves 1)
$k_1d = 4.0$ (相對水深)
$k_1a_1 = 0.2$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2d = 1.0$ (相對水深)
$k_2a_2 = 0.2$ (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 90°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-10d
Fig. 5-10d



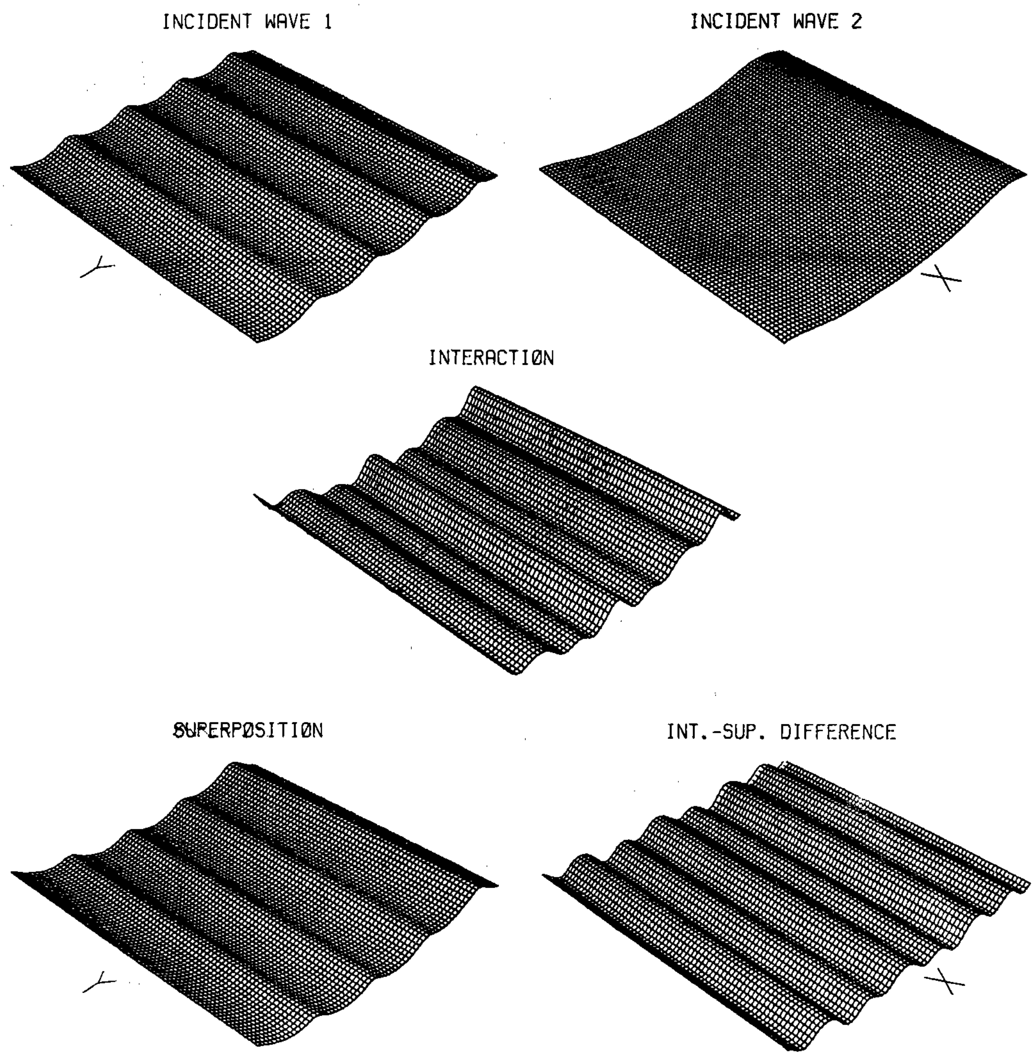
第一列來源波列(incident waves 1)
$k_1d = 4.0$ (相對水深)
$k_1a_1 = 0.2$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2d = 1.0$ (相對水深)
$k_2a_2 = 0.2$ (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 135°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-10e
Fig. 5-10e



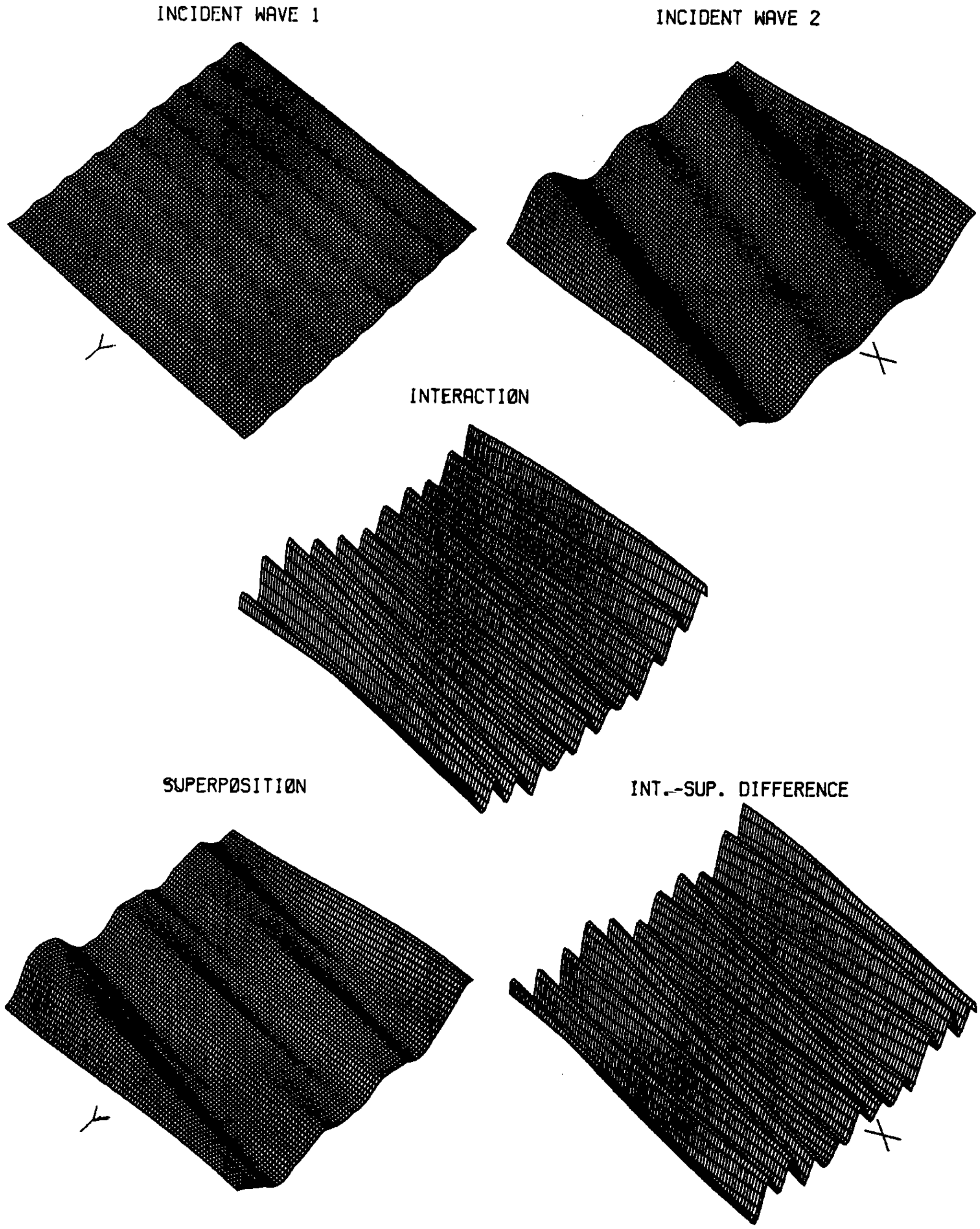
第一列來源波列(incident waves 1)
$k_1d = 4.0$ (相對水深)
$k_1a_1 = 0.2$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2d = 1.0$ (相對水深)
$k_2a_2 = 0.2$ (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 180°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-10f
Fig. 5-10f



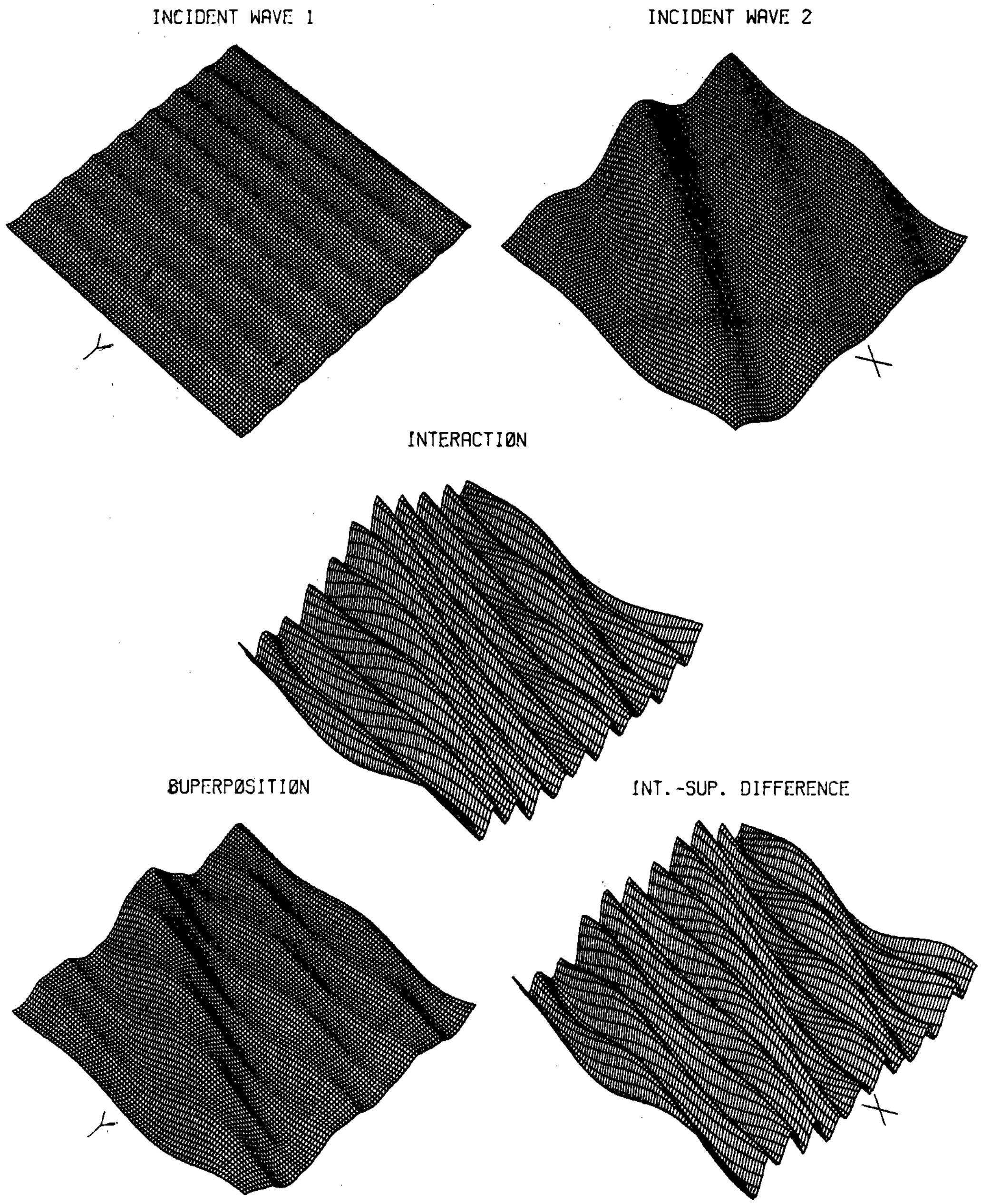
第一列來源波列(incident waves 1)
k_1d = 4.0 (相對水深)
k_1a_1= 0.2 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.5 (相對水深)
k_2a_2= 0.2 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 10°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-11a
Fig. 5-11a



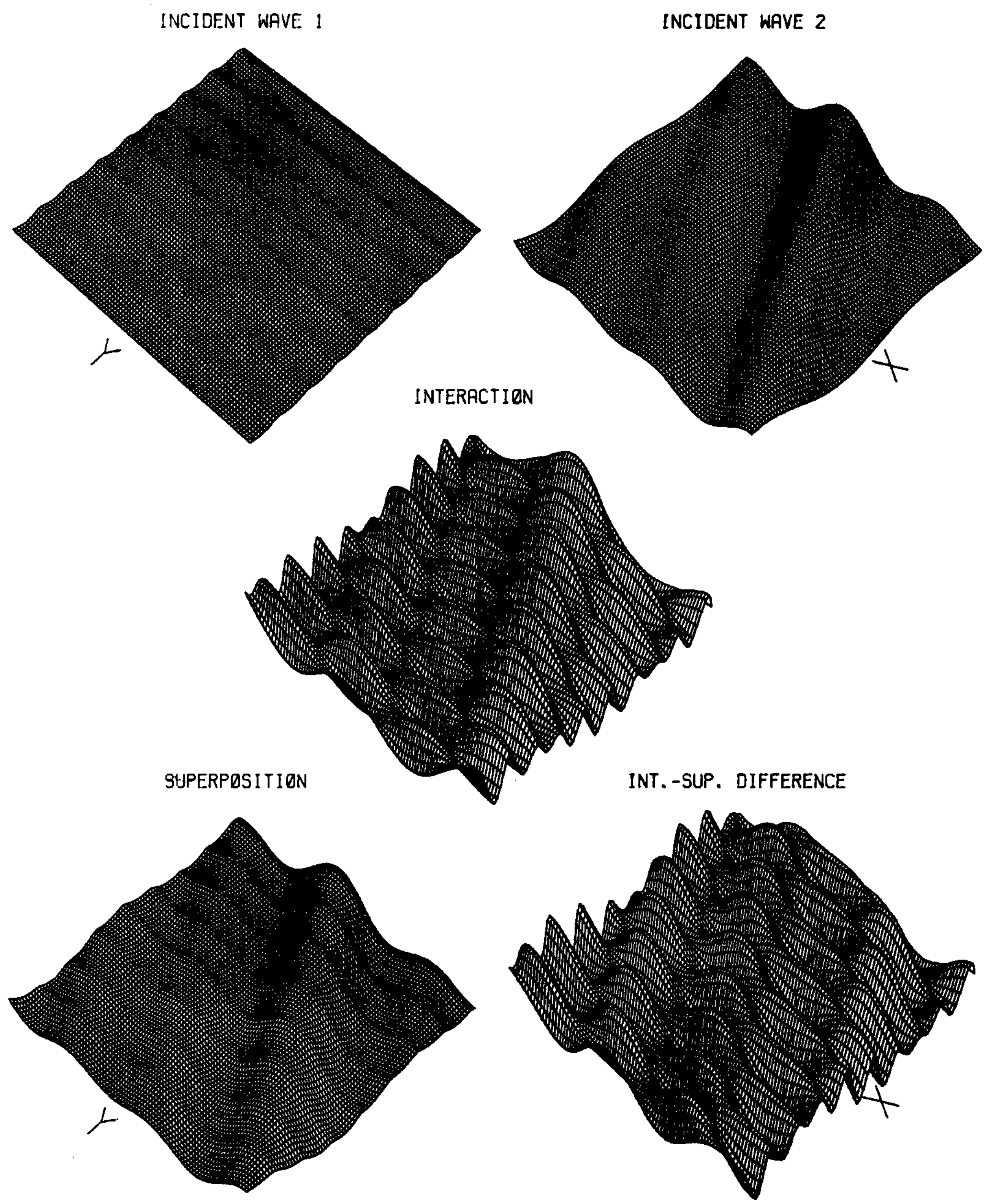
第一列來源波列(incident waves 1)
$k_1d = 4.0$ (相對水深)
$k_1a_1 = 0.2$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2d = 0.5$ (相對水深)
$k_2a_2 = 0.2$ (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 30°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-11b
Fig. 5-11b



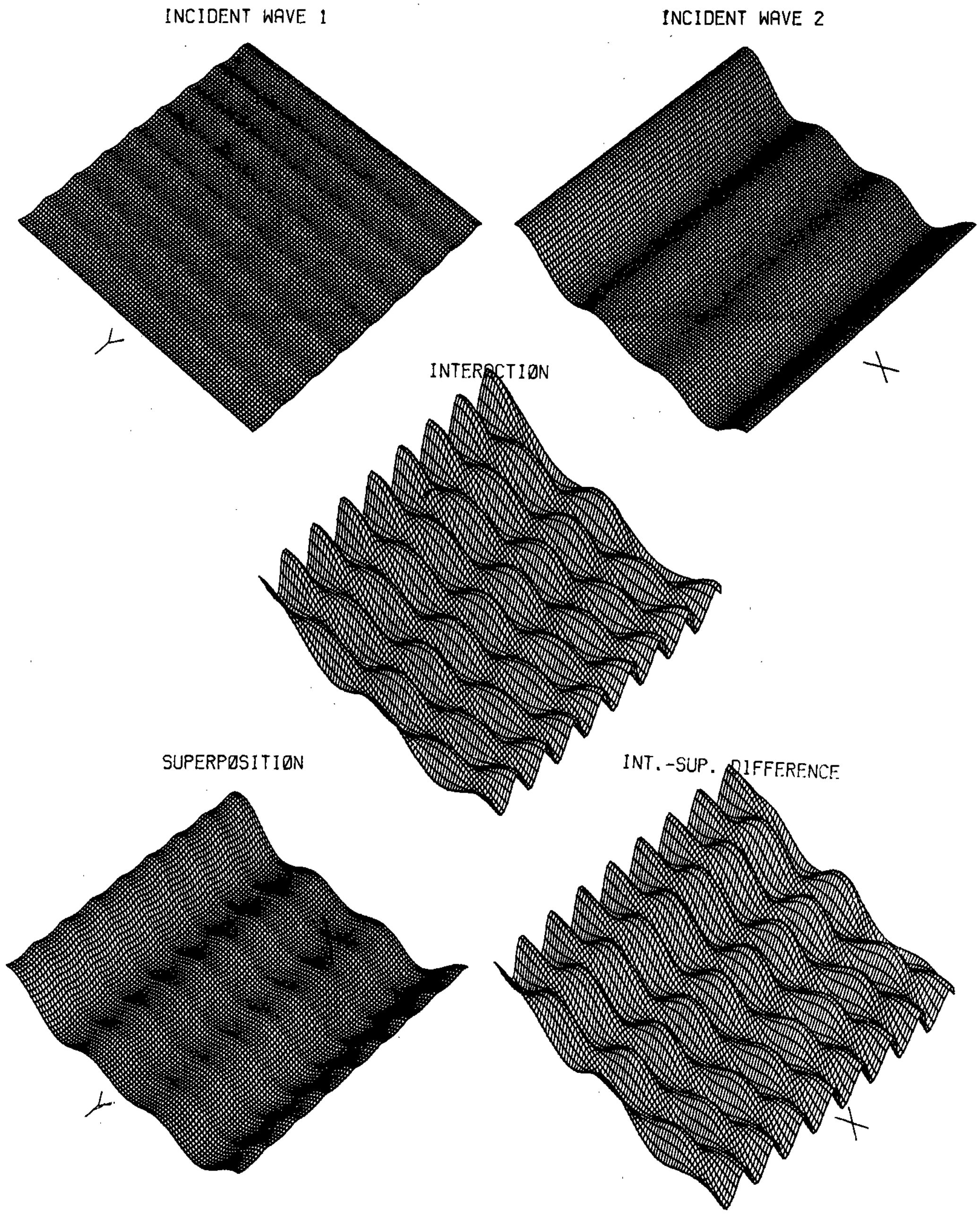
第一列來源波列(incident waves 1)
$k_1d = 4.0$ (相對水深)
$k_1a_1 = 0.2$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2d = 0.5$ (相對水深)
$k_2a_2 = 0.2$ (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 60°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-11c
Fig. 5-11c



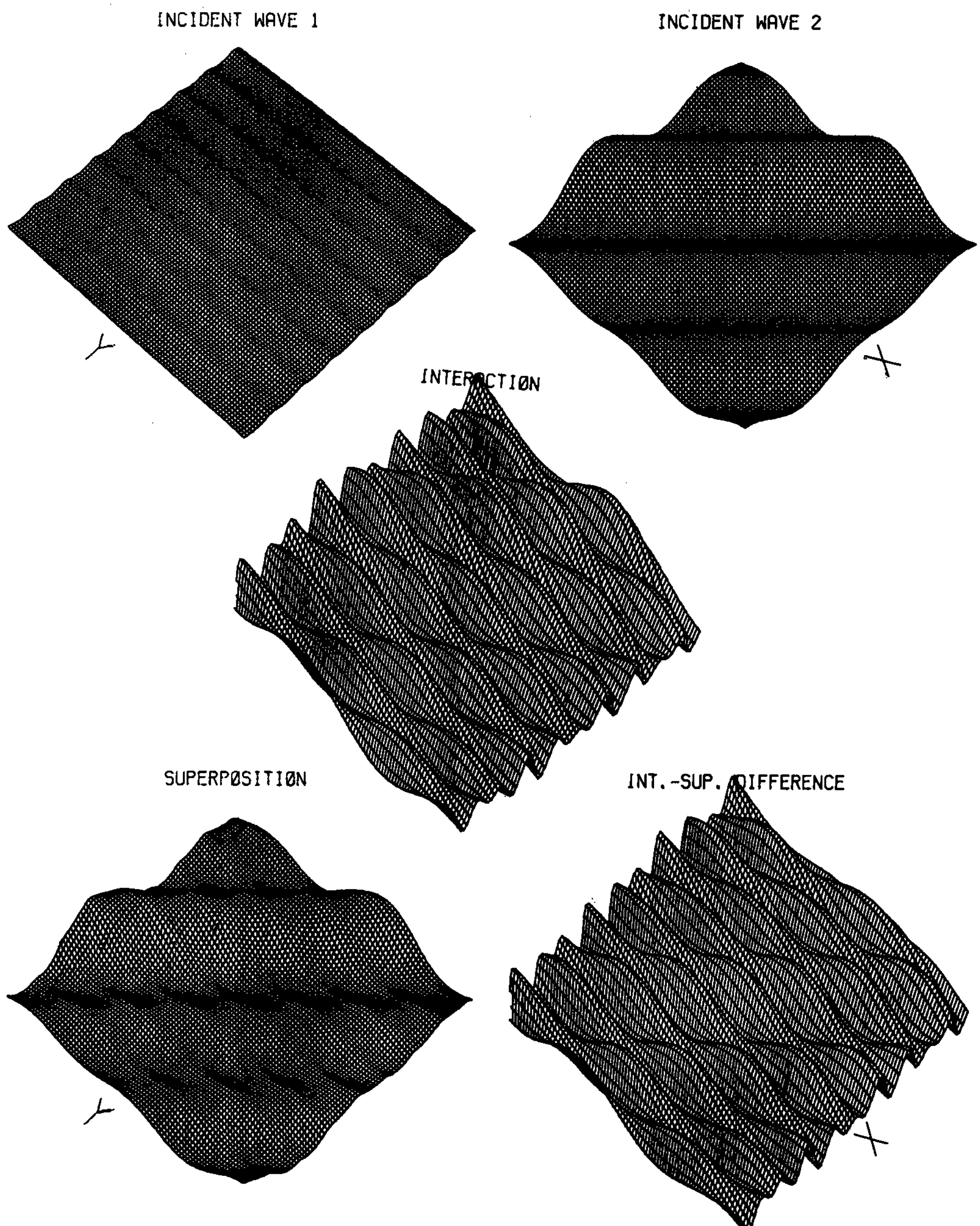
第一列來源波列(incident waves 1)
k_1d = 4.0 (相對水深)
k_1a_1= 0.2 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.5 (相對水深)
k_2a_2= 0.2 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2=90°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-11d
Fig. 5-11d



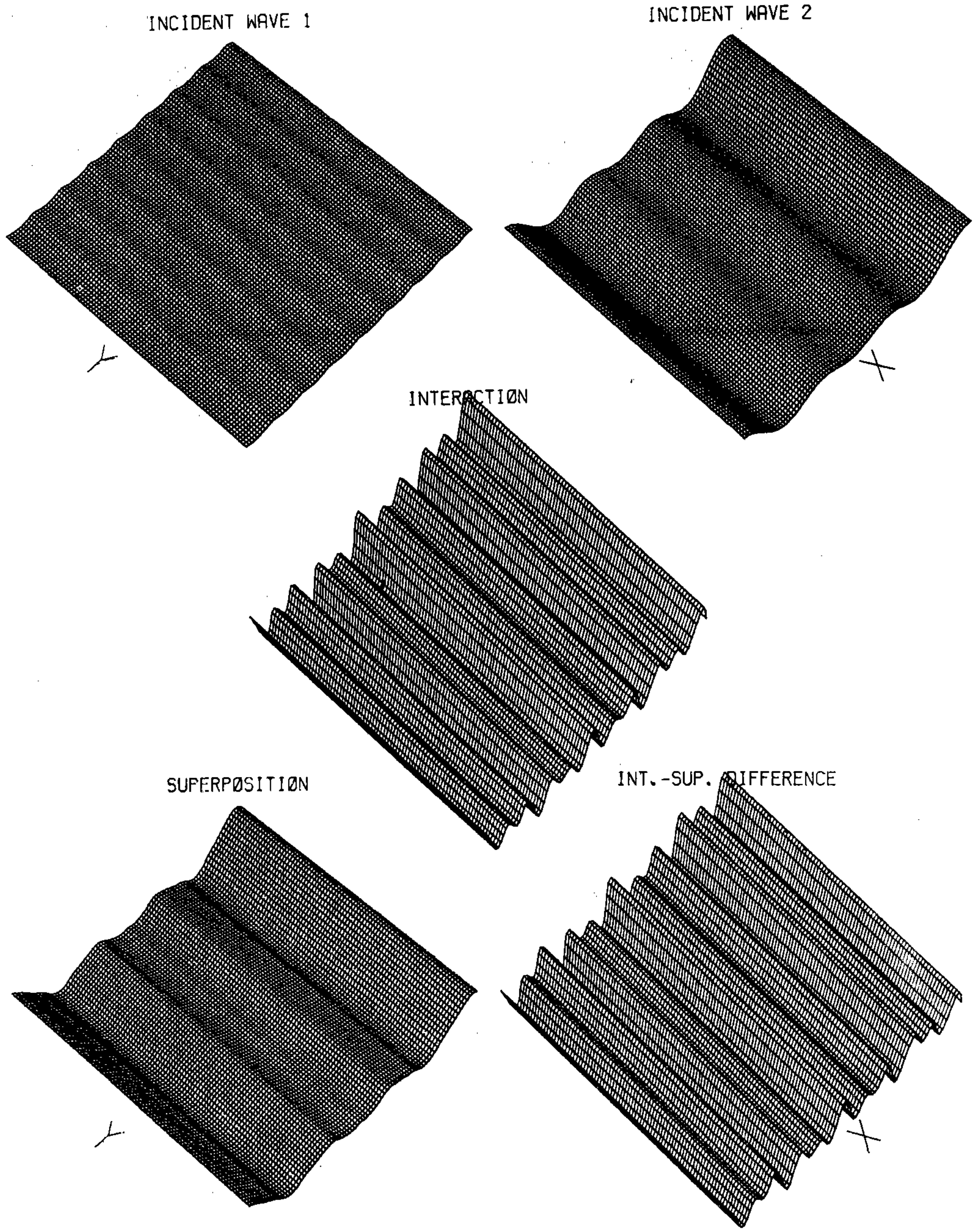
第一列來源波列(incident waves 1)
k_1d = 4.0 (相對水深)
k_1a_1= 0.2 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.5 (相對水深)
k_2a_2= 0.2 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2$= 135°

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-11e
Fig. 5-11e



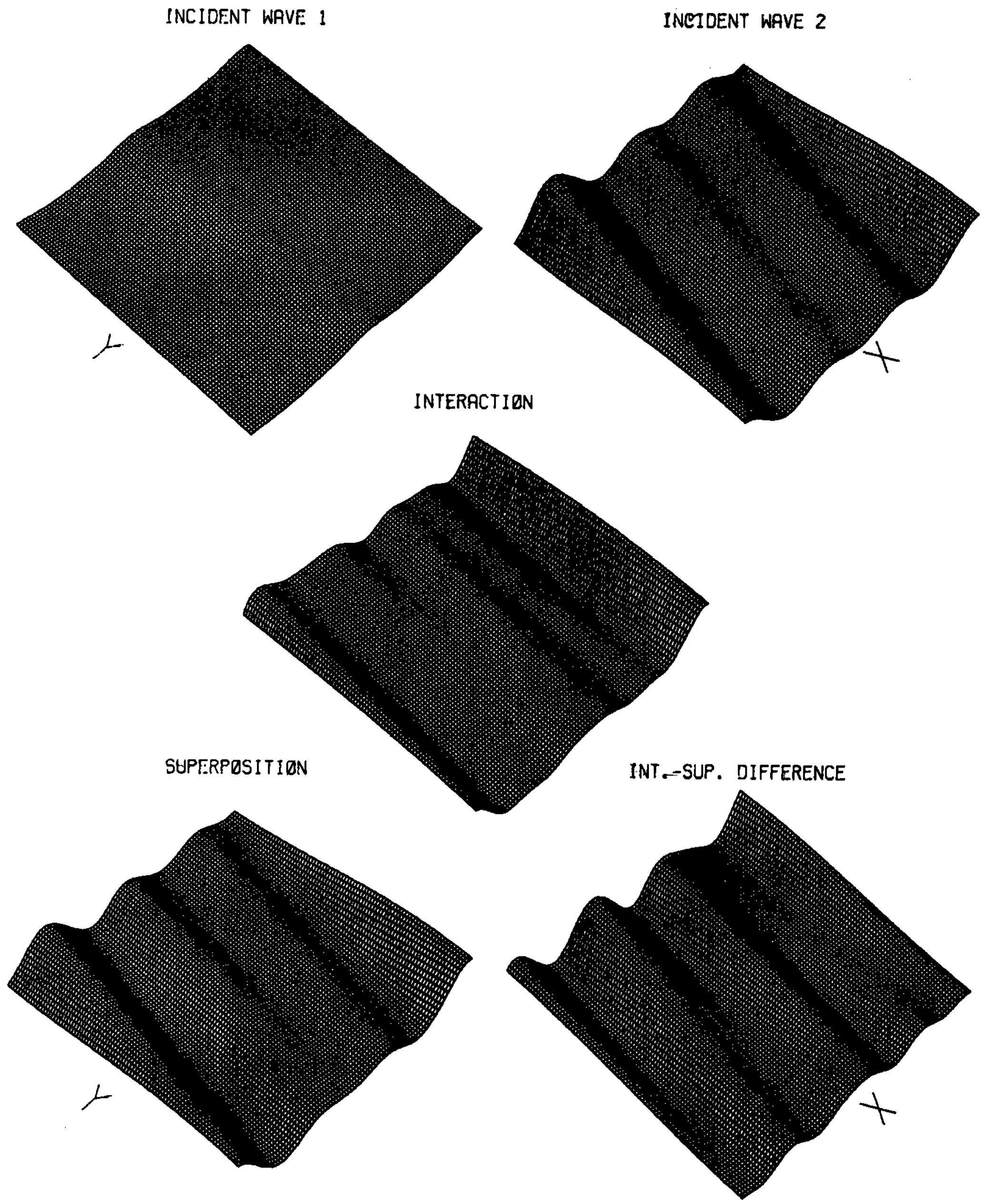
第一列來源波列(incident waves 1)
$k_1d = 4.0$ (相對水深)
$k_1a_1 = 0.2$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2d = 0.5$ (相對水深)
$k_2a_2 = 0.2$ (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 180°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-11f
Fig. 5-11f



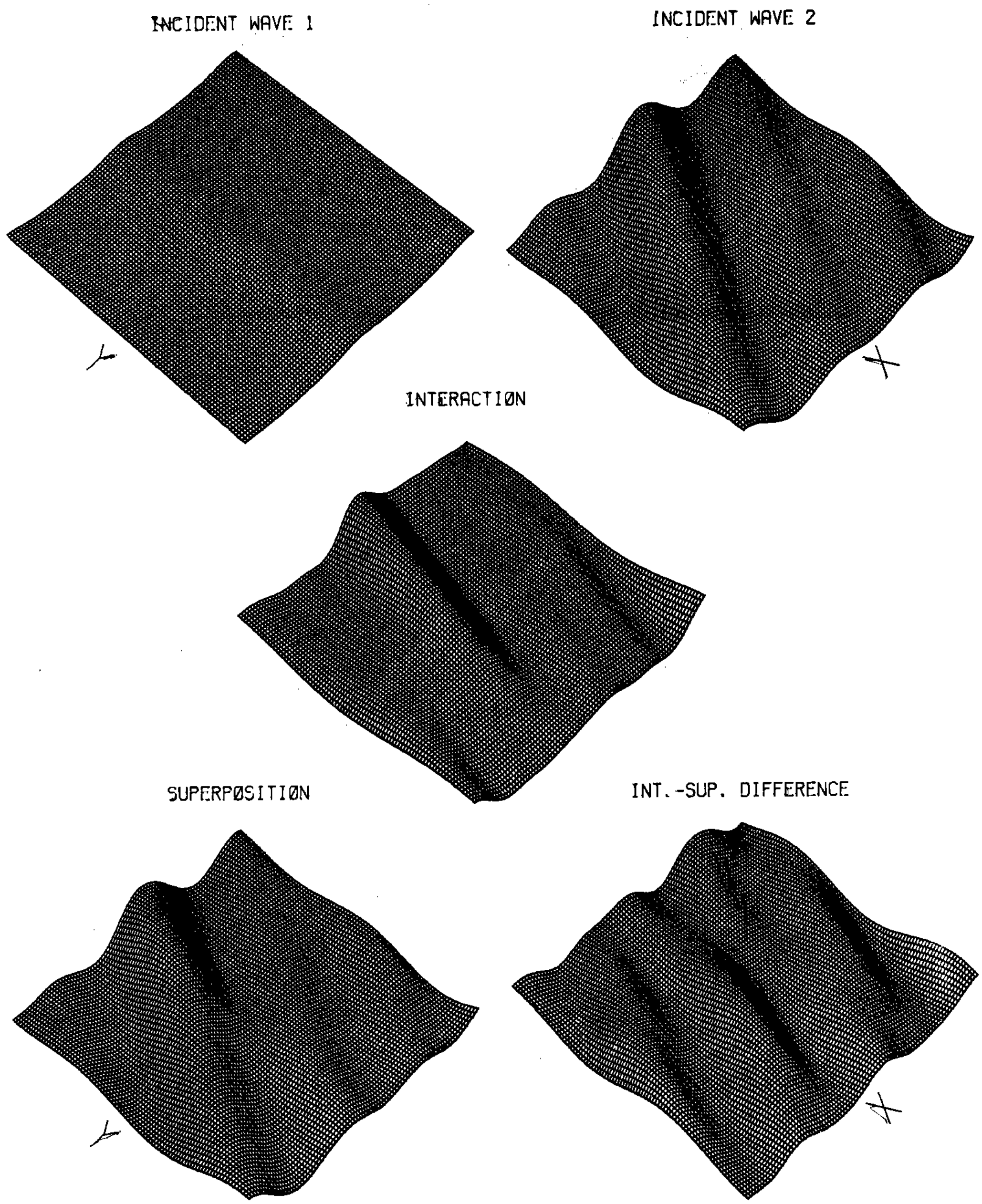
第一列來源波列(incident waves 1)
k_1d = 1.0 (相對水深)
k_1a_1= 0.2 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.5 (相對水深)
k_2a_2= 0.2 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2$= 10°

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-12a
Fig. 5-12a



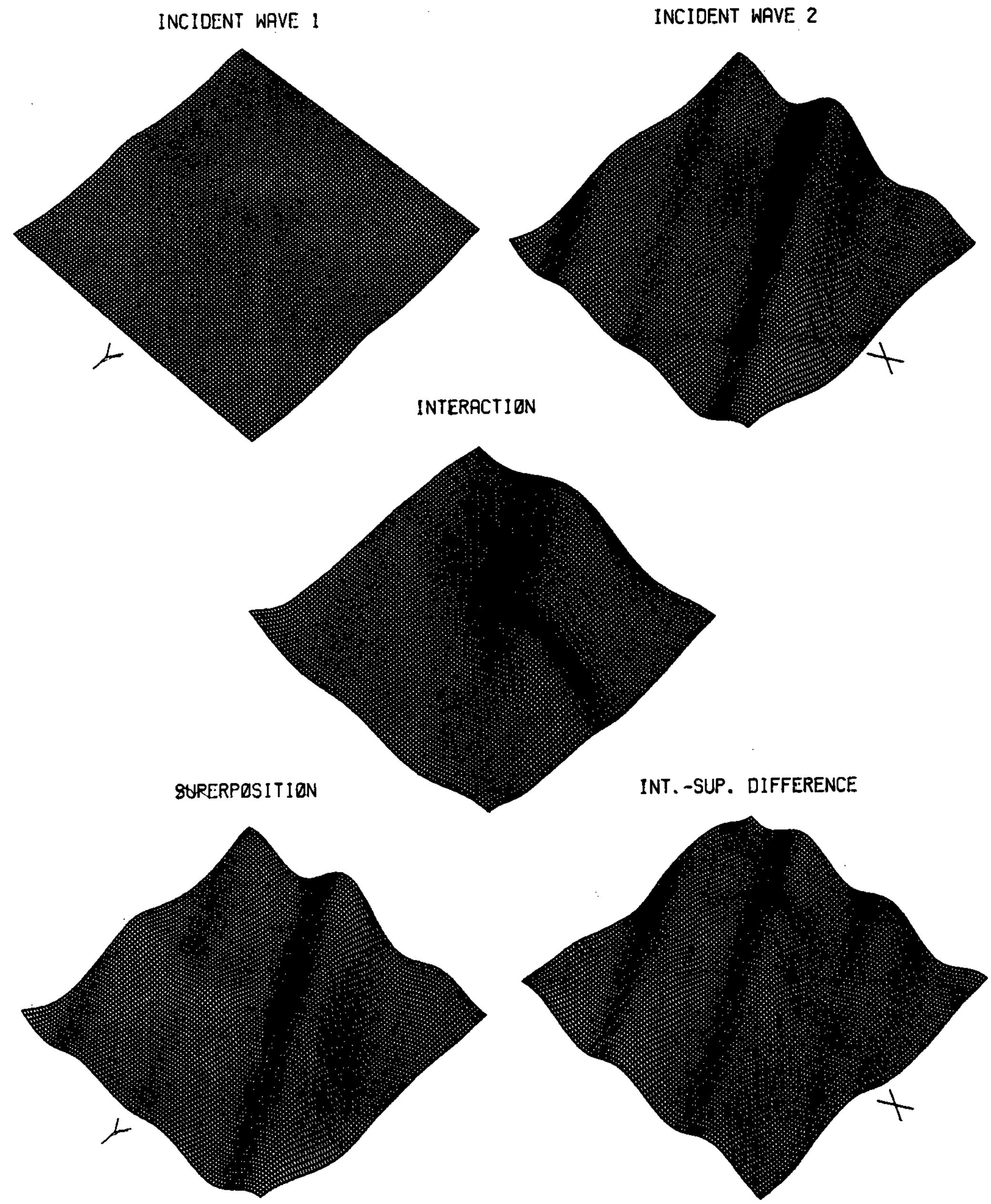
第一列來源波列(incident waves 1)
$k_1d = 1.0$ (相對水深)
$k_1a_1 = 0.2$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2d = 0.5$ (相對水深)
$k_2a_2 = 0.2$ (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 30°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-12b
Fig. 5-12b



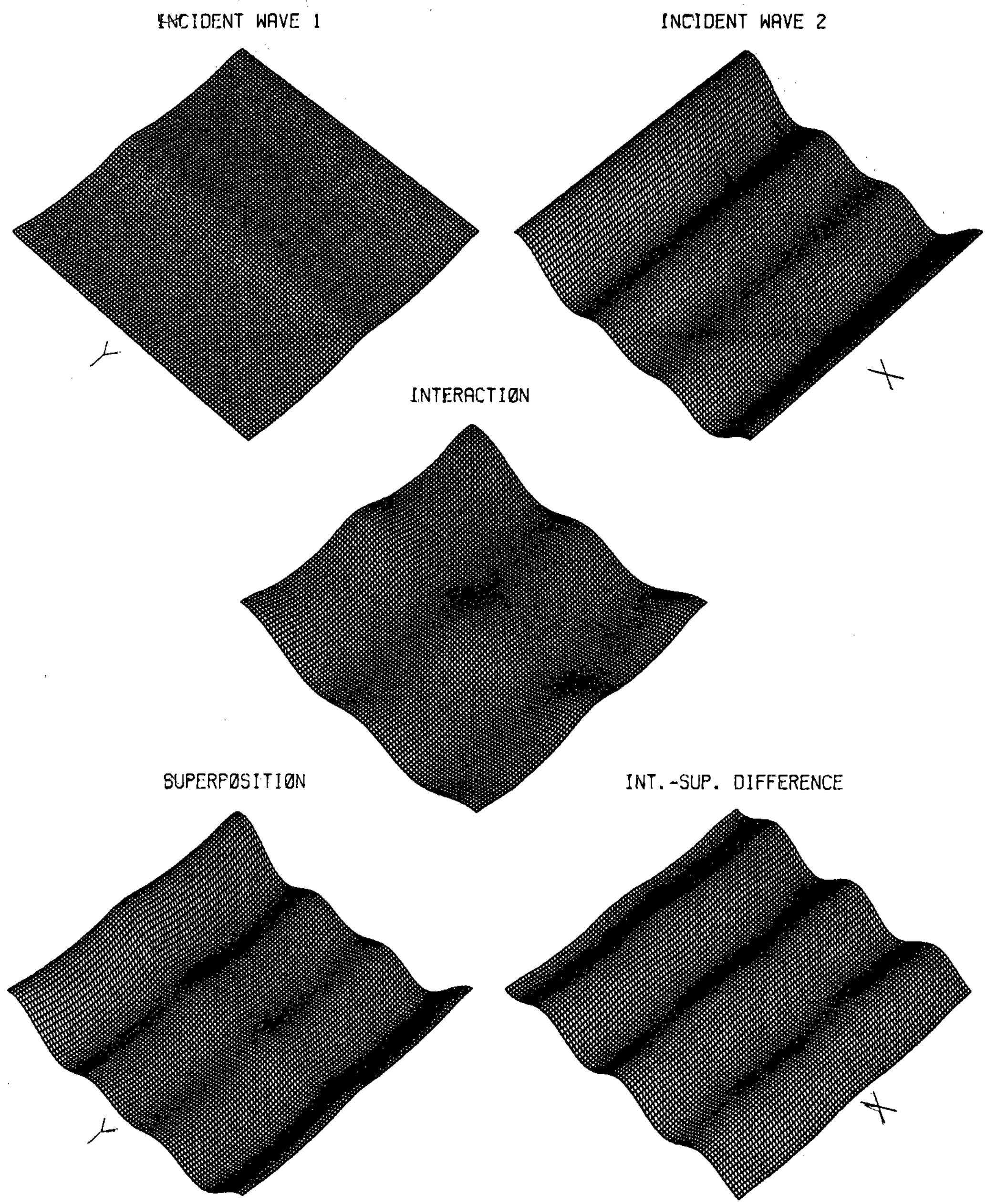
第一列來源波列(incident waves 1)
k_1d = 1.0 (相對水深)
k_1a_1= 0.2 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.5 (相對水深)
k_2a_2= 0.2 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 60°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-12c
Fig. 5-12c



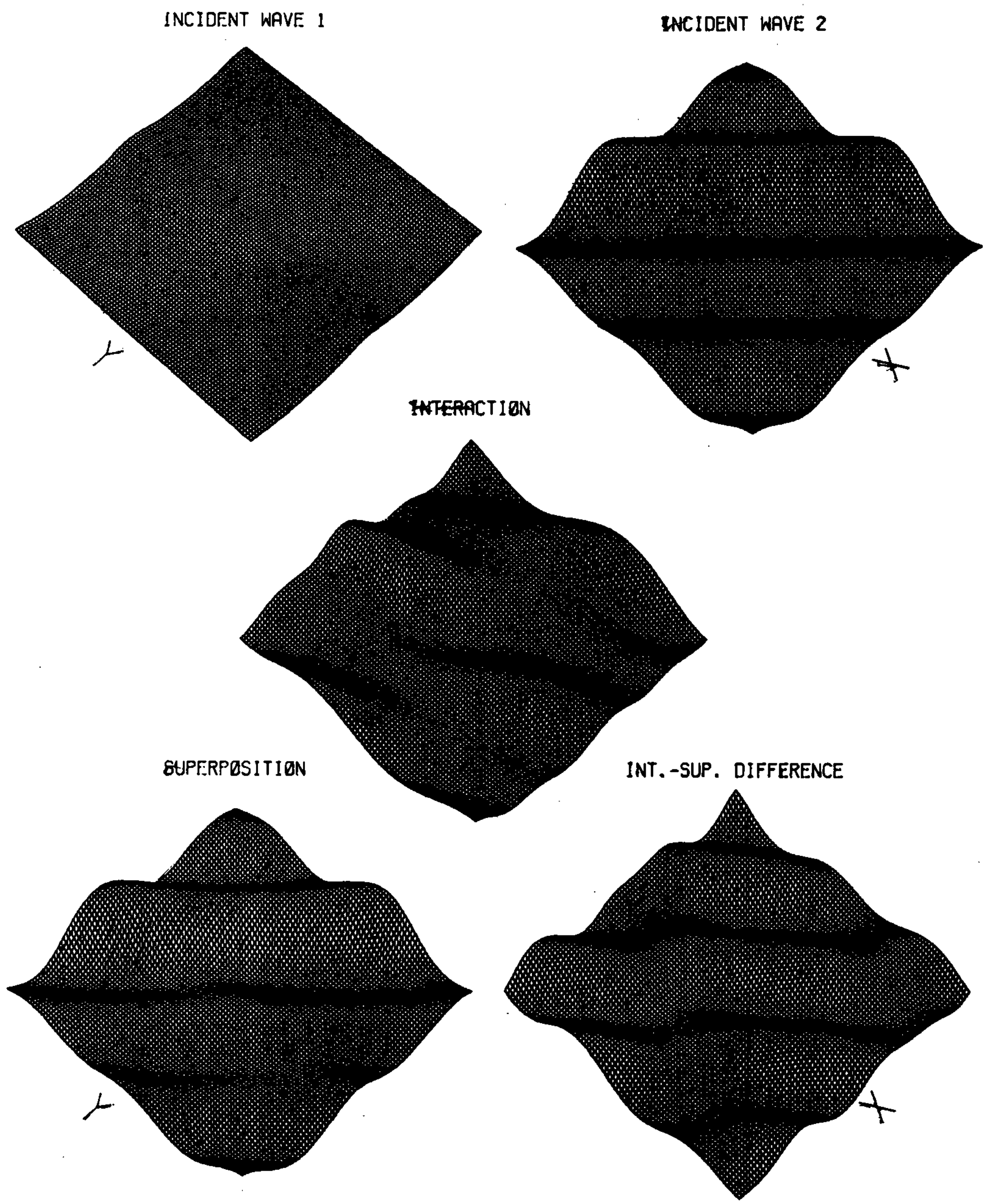
第一列來源波列(incident waves 1)
$k_1 d = 1.0$ (相對水深)
$k_1 a_1 = 0.2$ (波浪尖銳度)

第二列來源波列(incident waves 2)
$k_2 d = 0.5$ (相對水深)
$k_2 a_2 = 0.2$ (波浪尖銳度)

二波列交會夾角 $\theta = \theta_1 - \theta_2 = 90°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-12d
Fig. 5-12d



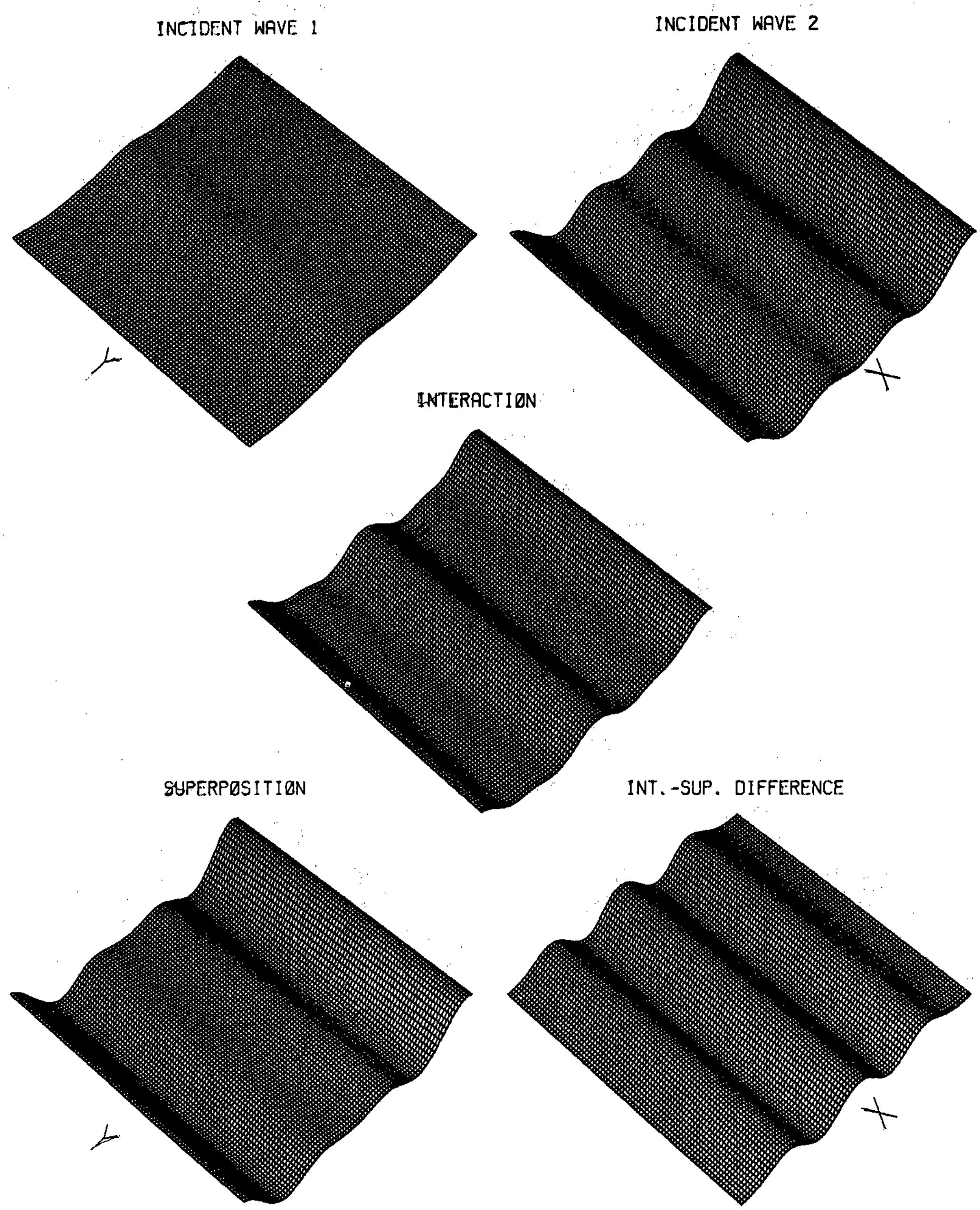
第一列來源波列(incident waves 1)
k_1d = 1.0 (相對水深)
k_1a_1= 0.2 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.5 (相對水深)
k_2a_2= 0.2 (波浪尖銳度)

二波列交會夾角 $\theta=\theta_1-\theta_2= 135°$

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-12e
Fig. 5-12e



第一列來源波列(incident waves 1)
k_1d = 1.0 (相對水深)
k_1a_1= 0.2 (波浪尖銳度)

第二列來源波列(incident waves 2)
k_2d = 0.5 (相對水深)
k_2a_2= 0.2 (波浪尖銳度)

二波列交會夾角$\theta=\theta_1-\theta_2$= 180°

INTERACTION = 二波列交會發生非線性交互作用的結果
SUPERPOSITION= 二波列直接線性疊加
INT.-SUP. DIFFERENCE = (INTERACTION)-(SUPERPOSITION)

圖5-12f
Fig. 5-12f

故是以致之，而於 θ 等於 $0°$ 或 $180°$ 時，則恰完全成同向與反向作用者而爲兩頭極端之最大，當然，此之交會夾角因素對各來源波列之週波率影響的情況亦符合此力學作用觀點，見上小節所述，至於 $\theta=0°$ 時會產生所謂的共振現象，因此亦即刻顯然可得知的，蓋因此乃同步作用的結果所致之故。(4)再者，就相交會的兩來源波列間的波長比因素而言，在其他條件保持相同情況下，所考慮的兩波列相交會所形成之波動系統，其整體呈現出的脈動波形，單就受此兩來源波列之波長比因素而造成之扭曲與增異量，是隨此比值之由1的增大而明顯地增強，此可由圖5－1a～f 、圖5－10a～f與圖5－11a～f及圖5－12a～f 等之各分圖的對照比較直接發現之（當然這除了共振之奇特情況爲例外）；事實上，對此現象亦可依據作用力學之觀點來對其說明的，在僅波長因素的考量下，當波長相同的兩來源波列相交會時，則因其具有等量的波能力通量，故兩者彼此間受它們交會相互作用之影響爲等量，因此，不致產生其中之一者受另一者有較強烈的作用影響而扭曲之，然，當兩來源波列之波長比較偏離1時，即爲一較短的波駕在一較長的波上，這猶如一者（即短波）被另一者（即長波）拖著走，因此造成其中能力較弱的波（即短波）受到另一能力較強的波（即長波）之較強烈的交會衝擊而生較大的扭曲與增異量。

5-5 波壓之解析

這是一般所熟知的，水波波動對水中結構物如防波堤者之衝擊，是以其波壓爲一較具體呈現的作用力。且由於防波堤附近常有兩波列交會的出現，如前言所述，因此，探討兩波列交會之波場內的波壓是有其必要的，再者，亦可藉於試驗室中量測此可具體呈現之波壓來進一步驗證理論解析結果的適足性。有關本文所考慮的兩波列交會所形成之波動流場內的波壓p之解析，如下。依Bernoulli equation之應用，則波動場內之波壓p可得之，爲。

$$p/\rho=-\left[\frac{\partial\phi}{\partial t}+gz+\frac{1}{2}\left(\phi_x^2+\phi_y^2+\phi_z^2\right)\right]，\rho \text{爲流體密度} \tag{5.34}$$

因此，於(5.27)～(5.30)之代入下，可解得至第三階的兩波交會流場內之波壓解（動壓力 $\frac{p}{\rho}+gz$ ）爲，

$$\frac{p}{\rho}+gz=\{ga_1\frac{\cosh k_1(d+z)}{\cosh k_1 d}+gk_1^2a_1^3\Big[\frac{\sigma_2^{(1)}}{k_1^2a_1^2\sigma_0^{(1)}}\frac{\cosh k_1(d+z)}{\cosh k_1 d}$$

$$-\frac{3}{4}\frac{\cosh 3k_1(d+z)}{\sinh^3k_1d\cdot\sinh 2k_1d}-\frac{1}{2}\frac{a_2^2\sigma_0^{(2)}}{gk_1^2a_1^3}(k_1\cos\theta+k_2)A_{11}^{+}$$

$$\times\frac{\cosh k_2(d+z)\cdot\cosh|\vec{k}_1+\vec{k}_2|(d+z)}{\sinh k_2d\cdot\cosh|\vec{k}_1+\vec{k}_2|d}-\frac{1}{2}\frac{a_2\sigma_0^{(2)}}{gk_1^2a_1^3}$$

$$\times(k_1\cos\theta-k_2)A_{11}^{-}\frac{\cosh k_2(d+z)\cdot\cosh|\vec{k}_1-\vec{k}_2|(d+z)}{\sinh k_2d\cdot\cosh|\vec{k}_1-\vec{k}_2|d}$$

$$-\frac{1}{2}\frac{a_2\sigma_0^{(2)}}{gk_1^2a_1^3}|\vec{k}_1+\vec{k}_2|A_{11}^{+}$$

$$\times\frac{\sinh k_2(d+z)\cdot\sinh|\vec{k}_1+\vec{k}_2|(d+z)}{\sinh k_2d\cdot\cosh|\vec{k}_1+\vec{k}_2|d}$$

$$+\frac{1}{2}\frac{a_2\sigma_0^{(2)}}{gk_1^2a_1^3}|\vec{k}_1-\vec{k}_2|A_{11}^{-}$$

$$\times\frac{\sinh k_2(d+z)\cdot\sinh|\vec{k}_1-\vec{k}_2|(d+z)}{\sinh k_2d\cdot\cosh|\vec{k}_1-\vec{k}_2|d}\Big]\}\cos S_1$$

$$+\{ga_2\frac{\cosh k_2(d+z)}{\cosh k_2d}$$

$$+gk_2^2a_2^3\Big[\frac{\sigma_2^{(2)}}{k_2^2a_2^2\sigma_0^{(2)}}\frac{\cosh k_2(d+z)}{\cosh k_2d}$$

$$-\frac{3}{4}\frac{\cosh 3k_2(d+z)}{\sinh^3k_2d\cdot\sinh 2k_2d}-\frac{1}{2}\frac{a_1\sigma_0^{(1)}}{gk_2^2a_2^3}(k_1+k_2\cos\theta)A_{11}^{+}$$

$$\times\frac{\cosh k_1(d+z)\cdot\cosh|\vec{k}_1+\vec{k}_2|(d+z)}{\sinh k_1d\cdot\cosh|\vec{k}_1+\vec{k}_2|d}-\frac{1}{2}\frac{a_1\sigma_0^{(1)}}{gk_2^2a_2^3}$$

$$\times(k_1-k_2\cos\theta)A_{11}^{-}\frac{\cosh k_1(d+z)\cdot\cosh|\vec{k}-\vec{k}_2|(d+z)}{\sinh k_1d\cdot\cosh|\vec{k}_1-\vec{k}_2|d}$$

$$-\frac{1}{2}\frac{a_1\sigma_0^{(1)}}{gk_2^2a_2^3}|\vec{k}_1+\vec{k}_2|A_{11}^{+}$$

$$\times \frac{\sinh k_1(d+z)\cdot \sinh|\vec{k}_1+\vec{k}_2|(d+z)}{\sinh k_1 d\cdot \cosh|\vec{k}_1+\vec{k}_2|d}$$

$$-\frac{1}{2}\frac{a_1\sigma_0^{(1)}}{g k_2^2 a_2^3}|\vec{k}_1-\vec{k}_2|A_{11}^-$$

$$\times \frac{\sinh k_1(d+z)\cdot \sinh|\vec{k}_1-\vec{k}_2|(d+z)}{\operatorname{sink} k_1 d\cdot \cosh|\vec{k}_1-\vec{k}_2|d}]\}\cos S_2$$

$$+g[-k_1 a_1^2\frac{\sinh^2 k_1(d+z)}{\sinh 2k_1 d}-k_2 a_2^2\frac{\sinh k_2(d+z)}{\sinh 2k_2 d}]$$

$$+g k_1 a_1^2[\frac{3}{2}\frac{\cosh 2k_1(d+z)}{\sinh^2 k_1 d\cdot \sinh 2k_1 d}-\frac{1}{2}\frac{1}{\operatorname{sink} 2k_1 d}]\cos 2S_1$$

$$+g k_2 a_2^2[\frac{3}{2}\frac{\cosh 2k_2(d+z)}{\sinh^2 k_2 d\cdot \sinh 2k_2 d}-\frac{1}{2}\frac{1}{\sinh 2k_2 d}]\cos 2S_2$$

$$+g k_1 a_1 a_2[\frac{(\sigma_0^{(1)}+\sigma_0^{(2)})}{g k_1 a_1 a_2}A_{11}^+\frac{\cosh|\vec{k}_1+\vec{k}_2|(d+z)}{\cosh|\vec{k}_1+\vec{k}_2|d}$$

$$+\frac{1}{2}\frac{g k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}\frac{\sinh k_1(d+z)\cdot \sinh k_2(d+z)}{\cosh k_1 d\cdot \cosh k_2 d}$$

$$-\frac{1}{2}\frac{g k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}\frac{\cosh k_1(d+z)\cdot \cosh k_2(d+z)}{\cosh k_1 d\cdot \cosh k_2 d}\cos\theta]$$

$$\times\cos(S_1+S_2)+g k_1 a_1 a_2[\frac{(\sigma_0^{(1)}-\sigma_0^{(2)})}{g k_1 a_1 a_2}A_{11}^-$$

$$\times\frac{\cosh|\vec{k}_1-\vec{k}_2|(d+z)}{\cosh|\vec{k}_1-\vec{k}_2|d}-\frac{1}{2}\frac{g k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}$$

$$\times\frac{\sinh k_1(d+z)\cdot \sinh k_2(d+z)}{\cosh k_1 d\cdot \cosh k_2 d}-\frac{1}{2}\frac{g k_2}{\sigma_0^{(1)}\sigma_0^{(2)}}$$

$$\times\frac{\cosh k_1(d+z)\cdot \cosh k_2(d+z)}{\cosh k_1 d\cdot \cosh k_2 d}\cdot\cos\theta]\cos(S_1-S_2)$$

$$+g k_1^2 a_1^3[\frac{1}{64}(9\tanh^{-6}k_1 d+5\tanh^{-4}k_1 d-53\tanh^{-2}k_1 d$$

$$+39)\frac{\cosh 3k_1(d+z)}{\cosh 3k_1 d}-\frac{3}{4}\frac{\cosh k_1(d+z)}{\sinh^3 k_1 d\cdot \sinh 2k_1 d}]\cos 3S_1$$

$$+ g k_2^2 a_2^3 \left[\frac{3}{64} (9 \tanh^{-6} k_2 d + 5 \tanh^{-4} k_2 d - 53 \tanh^{-2} k_2 d \right.$$

$$\left. + 39) \frac{\cosh 3k_2 (d+z)}{\cosh 3k_2 d} - \frac{3}{4} \frac{\cosh k_2 (d+z)}{\sinh^3 k_2 d \cdot \sinh 2k_2 d} \right] \cos 3S_2$$

$$+ g k_1^2 a_1^2 a_2 \left\{ \frac{(2\sigma_0^{(1)} + \sigma_0^{(2)})}{g k_1^2 a_1^2 a_2} \right.$$

$$\times \frac{\mu_{21}^{+}}{(2\sigma_0^{(1)} + \sigma_0^{(2)})^2 - g \mid 2\vec{k}_1 + \vec{k}_2 \mid \cdot \tanh \mid 2\vec{k}_1 + \vec{k}_2 \mid d}$$

$$\times \frac{\cosh \mid 2\vec{k}_1 + \vec{k}_2 \mid (d+z)}{\cosh \mid 2\vec{k}_1 + \vec{k}_2 \mid d} - \frac{1}{2} \frac{\sigma_0^{(1)}}{g k_1^2 a_1 a_2} (k_1 + k_2 \cos\theta) A_{11}^{+}$$

$$\times \frac{\cosh k_1 (d+z) \cdot \cosh \mid \vec{k}_1 + \vec{k}_2 \mid (d+z)}{\sinh k_1 d \cdot \cosh \mid \vec{k}_1 + \vec{k}_2 \mid d}$$

$$+ \frac{1}{2} \frac{\sigma_0^{(1)}}{g k_1^2 a_1 a_2} \mid \vec{k}_1 + \vec{k}_2 \mid A_{11}^{+}$$

$$\times \frac{\sinh k_1 (d+z) \cdot \sinh \mid \vec{k}_1 + \vec{k}_2 \mid (d+z)}{\sinh k_1 d \cdot \cosh \mid \vec{k}_1 + \vec{k}_2 \mid d} - \frac{3}{8} \frac{\sigma_0^{(1)} \sigma_0^{(2)}}{g k_1}$$

$$\left. \times \frac{[\cosh 2k_1(d+z) \cdot \cosh k_2(d+z) \cdot \cos\theta - \sinh 2k_1(d+z) \sinh k_2(d+z)]}{\sinh^4 k_1 d \cdot \sinh k_2 d} \right\}$$

$$\times \cos(2S_1 + S_2) + g k_1^2 a_1^2 a_2 \left\{ \frac{(2\sigma_0^{(1)} - \sigma_0^{(2)})}{g k_1^2 a_1^2 a_2} \right.$$

$$\times \frac{\mu_{21}^{-}}{(2\sigma_0^{(1)} - \sigma_0^{(2)})^2 - g \mid 2\vec{k}_1 - \vec{k}_2 \mid \cdot \tanh \mid 2\vec{k}_1 - \vec{k}_2 \mid d}$$

$$\times \frac{\cosh \mid 2\vec{k}_1 - \vec{k}_2 \mid (d+z)}{\cosh \mid 2\vec{k}_1 - \vec{k}_2 \mid d} - \frac{1}{2} \frac{\sigma_0^{(1)}}{g k_1^2 a_1 a_2} (k_1 - k_2 \cos\theta) A_{11}^{-}$$

$$\times \frac{\cosh k_1 (d+z) \cdot \cosh \mid \vec{k}_1 - \vec{k}_2 \mid (d+z)}{\sinh k_1 d \cdot \cosh \mid \vec{k}_1 - \vec{k}_2 \mid d} + \frac{1}{2} \frac{\sigma_0^{(1)}}{g k_1^2 a_1 a_2}$$

$$\times \mid \vec{k}_1 - \vec{k}_2 \mid A_{11}^{-} \frac{\sinh k_1 (d+z) \cdot \sinh \mid \vec{k}_1 - \vec{k}_2 \mid (d+z)}{\sinh k_1 d \cdot \cosh \mid \vec{k}_1 - \vec{k}_2 \mid d}$$

$$-\frac{3}{8}\frac{\sigma_0^{(1)}\sigma_0^{(2)}}{gk_1}$$

$$\times\frac{[\cosh 2k_1(d+z)\cdot\cosh k_2(d+z)\cdot\cos\theta+\sinh 2k_1(d+z)\cdot\sinh k_2(d+z)]}{\sinh^4 k_1 d\cdot\sinh k_2 d}\}$$

$$\times\cos(2S_1-S_2)+gk_2^2\, a_1 a_2^2\ \{\frac{(\sigma_0^{(1)}+2\sigma_0^{(2)})}{gk_2^2\ a_1 a_2^2}$$

$$\times\frac{\mu_{12}^{+}}{(\sigma_0^{(1)}+2\sigma_0^{(2)})^2-g\,|\vec{k}_1+2\vec{k}_2|\cdot\tanh|\vec{k}_1+2\vec{k}_2|\,d}$$

$$\times\frac{\cosh|\vec{k}_1+2\vec{k}_2|(d+z)}{\cosh|\vec{k}_1+2\vec{k}_2|\,d}-\frac{1}{2}\frac{\sigma_0^{(2)}}{gk_2^2\ a_1 a_2}(k_1\cos\theta+k_2)A_{11}^{+}$$

$$\times\frac{\cosh k_2(d+z)\cdot\cosh|\vec{k}_1+\vec{k}_2|(d+z)}{\sinh k_2 d\cdot\cosh|\vec{k}_1+\vec{k}_2|\,d}$$

$$+\frac{1}{2}\frac{\sigma_0^{(2)}}{gk_2^2\ a_1 a_2}|\vec{k}_1+\vec{k}_2|\ A_{11}^{+}$$

$$\times\frac{\sinh k_2(d+z)\cdot\sinh|\vec{k}_1+\vec{k}_2|(d+z)}{\sinh k_2 d\cdot\cosh|\vec{k}_1+\vec{k}_2|\,d}-\frac{3}{8}\frac{\sigma_0^{(1)}\sigma_0^{(2)}}{gk_2}$$

$$\times\frac{[\cosh k_1(d+z)\cdot\cosh 2k_2(d+z)\cdot\cos\theta-\sinh k_1(d+z)\cdot\sinh 2k_2(d+z)]}{\sinh k_1 d\cdot\sinh^4 k_2 d}\}$$

$$\times\cos(S_1+2S_2)+gk_2^2\ a_1 a_2^2\ \{\frac{(\sigma_0^{(1)}-2\sigma_0^{(2)})}{g\,k_2^2 a_1 a_2^2}$$

$$\times\frac{\mu_{12}^{-}}{(\sigma_0^{(1)}-2\sigma_0^{(2)})^2-g\,|\vec{k}_1-2\vec{k}_2|\cdot\tanh|\vec{k}_1-2\vec{k}_2|\,d}$$

$$\times\frac{\cosh|\vec{k}_1-2\vec{k}_2|(d+z)}{\cosh|\vec{k}_1-2\vec{k}_2|\,d}-\frac{1}{2}\frac{\sigma_0^{(2)}}{g\,k_2^2 a_1 a_2}(k_1\cos\theta-k_2)A_{11}^{-}$$

$$\times\frac{\cosh k_2(d+z)\cdot\cosh|\vec{k}_1-\vec{k}_2|(d+z)}{\sinh k_2 d\cdot\cosh|\vec{k}_1-\vec{k}_2|\,d}$$

$$-\frac{1}{2}\frac{\sigma_0^{(2)}}{gk_2^2\ a_1 a_2}|\vec{k}_1-\vec{k}_2|\ A_{11}^{-}$$

$$\times\frac{\sinh k_2(d+z)\cdot\sinh|\vec{k}_1-\vec{k}_2|(d+z)}{\sinh k_2 d\cdot\cosh|\vec{k}_1-\vec{k}_2|\,d}-\frac{3}{8}\frac{\sigma_0^{(1)}\sigma_0^{(3)}}{gk}$$

$$\times \frac{\left[\cosh k_1(d+z)\cdot\cosh 2k_2(d+z)\cdot\cos\theta+\sinh k_1(d+z)\cdot\sinh 2k_2(d+z)\right]}{\sinh k_1 d\cdot\sinh^4 k_2 d}\}$$

$$\times\cos\left(S_1-2S_1\right) \tag{5.35}$$

式中之$A^{\pm}_{11}$ 、$\mu^{\pm}_{12}$ 與$\mu^{\pm}_{21}$ 等量如上節所述。有關（5.35）式所示之本文所考慮的兩波列交會之波場內的波壓之變化特性，限於篇幅起見，詳情請參見筆者（Ho & Chen, 1990）之論述；然由此所得之波壓值與試驗結果的比較，則見之於本文之第七節中。

六、兩波交會流場之一例：短峰波情況

6-1 概述與短峰波波動系統

(1)概　述

由於從外海傳向近岸的單一規則前進重力波列，常會受岸壁或結構物（防波堤等）的反射而形成重複波（clapotis），或所謂的短峯波情況。因此，欲對海面波動實況有較逼眞的探究與確切地掌握，當然，得對這常見於近岸海域的短峯波情況予之必要的考量，如參見 Jeffrays（1924）[1] 之述。

往昔迄今有關對短峯波波動現象的研究，可見到的文獻，爲數並不很多，蓋因它涉及到三度空間之時空運動的複雜性與頗繁長的解析過程。Fuchs（1952）首先以有因次形式的攝動法展開之第二階解，較具體地描述短峯波波動流場之物理特性；爾後，Chappelear（1961）將之推展至第三階的解析。Hsu、Tsuchiya & Silvester（1979）爲尋求能涵蓋短峯波各種情況的較廣泛性流場解，引入一特定的攝動參數，在無因次形式下，展開至第三階解；接著Hsu et al（1980）利用其所得的解析解，更進一步地描述因短峯波波動所導致的底部處流場特性。另，由於近代高速率的電腦發展，一些有效的數值計算法已被應用來求算更具精度的短峯波波動流場特性。如Bryant（1982）已對相對淺水中的短峯波流暢進行數值解析，可是其精度却僅至O（a／h）而已；此處a爲波動振幅而h爲平均水深。相對地，Roberts & Schwartz（1983）與Roberts（1983）對較深水中的短峰波波動已數值計算到相當高的階次，並描述其流場中的一些特性，尤其是對產生共振現象的探討；隨後，林銘崇等（1987）基此模式更將之推展到一般有限水深的情況。最近，鄭東昇與蔡清標（1988）以保留波形爲隱函數形式，將之等間隔，亦對深海中之短峯波波動流場進行數值求解。

由上述有關文獻的學習與探討，這是可明確地斷言的是，不管諸學者們所取用的解析法爲何，以理論推導或數值計算，他們直接都以Fuchs（1952）之解的型式再行推展到較高階時的較多項級數解而已，其間頂多將有因次的形式轉換成某特定的無因次形式以利解析罷！然而此值得愼思的一事是，對眞實物理現象的探究，若事先無確切地掌握其本質特性就直接限定其解的形式，則顯然地當會對該場中的一些原

眞性造成漏失描述之虞，甚或出現與事實相左的不合理結果。就此嚴謹的觀點來考察，往昔直接以這事先就限定流場解的形式，來對短峯波波動特性進行解析，似是有點不妥而未能圓融具足，這可由往昔所得的解析解未能完全滿足地退化成單一前進波解而得知，詳見下文對Hsu et al（1979）者之比較討論；甚而對某些特異現象的產生亦無法了解洞澈其原由，如Roberts（ 1983 ）之數值結果全然於短峯波退化成單一前進波時發生共振情況。當然，於此種的偏失下，亦將可能造致對短峯波波動流場有關特性描述的不足，如見 Fenton（ 1985 ）檢核 Hsu et al（1979）所得之波壓結果的缺失。

其實，依本質而言，短峯波是由兩個完全相同性質的前進波列，即於等水深中具相同波長與振幅者，交會所形成的波動；當然，於實際波動現象中，短峯波亦會因單一規則前進波列經一直立堤壁的完全反射而形成，不過，於此情況中，得需注意的一點是，當入射的單一規則前進波列是沿著堤壁前進時，則此時的波動系統並沒有反射波列，而是僅存在一個入射的前進波列，即退化成單一波列的情況，由此可明顯地得知，短峯波波動現象僅是任二規則前進波列相交會所構成的波動系統中之一特例而已。因此，欲較全盤完整地對短峯波波動流場給予詳盡的闡述，則自當需先對兩波交會所形成的波動流場結構有充分的掌握與瞭解方可。基此，本文將以兩波交會的觀點，及利用其已被解出的流場解，參見上節之論述，來對短峯波波動流場特性做較廣度的描述，由深海至相當淺水情況；同時，藉此及試驗結果，再對所得的兩波交會流場解析解做一有效的佐證。

(2)短峯波波動系統

於三度空間中，等深 d 之廣大海域上，考慮性質完全相同的二自由表面規則前進重力波列相交會所構成的波動系統，如圖 6 所示，謂之短峯波（ short crested waves ）者。對此波動系統，於圖 6 中，是選用三度空間卡氏直角座標架構來對它描述的，其中 x－y 平面恰取爲平均靜止的水平面，而 z 軸垂直於 x－y 平面且向上取正。在此參考座標架下，所考慮的兩完全相同性質的來源成份波列，其前進方向上的波數向量（ wave-number vector ）各爲 $\overrightarrow{k_I}$ 與 $\overrightarrow{k_R}$ ，而與 y 軸間的夾角分別成 $\pi-\alpha$ 與 α ；於此，此兩波列之前進方向間的交角 $\theta=\pi-2\alpha$ ，見圖 6 所示。

圖 6 所描述的波動系統，對常見於岸壁或海中結構物附近水域處，單一規則前進

重力波列遭到反射而形成之短峯波波動現象，亦含括在其中；於此情況時，x軸可取爲是沿著直立堤的壁面，波數向量$\vec{k}_I$的前進波列可視爲入射波（ incident

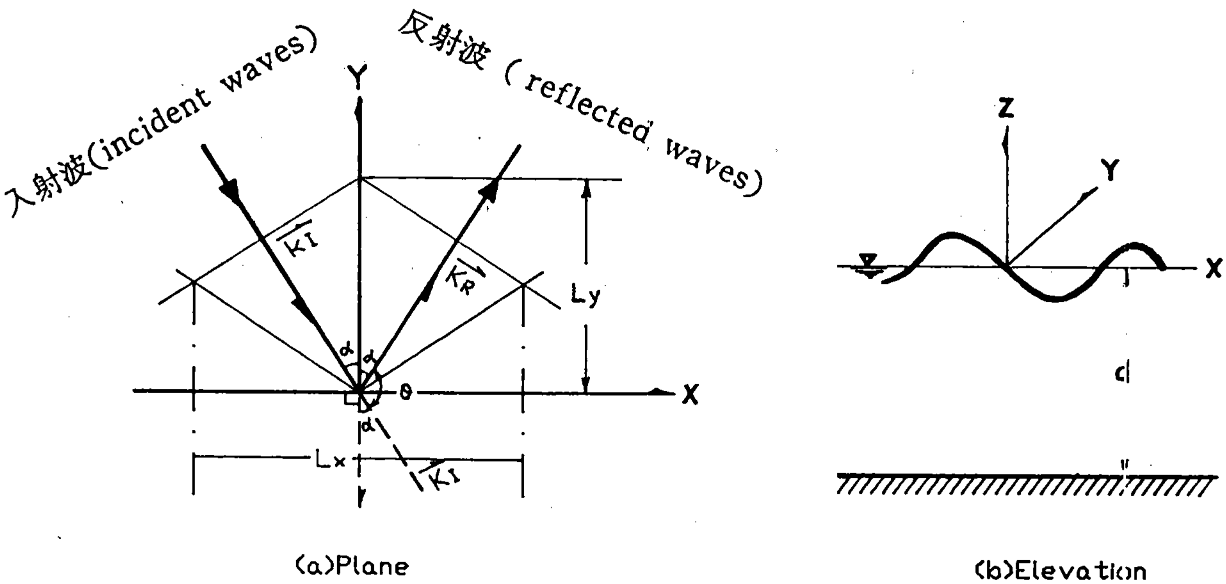


圖 6　短峰波波動系統示意圖

Fig. 6　Definition sketch for short-crested waves system.

waves ），而$\vec{k}_R$者則被視爲完全反射波（ completely reflected waves ）。不過，在此情況下，得需注意的一點是，當入射波列是沿著堤壁面（即x軸）前進時，則此時的波動系統僅爲入射波者，並無反射波列的存在，亦即退化成單一前進重力波列的情況；而非上段無堤時變成二完全相同性質的前進波列同時完全疊合在一起的狀態。以上 $|\vec{k}_I| = |\vec{k}_R| = 2\pi / L$；L爲二波列之波長。

如上節對兩波列交會問題之解析般，定義出一流速勢函數（ velocity potential ）ϕ_s，使得所考慮的短峯波波動流場中的水粒子的速度與其分量可被表示爲

$$\vec{V}_s = \nabla\phi_s = \left(\frac{\partial\phi_s}{\partial x}, \frac{\partial\phi_s}{\partial y}, \frac{\partial\phi_s}{\partial z}\right) = (u_s, v_s, w_s) \tag{6.1}$$

此處流速勢函數 $\phi_s = \phi_s(x, y, z, t)$ 須滿足Laplace's equation而爲短峯波波動流場的基本控制方程式，即

$$\nabla^2\phi_s = \frac{\partial^2\phi_s}{\partial x^2} + \frac{\partial^2\phi_s}{\partial y^2} + \frac{\partial^2\phi_s}{\partial z^2} = 0 \tag{6.2}$$

至於短峯波波動流場所必要滿足的邊界條件有：

(A)沿著 x 軸上（或直立堤的壁面處）

$$v_s=\frac{\partial \phi_s}{\partial y}=0 \quad , \quad y=0 \tag{6.3}$$

其實，此條件於等水深中任兩波列交會的通案性情形裡，是由於交會的兩波其特性（即振幅與波長）完全相同，而取 x 軸將它們前進方向間的夾角等半分割所致，即交會的兩完全相同性質的來源前進波列，其波動是被描述在對稱於 x 軸的座標系統中，當然，這亦包括入射波經直立堤壁面完全反射的情況。是故，短峯波波動者僅是等深水中任兩規則前進波列交會所形成之波動系統中之一特例而已。

(B)在底部處

$$w_s=\frac{\partial \phi_s}{\partial z}=0 \quad , \quad z=-d \tag{6.4}$$

(C)在短峯波波動表面處，有

(a)運動邊界條件

$$\frac{\partial \phi_s}{\partial z}=\frac{d\eta_s}{dt}=\frac{\partial \eta_s}{\partial t}+\frac{\partial \phi_s}{\partial x}\frac{\partial \eta_s}{\partial x}+\frac{\partial \phi_s}{\partial y}\frac{\partial \eta_s}{\partial y} \quad , \quad z=\eta_s \tag{6.5}$$

(b)動力邊界條件（假設處於常壓下之自由表面者）

$$\frac{\partial \phi_s}{\partial t}+g\eta_s+\frac{1}{2}\left[\left(\frac{\partial \phi_s}{\partial x}\right)^2+\left(\frac{\partial \phi_s}{\partial y}\right)^2+\left(\frac{\partial \phi_s}{\partial z}\right)^2\right]=0 \quad , \quad z=\eta_s \tag{6.6}$$

上式中 $\eta_s=\eta_s(x, y, t)$ 爲短峯波波動表面水位，g 爲重力加速度，t 爲時間；注意，關於短峯波波動流場解中僅涉及到時間 t 的項（包括任意常數）已被併入在 ϕ_s 中，如上節所述般的。

另，由於 x－y 平面是被取爲平均靜止的水平面，因此，如上節者，依質量守衡及短峯波波動系統的空間週期性，再有一必要的條件式爲

$$\int_0^{2\pi}\int_0^{2\pi}\eta_s(x, y, t)\,d(\vec{k}_I\cdot\vec{X})\,d(\vec{k}_R\cdot\vec{X})=0 \tag{6.7}$$

此處 $\vec{X}=\vec{i}x+\vec{j}y$ 表示在 x－y 平面上的位置向量。注意：(2.7) 式的積分已被轉換對無因次量 $\vec{k}_I\cdot\vec{X}$ 與 $\vec{k}_R\cdot\vec{X}$ 之積分形式。

這是如同所有相關的水波問題之處理般，對本節所考慮的短峯波波動系統之求解，旨在解析出滿足（6.2）至（6.7）式之所有控制方程式下的流場結構解 ϕ_s 與 η_s 者，然其關連到非線性邊界條件的困難處理是爲眾所皆知的主要障礙。有關克服此種困難而以攝動展開法，來對其波動流場之逐階化的求解過程，詳情已被頗清楚地列述於等深水中任兩規則前進重力波列相交會所構成之波動系統的通案性流場結構解之解析中，參見上節述之論述，於此不再給予贅述；蓋因短峯波波動現象僅是所言的任兩前進波列相交會所成之波動系統中之一特例而已，故僅需將所得的任兩前進波列相交會所構成之波動系統的流場，退化成兩完全相同性質之波列相交會的情況即可，如下所述。

6-2 短峰波流場解及其波壓

如所知，由於短峯波波動是爲等深水中任兩規則前進波列相交會所構成的波動系統之一特例，因此，其波動流場解與波壓當然可由所言的任兩波交會系統之通案性的流場解與波壓直接簡化求得。對此，今分述之於下。

(1)短峯波之流場解

由上所述，這是顯然地，欲寫出短峯波波動流場解，至第三階次，則僅需將短峯波情況時之特例條件，代之已被解出的所言之任兩波列交會所成之波動系統的通案性流場解即可；即將上節所解出的（5.27）至（5.30）式之等深水中兩波交會系統的通解退化成短峯波情況者。做此解析，今可依上小節所描述的短峯波波動系統，則於2～5節中所言的等深水中任兩規則前進重力波列相交會所構成的波動系統，退化成短峯波波動情況時，其條件爲下。

當令 $\vec{k}_I=\vec{k}_1$，$\vec{k}_R=\vec{k}_2$ 時（或 $\vec{k}_I=\vec{k}_2$，$\vec{k}_R=\vec{k}_1$ 時亦可） (6.8)

則有 $|\vec{k}_1|=|\vec{k}_2|=|\vec{k}_I|=|\vec{k}_R|=k_1=k_2=k=2\pi/L$ (6.9)

及 $a_1=a_2=a$ ， $\sigma_1=\sigma_2=\sigma$ (6.10)

又依圖6可知 $\theta=\pi-2\alpha$ ， $\alpha=(\pi-\theta)/2$ (6.11)

且令 $m=\sin\alpha$ ， $n=\cos\alpha$ (6.12)

另，由於短峯波波動現象發生時，交會的兩波於座標原點處具有同水位，故，方便上，可取定

$$\epsilon_1=\epsilon_2=0 \tag{6.13}$$

因此，在此短峯波波動系統的描述下，則相交會的兩來源前進波列其脈動基本相位 S_1 與 S_2，可分別被寫爲（參照圖6所示，及利用(6.8)～(6.13)式）

$$\left.\begin{aligned} S_1 &= \vec{k}_1 \cdot \vec{X} - \sigma_1 t = k_1\left(x\cos\frac{\theta}{2} - y\sin\frac{\theta}{2}\right) - \sigma_1 t \\ &= k\left(x\sin\alpha - y\cos\alpha\right) - \sigma t = mkx - nky - \sigma t \\ S_2 &= \vec{k}_2 \cdot \vec{X} - \sigma_2 t = k_2\left(x\cos\frac{\theta}{2} + y\sin\frac{\theta}{2}\right) - \sigma_2 t \\ &= k\left(x\sin\alpha + y\cos\alpha\right) - \sigma t = mkx + nky - \sigma t \end{aligned}\right\} \quad (6.14)$$

依上式，(5.27)至(5.30)式所示之等深水中任兩規則前進重力波列相交會所構成之波動系統的通案性流場解，退化成短峯波波動流場解之特例時，至第三階次量下，其中有關的各參數，則可簡化成

$$\left.\begin{aligned} |\vec{k}_1 + \vec{k}_2| &= \left(k_1^2 + 2k_1k_2\cos\theta + k_2^2\right)^{1/2} \\ &= \left(2k^2 - 2k^2\cos 2\alpha\right)^{1/2} = 2k\sin\alpha = 2mk \\ |\vec{k}_1 - \vec{k}_2| &= \left(k_1^2 - 2k_1k_2\cos\theta + k_2^2\right)^{1/2} \\ &= \left(2k^2 + 2k^2\cos 2\alpha\right)^{1/2} = 2k\cos\alpha = 2nk \\ |2\vec{k}_1 + \vec{k}_2| &= \left(4k_1^2 + 4k_1k_2\cos\theta + k_2^2\right)^{1/2} \\ &= \left(5k^2 - 4k^2\cos 2\alpha\right)^{1/2} = k\left(1 + 8\sin^2\alpha\right)^{1/2} \\ &= \left(1 + 8m^2\right)^{1/2} k = |\vec{k}_1 + 2\vec{k}_2| \\ |2\vec{k}_1 - \vec{k}_2| &= \left(4k_1^2 - 4k_1k_2\cos\theta + k_2^2\right)^{1/2} \\ &= \left(5k^2 + 4k^2\cos 2\alpha\right)^{1/2} = k\left(1 + 8\cos^2\alpha\right)^{1/2} \\ &= \left(1 + 8n^2\right)^{1/2} k = |\vec{k}_1 - 2\vec{k}_2| \end{aligned}\right\} \quad (6.15)$$

及

$$\left.\begin{aligned} &S_1 + S_2 = 2mkx - 2\sigma t, \quad S_1 - S_2 = -2nky, \\ &2S_1 + S_2 = 3mkx - nky - 3\sigma t, \quad 2S_1 - S_2 = mkx - 3nky - \sigma t, \\ &S_1 + 2S_2 = 3mkx + nky - 3\sigma t, \quad S_1 - 2S_2 = -mkx - 3nky + \sigma t \end{aligned}\right\} \quad (6.16)$$

再者，至於(5.27)與(5.28)式中所包含的有關係數 $A_{11}^{\pm}$、$\mu_{ij}^{\pm}$ 與 $\zeta_{ij}^{\pm}$，即(5.8)、(5.20c－f)與(5.26a－h)式中者，於波動系統退化成短峯波情形下，亦可經運算整理而簡化成

$$A_{11}^- = 0 , \quad \sigma_0^{(1)} = \sigma_0^{(2)} = \sigma_0 , \quad \sigma_0^2 = gk \tanh kd \tag{6.17}$$

$$A_{11}^+ = a^2 \left(-\frac{g^2k^2}{\sigma_0} - 3\sigma_0^3 + 4m^2 \frac{g^2k^2}{\sigma_0} \right) / \left(4\sigma_0^2 - 2mgk \tanh 2mkd \right) \tag{6.18}$$

$$\begin{aligned} \mu_{21}^+ = gk^2a^3\sigma_0 \{ & \frac{1}{8} (-9\tanh^{-4}kd - 60\tanh^{-2}kd + 35 - 6\tanh^2 kd) \\ & + \frac{1}{2} m^2 (9\tanh^{-4}kd - \tanh^{-2}kd + 4) - m^2 \tanh^{-2}kd \\ & + (1 - \tanh^2 kd + 8m^2 - 10m \tanh kd \cdot \tanh 2mkd) (4m^2 - 1 \\ & - 3\tanh^2 kd) \tanh^{-1}kd \div (4\tanh kd - 2m \tanh 2mkd) \} \\ = \mu_{12}^+ & \end{aligned} \tag{6.19}$$

$$\begin{aligned} \mu_{21}^- = gk^2a^3\sigma_0 \{ & \frac{1}{8} (-3\tanh^{-4}kd + 5 - 2\tanh^2 kd) \\ & + \frac{1}{2} m^2 (3\tanh^{-4}kd - 5\tanh^{-2}kd + 2) + m^4 \tanh^{-2}kd \} \\ = -\mu_{12}^- & \end{aligned} \tag{6.20}$$

與 $$\begin{aligned} \zeta_{30}^{(1)} = k^2a^3 \{ & \frac{1}{16} (3\tanh^{-4}kd + 4\tanh^{-2}kd - 5 + 20\tanh^2 kd) \\ & + m^2 (\tanh^{-2}kd - 1) - m^4 \tanh^{-2}kd - \frac{1}{2} (1 - 2m^2 \\ & - 3\tanh^2 kd) (4m^2 - 1 - 3\tanh^2 kd) \tanh^{-1}kd \\ & / (4\tanh kd - 2m \tanh 2mkd) \} = \zeta_{03}^{(1)} \end{aligned} \tag{6.21}$$

$$\begin{aligned} \zeta_{30}^{(3)} & = \frac{1}{64} k^2a^3 (27\tanh^{-6}kd - 9\tanh^{-4}kd + 9\tanh^{-2}kd - 3) \\ & = \zeta_{03}^{(3)} \end{aligned} \tag{6.22}$$

$$\begin{aligned} \zeta_{21}^+ = k^2a^3 \{ & \frac{1}{8} (3\tanh^{-4}kd + 21\tanh^{-2}kd - 9 + 6\tanh^2 kd) \\ & - \frac{1}{4} m^2 (3\tanh^{-4}kd + 3) - (m^2 - \tanh^2 kd \end{aligned}$$

$$-3m\tanh kd\cdot\tanh 2mkd\,)(\,4m^2-1-3\tanh^2 kd\,)\tanh^{-1}kd$$

$$\div(\,4\tanh kd-2m\tanh 2mkd\,)+3\left[\,\frac{1}{8}(-9\tanh^{-3}kd\right.$$

$$-60\tanh^{-1}kd+35\tanh kd-6\tanh^3 kd\,)+\frac{1}{2}m^2(\,9\tanh^{-3}kd$$

$$-\tanh^{-1}kd+4\tanh kd\,)-m^4\tanh^{-1}kd+(\,1+8m^2-\tanh^2 kd$$

$$-10m\tanh kd\cdot\tanh 2mkd\,)(\,4m^2-1-3\tanh^2 kd\,)$$

$$/(\,4\tanh kd-2m\tanh 2mkd\,)\,]/[\,9\tanh kd$$

$$-(\,1+8m^2\,)^{1/2}\tanh(\,1+8m^2\,)^{1/2}kd\,]\,\}=\zeta_{12}^{+} \qquad (6.23)$$

$$\zeta_{21}^{-}=k^2a^3\left\{\frac{1}{8}(\,9\tanh^{-2}kd-8+2\tanh^2 kd\,)+\frac{3}{8}(\,1-2m^2\,)\right.$$

$$(\,\tanh^{-4}kd+1\,)+\left[\,\frac{1}{8}(-3\tanh^{-3}kd+5\tanh kd\right.$$

$$-2\tanh^3 kd\,)+\frac{1}{2}m^2(\,3\tanh^{-3}kd-5\tanh^{-1}kd+2\tanh kd\,)$$

$$\left.+m^4\tanh^{-1}kd\,]/[\,\tanh kd-(\,1+8n^2\,)^{1/2}\tanh(\,1+8n^2\,)^{1/2}kd\,]\right\}$$

$$=\zeta_{12}^{-} \qquad (6.24)$$

至此，這是很顯然的，所描述的短峯波波動流場解，至第三階次量，即刻可由(6.8)至(6.24)式所示之短峯波情況下的條件式代之(5.27)至(5.30)式中而求得。然，如上小節中所述，出現於等深水中的短峯波波動現象，可由廣大水域上的兩完全相同性質之規則前進波列相交會而成，亦可由單一規則前進波列經一很長的直立堤壁面之完全反射所致。惟得注意的是，由此兩種情況所構成的短峯波波動系統，其間稍有一點的差異；即後者當其沿著堤壁面前進時，則此時整個波動系統僅有單一波列的存在，或言短峯波退化成單一前進波列，這是往昔學者所常考慮的情況；至於前者而言，其本質皆是兩波列的存在（惟有事先就給定其中的一波列消失之特例情況除外），當它們同向前進時，則此時的波動系統變成兩完全相同的波列同時正好疊合在一起的交會情況。基此短峯波波動現象之形成的兩種稍有差別的情況，就其流場結構特性詳加探究起見，於此將針對此兩種情況分別列述其流場解，至第三階次，如下。

(A)無堤情況

由於此情況的短峯波波動場皆是存在著兩則進波列，故其流場解$\phi=\phi_s$、$\eta=\eta_s$、$\sigma=\sigma_1=\sigma_2$，直接可由(6.8)至(6.24)式代入(5.27)至(5.30)式而求得，爲

$$
\begin{aligned}
\phi_s =& \frac{2ag}{\sigma_0}\frac{\cosh k(d+z)}{\cosh kd}\sin(mkx-\sigma t)\cos nky \\
&+\frac{3}{4}a^2\sigma_0\frac{\cosh 2k(d+z)}{\sinh^4 kd}\sin 2(mkx-\sigma t)\cos 2nky \\
&-\frac{1}{2}\frac{a^2\sigma_0^2 t}{\sinh^2 kd}+a^2\sigma_0\frac{(4m^2-1-3\tanh^2 kd)\tanh^{-1}kd}{4\tanh kd-2m\tanh 2mkd} \\
&\times\frac{\cosh 2mk(d+z)}{\cosh 2mkd}\sin 2(mkx-\sigma t) \\
&+\frac{1}{32}ka^3\sigma_0(9\tanh^{-7}kd+5\tanh^{-5}kd-53\tanh^{-3}kd \\
&+39\tan^{-1}kd)\frac{\cosh 3k(d+z)}{\cosh 3kd}\sin 3(mkx-\sigma t)\cos 3nky \\
&+ka^3\sigma_0\{[\frac{1}{4}(-9\tanh^{-4}kd-60\tanh^{-2}kd+35-6\tanh^2 kd) \\
&+m^2(9\tanh^{-4}kd-\tanh^{-2}kd+4)-2m^4\tanh^{-2}kd \\
&+(1+8m^2-\tanh^2 kd-10m\tanh kd\cdot\tanh 2mkd) \\
&\times\frac{(4m^2-1-3\tanh^2 kd)\tanh^{-1}kd}{2\tanh kd-m\tanh 2mkd}]/[9\tanh kd \\
&-(1+8m^2)^{1/2}\tanh(1+8m^2)^{1/2}kd]\}\frac{\cosh(1+8m^2)^{1/2}k(d+z)}{\cosh(1+8m^2)^{1/2}kd} \\
&\times\sin 3(mkx-\sigma t)\cos nky+ka^3\sigma_0\{[\frac{1}{4}(-3\tanh^{-4}kd+5 \\
&-2\tanh^2 kd)+2m^4\tanh^{-2}kd+m^2(3\tanh^{-4}kd-5\tanh^{-2}kd \\
&+2)]/[\tanh kd-(1+8n^2)^{1/2}\tanh(1+8n^2)^{1/2}kd]\} \\
&\times\frac{\cosh(1+8n^2)^{1/2}k(d+z)}{\cosh(1+8n^2)^{1/2}kd}\sin(mkx-\sigma t)\cos 3nky \qquad (6.25)
\end{aligned}
$$

$$\eta_s = 2a\cos(mkx-\sigma t)\cos nky + \frac{1}{2}(3\tanh^{-3}kd - \tanh^{-1}kd)$$

$$\times ka^2\cos 2(mkx-\sigma t)\cos 2nky + \frac{1}{2}[(1-2m^2)\tanh^{-1}kd$$

$$+\tanh kd]ka^2\cos 2nky + \frac{1}{2}\{(1-2m^2)\tanh^{-1}kd + 3\tanh kd$$

$$+\frac{8m^2-2-6\tanh^2 kd}{2\tanh kd - m\tanh 2mkd}\}ka^2\cos 2(mkx-\sigma t)$$

$$+\{\frac{1}{8}(3\tanh^{-4}kd + 4\tanh^{-2}kd - 5 + 20\tanh^2 kd)$$

$$+2m^2(\tanh^{-2}kd - 1) - 2m^4\tanh^{-2}kd - (1-2m^2-3\tanh^2 kd)$$

$$\times\frac{(4m^2-1-3\tanh^2 kd)\tanh^{-1}kd}{4\tanh kd - 2m\tanh 2mkd}\}k^2a^3\cos(mkx-\sigma t)$$

$$\times\cos nky + \{\frac{1}{4}(3\tanh^{-4}kd + 21\tanh^{-2}kd - 9 + 6\tanh^2 kd)$$

$$-\frac{3}{2}m^2(\tanh^{-4}kd + 1) - (m^2 - \tanh^2 kd - 3m\tanh kd\cdot\tanh 2mkd)$$

$$\times\frac{(4m^2-1-3\tanh^2 kd)\tanh^{-1}kd}{2\tanh kd - m\tanh 2mkd} + 3[\frac{1}{4}(-9\tanh^{-3}kd$$

$$-60\tanh^{-1}kd + 35\tanh kd - 6\tanh^3 kd) + m^2(9\tanh^{-3}kd$$

$$-\tanh^{-1}kd + 4\tanh kd) - 2m^4\tanh^{-1}kd + (1+8m^2-\tanh^2 kd$$

$$-10m\tanh kd\cdot\tanh 2mkd)\frac{4m^2-1-3\tanh^2 kd}{2\tanh kd - m\tanh 2mkd}]$$

$$/[9\tanh kd - (1+8m^2)^{1/2}\tanh(1+8m^2)^{1/2}kd]\}$$

$$\times k^2a^3\cos 3(mkx-\sigma t)\cos nky + \{\frac{1}{4}(3\tanh^{-4}kd + 9\tanh^{-2}kd$$

$$-5+2\tanh^2 kd) - \frac{3}{2}m^2(\tanh^{-4}kd + 1) + [\frac{1}{4}(-3\tanh^{-3}kd$$

$$+5\tanh kd - 2\tanh^3 kd) + m^2(3\tanh^{-3}kd - 5\tanh^{-1}kd$$

$$+2\tanh kd)+2m^4\tanh^{-1}kd]/[\tanh kd$$
$$-(1+8n^2)^{1/2}\tanh(1+8n^2)^{1/2}kd]\}k^2a^3\cos(mkx-\sigma t)$$
$$\times\cos 3nky \tag{6.26}$$

$$\sigma=\sigma_0+\{\frac{1}{16}(9\tanh^{-4}kd-14\tanh^{-2}kd-7+4\tanh^2kd)$$
$$+m^2(\tanh^{-2}kd+2)-m^4\tanh^{-2}kd+\frac{1}{2}(4m^2-1-3\tanh^2kd)$$
$$\times\tanh^{-1}kd(4m^2-1+\tanh^2kd-2m\tanh kd\cdot\tanh 2mkd)$$
$$/(4\tanh kd-2m\tanh 2mkd)\}k^2a^2\sigma_0 \tag{6.27}$$

對上式所示之完全相同性質的兩規則前進波列，於等深水中，相交會而成的短峯波波動流場解，便於與往昔所得者之比較起見，將它無因化形式表示之。今對各有關的物理量無因次化，爲

$$\left.\begin{aligned}&\varepsilon=2\epsilon=2ka,\ \bar{x}=kx,\ \bar{y}=ky,\ \bar{z}=kz,\ \bar{d}=kd,\ \bar{t}=\sigma t,\\&\bar{\phi}_s=(k^3/g)^{1/2}\phi_s,\ \bar{\eta}_s=k\eta_s,\ \omega=\sigma/(gk)^{1/2},\\&\omega_0^2=\tanh kd,\ \omega_m^2=\tanh mkd\end{aligned}\right\} \tag{6.28}$$

因此，(6.25)至(6.27)式於(6.28)式之無因次化下的表示爲(至 ϵ^3 階)

$$\bar{\phi}_s=\varepsilon\omega_0^{-1}\frac{\cosh(\bar{d}+\bar{z})}{\cosh\bar{d}}\sin(m\bar{x}-\bar{t})\cos n\bar{y}-\frac{1}{8}\varepsilon^2(\omega_0^{-3}-\omega_0)\bar{t}$$
$$+\frac{1}{16}\varepsilon^2(\omega_0^{-7}-\omega_0)\frac{\cosh 2(\bar{d}+\bar{z})}{\cosh 2\bar{d}}\sin 2(m\bar{x}-\bar{t})\cos 2n\bar{y}$$
$$+\frac{1}{16}\varepsilon^2(1+\omega_m^4)\frac{(4m^2-1)\omega_0^{-3}-3\omega_0}{(1+\omega_m^4)-m\omega_m^2/\omega_0^2}$$
$$\times\frac{\cosh 2m(\bar{d}+\bar{z})}{\cosh 2m\bar{d}}\sin 2(m\bar{x}-\bar{t})+\frac{1}{256}\varepsilon^3(9\omega_0^{-13}+5\omega_0^{-9}$$
$$-53\omega_0^{-5}+39\omega_0^{-1})\frac{\cosh 3(\bar{d}+\bar{z})}{\cosh 3\bar{d}}\sin 3(m\bar{x}-\bar{t})\cos 3n\bar{y}$$

$$+\frac{1}{8}\varepsilon^3\{[\frac{1}{4}(-3\omega_0^{-7}+5\omega_0-2\omega_0^5)+m^2(3\omega_0^{-7}-5\omega_0^{-3}+2\omega_0)$$

$$+2m^4\omega_0^{-3}]/[\omega_0^2-(1+8n^2)^{1/2}\tanh(1+8n^2)^{1/2}\bar{d}]\}$$

$$\times\frac{\cosh(1+8n^2)^{1/2}(\bar{d}+\bar{z})}{\cosh(1+8n^2)^{1/2}\bar{d}}\sin(m\bar{x}-\bar{t})\cos 3n\bar{y}$$

$$+\frac{1}{8}\varepsilon^3\{\frac{1}{4}(-9\omega_0^{-7}-60\omega_0^{-3}+35\omega_0-6\omega_0^5)+m^2(9\omega_0^{-7}-\omega_0^{-3}$$

$$+4\omega_0)-2m^4\omega_0^{-3}+\frac{1}{2}[(1+8m^2-\omega_0^4)(1+\omega_m^4)$$

$$-20m\omega_0^2\omega_m^2]\frac{(4m^2-1)\omega_0^{-3}-3\omega_0}{(1+\omega_m^4)-m\omega_m^2/\omega_0^2}\}/[9\omega_0^2$$

$$-(1+8m^2)^{1/2}\tanh(1+8m^2)^{1/2}\bar{d}]\frac{\cosh(1+8m^2)^{1/2}(\bar{d}+\bar{z})}{\cosh(1+8m^2)^{1/2}\bar{d}}$$

$$\times\sin 3(m\bar{x}-\bar{t})\cos n\bar{y} \tag{6.29}$$

$$\bar{\eta}_s=\varepsilon\cos(mx-\bar{t})\cos n\bar{y}+\frac{1}{8}\varepsilon^2(3\omega_0^{-6}-\omega_0^{-2})\cos 2(m\bar{x}-\bar{t})\cos 2n\bar{y}$$

$$+\frac{1}{8}\varepsilon^2[(1-2m^2)\omega_0^{-2}+\omega_0^2]\cos 2n\bar{y}+\frac{1}{8}\varepsilon^2\{(1-2m^2)\omega_0^{-2}$$

$$+3\omega_0^2+(1+\omega_m^4)[(4m^2-1)\omega_0^{-2}-3\omega_0^2]/[(1+\omega_m^4)$$

$$-m(\omega_m/\omega_0)^2]\}\cos 2(m\bar{x}-\bar{t})+\frac{1}{8}\varepsilon^3\{\frac{1}{8}(3\omega_0^{-8}+4\omega_0^{-4}$$

$$-5+20\omega_0^4)+2m^2(\omega_0^{-4}-1)-2m^4\omega_0^{-4}-\frac{1}{4}(1-2m^2-3\omega_0^4)$$

$$\times(1+\omega_m^4)\frac{(4m^2-1)\omega_0^{-4}-3}{(1+\omega_m^4)-m\omega_m^2/\omega_0^2}\}\cos(m\bar{x}-\bar{t})\cos n\bar{y}$$

$$+\frac{1}{8}\varepsilon^2\{\frac{1}{4}(3\omega_0^{-8}+21\omega_0^{-4}-9+6\omega_0^4)-\frac{3}{2}m^2(\omega_0^{-8}+1)$$

$$-\frac{1}{2}[(m^2-\omega_0^4)(1+\omega_m^4)-6m\omega_0^2\omega_m^2]\frac{(4m_2^2-1)\omega_0^{-4}-3}{(1+\omega_m^4)-m\omega_m^2/\omega_0^2}$$

$$+\left[\frac{3}{4}(-9\omega_0^{-6}-60\omega_0^{-2}+35\omega_0^2-6\omega_0^6)+3m^2(9\omega_0^{-6}-\omega_0^{-2}\right.$$

$$\left.+4\omega_0^2)-6m^4\omega_0^{-2}\right]/\left[9\omega_0^2-(1+8m^2)^{1/2}\tanh(1+8m^2)^{1/2}\bar{d}\right]$$

$$+\frac{3}{2}\left[(1+8m^2-\omega_0^4)(1+\omega_m^4)-20m\omega_0^2\omega_m^2\right]\left[(4m^2-1)\omega_0^{-2}\right.$$

$$\left.-3\omega_0^2\right]/\left[9\omega_0^2-(1+8m^2)^{1/2}\tanh(1+8m_2)^{1/2}\bar{d}\right]$$

$$/\left[(1+\omega_m^4)-m(\omega_m/\omega_0)^2\right]\}\cos 3(m\bar{x}-\bar{t})\cos n\bar{y}$$

$$+\frac{1}{8}\varepsilon^3\{\frac{1}{4}(3\omega_0^{-8}+9\omega_0^{-4}-5+2\omega_0^4)-\frac{3}{2}m^2(\omega_0^{-8}+1)$$

$$+\left[\frac{1}{4}(-3\omega_0^{-6}+5\omega_0^2-2\omega_0^6)+m^2(3\omega_0^{-6}-5\omega_0^{-2}+2\omega_0^2)\right.$$

$$\left.+2m^4\omega_0^{-2}\right]/\left[\omega_0^2-(1+8n^2)^{1/2}\tanh(1+8n^2)^{1/2}\bar{d}\right]\}$$

$$\times\cos(m\bar{x}-\bar{t})\cos 3n\bar{y}+\frac{1}{256}\varepsilon^3(27\omega_0^{-12}-9\omega_0^{-8}+9\omega_0^{-4}-3)$$

$$\times\cos 3(m\bar{x}-\bar{t})\cos 3n\bar{y} \tag{6.30}$$

$$\omega=\omega_0+\frac{1}{4}\varepsilon^2\{\frac{1}{16}(9\omega_0^{-7}-14\omega_0^{-3}-7\omega_0+4\omega_0^5)+m^2(\omega_0^{-3}+2\omega_0)$$

$$-m^4\omega_0^{-3}+\frac{1}{8}\left[(4m^2-1+\omega_0^4)(1+\omega_m^4)-4m\omega_0^2\omega_m^2\right]$$

$$\times\frac{(4m^2-1)\omega_0^{-3}-3\omega_0}{(1+\omega_m^4)-m\omega_m^2/\omega_0^2}\} \tag{6.31}$$

(B)有堤情況

此情況的短峯波波動場是因單一規則前進波列，受一無限長的直立堤壁面之完全反射，而形成於半無限空間（設 y ＞ 0 者）之海域者。對此情況的短峯波波動場，得需注意的一點是，其有僅爲來源的單一視則前進波列之波動情況的特例發生；即是當入射的單一規則前進波列沿著提壁面前進時，則並無反射波列的產生。因此，欲適足地描述此情況之短峯波波動流場機構特性，得需對其僅有入射波列之特例情況給予涵括並分類出。換言之，對此短峯波之波動流場解，得需分別歸類出入射波列與反射波

列者，以及由此兩波列相交會間之相互作用而生者。做此解析，今可以單位階梯函數（ unit step function ）引入的輔助，來對其詳論於後。

首先

$$令 \quad U(m)=\begin{cases} 0 , & m=1 , \\ 1 , & 0 \le m < 1 , \end{cases} \qquad m=\sin\alpha \,,\ 0 \le \alpha \le 90^\circ \qquad (6.32)$$

然後直接由 (5.27) 至 (5.30) 式所式之等深水中任兩規則前進波列相交會所構成之波動系統的通案性流場解中，分別歸類出兩來源波列自身者，以及它們間相互作用而生者。之後，再以 (6.8) 至 (6.24) 式之短峯波時的條件代入，同時以單位階梯函數U(m)的引入給予區別之。因此，可得於等深水中，單一規則前進重力波列經一無限長的直立堤壁面之完全反射而形成之短峯波波動，其流場解（至第三階）為

$$\phi_s=\frac{ag}{\sigma_0}\frac{\cosh k(d+z)}{\cosh kd}\sin(mkx-nky-\sigma t)$$

$$+\frac{3}{8}a^2\sigma_0\frac{\cosh 2k(d+z)}{\sinh^4 kd}\sin 2(mkx-nky-\sigma t)$$

$$-\frac{1}{4}\frac{a^2\sigma_0^2 t}{\sinh^2 kd}+\frac{1}{64}ka^3\sigma_0(9\tanh^{-7}kd+5\tanh^{-5}kd$$

$$-53\tanh^{-3}kd+39\tanh^{-1}kd)\frac{\cosh 3k(d+z)}{\cosh 3kd}\sin 3(mkx$$

$$-nky-\sigma t)+\{\frac{ag}{\sigma_0}\frac{\cosh k(d+z)}{\cosh kd}\sin(mkx+nky-\sigma t)$$

$$+\frac{3}{8}a^2\sigma_0\frac{\cosh 2k(d+z)}{\sinh^4 kd}\sin 2(mkx+nky-\sigma t)-\frac{1}{4}\frac{a^2\sigma_0^2 t}{\sinh^2 kd}$$

$$+a^2\sigma_0\frac{(4m^2-1-3\tanh^2 kd)\tanh^{-1}kd}{4\tanh kd-2m\tanh 2mkd}\frac{\cosh 2mk(d+z)}{\cosh 2mkd}$$

$$\times\sin 2(mkx-\sigma t)+\frac{1}{64}ka^3\sigma_0(9\tanh^{-7}kd+5\tanh^{-5}kd$$

$$-53\tanh^{-3}kd+39\tanh^{-1}kd)\frac{\cosh 3k(d+z)}{\cosh 3kd}$$

$$\times\sin 3(mkx+nky-\sigma t)+ka^3\sigma_0[\frac{1}{4}(-9\tanh^{-4}kd$$

$$-60\tanh^{-2}kd+35-6\tanh^2 kd)+m^2(9\tanh^{-4}kd-\tanh^{-2}kd$$

$$+4)-2m^4\tanh^{-2}kd+(1+8m^2-\tanh^2 kd$$

$$-10m\tanh kd\cdot\tanh 2mkd)\frac{(4m^2-1-3\tanh^2 kd)\tanh^{-1}kd}{2\tanh kd-m\tanh 2mkd}]$$

$$/[9\tanh kd-(1+8m^2)^{1/2}\tanh(1+8m^2)^{1/2}kd]$$

$$\times\frac{\cosh(1+8m^2)^{1/2}k(d+z)}{\cosh(1+8m^2)^{1/2}kd}\sin 3(mkx-\sigma t)\cos nky$$

$$+ka^3\sigma_0[\frac{1}{4}(-3\tanh^{-4}kd+5-2\tanh^2 kd)+2m^4\tanh^{-2}kd$$

$$+m^2(3\tanh^{-4}kd-5\tanh^{-2}kd+2)]/[\tanh kd-(1+8n^2)^{1/2}$$

$$\times\tanh(1+8n^2)^{1/2}kd]\frac{\cosh(1+8n^2)^{1/2}k(d+z)}{\cosh(1+8n^2)^{1/2}kd}$$

$$\times\sin(mkx-\sigma t)\cos 3nky\}U(m) \qquad (6.33)$$

$$\eta_s=a\cos(mkx-nky-\sigma t)+\frac{1}{4}(3\tanh^{-3}kd-\tanh^{-1}kd)$$

$$\times ka^2\cos 2(mkx-nky-\sigma t)+\frac{1}{16}(3\tanh^{-4}kd+8\tanh^{-2}kd$$

$$-9)k^2a^3\cos(mkx-nky-\sigma t)+\frac{1}{64}(27\tanh^{-6}kd-9\tanh^{-4}kd$$

$$+9\tanh^{-2}kd-3)k^2a^3\cos 3(mkx-nky-\sigma t)+\Big\{a\cos(mkx+nky$$

$$-\sigma t)+\frac{1}{4}(3\tanh^{-3}kd-\tanh^{-1}kd)ka^2\cos 2(mkx+nky-\sigma t)$$

$$+\frac{1}{16}(3\tanh^{-4}kd+8\tanh^{-2}kd-9)k^2a^3\cos(mkx+nky-\sigma t)$$

$$+\frac{1}{2}[(1-2m^2)\tanh^{-1}kd+\tanh kd]ka^2\cos 2nky$$

$$+\frac{1}{2}\{(1-2m^2)\tanh^{-1}kd+3\tanh kd+\frac{8m^2-2-6\tanh^2 kd}{2\tanh kd-m\tanh 2mkd}\}$$

$$\times ka^2 \cos 2(mkx-\sigma t)+\{\frac{1}{8}(-4\tanh^{-2}kd+4+20\tanh^2 kd)$$

$$+2m^2(\tanh^{-2}kd-1)-2m^4\tanh^{-2}kd-(1-2m^2-3\tanh^2 kd)$$

$$\times\frac{(4m^2-1-3\tanh^2 kd)\tanh^{-1}kd}{4\tanh kd-2m\tanh 2mkd}\}k^2a^3\cos(mkx-\sigma t)\cos nky$$

$$+\{\frac{1}{4}(3\tanh^{-4}kd+21\tanh^{-2}kd-9+6\tanh^2 kd)$$

$$-\frac{3}{2}m^2(\tanh^{-4}kd+1)-(m^2-\tanh^2 kd-3m\tanh kd\cdot\tanh 2mkd)$$

$$\times\frac{(4m^2-1-3\tanh^2 kd)\tanh^{-1}kd}{2\tanh kd-m\tanh 2mkd}+3[\frac{1}{4}(-9\tanh^{-3}kd$$

$$-60\tanh^{-1}kd+35\tanh kd-6\tanh^3 kd)+m^2(9\tanh^{-3}kd$$

$$-\tanh^{-1}kd+4\tanh kd)-2m^4\tanh^{-1}kd+(1+8m^2-\tanh^2 kd$$

$$-10m\tanh kd\cdot\tanh 2mkd)\frac{4m^2-1-3\tanh^2 kd}{2\tanh kd-m\tanh 2mkd}]$$

$$/[9\tanh kd-(1+8m^2)^{1/2}\tanh(1+8m^2)^{1/2}kd]\}$$

$$\times k^2a^3\cos 3(mkx-\sigma t)\cos nky+\{\frac{1}{4}(3\tanh^{-4}kd+9\tanh^{-2}kd$$

$$-5+2\tanh^2 kd)-\frac{3}{2}m^2(\tanh^{-4}kd+1)+[\frac{1}{4}(-3\tanh^{-3}kd$$

$$+5\tanh kd-2\tanh^3 kd)+m^2(3\tanh^{-3}kd-5\tanh^{-1}kd$$

$$+2\tanh kd)+2m^4\tanh^{-1}kd]/[\tanh kd-(1+8n^2)^{1/2}$$

$$\times \mathrm{than}(1+8n^2)^{1/2}kd]\}k^2a^3\cos(mkx-\sigma t)\cos 3nky\frac{1}{64}(27\tanh^{-6}kd$$

$$-9\tanh^{-4}kd+9\tanh^{-2}kd-3)k^2a^3\cos 3(mkx+nky-\sigma t)\Bigg\}U(m) \tag{6.34}$$

$$\sigma=\sigma_0+\frac{1}{16}(9\tanh^{-4}kd-10\tanh^{-2}kd+9)k^2a^2\sigma_0$$

$$+\{\left[\frac{1}{4}(-\tanh^{-2}kd-4+\tanh^{2}kd)+m^{2}(\tanh^{-2}kd+2)\right.$$

$$-m^{4}\tanh^{-2}kd+\frac{1}{4}(4m^{2}-1-3\tanh^{2}kd)\tanh^{-1}kd\cdot(4m^{2}-1$$

$$\left.+\tanh^{2}kd-2m\tanh kd\cdot\tanh 2mkd)/(2\tanh kd-m\tanh 2mkd)\right]$$

$$\times k^{2}a^{2}\sigma_{0}\}U(m) \tag{6.35}$$

這是即刻可容易地求知的，當$U(m)=1$時，即$0\leq\alpha<90°$時，此時的波動場同時存在有入射與完全反射的兩波列，則(6.33)至(6.35)式之短峯波波動流場解是完全與(6.25)至(6.27)式者一致之。另，當$U(m)=0$時，即$\alpha=90°$時，此時的波動場僅有入射的單一波列存在，則(6.33)至(6.35)式之短峯波波動流場解完全退化成單一前進波列者。此乃往昔解析所不足之處，詳見下節之印證與比較。

如同上小節之無堤情況者，有堤情況下之短峯波波動流場解亦可引用(6.28)式之無因次量將其無因化之，因此，(6.33)至(6.35)式之解可表示爲

$$\overline{\phi}_{s}=\epsilon\omega_{0}^{-1}\frac{\cosh(\overline{d}+\overline{z})}{\cosh\overline{d}}\sin(m\overline{x}-n\overline{y}-\overline{t})-\frac{1}{4}\epsilon^{2}(\omega_{0}^{-3}-\omega_{0})\overline{t}$$

$$+\frac{3}{8}\epsilon^{2}(\omega_{0}^{-7}-\omega_{0})\frac{\cosh 2(\overline{d}+\overline{z})}{\cosh 2\overline{d}}\sin 2(m\overline{x}-n\overline{y}-\overline{t})$$

$$+\frac{1}{64}\epsilon^{3}(9\omega_{0}^{-13}+5\omega_{0}^{-9}-53\omega_{0}^{-5}+39\omega_{0}^{-1})$$

$$\times\frac{\cosh 3(\overline{d}+\overline{z})}{\cosh 3\overline{d}}\sin 3(m\overline{x}-n\overline{y}-\overline{t})+\Bigg(\epsilon\omega_{0}^{-1}\frac{\cosh(\overline{d}+\overline{z})}{\cosh\overline{d}}$$

$$\times\sin(m\overline{x}+n\overline{y}-\overline{t})-\frac{1}{4}\epsilon^{2}(\omega_{0}^{-3}-\omega_{0})\overline{t}+\frac{3}{8}\epsilon^{2}(\omega_{0}^{-7}-\omega_{0})$$

$$\times\frac{\cosh 2(\overline{d}+\overline{z})}{\cosh 2\overline{d}}\sin 2(m\overline{x}+n\overline{y}-\overline{t})+\frac{1}{64}\epsilon^{3}(9\omega_{0}^{-13}+5\omega_{0}^{-9}$$

$$-53\omega_{0}^{-5}+39\omega_{0}^{-1})\frac{\cosh 3(\overline{d}+\overline{z})}{\cosh 3\overline{d}}\sin 3(m\overline{x}+n\overline{y}-\overline{t})$$

$$+\frac{1}{4}\epsilon^{2}(1+\omega_{m}^{4})\frac{(4m^{2}-1)\omega_{0}^{-3}-3\omega_{0}}{(1+\omega_{m}^{4})-m\omega_{m}^{2}/\omega_{0}^{2}}\frac{\cosh 2m(\overline{d}+\overline{z})}{\cosh 2m\overline{d}}$$

$$\times \sin 2(m\bar{x}-\bar{t}) + \epsilon^3 \{ [\frac{1}{4} (-3\omega_0^{-7} + 5\omega_0 - 2\omega_0^5)$$

$$+ m^2 (3\omega_0^{-7} - 5\omega_0^{-3} + 2\omega_0) + 2m^4 \omega_0^{-3}]$$

$$/ [\omega_0^2 - (1+8n^2)^{1/2} \tanh (1+8n^2)^{1/2} \bar{d}] \}$$

$$\times \frac{\cosh (1+8n^2)^{1/2} (\bar{d}+\bar{z})}{\cosh (1+8n^2)^{1/2} \bar{d}} \sin (m\bar{x} - \bar{t}) \cos 3n\bar{y}$$

$$+ \epsilon^3 \{ \frac{1}{4} (-9\omega_0^{-7} - 60\omega_0^{-3} + 35\omega_0 - 6\omega_0^5) + m^2 (9\omega_0^{-7} - \omega_0^{-3}$$

$$+ 4\omega_0) - 2m^4 \omega_0^{-3} + \frac{1}{2} [(1+8m^2 - \omega_0^4)(1+\omega_m^4)$$

$$- 20m\omega_0^2 \omega_m^2] \frac{(4m^2-1)\omega_0^{-3} - 3\omega_0}{(1+\omega_m^4) - m\omega_m^2 / \omega_0^2} \} / [9\omega_0^2 - (1+8m^2)^{1/2}$$

$$\times \tanh (1+8m^2)^{1/2} \bar{d}] \frac{\cosh (1+8m^2)^{1/2} (\bar{d}+\bar{z})}{\cosh (1+8m^2)^{1/2} \bar{d}}$$

$$\times \sin 3(m\bar{x}-\bar{t}) \cos n\bar{y} \Big) U(m) \qquad (6.36)$$

$$\bar{\eta}_s = [\epsilon + \frac{1}{16} \epsilon^3 (3\omega_0^{-8} + 8\omega_0^{-4} - 9)] \cos (m\bar{x} - n\bar{y} - \bar{t})$$

$$+ \frac{1}{4} \epsilon^2 (3\omega_0^{-6} - \omega_0^{-2}) \cos 2(m\bar{x} - n\bar{y} - \bar{t}) + \frac{1}{64} \epsilon^3 (27\omega_0^{-12}$$

$$- 9\omega_0^{-8} + 9\omega_0^{-4} - 3) \cos 3(m\bar{x} - n\bar{y} - \bar{t}) + \Big([\epsilon + \frac{1}{16} \epsilon^3 (3\omega_0^{-8}$$

$$+ 8\omega_0^{-4} - 9)] \cos (m\bar{x} + n\bar{y} - \bar{t}) + \frac{1}{4} \epsilon^2 (3\omega_0^{-6} - \omega_0^{-2})$$

$$\times \cos 2(m\bar{x} + n\bar{y} - \bar{t}) + \frac{1}{64} \epsilon^3 (27\omega_0^{-12} - 9\omega_0^{-8} + 9\omega_0^{-4} - 3)$$

$$\times \cos 3(m\bar{x} + n\bar{y} - \bar{t}) + \frac{1}{2} \epsilon^2 [(1-2m^2)\omega_0^{-2} + \omega_0^2] \cos 2n\bar{y}$$

$$+ \frac{1}{2} \epsilon^2 \{ (1-2m^2)\omega_0^{-2} + 3\omega_0^2 + (1+\omega_m^4) [(4m^2-1)\omega_0^{-2}$$

$$-3\omega_0^2]/[(1+\omega_m^4)-m\omega_m^2/\omega_0^2\}\cos 2(m\bar{x}-\bar{t})$$

$$+\epsilon^3\{\frac{1}{2}[-\omega_0^{-4}+1+5\omega_0^4)+2m^2(\omega_0^{-4}-1)-2m^4\omega_0^{-4}$$

$$-\frac{1}{4}(1-2m^2-3\omega_0^4)(1+\omega_m^4)[(4m^2-1)\omega_0^{-4}-3]$$

$$/[(1+\omega_m^4)-m\omega_m^2/\omega_0^2]\}\times\cos(m\bar{x}-\bar{t})\cos n\bar{y}+\epsilon^3\{\frac{1}{4}(3\omega_0^{-8}$$

$$+21\omega_0^{-4}-9+6\omega_0^4)-\frac{3}{2}m^2(\omega_0^{-8}+1)-\frac{1}{2}[(m^2-\omega_0^4)(1+\omega_m^4)$$

$$-6m\omega_m^2\omega_0^2][(4m^2-1)\omega_0^{-4}-3]/[(1+\omega_m^4)-m\omega_m^2/\omega_0^2]$$

$$+[\frac{3}{4}(-9\omega_0^{-6}-60\omega_0^{-2}+35\omega_0^2-6\omega_0^6)+3m^2(9\omega_0^{-6}-\omega_0^{-2}$$

$$+4\omega_0^2)-6m^4\omega_0^{-2}]/(9\omega_0^2-(1+8m^2)^{1/2}\tanh(1+8m^2)^{1/2}\bar{d}]$$

$$+\frac{3}{2}[(1+8m^2-\omega_0^4)(1+\omega_m^4)-20m\omega_m^2\omega_0^2][(4m^2-1)\omega_0^{-2}$$

$$-3\omega_0^2]/[9\omega_0^2-\sqrt{1+8m^2}\tanh\sqrt{1+8m^2}\bar{d}]\div[(1+\omega_m^4)$$

$$-m\omega_m^2/\omega_0^2]\}\cos 3(m\bar{x}-\bar{t})\cos n\bar{y}$$

$$+\epsilon^3\{\frac{1}{4}(3\omega_0^{-8}+9\omega_0^{-4}-5+2\omega_0^4)-\frac{3}{2}m^2(\omega_0^{-8}+1)$$

$$+[\frac{1}{4}(-3\omega_0^{-6}+5\omega_0^2-2\omega_0^6)+m^2(3\omega_0^{-6}-5\omega_0^{-2}+2\omega_0^2)$$

$$+2m^4\omega_0^{-2}]/[\omega_0^2-\sqrt{1+8n^2}\tanh\sqrt{1+8n^2}\bar{d}]\}$$

$$\times\cos(m\bar{x}-\bar{t})\cos 3n\bar{y}\Big)U(m) \tag{6·37}$$

$$\omega=\omega_0+\frac{1}{16}\epsilon^2(9\omega_0^{-7}-10\omega_0^{-3}+9\omega_0)+\varepsilon^2\{[\frac{1}{4}(-\omega_0^{-3}-4\omega_0+\omega_0^5)$$

$$+m^2(\omega_0^{-3}+2\omega_0)-m^4\omega_0^{-3}]+\frac{1}{8}[(4m^2-1+\omega_0^4)(1+\omega_m^4)$$

$$-4m\omega_m^2\omega_0^2][(4m^2-1)\omega_0^{-3}-3\omega_0]\div[(1+\omega_m^4)$$

$$-m\omega_m^2/\omega_0^2\,]\}\,U(m) \tag{6.38}$$

如上之有因次解的比較般，當U(m)＝1時，有堤情況之短峯波波動場有入射與完全反射兩波列的同時存在，故其無因次化的流場解（6.36）至（6.38）式是與（6.29）至（6.31）式之無堤情況者完全一致；而當U(m)＝0時，波動場僅有入射的單一波列，因此（6.36）至（6.38）式之解是退化成單一規則前進重力波者。

(2)波壓之解析

基於對本節以上所導述的短峯波波動流場解做較確切的檢核，此處將描述可試驗量測得其流場內所會呈現的動力特性，即波壓者，以爲往後對它進行具體的驗證。由於短峯波場內之波壓的量測，方便上，是造一單一規則前進的重力波列入射，經一直立堤壁面完全反射而形成短峯波場後，再由安放在堤壁面上的波壓器讀取之。因此，對短峯波場內波壓的描述，吾人通常無考慮其退化成單一規則前進波列的情況，即對上小節之流場解取U(m)＝1來解析短峯波場中之波壓，故依（6.25）至（6.27）式（若由（6.33）至（6.35）式，則取U(m)＝1）代入Bernoulli's 方程式

$$\frac{\partial\phi_s}{\partial t}+\frac{p_s}{\rho}+gz+\frac{1}{2}\left[\left(\frac{\partial\phi_s}{\partial x}\right)^2+\left(\frac{\partial\phi_s}{\partial y}\right)^2+\left(\frac{\partial\phi_s}{\partial z}\right)^2\right]=0 \tag{6.39}$$

ρ 爲流體之密度，經細心的運算展開及歸類整理，可得短峯波場中之壓力 p_s，至第三階次下之解析結果，爲

$$\frac{p_s}{\rho}+gz=2ag\frac{\cosh k(d+z)}{\cosh kd}\cos(mkx-\sigma t)\cos nky$$

$$-\frac{gka^2}{\omega_0^2}\frac{\sinh^2 k(d+z)}{\cosh^2 kd}-\frac{1}{2}\frac{gka^2}{\omega_0^2}\Big[(2m^2-1)(1-\omega_0^4)$$

$$+2m^2\frac{\sinh^2 k(d+z)}{\cosh^2 kd}\Big]\cos 2nky+\frac{1}{2}\frac{gka^2}{\omega_0^2}\Big[(1-2m^2)$$

$$\times(1-\omega_0^4)+2(1-m^2)\frac{\sinh^2 k(d+z)}{\cosh^2 kd}+(1+\omega_m^4)$$

$$\times\frac{(4m^2-1)-3\omega_0^4}{(1+\omega_m^4)-m\omega_m^2/\omega_0^2}\frac{\cosh 2mk(d+z)}{\cosh 2mkd}\Big]$$

$$\times\cos 2(mkx-\sigma t)+\frac{1}{2}\frac{gka^2}{\omega_0^2}\big[-(1-\omega_0^4)+3(\omega_0^{-4}-\omega_0^4)$$

$$\times\frac{\cosh 2k(d+z)}{\cosh 2kd}\Big]\cos 2(mkx-\sigma t)\cos 2nky$$

$$+gk^2a^3\Big\{\Big[\frac{1}{8}(9\omega_0^{-8}-14\omega_0^{-4}-7+4\omega_0^4)+2m^2(\omega_0^{-4}+2)$$

$$-2m^4\omega_0^{-4}+\frac{1}{4}\big((4m^2-1+\omega_0^4)(1+\omega_m^4)-4m\omega_0^2\omega_m^2\big)$$

$$\times\frac{(4m^2-1)\omega_0^{-4}-3}{(1+\omega_m^4)-m\omega_m^2/\omega_0^2}\Big]\frac{\cosh k(d+z)}{\cosh kd}-\frac{3}{4}(\omega_0^{-8}-1)$$

$$\times\frac{\cosh 3k(d+z)}{\cosh kd\cdot\cosh 2kd}-\frac{(1+\omega_m^4)}{4}\frac{(4m^2-1)\omega_0^{-4}-3}{(1+\omega_m^4)-m\omega_m^2/\omega_0^2}$$

$$\times\Big[(m^2+m)\frac{\cosh(1+2m)k(d+z)}{\cosh kd\cdot\cosh 2mkd}$$

$$+(m^2-m)\frac{\cosh(1-2m)k(d+z)}{\cosh kd\cdot\cosh 2mkd}\Big]\Big\}\cos(mkx-\sigma t)\cos nky$$

$$+gk^2a^3\Big\{\Big[\frac{1}{4}(-3\omega_0^{-6}+5\omega_0^2-2\omega_0^6)+m^2(3\omega_0^{-6}-5\omega_0^{-2}$$

$$+2\omega_0^2)+2m^4\omega_0^{-2}\Big]/\big[\omega_0^2-(1+8n^2)^{1/2}\tanh(1+8n^2)^{1/2}kd\big]$$

$$\times\frac{\cosh(1+8n^2)^{1/2}k(d+z)}{\cosh(1+8n^2)^{1/2}kd}-\frac{3}{4}(\omega_0^{-8}-1)$$

$$\times\Big[m^2\frac{\cosh 3k(d+z)}{\cosh kd\cdot\cosh 2kd}+(m^2-1)\frac{\cosh k(d+z)}{\cosh kd\cdot\cosh 2kd}\Big]\Big\}$$

$$\times\cos(mkx-\sigma t)\cos 3nky+gk^2a^3\Big\{\Big[\frac{3}{4}(-9\omega_0^{-6}-60\omega_0^{-2}$$

$$+35\omega_0^2-6\omega_0^6)+3m^2(9\omega_0^{-6}-\omega_0^{-2}+4\omega_0^2)-6m^4\omega_0^{-2}$$

$$+\frac{3}{2}\big((1+8m^2-\omega_0^4)(1+\omega_m^4)-20m\omega_m^2\omega_0^2\big)$$

$$\times\frac{(4m^2-1)\omega_0^{-2}-3\omega_0^2}{(1+\omega_m^4)-m\omega_m^2/\omega_0^2}\Big]/\big[9\omega_0-(1+8m^2)^{1/2}$$

$$\times \tanh\left(1+8m^2 \right)^{1/2} kd \,\big]\, \frac{\cosh\left(1+8m^2 \right)^{1/2} k\left(d+z \right)}{\cosh\left(1+8m^2 \right)^{1/2} kd}$$

$$-\frac{3}{4}\left(\omega_0^{-8}-1 \right)\left[\left(m^2-1 \right)\frac{\cosh 3k\left(d+z \right)}{\cosh kd \cdot \cosh 2kd}\right.$$

$$\left. +m^2\frac{\cosh k\left(d+z \right)}{\cosh kd \cdot \cosh 2kd}\right]-\frac{\left(1+\omega_m^4 \right)}{4}\frac{\left(4m^2-1 \right)\omega_0^{-4}-3}{\left(1+\omega_m^4 \right)-m\omega_m^2/\omega_0^2}$$

$$\times\left[\left(m^2-m \right)\frac{\cosh\left(1+2m \right)k\left(d+z \right)}{\cosh kd \cdot \cosh 2mkd}+\left(m^2+m \right)\right.$$

$$\left.\times\frac{\cosh\left(1-2m \right)k\left(d+z \right)}{\cosh kd \cdot \cosh 2mkd}\right]\Big\} \cos 3\left(mkx-\sigma t \right)\cos nky$$

$$+gk^2a^3\Big\{ \frac{3}{32}\left(9\omega_0^{-12}+5\omega_0^{-8}-53\omega_0^{-4}+39 \right)\frac{\cosh 3k\left(d+z \right)}{\cosh 3kd}$$

$$-\frac{3}{4}\left(\omega_0^{-8}-1 \right)\frac{\cosh k\left(d+z \right)}{\cosh kd \cdot \cosh 2kd}\Big\} \cos 3\left(mkx-\sigma t \right)\cos 3nky$$

(6.40)

6-3 短峰波場之一些特性

在等深水中，純由兩完全相同性質之規則前進波列於無受限之廣大海域中相交會而成的短峯波波動，亦抑單一規則前進波列因受一直立堤壁面之完全反射而形成的，其流場之整體特性，包括這兩完全相同性質波列自身原具有的非線性本質及由其間之相互作用而衍生出之效應量等，至第三階量，雖然可由上小節中之有堤與無堤情況所述之流場解給予其全盤性詳盡的論述，然此處限於篇幅起見，僅將其中較爲顯著的效應與具有迷惑性之極端者給予詳加描述，於下。

(1)週波率影響效應

等深水中，任兩規則前進重力波列相交會所構成之波動系統，其流場內會發生的週波率影響效應雖已被整體性的描述，如見於5.4節中之說明，由此當然已涵括了其特例之短峯波波動情況者，然爲更詳盡地細究這短峯波場中各種情況時之週波率影響效應的變化特性，於此將特予闡論之。

這是眾所知曉的，於任一均勻等深水中，單一自由表面規則前進重力波之週波率皆隨振幅（或波浪尖銳度）之增加而增大；然對重力駐波者，其於較深水中是隨其振幅（或波浪尖銳度）之增加反而減少，而在較淺水中（三階解時，水深對波長比約小於 0.17下）才變爲隨之增大，謂之重力駐波週波率之逆變特性。由於單一前進波列與駐波是短峯波波動中之兩個極端特例情況，因此，於短峯波波動場中必會存在有一影響週波率的效應，謂之週波率影響效應，如此才會致使波動的週波率有正逆變化之特性出現。這如 5.4節中之對任兩波列相交會所構成之波動場的探討般，發生於短峯波波動場中的此種週波率影響效應，是可由所造致的短峯波波動之週波率與來源的單一自由表面規則前進重力波列者之相減而得之，即

$$\frac{\sigma}{\sigma_0}-1-\frac{1}{16}(9\omega_0^{-8}-10\omega_0^{-4}+9)k^2a^2=f_s(kd,\alpha)k^2a^2 \tag{6·41}$$

此處 $f_s(kd,\alpha)$ 稱爲短峯波波動場中之週波率影響效應函數（ the effect function by interaction on frequency ），其可依無堤與有堤情況而分別由 (6.27) 式（或 (6.31) 式）與 (6.35) 式（或 (6.38) 式）代入 (6.41) 式後，得爲

$$\begin{aligned} f_s(kd;\alpha)=&\frac{1}{4}(-\omega_0^{-4}-4+\omega_0^4)+m^2(\omega_0^{-4}+2)-m^4\omega_0^{-4} \\ &+\frac{1}{8}[(4m^2-1+\omega_0^4)(1+\omega_m^4)-4m\omega_m^2\omega_0^2] \\ &\times\frac{(4m^2-1)\omega_0^{-4}-3}{(1+\omega_m^4)-m\omega_m^2/\omega_0^2} \end{aligned} \tag{6.42}$$

與

$$\begin{aligned} f_s(kd,\alpha)=&\{\frac{1}{4}(-\omega_0^{-4}-4+\omega_0^4)+m^2(\omega_0^{-4}+2)-m^4\omega_0^{-4} \\ &+\frac{1}{8}[(4m^2-1+\omega_0^4)(1+\omega_m^4)-4m\omega_m^2\omega_0^2] \\ &\times\frac{(4m^2-1)\omega_0^{-4}-3}{(1+\omega_m^4)-m\omega_m^2/\omega_0^2}\}U(m) \end{aligned} \tag{6.43}$$

由上顯然可知，只要 f_s 之變化特性被掌握，則短峯波波動之週波率當即可被描

述。此短峯波之週波率影響效應函數 f_s，於各種相對水深中，隨入射波列之入射角 α 變化的情況如圖7所示。

依圖7所示，當角度 α 由 $0°$ 增至 $90°$ 時，則於各相對水深中之 f_s 值皆由負值漸變成正值，且幾皆於 $45°$ 處爲零。此種變化特性是符合作用力學之原則的，蓋因形成短峯波波動之兩完全相同性質的規則前進重力波列，於圖6的座標系統描述下，其相交間之夾角爲 $\theta=\pi-2\alpha$，故當 $0°\leq\alpha\leq 45°$ 時，則此兩相交會的波列是在逆向相衝擊的作用範圍內（因此時其間之交角爲 $180°\geq\theta\geq 90°$），所以造致對各波列之週波率的影響效應是使之減小者，即此時之 f_s 值爲在負的增長；反之，當 $45°\leq\alpha\leq 90°$ 時，則此兩相交會之波列是在正向相衝擊的作用範圍內（因此時其間之交角爲 $90°\geq\theta\geq 0°$，有堤情況時退化成單一前進波列者除外），因此，對各波列之週波

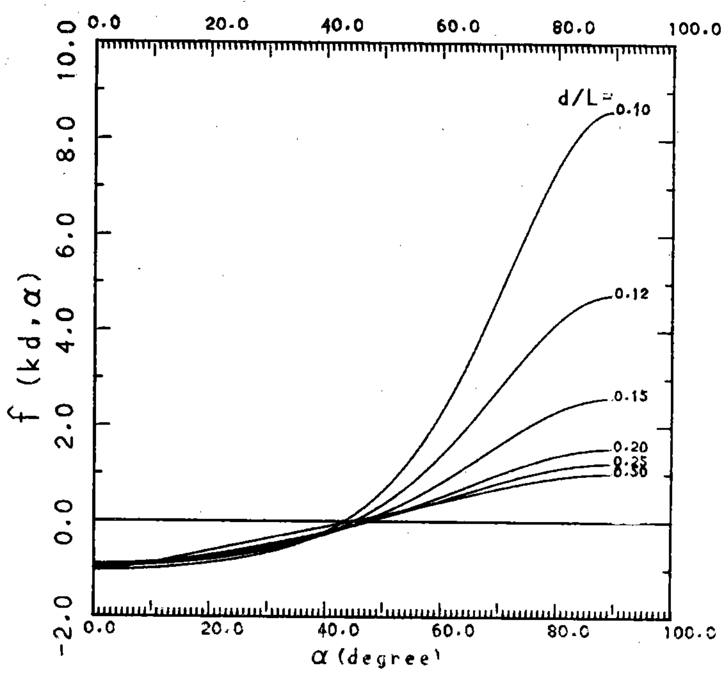


圖 7　短峰波之週波率影響效應函數f，隨相對水深d/L及入射角 α 之變化
Fig. Frequency effect function f, of short-crested waves relative to water depth d/L and incident wave angle α.

率造成的影響效應是使之增大者，即此時之f_s值爲正的遞增；可想得到的f_s值幾以$\alpha=45°$時做正負分界，蓋因此時之$\theta=90°$，即相交會的兩波列是相垂直前進穿過之故。又由圖7所示可知，f_s之變化隨所在的相對水深d/L之變淺而趨強烈，此現象亦與波動的本質特性一致的，蓋因如吾們所共熟知的，當相對水深愈淺時，則波動之非線性本質愈爲顯著，由此於短峯波波動場中，因而波列相交會之相互作用而衍生出此非線性影響效應量f_s，當然亦隨相對水深之變淺而轉顯著的變化。

至於短峯波波動場中，其週波率ω隨所在相對水深及其波浪尖銳度之整個變化的特性，當然是可直接由(6.41)至(6.43)式描述之。然，於此，爲與Roberts（1983）之深海情況的高階數值計算結果做比較起見，僅對深海情況者做必要的列述。首先，由上三式得知，於無堤與有堤情況中各有

當$d\to\infty$，$\omega_0=1$時

$$\omega=1+\left[-\frac{1}{2}+3m^2-m^4+2\left(m^2+m^2\omega_m^4-m\omega_m^2\right)\frac{m^2-1}{1+\omega_m^4-m\omega_m^2}\right]k^2a^2 \tag{6.44}$$

與 $$\omega=1+\frac{1}{2}k^2a^2+\left\{\left[-1+3m^2-m^4+2\left(m^2+m^2\omega_m^4-m\omega_m^2\right)\times\frac{m^2-1}{1+\omega_m^4-m\omega_m^2}\right]k^2a^2\right\}U(m) \tag{6.45}$$

爲與Roberts（1983）所引用的波浪尖銳度參數一致，即其以$\frac{1}{2}kH_s$者，H_s爲短峯波之波高，則上兩式中之參數ka得需轉換成與kH_s之關連，此可由(6.26)式（或(6.30)式）與(6.34)式（或(6.37)式）得之，今取$mkx=nky=\sigma t=0$之波峯點與$mkx=\sigma t=0$，$nky=\pi$之波谷點兩者高程差之半，故各有

當$d\to\infty$，$\omega_0=1$時

$$\begin{aligned}h=\frac{\pi H_s}{L}=\frac{1}{2}k\left[\eta(0,0,0)-\eta(0,\pi,0)\right]\\
&=2ka+\{7-2m^2-2m^4-8m(m^2-1)\omega_m^2\div(1+\omega_m^4-m\omega_m^2)\\
&-2(15+6m^2-21m^4-75m\omega_m^2+6m^2\omega_m^4+78m^3\omega_m^2\\
&\quad-21m^4\omega_m^4-3m^5\omega_m^2+15\omega_m^4)/\left[(9-\sqrt{1+8m^2})\right.\\
&\quad\left.\times(1+\omega_m^4-m\omega_m^2)\right]+2m^4/(1-\sqrt{1+8n^2})\}k^3a^3\end{aligned} \tag{6.46}$$

與 $h=ka+\frac{1}{2}k^3a^3+\{ka+[\frac{13}{2}-2m^2-2m^4-8m(m^2-1)\omega_m^2$

$/(1+\omega_m^4-m\omega_m^2)-2(15+6m^2-21m^4-75m\omega_m^2+6m^2\omega_m^4$

$+78m^3\omega_m^2-21m^4\omega_m^4-3m^5\omega_m^2+15\omega_m^4)/(9-\sqrt{1+8m^2})$

$/(1+\omega_m^4-m\omega_m^2)+2m^4/(1-\sqrt{1+8n^2})]k^3a^3\}U(m)$

(6.47)

依(6.46)與(6.47)兩式即刻可得知，於深水中無堤與有堤情況之短峯波波動場內，來源波之ka（或換算成其波浪尖銳度）與所形成的短峯波之波浪尖銳度H_s/L之變化關係，至第三階次。然後再由(6.44)與(6.45)兩式之應用，則所要描述的短峯波波動之週波率ω與其波浪尖銳度H_s/L之變化關係就可被列下，示之於圖8a。

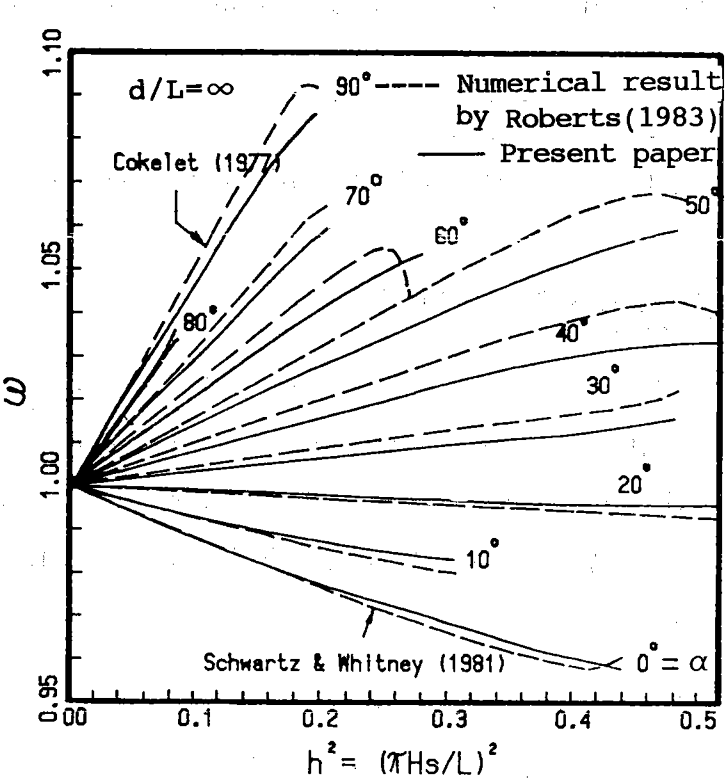


圖8a 深海中，短峯波週波率ω(無因次)隨入射角α及其波浪尖銳度平方h^2之變化，及與高階數值結果之比較

Fig. 8a In deep water, the frequency of short-crested waves as a function of the wave steepness square of fixed incident wave angle α, and the comparison with higher-order numerical calculation result.

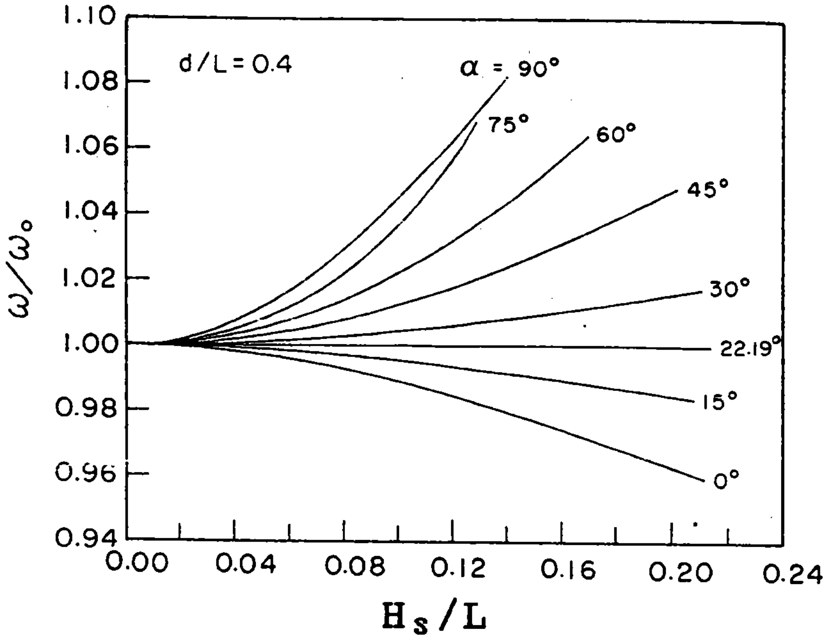


圖8b　相對水深0.4時，短峰波三階解之週波率比值與尖銳度之關係
Fig. 8b　in d/L=0.4 case, the frequency of short-crested waves with H_s/L.

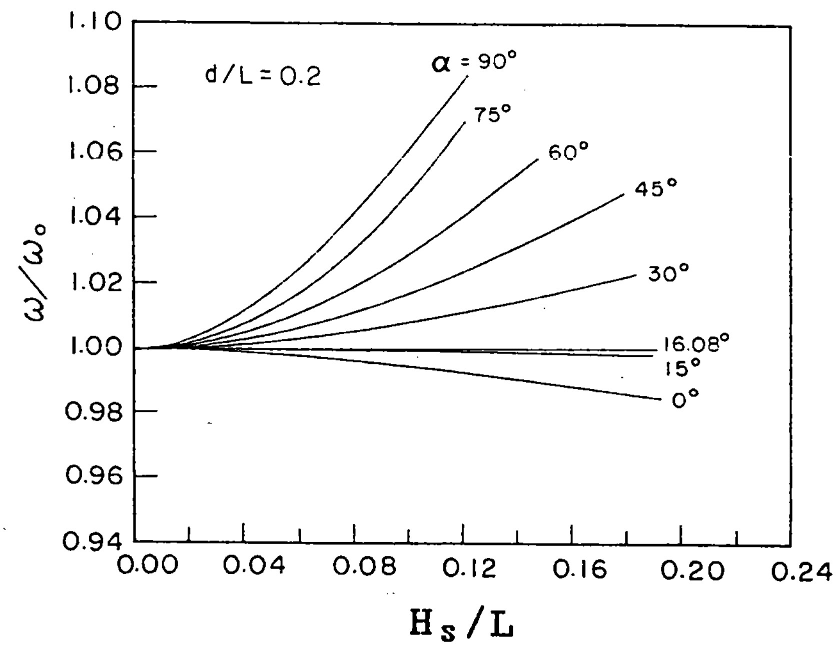


圖8c　相對水深0.2時，短峰波三階解之週波率比值與尖銳度之關係
Fig. 8c　in d/L=0.2 case, the frequency of short-crested waves with H_s/L.

由圖7a之比較可知，Roberts（1983）高階數值計算結果與本文之三階解析者相差無幾，至多在1%左右，且短峯波波動的週波率ω，於深海情況，由小於1轉變成大於1之角度（或於有堤情況時之單一前進規則波列之入射角）α同是發生於近21.97°時，此鄭與蔡（1988）者亦是雷同。然，此處得稍給予評論的是，以數值模式來求算者，是無法涵括短峯波波動之兩個極端特例情況，即其於$\alpha=0°$時退化成重力駐波與於$\alpha=90°$時退化成單一前進規則波者，這可由圖8a中Roberts（1983）引用他人的此二情況的結果而得知；反之，本文之解析結果是可全面涵的。

(2)共振情況

於往昔迄今對短峯波波動研究中，不管以理論解析或用數值計算，所得的結果皆希望能符合眞實的物理實況，以做爲最基本的檢核與驗證。基此，往昔研究者們根據構成短峯波波動的本質，即以它可退化成駐波與單一規則前進波之兩特例情況，來印驗其所得的結果，然，考察這些的檢核，往昔所得的結果似是未具圓滿性的；諸如Hus et al（1979）之理論解析解未能成功地退化成一單規則前進波，及Roberts（1983）之數值計算結果於退化成前進波情況時全然發生共振現象等。其實，往昔研究之所以會產生此種困擾，乃因疏忽形成短峯波波動之根本源由所致。這是在本節概述中已述及的，往昔者皆事先就限定短峯波波動流場解的形式以進行解析。事實上，這由上二小節之解析中，很清楚地得知，往昔者之此限定解的形式，是由兩完全相同性質之規則前進波列相交會所造成。因此，往昔之此結果的成因，是屬兩波列相交會作用的範疇，非是純由單一規則前進的入射波列受一直立堤壁面之完全反射而形成的短峯波者所能詮釋；即得依6.2(1)小節中之無堤情況下純由兩波列相交會而成者探究之。

於無堤情況之短峯波波動流場解(6.25)與(6.26)式中，這是顯然地，當兩完全相同性質的規則前進重力波列同向前進交會在一起時，則會發生所謂的共振現象，即於本文圖6的座標系統描述下，由此二波列相交會所形成的短峯波波動，其產生共振之條件爲

$$\alpha=90°\ ,\ m=\sin\alpha=1\ ,\ n=\cos\alpha=0$$

$$\tanh kd-(1+8n^2)^{1/2}\tanh(1+8n^2)^{1/2}kd=0 \qquad (6.48)$$

此時即兩完全相同性質的規則前進重力波列完全疊合在一起前進，產生同步效應所致。

θ=45° (intersection angle), T_1=T_2=1.20sec(incident wave period 1 & 2)
d =50cm(water depth), H_1=9.57cm(incident wave height 1)
H_2=2.84cm(incident wave height 2)

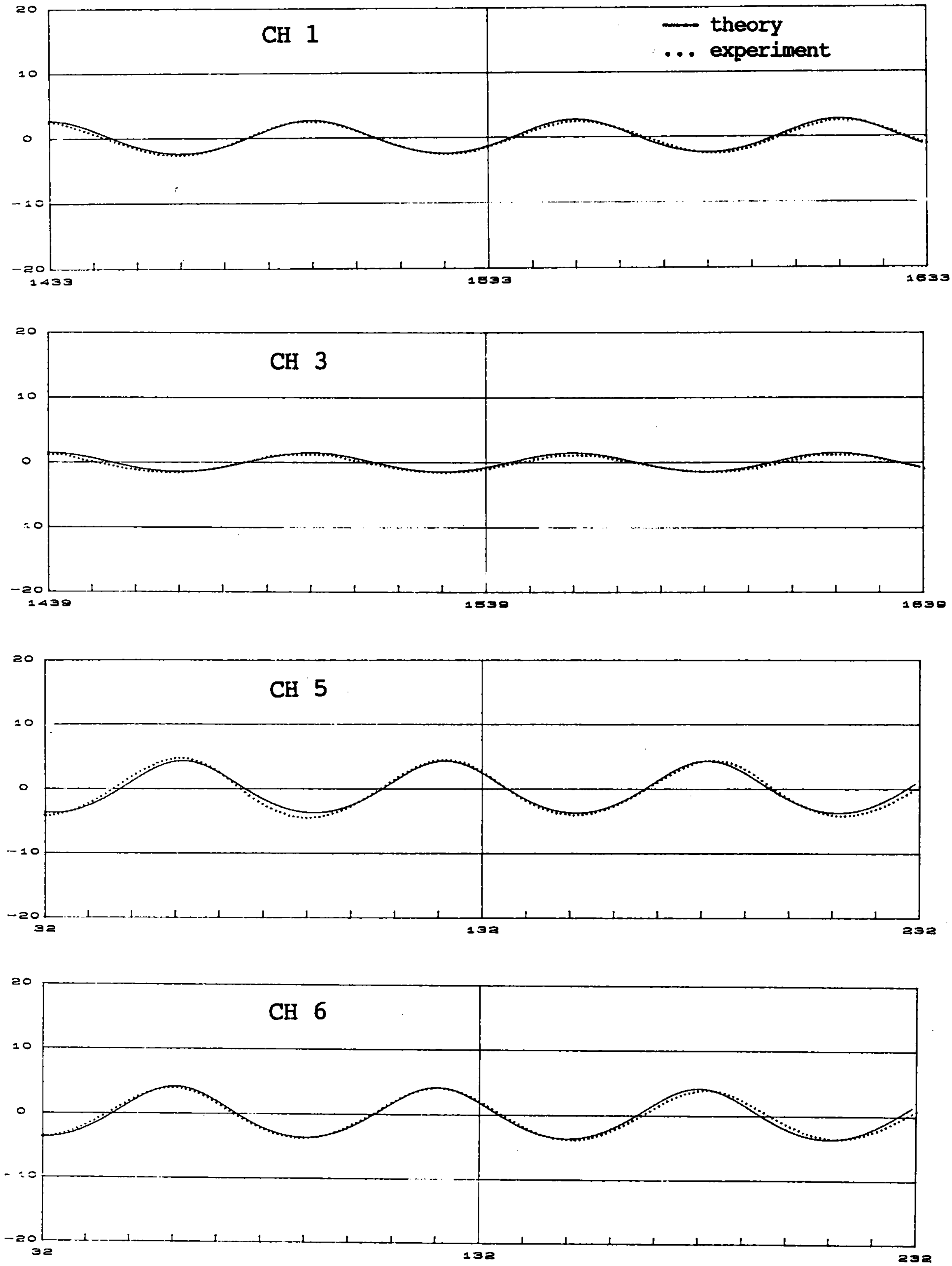


圖10-19

七、理論印證與試驗檢核

爲說明本文對所考慮的兩波列交會之波動系統所解出的波動流場結構解及其退化成特例下之短峰波流場結構解的正確性與適足性，則當要的理論印證與試驗檢核得需給予陳述，對此謹將列述於本節，如下。

7-1 理論驗印證

(1)兩波列交會之通案性情況

茲爲印證（5.27）～（5.30）式所描述的兩波列交會波動系統之通案性的波動流場解及其波壓，今以兩個特例情況來對其檢核之，於後。

(A)當所考慮的波動系統中之一來源成份波列消失時，即此時的波動場退化成僅有一自由表面規則前進重力波列的存在；或言以 $a_2=\sigma_2=k_2=0$、$a_1=a$、$\sigma_1=\sigma$、$k_1=k$ 時，則波動流場解（5.27）～（5.30）式及其波壓（5.35）中之各相關係數 $A_{11}{}^{\pm}$、$\mu_{ij}{}^{\pm}$ 與 $\zeta_{ij}{}^{\pm}$ 變爲

$$\left.\begin{aligned}
&\text{當}\quad a_2=\sigma_2=k_2=0\text{，}a_1=a\text{，}\sigma_1=\sigma\text{，}k_1=k\text{，}S_1=S\text{ 時，}\\
&\text{則}\quad A_{11}{}^{+}=A_{11}{}^{-}=\mu_{21}{}^{+}=\mu_{21}{}^{-}=\mu_{12}{}^{+}=\mu_{12}{}^{-}=\zeta_{21}{}^{+}=\zeta_{21}{}^{-}=\zeta_{12}{}^{+}=\zeta_{12}{}^{-}\\
&\qquad=\zeta_{03}{}^{(1)}=\zeta_{03}{}^{(3)}=0\text{，}\\
&\text{而}\quad \zeta_{30}{}^{(1)}=\frac{1}{16}k_1^2\,a_1^3\left(3\tanh^{-4}k_1d+8\tanh^{-2}k_1d-9\right)\text{，}\\
&\qquad \zeta_{30}{}^{(3)}=\frac{1}{64}k_1^2\,a_1^3\left(27\tanh^{-6}k_1d-9\tanh^{-4}k_1d\right.\\
&\qquad\qquad \left.+9\tanh^{-2}k_1d-3\right)
\end{aligned}\right\}\qquad(7.1)$$

因此，波動流場解退化成

$$\begin{aligned}
\phi=&\frac{ag}{\sigma_0}\frac{\cosh k(d+z)}{\cosh kd}\sin S+\frac{3}{8}a^2\sigma_0\frac{\cosh 2k(d+z)}{\sinh^4 kd}\sin 2S\\
&-\frac{1}{4}\frac{a^2\sigma_0{}^2t}{\sinh^2 kd}+\frac{1}{64}(9\tanh^{-7}kd+5\tanh^{-5}kd
\end{aligned}$$

$$
\left.
\begin{aligned}
&-53\tanh^{-3}kd+39\tanh^{-1}kd)ka^3\sigma_0\frac{\cosh 3k(d+z)}{\cosh 3kd}\sin 3S\\
&\eta=\left[a+\frac{1}{16}k^2a^3(3\tanh^{-4}kd+8\tanh^{-2}kd-9)\right]\cos S\\
&+\frac{1}{4}ka^2\frac{\cosh kd}{\sinh^3 kd}(2\sinh^2 kd+3)\cos 2S\\
&+\frac{1}{64}k^2a^3(27\tanh^{-6}kd-9\tanh^{-4}kd+9\tanh^{-2}kd\\
&-3)\cos 3S\\
&\sigma=\sigma_0+\frac{1}{16}(9-10\tanh^{-2}kd+9\tanh^{-4}kd)k^2a^2\sigma_0\ ,\\
&\sigma_0^2=gk\tanh kd
\end{aligned}
\right\}\quad(7.2)
$$

而其對應的波壓，則退化成

$$
\left.
\begin{aligned}
&\frac{p}{\rho}+gz=ag\frac{\cosh k(d+z)}{\cosh kd}\cos S-\frac{gka^2}{2\omega_0^2}\frac{\sinh^2 k(d+z)}{\cosh^2 kd}\\
&+\frac{1}{4}gka^2\left[-(\tanh^{-1}kd-\tanh kd)\right.\\
&\left.+3(\tanh^{-3}kd-\tanh kd)\frac{\cosh 2k(d+z)}{\cosh 2kd}\right]\\
&\cos 2S+gk^2a^3\left[\frac{1}{16}(9\tanh^{-4}kd-10\tanh^{-2}kd\right.\\
&+9)\frac{\cosh k(d+z)}{\cosh kd}-\frac{3}{8}(\tanh^{-4}kd-1)\\
&\left.\frac{\cosh 3k(d+z)}{\cosh kd\cdot\cosh 2kd}\right]\cos S+gk^2a^3\left[\frac{3}{64}(9\tanh^{-6}kd\right.\\
&+5\tanh^{-4}kd-53\tanh^{-2}kd+39)\frac{\cosh 3k(d+z)}{\cosh 3kd}\\
&\left.-\frac{3}{8}(\tanh^{-4}kd-1)\frac{\cosh k(d+z)}{\cosh kd\cdot\cosh 2kd}\right]\cos 3S
\end{aligned}
\right\}\quad(7.3)
$$

(7.2)與(7.3)式即單一自由表面規則前進重力波列(或謂Stokes waves)之第三階解及其波壓，見Chen(1988)。(7.2)式雖乍看之下似與往昔者於形式上稍有差異，然只要將其中的參數稍做些適當的轉換處理，則其間的等同是可被明確地獲得的，於下。

令 $$ka+\frac{1}{16}k^3a^3(3\tanh^{-4}kd+8\tanh^{-2}kd-9)=\epsilon_1-\frac{3}{64}\epsilon_1^3$$
$$\times(9\tanh^{-6}kd-3\tanh^{-4}kd+3\tanh^{-2}kd-1) \tag{7.4}$$

則至第三階有

$$ka=\epsilon_1-\frac{1}{64}\epsilon_1^3(27\tanh^{-6}kd+3\tanh^{-4}kd$$
$$+41\tanh^{-2}kd-39) \tag{7.5}$$

因此，(7.2)式可化為

$$\left.\begin{aligned}
&(k^3/g)^{1/2}\phi=\epsilon_1(\tanh kd)^{1/2}\frac{\cosh k(d+z)}{\sinh kd}\sin S\\
&\quad+\frac{3}{8}\epsilon_1^2(\tanh kd)^{1/2}\frac{\cosh 2k(d+z)}{\sinh^4 kd}\sin 2S\\
&\quad-\frac{1}{4}\epsilon_1^2(\tanh kd)^{1/2}\frac{\sigma_0 t}{\sinh^2 kd}-\frac{1}{64}\epsilon_1^3(\tanh kd)^{1/2}\\
&\quad\times(27\tanh^{-6}kd+3\tanh^{-4}kd+41\tanh^{-2}kd-39)\\
&\quad\times\frac{\cosh k(d+z)}{\sinh kd}\sin S+\frac{1}{64}\epsilon_1^3(\tanh kd)^{1/2}(9\tanh^{-6}kd\\
&\quad-31\tanh^{-4}kd+35\tanh^{-2}kd-13)\frac{\cosh 3k(d+z)}{\sinh kd}\sin 3S\\
&k\eta=\epsilon_1\cos S+\frac{1}{4}\epsilon_1^2\frac{\cosh kd}{\sinh^3 kd}(2\sinh^2 kd+3)\cos 2S\\
&\quad-\frac{3}{64}\epsilon_1^3(9\tanh^{-6}kd-3\tanh^{-4}kd+3\tanh^{-2}kd-1)\\
&\quad(\cos S-\cos 3S)
\end{aligned}\right\} \tag{7.6}$$

以$\sigma=\bar{u}k$，$\bar{u}$為波速，則

$$\left(\frac{k}{g}\right)^{1/2}\bar{u}=(\tanh kd)^{1/2}\left[1+\frac{1}{16}\epsilon_1^2(9-10\tanh^{-2}kd+9\tanh^{-4}kd)\right],\quad \sigma_0^2=gk\tanh kd$$

至於其波壓(7.3)式，在以ϵ_1表示下，可被寫爲

$$\begin{aligned}\frac{kp}{\rho g}+kz=&\epsilon_1\frac{\cosh k(d+z)}{\cosh kd}\cos S-\epsilon_1^2\frac{\sinh^2 k(d+z)}{\sinh 2kd}\\&+\frac{1}{4}\epsilon_1^2\left[-(\tanh^{-1}kd-\tanh kd)+3(\tanh^{-3}kd-\tanh kd)\frac{\cosh 2k(d+z)}{\cosh 2kd}\right]\cos 2S\\&-\epsilon_1^3\left[\frac{1}{64}(27\tanh^{-6}kd-33\tanh^{-4}kd+81\tanh^{-2}kd-75)\frac{\cosh k(d+z)}{\cosh kd}+\frac{3}{8}(\tanh^{-4}kd-1)\right.\\&\left.\times\frac{\cosh 3k(d+z)}{\cosh kd\cdot\cosh 2kd}\right]\cos S+\epsilon_1^3\left[\frac{3}{64}(9\tanh^{-6}kd+5\tanh^{-4}kd-53\tanh^{-2}kd+39)\frac{\cosh 3k(d+z)}{\cosh 3kd}\right.\\&\left.-\frac{3}{8}(\tanh^{-4}kd-1)\frac{\cosh k(d+z)}{\cosh kd\cdot\cosh 2kd}\right]\cos 3S\end{aligned}\qquad(7.7)$$

(7.6)式完全一致於Fenton（1985）以穩定化運動下解析單一自由表面規則前進重力波後再返回原眞的固定座標系統裡之第三階解，可評見Chen（1988）之論述。當然反之，若以$a_1=\sigma_1=k_1=0$、$a_2=a$、$\sigma_2=\sigma$、$k_2=k$、$S_2=S$情況時，則波動場所退化成的解亦如同上式所示。

(B)當所考慮的波動系統是由完全相同性質（即相同振幅、波長或週期者）但傳遞方向相反之二自由表面規則前進重力波列相交會所構成時，即此時的波動場變成爲所謂的重力駐波脈動情況者。便於描述(5.27)～(5.30)與(5.35)式所示的波動流場解及其波壓退化成此駐波情況下的結構，容易處理起見，則可

調整所描述的座標系統使之二來源成份波列其前進方向各沿 $+x$ 與 $-x$ 軸者，同時其中的有關參數符號可被寫為

$$a_1=a_2=a\text{ ，}\sigma_1=\sigma_2=\sigma\text{ ，}k_1=k_2=k\text{ ，}S_1=kx-\sigma t\text{ ，}$$
$$S_2=-kx-\sigma t \tag{7.8}$$

又，因此二來源成份波列反向傳遞，即它們的交角 $\theta=\pi$（因 $\vec{k}_1=-\vec{k}_2$），故 (5.27)～(5.30) 與 (5.35) 式所示之波動流場解及其波壓中的有關係數 $A_{11}^{\pm}$、$\mu_{ij}^{\pm}$ 與 $\zeta_{ij}^{\pm}$ 可容易地簡化之，依 (5.8)、(5.20a～h)、(5.26a～h) 與 (7.8) 式為

$$A_{11}^{+}=-\frac{1}{4}a^2(3+\tanh^{-2}kd)\sigma_0\text{ ，}A_{11}^{-}=0 \tag{7.9a}$$

$$\mu_{30}^{(1)}=ga\{-2\sigma_2^{(1)}+\frac{1}{8}(9\tanh^{-4}kd-12\tanh^{-2}kd-3-2\tanh^2kd)k^2a^2\sigma_0\}$$

$$=ga\{-2\sigma_2^{(2)}+\frac{1}{8}(9\tanh^{-4}kd-12\tanh^{-2}kd-3-2\tanh^2kd)k^2a^2\sigma_0\}$$

$$=\mu_{03}^{(1)} \tag{7.9b}$$

$$\mu_{30}^{(3)}=\frac{1}{8}gk^2a^3\sigma_0(27\tanh^{-4}kd-66\tanh^{-2}kd+39)=\mu_{03}^{(3)} \tag{7.9c}$$

$$\mu_{21}^{+}=\frac{1}{8}gk^2a^3\sigma_0(-9\tanh^{-4}kd-62\tanh^{-2}kd+31)=\mu_{12}^{+} \tag{7.9d}$$

$$\mu_{21}^{-}=\frac{1}{8}gk^2a^3\sigma_0(-3\tanh^{-4}kd+5-2\tanh^2kd)=-\mu_{12}^{-} \tag{7.9e}$$

$$\zeta_{30}^{(1)}=\frac{1}{16}k^2a^3(3\tanh^{-4}kd+6\tanh^{-2}kd-5+2\tanh^2kd)$$

$$=\zeta_{03}^{(1)} \tag{7.9f}$$

$$\zeta_{30}^{(3)}=\frac{1}{64}k^2a^3(27\tanh^{-6}kd-9\tanh^{-4}kd+9\tanh^{-2}kd-3)$$

$$=\zeta_{03}^{(3)} \tag{7.9g}$$

$$\zeta_{21}^{+}=\frac{1}{64}k^2a^3(-3\tanh^{-4}kd-18\tanh^{-2}kd+5)=\zeta_{12}^{+} \tag{7.9h}$$

$$\zeta_{21}^{-}=\frac{1}{64}k^2a^3(27\tanh^{-4}kd+81\tanh^{-2}kd-45+3\tanh^2 kd+6\tanh^4 kd)$$

$$=\zeta_{12}^{-} \tag{7.9i}$$

因此，（5.27）～（5.30）式之流場解退化爲

$$\left.\begin{aligned}
\phi=&-\frac{2ag}{\sigma_0}\frac{\cosh k(d+z)}{\cosh kd}\cos kx\sin\sigma t-\frac{1}{2}\frac{a^2\sigma_0^2 t}{\sinh^2 kd}\\
&+\frac{1}{4}a^2\sigma_0(3+\tanh^{-2}kd)\sin 2\sigma t-\frac{3}{4}a^2\sigma_0\frac{\cosh 2k(d+z)}{\sinh^4 kd}\\
&\cos 2kx\sin 2\sigma t+\frac{1}{32}ka^3\sigma_0(9\tanh^{-5}kd+62\tanh^{-3}kd\\
&-31\tanh^{-1}kd)\frac{\cosh k(d+z)}{\cosh kd}\cos kx\sin 3\sigma t\\
&-\frac{1}{32}ka^3\sigma_0(1+3\tanh^2 kd)(3\tanh^{-5}kd-5\tanh^{-1}kd\\
&+2\tanh kd)\frac{\cosh 3k(d+z)}{\cosh 3kd}\cos 3kx\sin\sigma t\\
&-\frac{1}{32}ka^3\sigma_0(9\tanh^{-7}kd+5\tanh^{-5}kd-53\tanh^{-3}kd\\
&+39\tanh^{-1}kd)\frac{\cosh 3k(d+z)}{\cosh 3kd}\cos 3kx\sin 3\sigma t\\
\eta=&2a\cos kx\cos\sigma t+\frac{1}{2}ka^2(\tanh kd+\tanh^{-1}kd)\cos 2kx\\
&+\frac{1}{2}ka^2\frac{\cosh kd}{\sinh^3 kd}(2\sinh^2 kd+3)\cos 2kx\cos 2\sigma t\\
&+\frac{1}{8}k^2a^3(3\tanh^{-4}kd+6\tanh^{-2}kd-5+2\tanh^2 kd)\\
&\times\cos kx\cos\sigma t+\frac{1}{32}k^2a^3(-3\tanh^{-4}kd-18\tanh^{-2}kd
\end{aligned}\right\} \tag{7.10}$$

$$+5)\cos kx\cos 3\sigma t+\frac{1}{32}k^2a^3(27\tanh^{-4}kd$$

$$+81\tanh^{-2}kd-45+3\tanh^2 kd+6\tanh^4 kd)\cos 3kx\cos\sigma t$$

$$+\frac{1}{32}k^2a^3(27\tanh^{-6}kd-9\tanh^{-4}kd+9\tanh^{-2}kd-3)$$

$$\times\cos 3kx\cos 3\sigma t$$

$$\sigma=\sigma_0+\frac{1}{16}k^2a^2\sigma_0(9\tanh^{-4}kd-12\tanh^{-2}kd-3-2\tanh^2 kd),$$

$$\sigma_0^2=gk\tanh kd$$

而其對應的波壓退化為

$$\frac{p}{\rho}+gz=2ag\frac{\cosh k(d+z)}{\cosh kd}\cos kx\cos\sigma t-2gka^2\frac{\sinh^2 k(d+z)}{\sinh 2kd}$$

$$+gka^2\left[-2\tanh kd+\tanh^{-1}kd\cdot\frac{\sinh^2 k(d+z)}{\cosh^2 kd}\right]\cos 2\sigma t$$

$$+\frac{1}{2}gka^2(\tanh^{-1}kd-\tanh kd)\cos 2kx+\frac{1}{2}gka^2$$

$$\times\left[-(\tanh^{-1}kd-\tanh kd)+3(\tanh^{-3}kd-\tanh kd)\right.$$

$$\left.\times\frac{\cosh 2k(d+z)}{\cosh 2kd}\right]\cos 2kx\cos 2\sigma t+8gk^2a^3\left[\frac{1}{64}(9\tanh^{-4}kd\right.$$

$$-12\tanh^{-2}kd-3-2\tanh^2 kd)\frac{\cosh k(d+z)}{\cosh kd}$$

$$\left.-\frac{3}{32}(\tanh^{-4}kd-1)\frac{\cosh 3k(d+z)}{\cosh kd\cdot\cosh 2kd}\right]\cos kx\cos\sigma t$$

$$+8gk^2a^3\left[\frac{3}{256}(-9\tanh^{-4}kd-62\tanh^{-2}kd+31)\right.$$

$$\times\frac{\cosh k(d+z)}{\cosh kd}+\frac{3}{32}(\tanh^{-4}kd-1)$$

$$\left.\times\frac{\cosh 3k(d+z)}{\cosh kd\cdot\cosh 2kd}\right]\cos kx\cos 3\sigma t+8gk^2a^3$$

$$
\left.
\begin{aligned}
&\times\Big[\frac{1}{256}(3\tanh^{-4}kd+9\tanh^{-2}kd-5-13\tanh^{2}kd\\
&+6\tanh^{4}kd)\frac{\cosh 3k(d+z)}{\cosh 3kd}+\frac{3}{32}(\tanh^{-4}kd-1)\\
&\times\frac{\cosh k(d+z)}{\cosh kd\cdot\cosh 2kd}\Big]\cos 3kx\cos\sigma t\\
&+8gk^{2}a^{3}\Big[\frac{3}{256}(9\tanh^{-6}kd+5\tanh^{-4}kd-53\tanh^{-2}kd\\
&+39)\frac{\cosh 3k(d+z)}{\cosh 3kd}-\frac{3}{32}(\tanh^{-4}kd-1)\\
&\times\frac{\cosh k(d+z)}{\cosh kd\cdot\cosh 2kd}\Big]\cos 3kx\cos 3\sigma t
\end{aligned}
\right\}(7.11)
$$

為更清楚地對所考慮的波動系統退化成重力駐波情況時，其流場結構解(7.10)式及其波壓(7.11)式與往昔所得之重力駐波解析解者有一致性的符合，於此將其有關的參數稍做些適當的處理，如下。即在無因次化下，

$$
\left.
\begin{aligned}
&\text{以}\quad 2ka=\varepsilon,\ kz=\bar{z},\ kx=\bar{x},\ kd=\bar{d},\ \sigma t=\bar{t},\ \tanh kd=\omega_0^{2},\\
&\qquad \bar{p}=\frac{kp}{\rho g}\ \text{時}\\
&\text{則}\quad \left(\frac{k^{3}}{g}\right)^{1/2}\phi=-\frac{\varepsilon}{\omega_0}\frac{\cosh(\bar{d}+\bar{z})}{\cosh\bar{d}}\cos\bar{x}\sin\bar{t}\\
&\qquad -\frac{1}{8}\varepsilon^{2}(\omega_0^{-3}-\omega_0)\bar{t}+\frac{1}{16}\varepsilon^{2}(\omega_0^{-3}+3\omega_0)\sin 2\bar{t}\\
&\qquad -\frac{3}{16}\varepsilon^{2}(\omega_0^{-7}-\omega_0)\frac{\cosh 2(\bar{d}+\bar{z})}{\cosh 2\bar{d}}\cos 2\bar{x}\sin 2\bar{t}\\
&\qquad +\frac{1}{256}\varepsilon^{3}(9\omega_0^{-9}+62\omega_0^{-5}-31\omega_0^{-1})\frac{\cosh(\bar{d}+\bar{z})}{\cosh\bar{d}}\cos\bar{x}\sin 3\bar{t}\\
&\qquad -\frac{1}{256}\varepsilon^{3}(1+3\omega_0^{4})(3\omega_0^{-9}-5\omega_0^{-1}+2\omega_0^{3})\frac{\cosh 3(\bar{d}+\bar{z})}{\cosh 3\bar{d}}
\end{aligned}
\right\}(7.12)
$$

$$
\begin{aligned}
&\times\cos 3\bar{x}\sin\bar{t}-\frac{1}{256}\varepsilon^3(1+3\omega_0^4)(9\omega_0^{-13}-22\omega_0^{-9}\\
&+13\omega_0^{-5})\frac{\cosh 3(\bar{d}+\bar{z})}{\cosh 3\bar{d}}\cos 3\bar{x}\sin 3\bar{t}\\
&k\eta=\varepsilon\cos\bar{x}\cos\bar{t}+\frac{1}{8}\varepsilon^2(\omega_0^{-2}+\omega_0^2)\cos 2\bar{x}+\frac{1}{8}\varepsilon^2(3\omega_0^{-6}\\
&-\omega_0^{-2})\cos 2\bar{x}\cos 2\bar{t}+\frac{1}{64}\varepsilon^3(3\omega_0^{-8}+6\omega_0^{-4}-5\\
&+2\omega_0^4)\cos\bar{x}\cos\bar{t}+\frac{1}{256}\varepsilon^3(-3\omega_0^{-8}-18\omega_0^{-4}\\
&+5)\cos\bar{x}\cos 3\bar{t}+\frac{1}{256}\varepsilon^3(27\omega_0^{-8}+81\omega_0^{-4}-45+3\omega_0^4\\
&+6\omega_0^8)\cos 3\bar{x}\cos\bar{t}+\frac{1}{256}\varepsilon^3(27\omega_0^{-12}-9\omega_0^{-8}+9\omega_0^{-4}\\
&-3)\cos 3\bar{x}\cos 3\bar{t}\\
&\frac{\sigma}{\sqrt{gk}}=\omega_0+\frac{1}{64}\varepsilon^2(9\omega_0^{-7}-12\omega_0^{-3}-3\omega_0-2\omega_0^5),\ \frac{\sigma_0}{\sqrt{gk}}=\omega_0
\end{aligned}
$$

$$
\left.
\begin{aligned}
&\bar{p}+\bar{z}=\varepsilon\frac{\cosh(\bar{d}+\bar{z})}{\cosh\bar{d}}\cos\bar{x}\cos\bar{t}-\frac{1}{4}\varepsilon^2\omega_0^{-2}\frac{\sinh^2(\bar{d}+\bar{z})}{\cosh^2\bar{d}}\\
&+\frac{1}{4}\varepsilon^2\left[-2\omega_0^2+\omega_0^{-2}\frac{\sinh^2(\bar{d}+\bar{z})}{\cosh^2\bar{d}}\right]\cos 2\bar{t}\\
&+\frac{1}{8}\varepsilon^2(\omega_0^{-2}-\omega_0^2)\cos 2\bar{x}+\frac{1}{8}\varepsilon^2\left[-(\omega_0^{-2}-\omega_0^2)\right.\\
&\left.+3(\omega_0^{-6}-\omega_0^2)\frac{\cosh 2(\bar{d}+\bar{z})}{\cosh 2\bar{d}}\right]\cos 2\bar{x}\cos 2\bar{t}\\
&+\varepsilon^3\left[\frac{1}{64}(9\omega_0^{-8}-12\omega_0^{-4}-3-2\omega_0^4)\frac{\cosh(\bar{d}+\bar{z})}{\cosh\bar{d}}\right.\\
&\left.-\frac{3}{32}(\omega_0^{-8}-1)\frac{\cosh 3(\bar{d}+\bar{z})}{\cosh\bar{d}\cdot\cosh 2\bar{d}}\right]\cos\bar{x}\cos\bar{t}
\end{aligned}
\right\}\quad(7.13)
$$

$$
\begin{aligned}
&+\varepsilon^3\Big[\frac{3}{256}(-9\omega_0^{-8}-62\omega_0^{-4}+31)\frac{\cosh(\bar{d}+\bar{z})}{\cosh\bar{d}}+\frac{3}{32}(\omega_0^{-8}-1)\\
&\times\frac{\cosh 3(\bar{d}+\bar{z})}{\cosh\bar{d}\cdot\cosh 2\bar{d}}\Big]\cos\bar{x}\cos 3\bar{t}+\varepsilon^3\Big[\frac{1}{256}(3\omega_0^{-8}+9\omega_0^{-4}-5\\
&-13\omega_0^{4}+6\omega_0^{8})\frac{\cosh 3(\bar{d}+\bar{z})}{\cosh 3\bar{d}}+\frac{3}{32}(\omega_0^{-8}-1)\frac{\cosh(\bar{d}+\bar{z})}{\cosh\bar{d}\cdot\cosh 2\bar{d}}\Big]\\
&\times\cos 3\bar{x}\cos\bar{t}+\varepsilon^3\Big[\frac{3}{256}(9\omega_0^{-12}+5\omega_0^{-8}-53\omega_0^{-4}+39)\\
&\times\frac{\cosh 3(\bar{d}+\bar{z})}{\cosh 3\bar{d}}-\frac{3}{32}(\omega_0^{-8}-1)\frac{\cosh(\bar{d}+\bar{z})}{\cosh\bar{d}\cdot\cosh 2\bar{d}}\Big]\cos 3\bar{x}\cos 3\bar{t}\Big\}
\end{aligned}
$$

(7.12) 與 (7.13) 兩式完全與往昔所得之任一均勻等水深中的重力駐波至第三階解相同，如見 Tadjbakhsh & Keller (1960)、Goda & Kakizaki (1966) 與 Chen (1988)。

(2)短峯波情況

雖然短峰波波動是等深水中任兩規則前進波列相交會所構成之波動系統中的一特例，然其亦包括單一規則波與駐波之兩特例情況，因此，對所得之短峰波波動流場結構解及其波壓之正確性與否，亦得需以此兩特例情況來對其印證之。

(A)當短峰波波動退化成單一規則前進重力波列情況：此情況是發生在有堤情況的短峰波波動場中，否則於無堤情況中乃為兩完全相同的前進波列同時疊合在一起。此時入射的單一前進波列是沿著堤壁面傳遞的，故於圖 6 所描述的波動座標系統下，有

$\theta=0^\circ$，$\alpha=90^\circ$，$m=\sin\alpha=1$，$n=\cos\alpha=0$，$U(m)=0$ (7.14)

因此，由 (6.33) 至 (6.35) 式，可得短峰波波動退化成單一規則前進重力波列時的流場解恰為 (7.2)式者，而其對應的波壓亦完全與 (7.4) 式相同之。

(B)當短峰波波動退化成駐波情況時：此時的波動場是由完全相同性質但反向傳遞之兩規則前進重力波列相交會所構成，故於第六節中圖 6 的座標系統描述下，有

$\theta=180°$，$\alpha=0$，$m=\sin\alpha=0$，$n=\cos\alpha=1$，$U(m)=1$ (7.15)

因此，由(6.25)至(6.27)式或(6.33)至(6.35)式，即刻可得短峰波波動退化成重力駐波時之流場解恰爲(7.10)式者，而其對應的波壓(6.40)式完全與(7.11)式相同之；當然，在所示之無因次化的表示下，它們各完全相同於(7.12)與(7.13)式者。

因此，依據本小節之以所考慮的波動系統退化成單一自由表面規則前進重力波列與重力駐波的二個特例驗證下，可確知本文所得之二自由表面規則前進重力波列相交會後的整個波動流場解與其特例之短峰波波動流場解，至第三階次量，即(5.27)～(5.30)與(6.33)～(6.35)式(或無堤情況下之(6.25)～(6.27)式)是正確而可全面性涵蓋的。當然，其對應的波壓亦復如是之。

7-2 試驗檢核

爲更具體地從定性與定量上，來檢核本文所述之兩波列交會波動系統的流場結構特性及其特例之短峰波波動流場情況者，本研究特在較大尺度的平面水池中進行試驗；由任意兩波列交會(非在共振情況)所測得之表面波形及場內波壓記錄來與理論結果進行比較，以驗證其間是否呈一致性。

(1)試驗佈置與過程

本試驗係於港灣技術研究所第一試驗場棚進行，其主要試驗設備與過程分述如下。

(A)試驗水池：試驗水池長60公尺、寬43公尺、深1公尺，池內配置平推式規則造波機4台，每台長10公尺，除各別造波外，亦可由連桿連接同步造波。

(B)量測儀器：波高計係爲日製KENEK CH-306型之容量型波高計，波壓計則爲KYOWA PGM-0.5KG型之感應器。

(C)資料處理系統：各測點之水位變化或壓力變化經由波高計或波壓計感應後傳入PC，再由PC內之A/D(analog-digital converter)轉換成數位訊號，儲存於PC之記憶體內做資料處理。

(D)試驗配置：爲量測兩波交會之最大變化量，有關波高計或波壓計皆佈置於兩波列會發生波峰與波峰、波谷與波谷或波峰與波谷之交會處，顯然地，這些測點

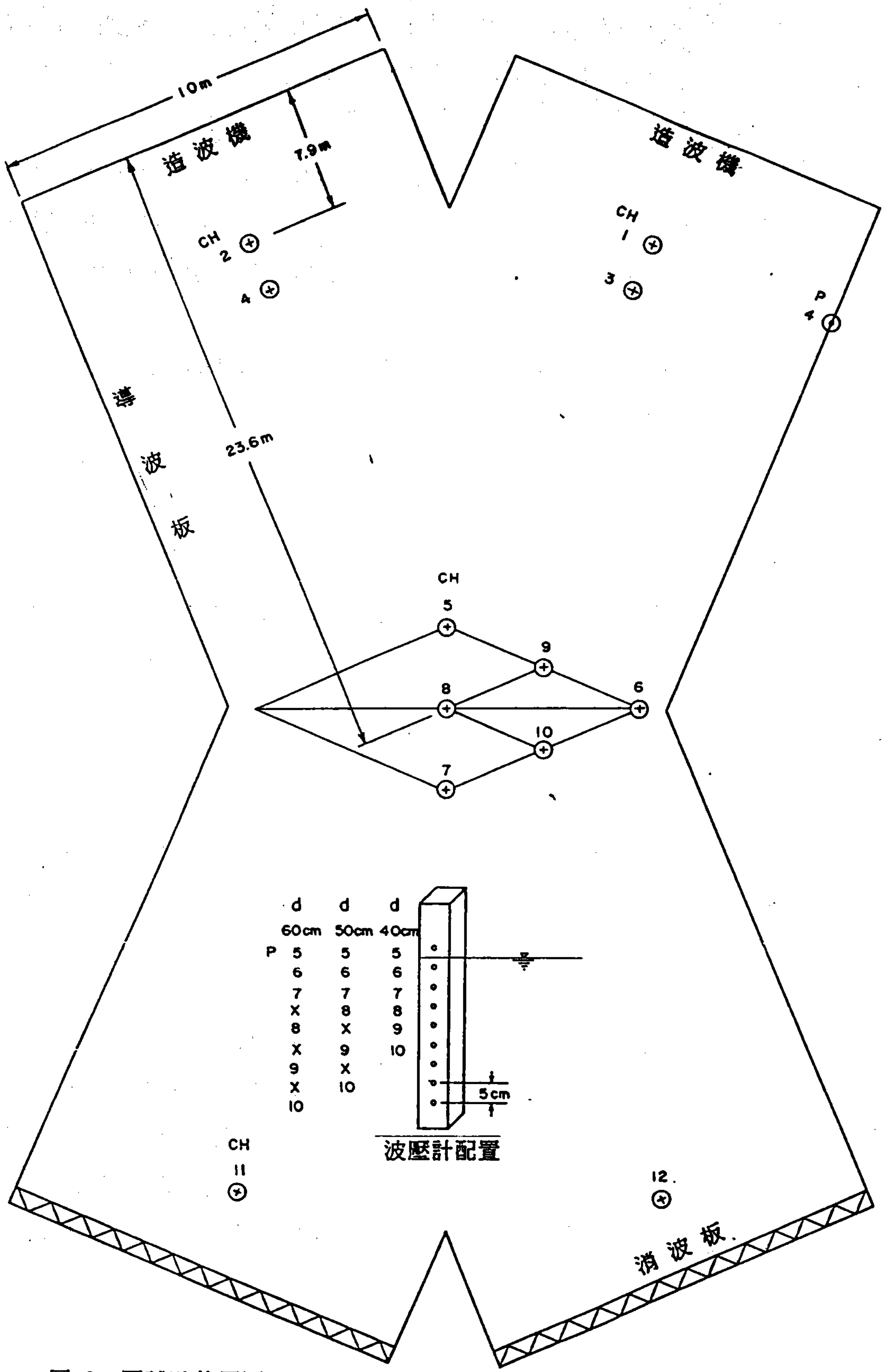


圖 9 圖試驗佈置圖

Fig. 9 Experimental sketch for wave & wave interaction
CH:wave gague position; P:pressure position under CH8 & CH9

是可由兩來源前進波交會所形成之平行四邊形中取定，相關之試驗配置如圖 9 所示。圖中符號CH者表示波高計擺設的位置，波高計CH1、CH3與CH2、CH4分別量測兩來源前進波之波形與波長，CH5～CH10則量測兩波交會時之波形變化，其中，以兩造波機中心線相交處佈置CH8，再依兩前進波之波長距離佈置CH5、CH6與CH7，而CH9與CH10則係於兩波列會發生波峰與波谷之交會處。CH11與CH12則量測兩波交會後各自波形之變化情況。

由於波壓計數量關係，僅能於CH1、CH8與CH9等處佈置波壓計，其中，波壓計P4係佈置於導波板側壁水面下10公分處，以量測來源前進波之壓波，P5～P10係佈置於CH8位置，以量測兩波列會發生波峰與波峰或波谷與波谷交會處之波壓，而P11～P16是佈置於CH9位置，來量測兩波列會發生波峰與波谷交會處之波壓。有關波壓計之配置如圖 9 中所示，係以一直立柱體由上至下每隔5公分挖一孔安放一個波壓計。由於佈置一鐵軌於CH9位置之底床處以利移動P11～P16波壓計關係，故CH9位置處安放波壓計之直立柱高於CH8位置者1.5公分。

(E)試驗步驟：本試驗係以兩來源前進波成 $\theta=45^\circ$ 交會角來從事檢核印證工作；至於僅一來源波列入射經一長直立堤壁面反射而形成之短峰波波動情況，則以入射波之入射角 $\alpha=30^\circ$、45°、60° 等三種爲之。有關試驗步驟詳述如下：

(a)選擇試驗水深d分別爲40 cm、50 cm與60 cm等三種，來源前進波週期爲0.8 sec～2.4 sec等數種，波高則由3.0公分左右調至兩波交會時幾近碎波爲止，選取其間之數種當作試驗之條件。

(b)試驗項目計有同週期、同波高與同週期、不同波高，其波高比約爲1：2～1：4等二種；試驗範圍則爲 d/L_i 約 0.072～0.60，H_i/L_i 約 0.005～0.062；L_i，H_i（$i=1,2$）各爲兩來源波列之波長與波高。

(c)由先行之試驗，選取連續三個成熟的來源前進波，取平均值下量測其於各種週期與波高下之對應的波長，俾利爾後試驗時移動波高計與波壓計至相關位置。

(d)以兩造波機相同位相時進行造波試驗，待兩來源前進波開始交會時，即由PC以50Hz速度紀錄所需資料。當然量測之佈置，則是依兩來源前進波完全交

會後所形成之結構，以出現波峰（或波谷）點之中央處CH8之波峰點時做爲取定交會區中CH5～CH10之基準點來進行的。

(e)對波壓計之不同擺置方向，是否會有不同之波壓值，因此，於交會區中之波壓計分別以正對一前進波方向與面對兩前進波分角線方向量測波壓。

(2)試驗結果與理論值比較

雖然Chen等之解析處理，於符合攝動解析法的本質精神下，經由chain rule之微分轉換，並考量非線性影響而較完整地列出波形與波壓至第三階解，由於理論解析結果之適足性與完整性仍有賴試驗加以檢核印證，故本節中，依據前節之理論解析與上述之試驗過程，將試驗結果與理論值作一比較如下：

(A)波形

如圖10所示各測點位置處所量測得之波形，可知其與理論值間之一致性頗相吻合。

(B)波壓

由於波壓計感應器係一小圓面金屬器而非一點，再加以裝置波壓計之直立柱體爲一小柱狀體，因此，量測前進波浪之波壓，其面對之不同方向，可因水粒子運動速度受阻之不同，而產生不同之量測壓力。於此，先行比較正對一來源前進波方向（PN）與面對兩波分角線方向（PC）等兩種情況之試驗波壓，並選取在最具兩波交會特性的CH8位置處量測之值，結果顯示PN與PC之波壓量測值幾爲相等。爲方便計以下有關波壓值（單位：gr/cm^2）皆以PC值爲之。短峰波情況時，則波壓計緊貼在直立堤壁之平面上。由圖11可知，對兩前進波列交會及其特例之短峰波情況之波壓理論值與試驗結果之比較亦相爲良好。

八、結論與討論

綜觀本文以上之整個論述，於此可給予幾點顯要的結語，來對所考慮的任兩自由表面規則前進重力波列相交會所構成之波動系統，其波動流場特性做較確然性的闡述，如下。

(1) 避免往昔解析兩波列交會之波動系統時僅以線性之分散關係（ linear dispersion ）做爲界定，致使其中之庸凡項（ secular term ）無法妥當處理，而造成波動流場解會全然地發生共振現象之不合理的情況下，本文適足地考量入兩來源成份波列原具有的非線性本質，及它們相交會會存有的非線性相互作用量，並於攝動展開法之解析中引入 chain rule 做適當的微分轉換處理下，已將此會造成對求解波動流場之合理解的庸凡項障礙做圓滿地解決，因而使波動系統其波動流場內所有的非線性特性各適其所，且整體性地呈現其全貌；亦因此，進而將往昔僅針對的深海情況之波動系統適足地推展至任一等深水中情況。

(2) 於任一均勻等深水中，在非線性之考量下，依兩來源成份波列本身各自逐階次的非線性量之脈動特性，則此兩波列相交會所衍生出之它們間的非線性相互作用機制，可由此等之兩來源成份波列本身各自逐階次的非線性量的相互交錯而明確地描述之，因而兩波列交會的波動系統，其整體波動流場的脈動機構可直接很清楚的被瞭解掌握與理出，如圖 2 所示；亦因此，控制兩波列交會之波動系統的波動流場脈動的所有必要方程式，於 chain rule 之適當的微分轉換處理下，可被直接系統化地逐階次展開而列出之，是故，對兩波列交會問題之解析，由往昔處理之冗長迂迴繁雜化簡化成簡單直接明確化。

(3) 依本文之處理，所解得之任一均勻等深水中之兩波列交會之波動系統的整體波動流場結構解，至第三階次量下，其可直接被退化成單一波列存在時與變成駐波時及短峯波發生時，等之特例情況下所對應的波動流場結構解，並且皆與往昔對此等特例之波動情況的個案研究所得之解完全相符合，此乃往昔對兩波列相交會之波動系統的研究結果所未達成者。

(4) 至於兩來源成份波列相交會的衝擊作用下，各波列的週波率 σ_i（ $i=1,2$ ）隨所在的（相對）水深 d 、兩來源波列各自的波浪尖銳度 $k_i a_i$ 與它們的波長比 k_i / k_j 等本質特性、及此兩來源波列交會夾角 θ 等之變化關係，已被整體性合理地描述。

因此，對至今仍感迷惑的重力駐波之週波率隨其波浪尖銳度的變化，會因（相對）水深之變淺而有逆變情況發生的不尋常現象，給予滿足地解釋在因兩來源成份波列逆向交會之衝擊下，所造成的相互作用結果。如以（5.31）式來對重力駐波之週波率此逆變現象做更明確地闡述時，則可先寫下

$$\frac{\sigma_i}{\sigma_0^{(i)}}-1-\frac{1}{16}(9\tanh^{-4}k_id-10\tanh^{-2}k_id+9)k_i^2a_i^2$$

$$=f_i(k_i,k_j,d,\theta)k_j^2a_j^2\ ;\ i,j=1,2,i\neq j \tag{5.31}$$

式中 $f_i(k_i,k_j,d,\theta)$ 如5.4(1)節中該式所表示的被定義爲週波率影響效應函數。當兩來源成份波列相交會所形成之波動系統退化成重力駐波情況時，即 $\sigma_1=\sigma_2=\sigma$ ，$k_1=k_2=k$ ，$a_1=a_2=a$ ，$\theta=180°$ 時，則 $f_i(k_i,k_j,d,\theta)$ 退化成

$$f=f_1=f_2=\frac{1}{2}\{-\frac{3}{2}-\frac{1}{2}(\mathrm{sech}^2kd+\tanh^{-2}kd)$$

$$-\frac{[\mathrm{csch}\,2kd-(1+\tanh^{-2}kd)\tanh kd]\,\mathrm{sech}^2kd}{2\tanh kd}\}$$

$$=-\frac{1}{8}\tanh^{-2}kd-\frac{3}{4}-\frac{1}{8}\tanh^2kd \tag{8.1}$$

因此，其週波率 σ 恰爲重力駐波者，如（7.10）式中所示；此亦即說明，當所言的兩來源成份波列相交會所形成之波動系統退化成重力駐波情況時，則因有此週波率影響效應的作用而致使其發生所謂的逆變現象。

是故，就此週波率影響效應函數 f_i 隨兩波列交會夾角 θ 之變化關係，如圖3所示，即可得知各來源成份波列之週波率受它們交會之相互作用的變化情況；當 $\theta<90°$ 時，即它們的傳遞脈動具有同向分量時，則 f_i 值爲正，此時各波列的週波率受交會衝擊的影響成增加者，反之，爲 $\theta>90°$ 時，即兩波列的傳遞脈動具有逆向分量時，則 f_i 值爲負，此時各波率的週波率受交會衝擊的影響而會減少，因此，於水深變淺下會有所謂的週波率逆變現象的發生。此結果是與一般力學作用的觀點相符合，即正衝時則效應加強，反衝時則爲倒逆而減低，且以 $\theta=90°$ 相交時爲其中間之分隔之。至於短峯波之特例情況者亦爲如此。以上爲往昔所無論究的。

（5）對整體波動系統之脈動水位影響而言，其隨兩波列相交會而衍生出的非線性相互作

用量而發生之變化，亦如同造致週波率之變化般，造成整個波動系統之脈動波形的扭曲現象與其增量，是隨著兩來源成份波列相交會之夾角偏離 90° 而增加，至 0° 與 180° 時為兩頭的極端，如圖 5 所示；另，此扭曲現象與其增量，亦隨兩波列間的波數比偏離 1 時而加強，即在較長波與較短波之相交會下，則愈明顯地呈現之，亦見之該圖中的比較。至於對整體波動系統之波動流場內的波壓情況，亦復如此。當然以上此等之波動流場特性，於大波與大波相交會時為更為增大加強顯現之，且在較淺水時愈為明顯。諸此特性之論述為往昔尚未探討的。

(**6**) 兩波列相交會所形成之波動系統，於某些特定的（相對）水深及相交會夾角下，會發生所謂的共振情況，這恰如同往昔研究所著重論述而得之對此特例的結果。

(**7**) 任一均匀等深水中之短峯波波動流場結構解，已依本文所解得之任兩波列相交會之波動系統的通案性流場結構解退化成此特例情況而獲得；至於短峯波波動流場之整體脈動特性，如其週波率會發生之逆變現象及其脈動水位與波壓變化情況等，亦皆因而可由之給予其來由的力學機構本質的論究而詳細明確地陳述之。是故，亦進一步地延伸往昔僅對短峯波個案情況的研究而做深入究底的探明。

(**8**) 在大尺度的平面水槽進行試驗下，對任一均匀等深水中任兩來源成份波列相交會所形成之波動系統，量測其表面脈動水位與波動場內之波壓等之結果，更具體地顯示，本文所論述的任兩波列相交會之波動系統其整體流場特性的良好適足性；這當然含括通案性的波動流場情況與其特例的短峯波波動流場情況皆為如此。

最後，對本文得需討論的是，本文是依攝動法原則展開解析的，僅求至第三階次量之解，因此，對更淺水長波情況，即是當 $d/L_i < 0.05$ 時，則需推求至更高階次量的解，以便使解析所得之結果更有滿足的精度，此外，至於更多來源成份波列相交會所形成之波動系統，當然可依本文所陳述之方式來對其進行解析與描述，唯其仍需進行更繁雜的數學運算處理，如三波列相交會者已由張憲國（1991）進行頗為詳盡的闡述，進而據此可得所謂波列相交會共振下之一般條件式，此結果與 Hasselmann（1962）者完全相同之，然其基本原由已被更清楚的瞭解與陳述。

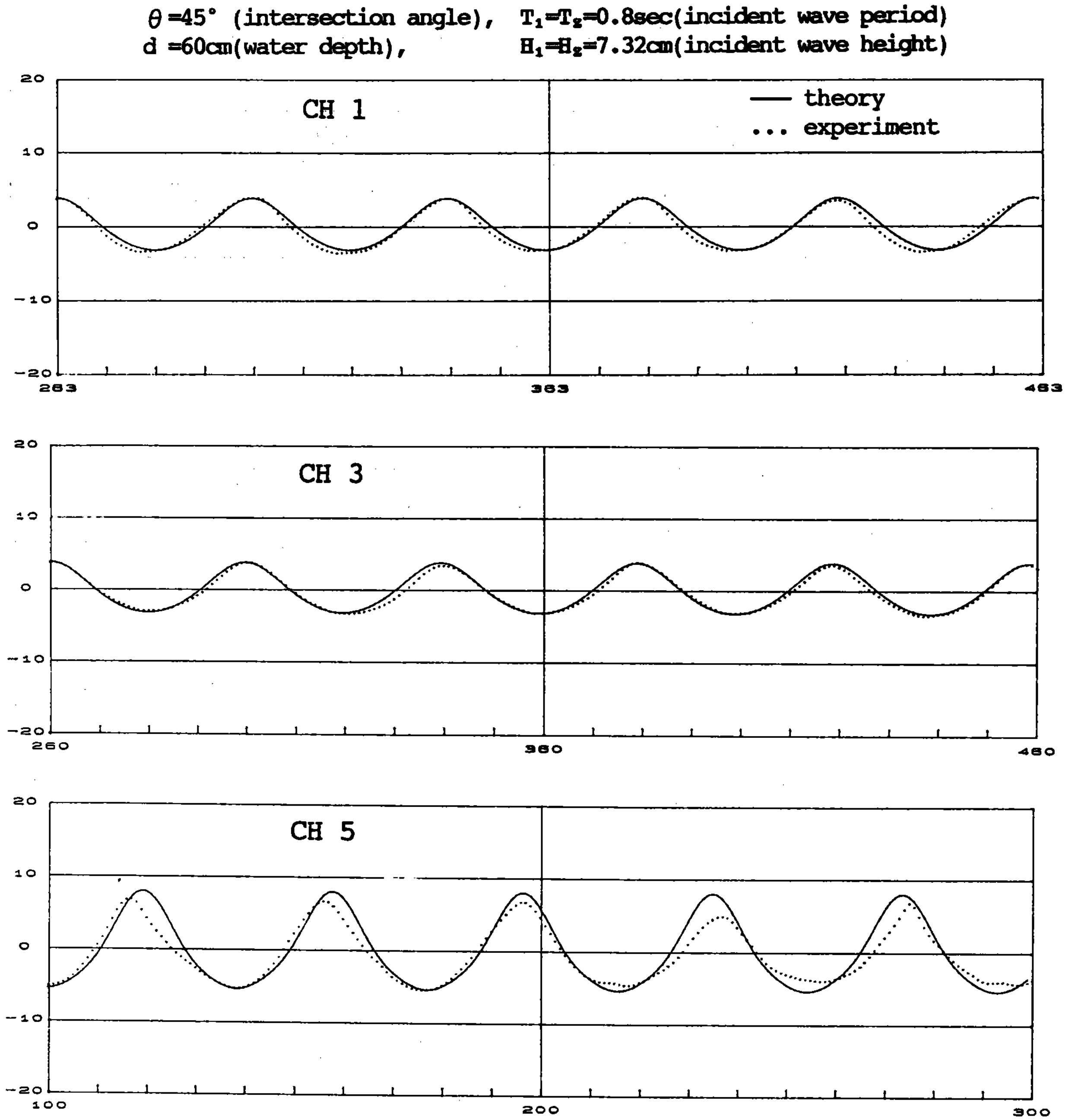


圖10-1

圖10 兩波交會相互作用所形成之自由表面波形試驗值與理論值比較

Fig. 10 The comparison of theoretical and experimental free-surface elevation resulted from the interaction between two progressive gravity waves trains.

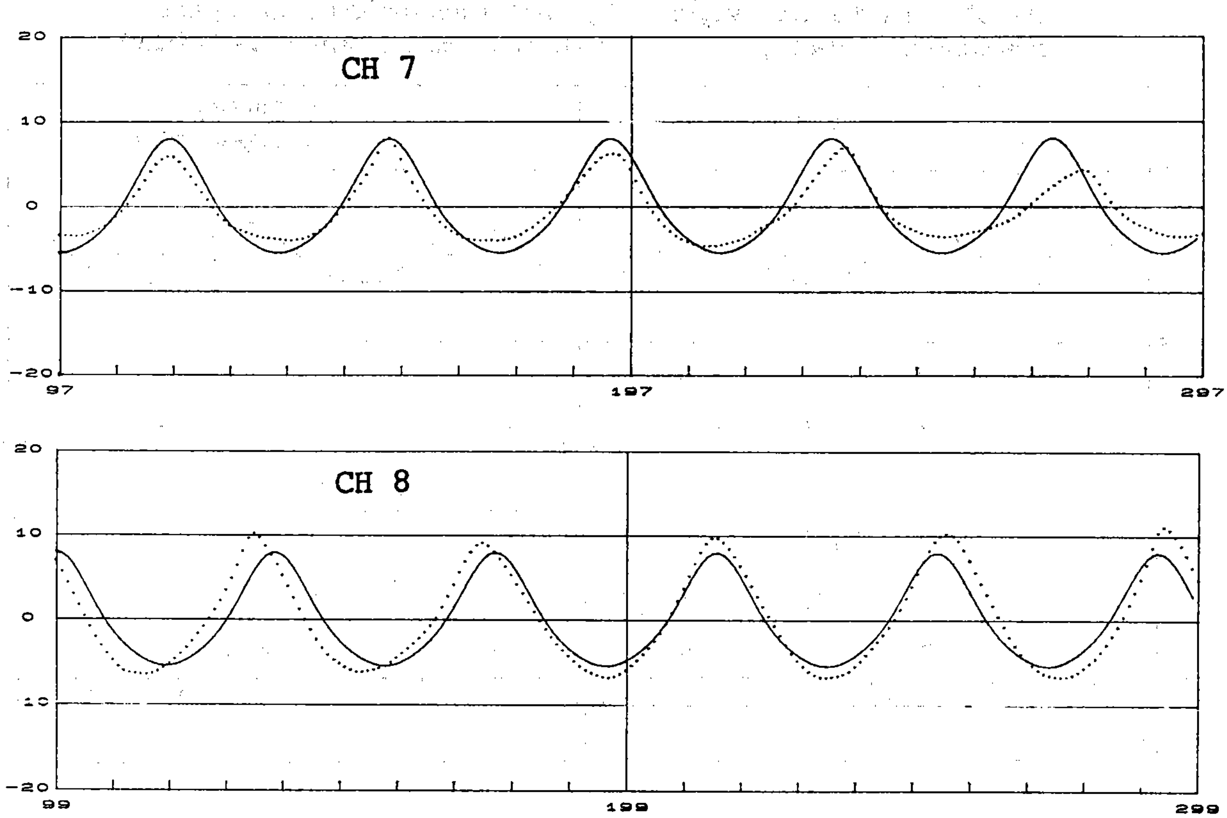


圖10-1

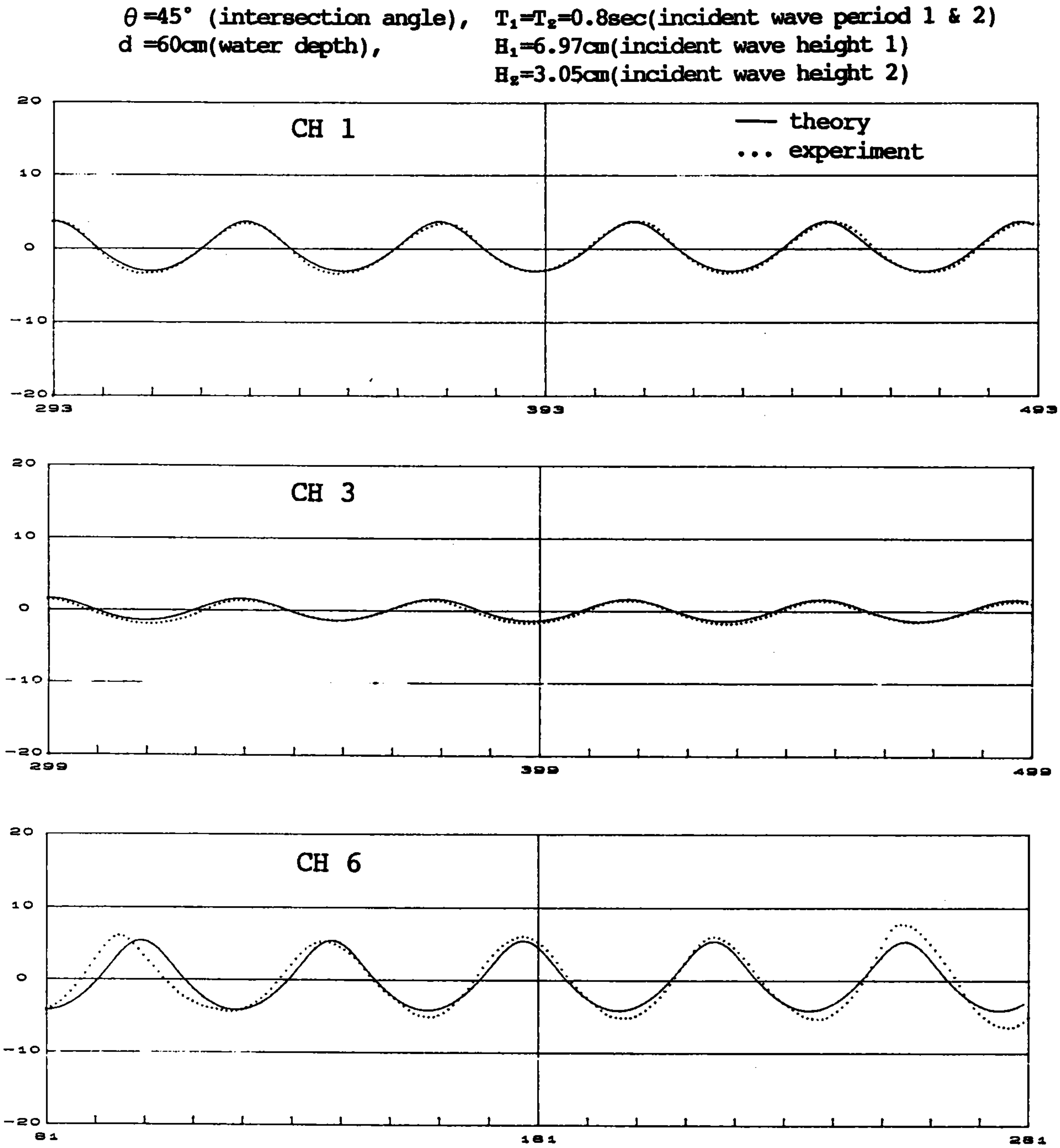

θ=45° (intersection angle), T₁=T₂=0.8sec(incident wave period 1 & 2)
d =60cm(water depth), H₁=6.97cm(incident wave height 1)
H₂=3.05cm(incident wave height 2)
CH 1
— theory
... experiment
20
10
0
−10
−20
293
393
493
CH 3
20
10
0
−10
−20
299
399
499
CH 6
20
10
0
−10
−20
81
181
281

圖10-2

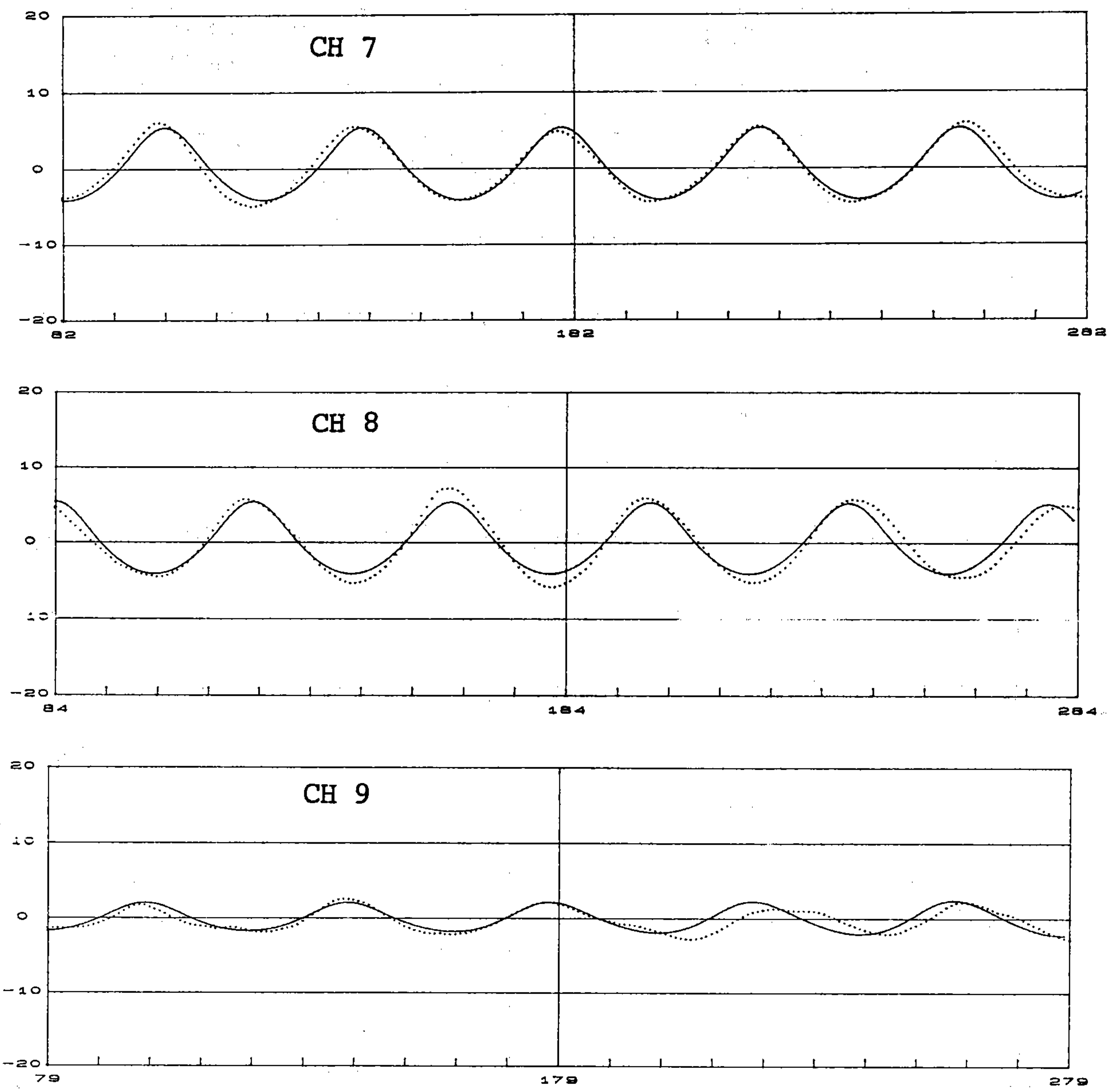

CH 7
20
10
0
-10
-20
82
182
282
CH 8
20
10
0
-10
-20
84
184
284
CH 9
20
10
0
-10
-20
79
179
279

圖10-2

θ=45° (intersection angle), $T_1=T_2$=1.20sec(incident wave period)
d =60cm(water depth), $H_1=H_2$=10.87cm(incident wave height)

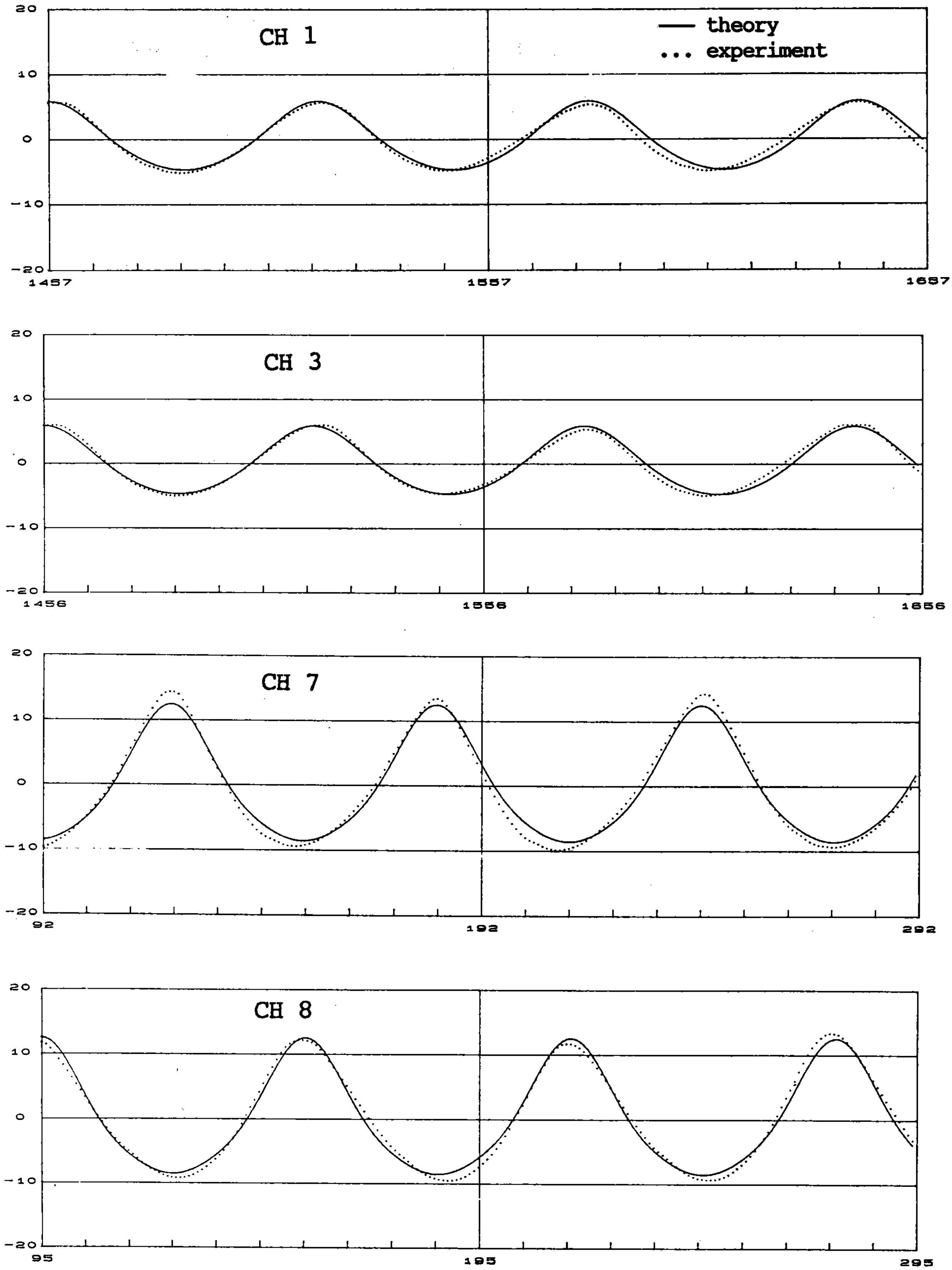


圖10-3

θ=45° (intersection angle), $T_1=T_2$=1.20sec(incident wave period 1 & 2)
d =60cm(water depth), H_1=6.06cm(incident wave height 1)
H_2=3.05cm(incident wave height 2)

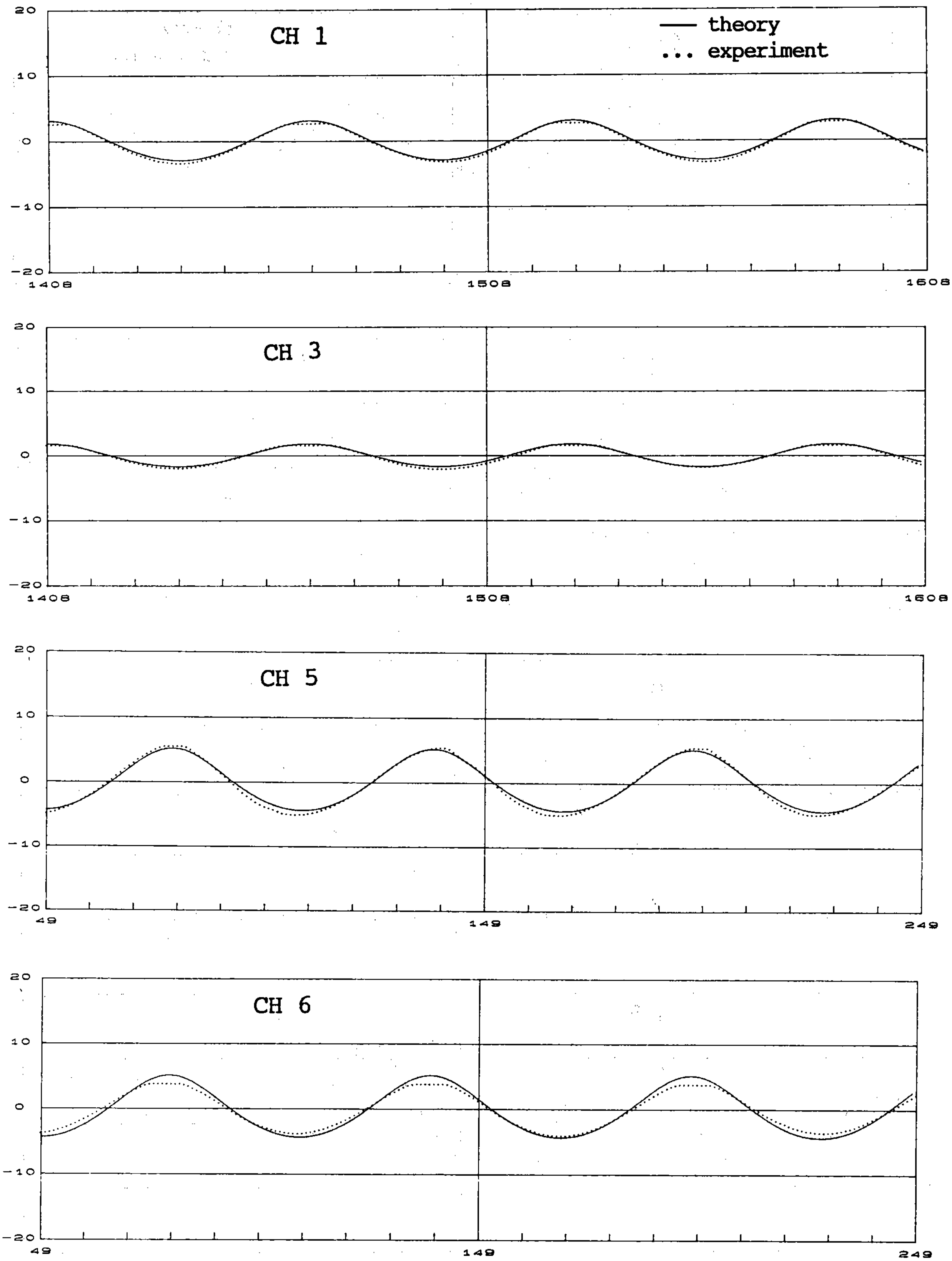


圖10-4

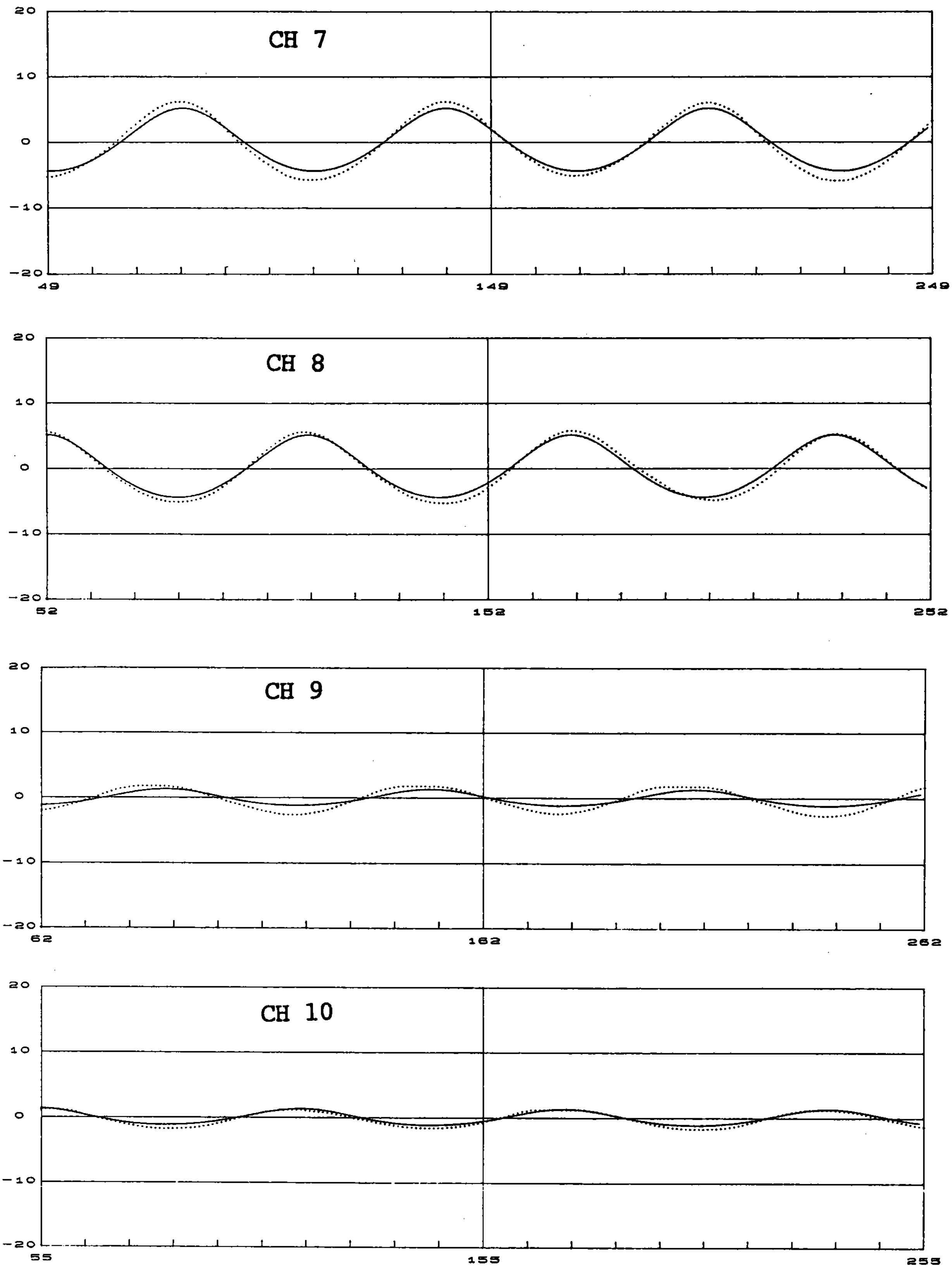

CH 7
20
10
0
−10
−20
49
149
249
CH 8
20
10
0
−10
−20
52
152
252
CH 9
20
10
0
−10
−20
62
162
262
CH 10
20
10
0
−10
−20
55
155
255

圖10-4

θ =45° (intersection angle), T_1=T_2=1.20sec(incident wave period 1 & 2)
d =60cm(water depth), H_1=10.83cm(incident wave height 1)
H_2=3.24cm(incident wave height 2)

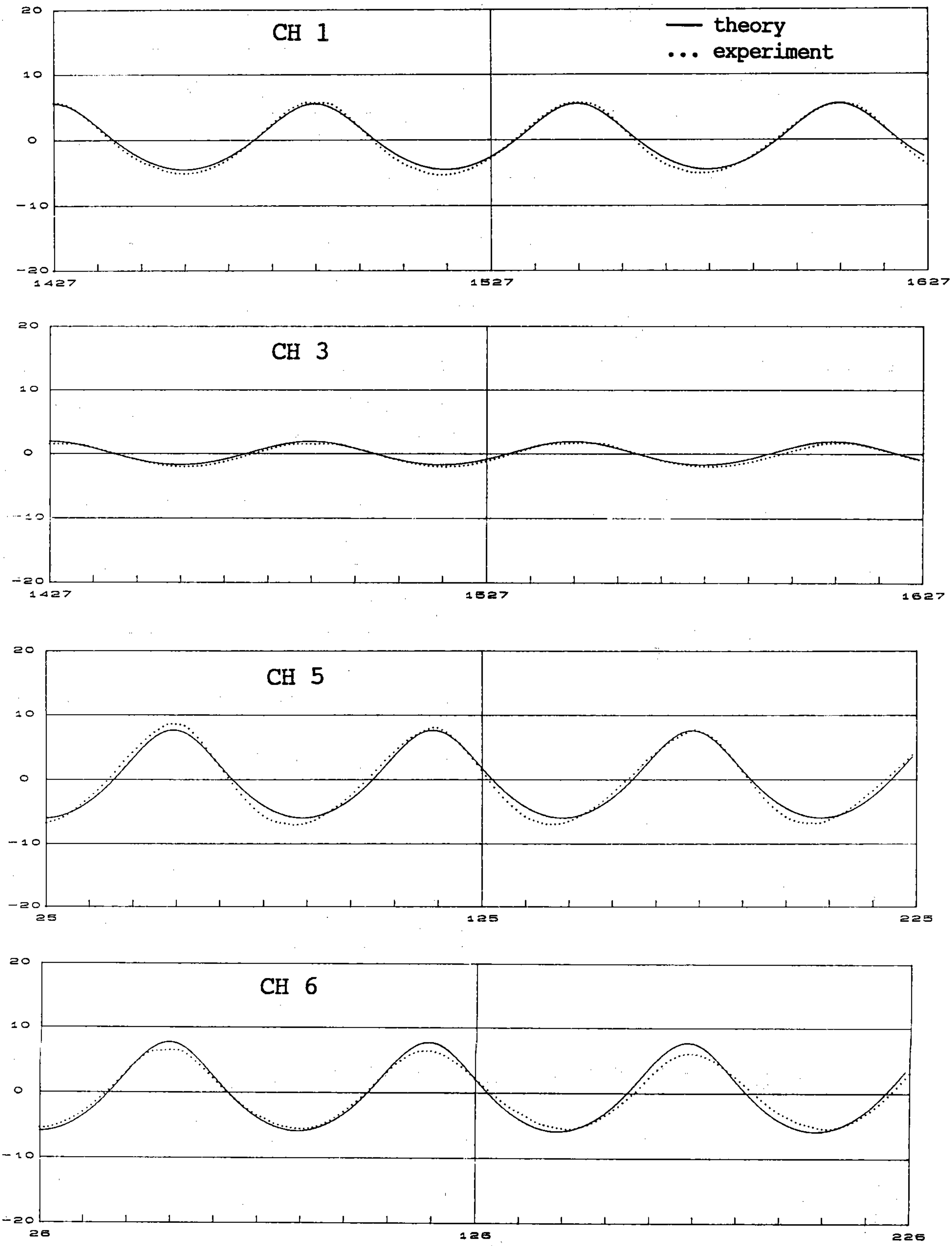


圖10-5

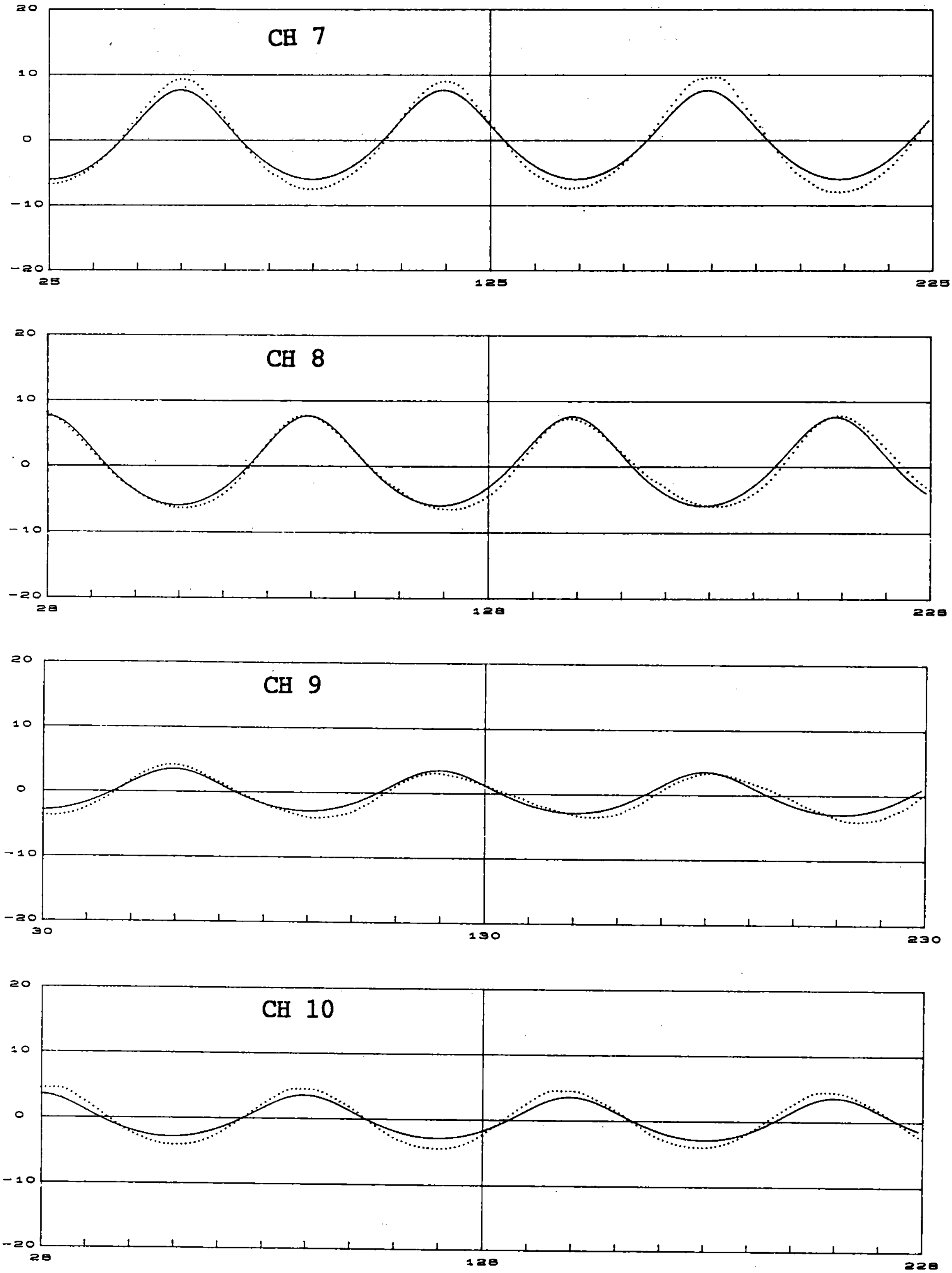

CH 7
20
10
0
-10
-20
25
125
225
CH 8
20
10
0
-10
-20
28
128
228
CH 9
20
10
0
-10
-20
30
130
230
CH 10
20
10
0
-10
-20
28
128
228

圖10-5

θ=45° (intersection angle), $T_1=T_2=1.60$sec(incident wave period)
d=60cm(water depth), $H_1=H_2=12.25$cm(incident wave height)

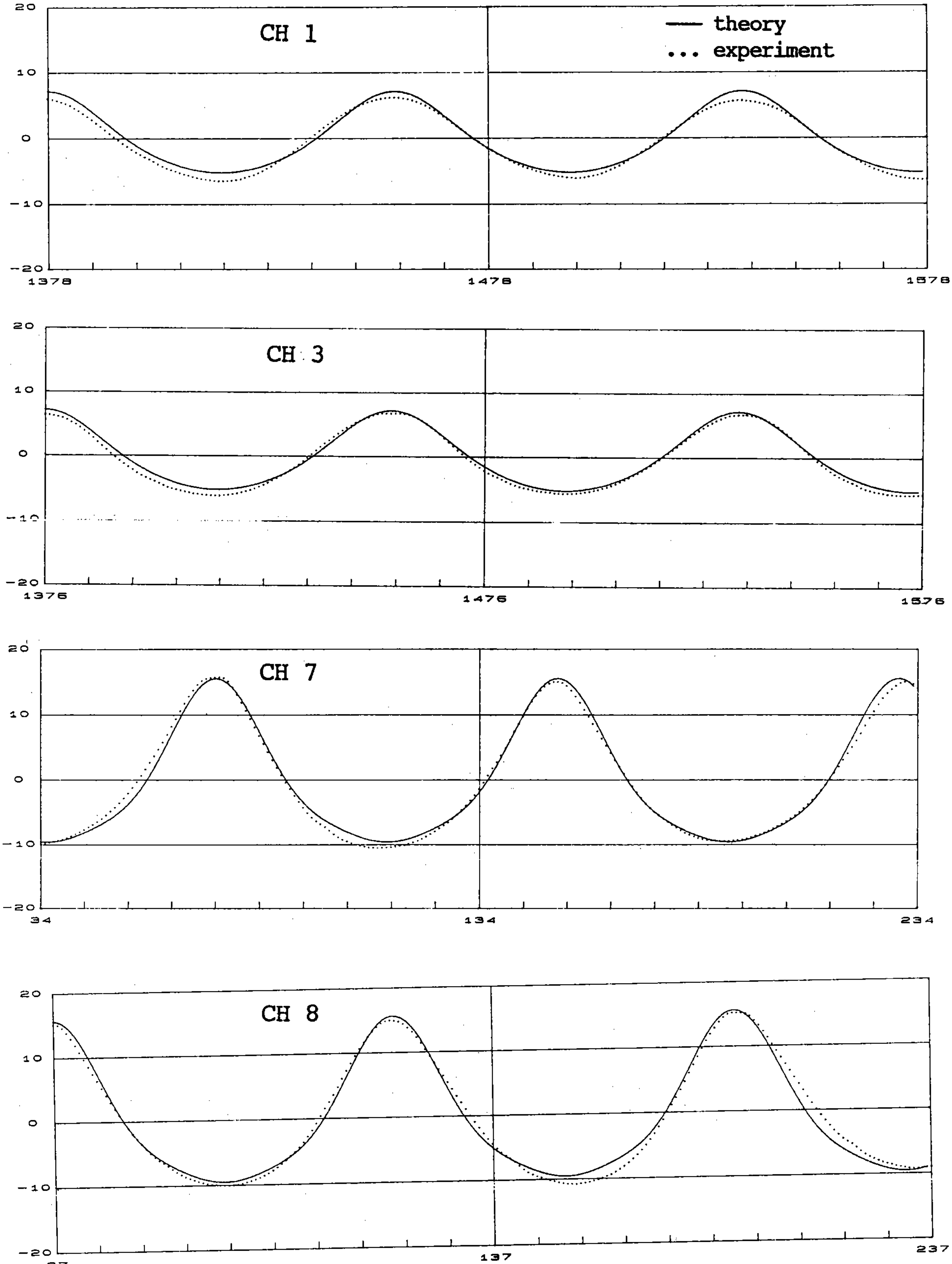


圖10-6

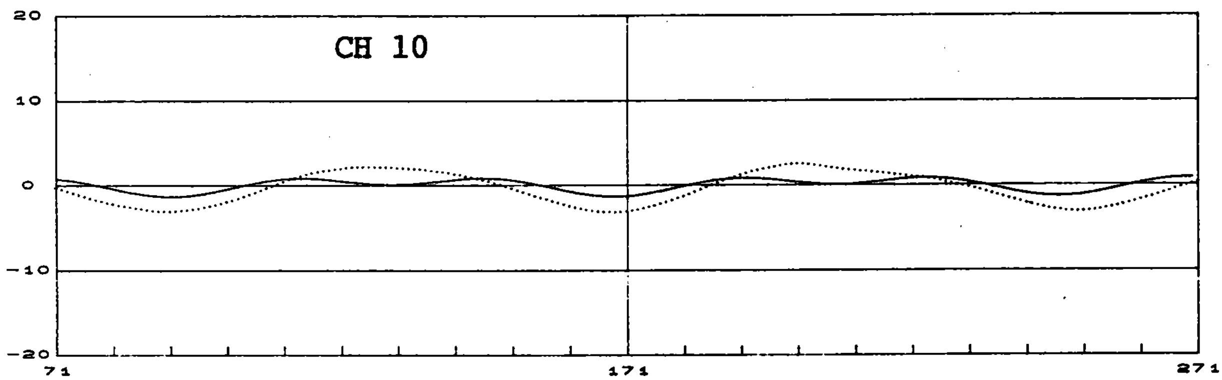


圖10-6

θ =45° (intersection angle), T_1=T_2=1.60sec(incident wave period 1 & 2)
d =60cm(water depth), H_1=6.87cm(incident wave height 1)
H_2=3.28cm(incident wave height 2)

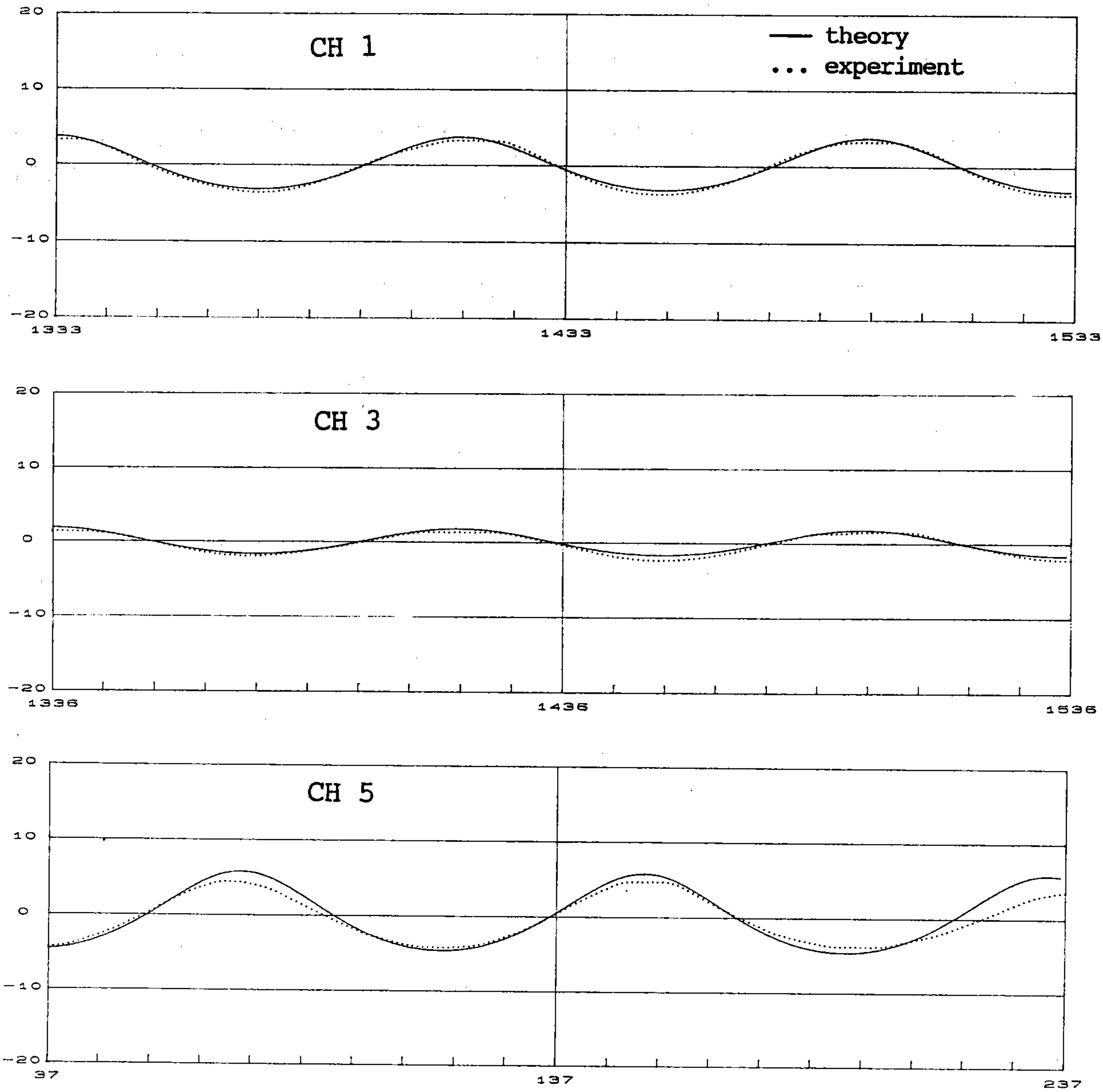
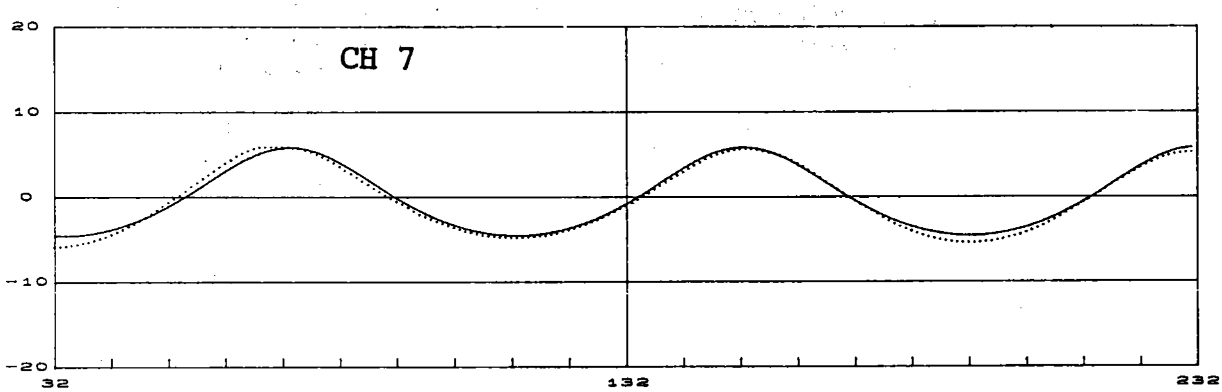
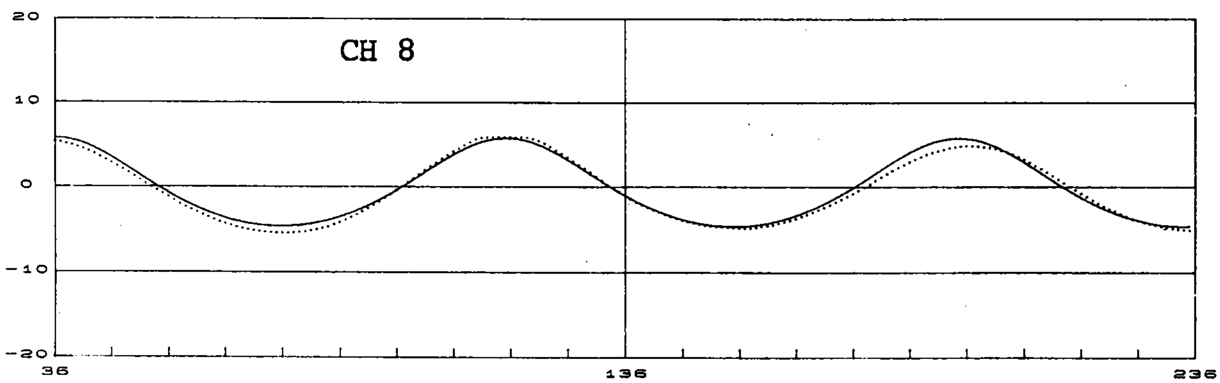
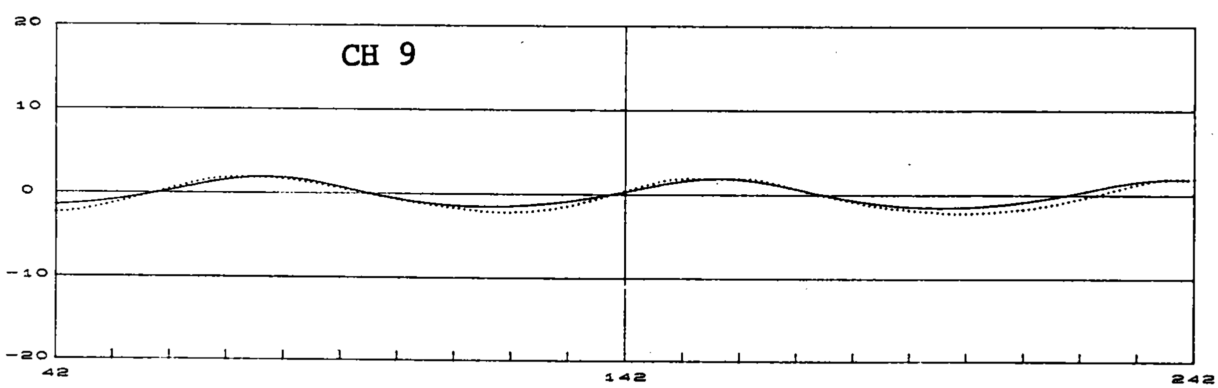


圖10-7


CH 7
20
10
0
-10
-20
32
132
232


CH 8
20
10
0
-10
-20
36
136
236


CH 9
20
10
0
-10
-20
42
142
242

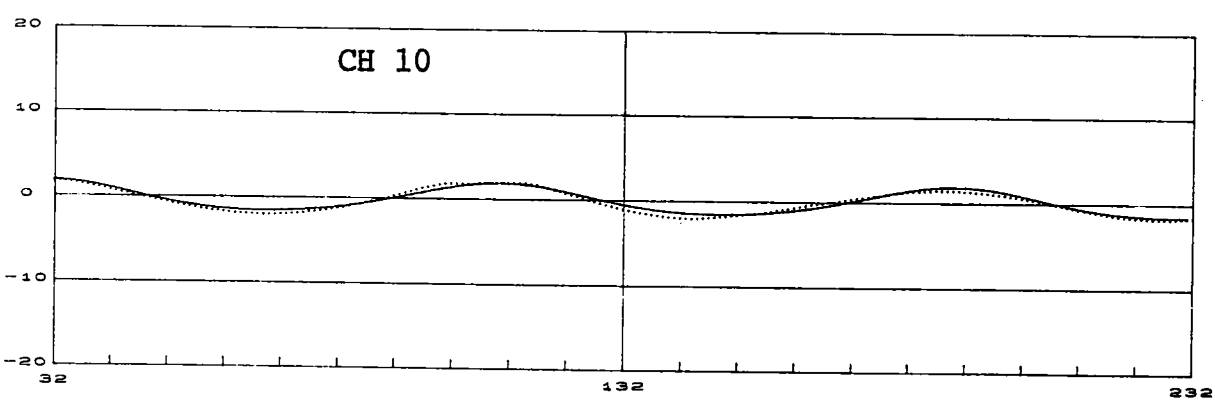

CH 10
20
10
0
-10
-20
32
132
232

圖10-7

θ =45° (intersection angle), T_1=T_2=1.60sec(incident wave period 1 & 2)
d =60cm(water depth), H_1=12.27cm(incident wave height 1)
H_2=3.37cm(incident wave height 2)

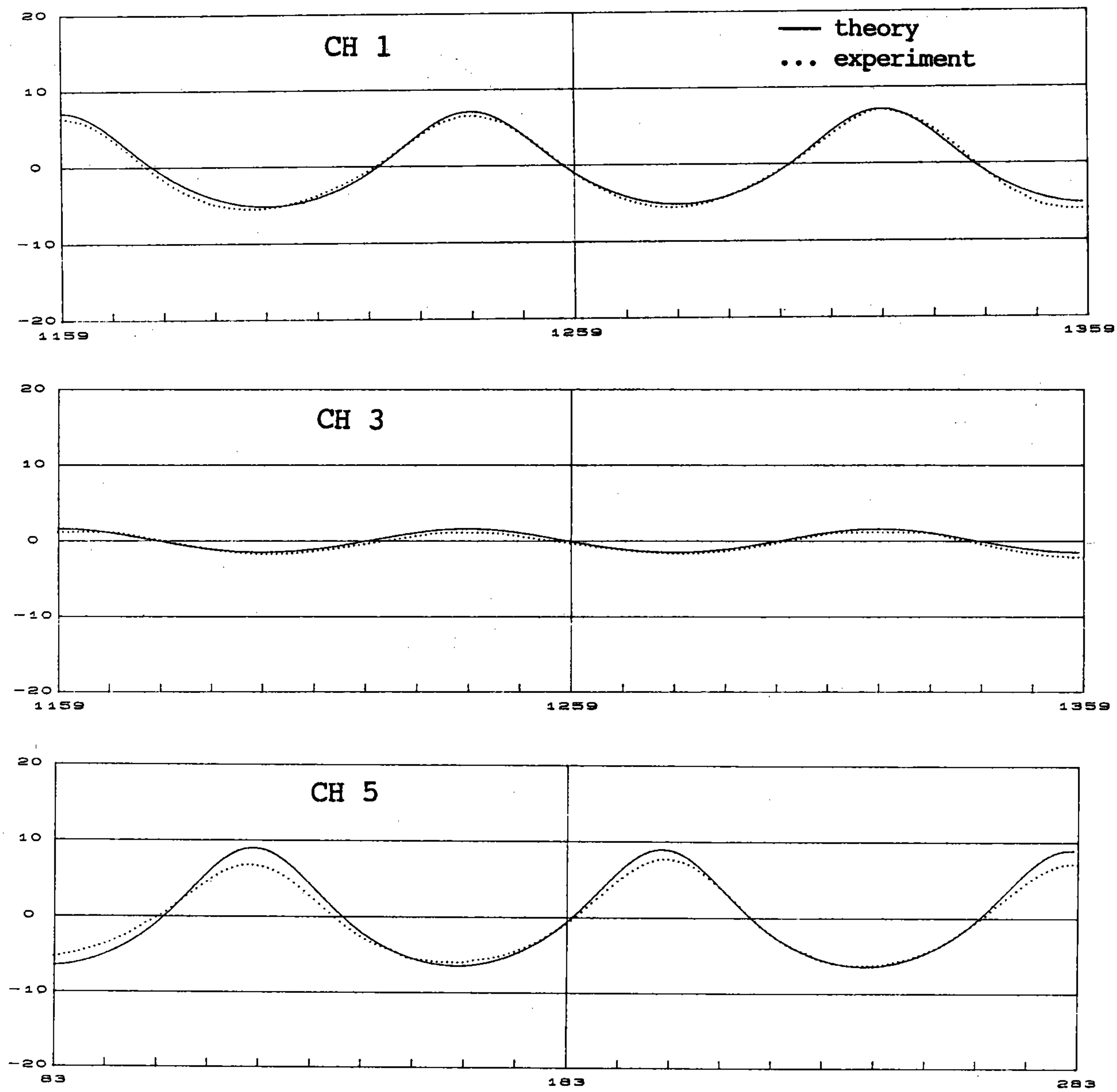


圖10-8

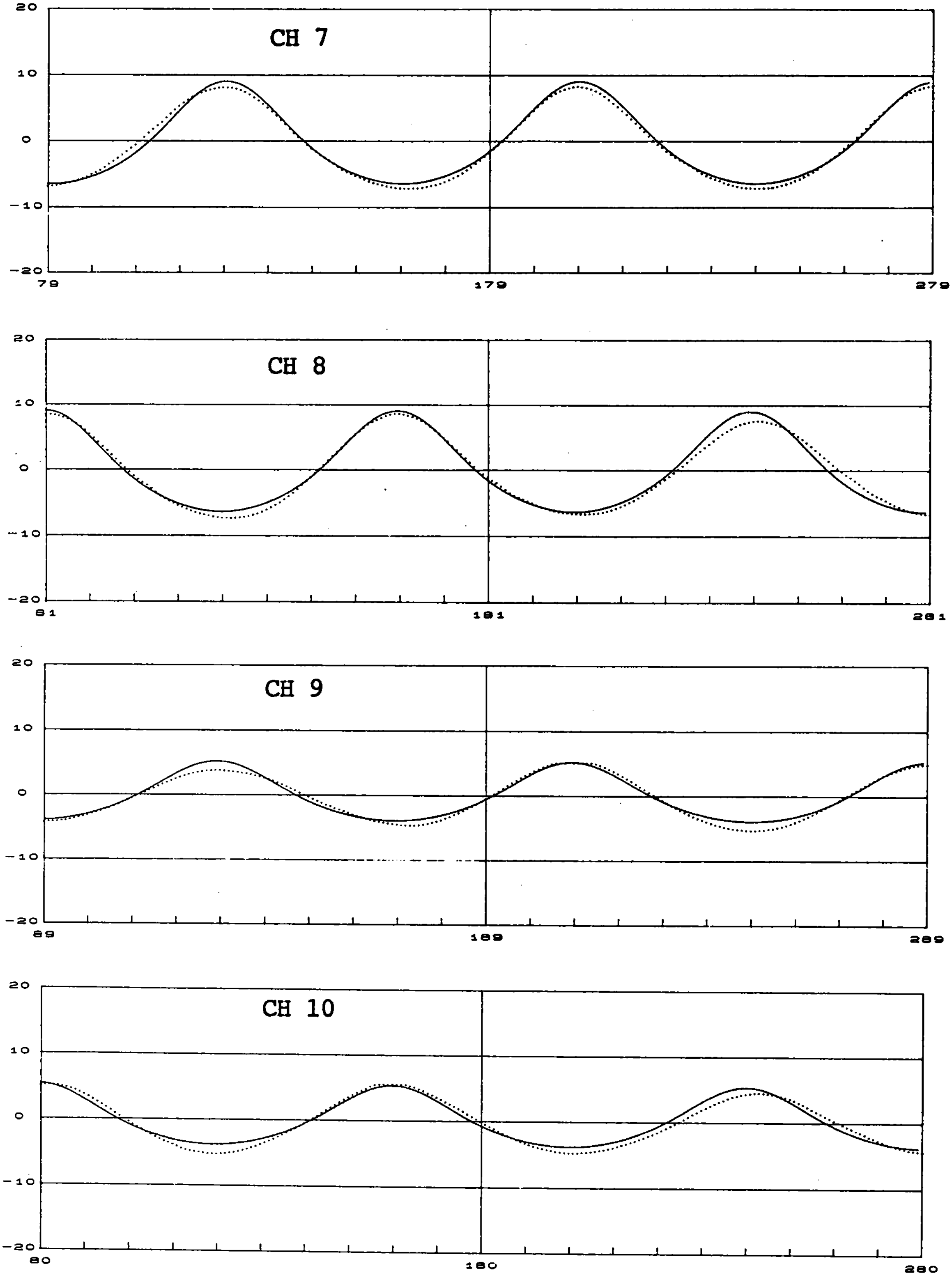

CH 7
20
10
0
−10
−20
79
179
279
CH 8
20
10
0
−10
−20
81
181
281
CH 9
20
10
0
−10
−20
89
189
289
CH 10
20
10
0
−10
−20
80
180
280

圖10-8

θ =45° (intersection angle), T_1=T_2=2.00sec(incident wave period)
d =60cm(water depth), H_1=H_2=13.17cm(incident wave height)

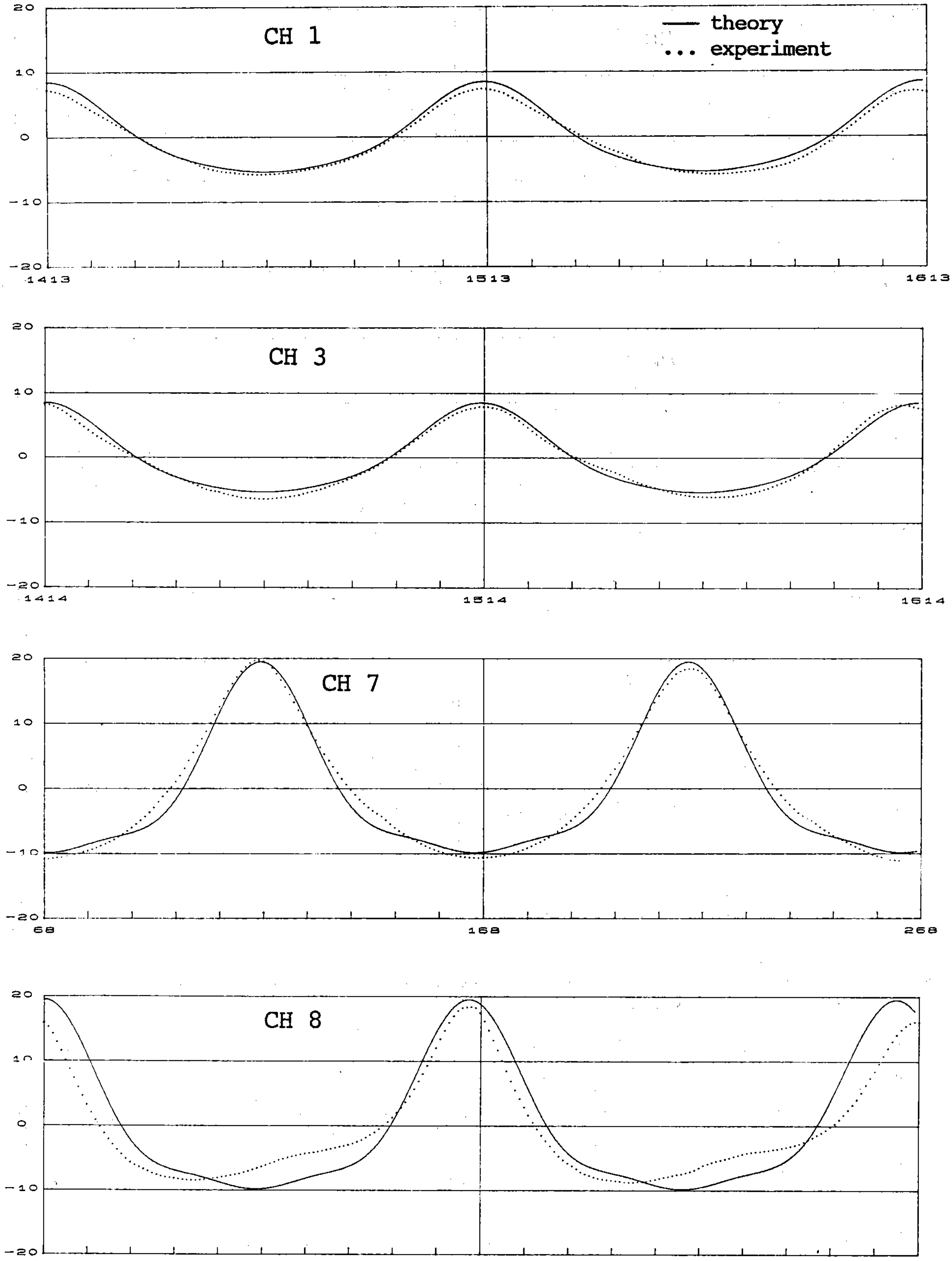


圖10-9

θ=45° (intersection angle), T_1=T_2=2.00sec(incident wave period 1 & 2)
d =60cm(water depth), H_1=6.75cm(incident wave height 1)
H_2=3.26cm(incident wave height 2)

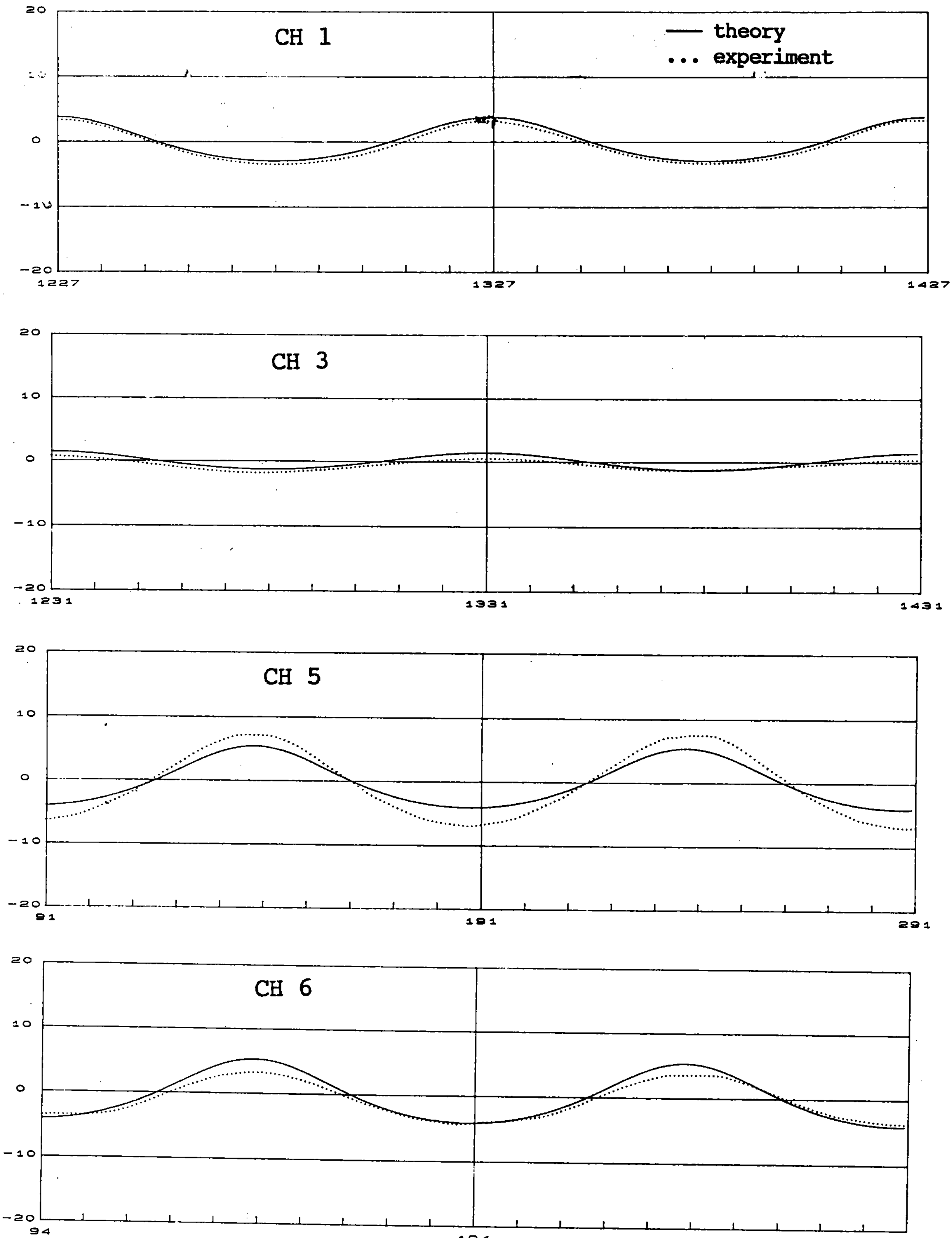


圖10-10

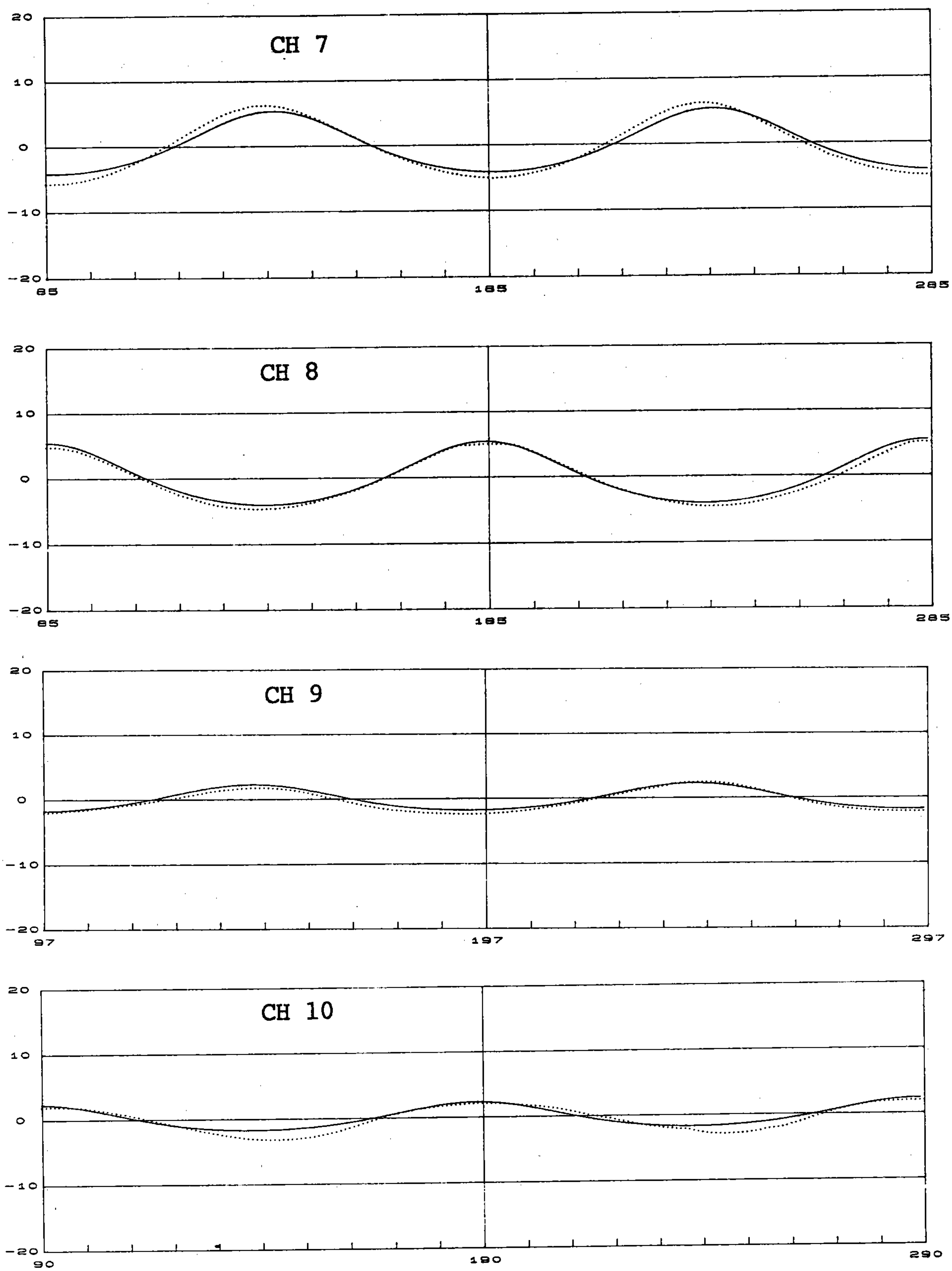

CH 7
20
10
0
-10
-20
85
185
285
CH 8
20
10
0
-10
-20
85
185
285
CH 9
20
10
0
-10
-20
97
197
297
CH 10
20
10
0
-10
-20
90
190
290

圖10-10

θ=45° (intersection angle), d =60cm(water depth),

T_1=T_2=2.00sec(incident wave period 1 & 2)

H_1=13.09cm(incident wave height 1)

H_2=3.48cm(incident wave height 2)

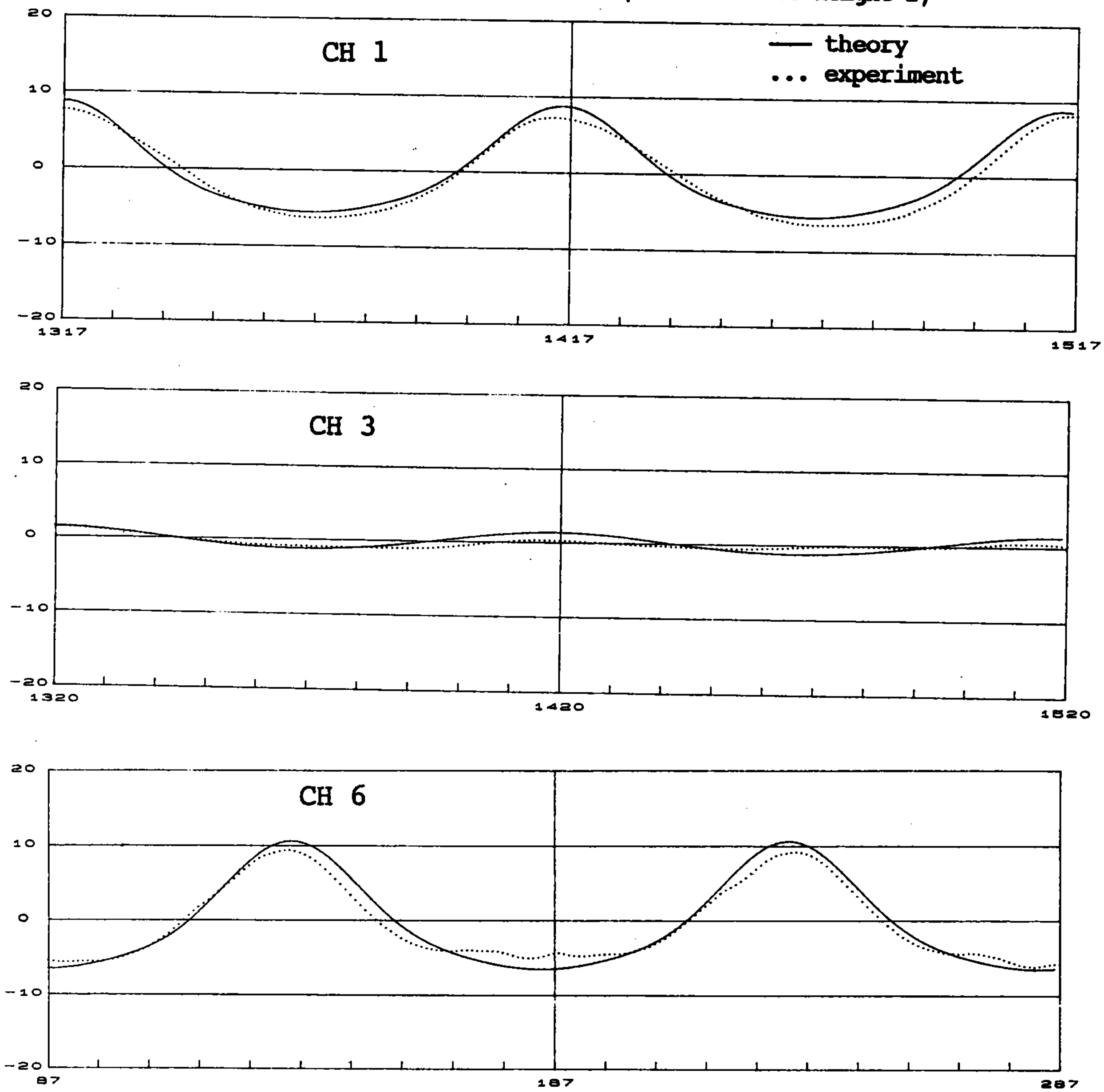


圖10-11

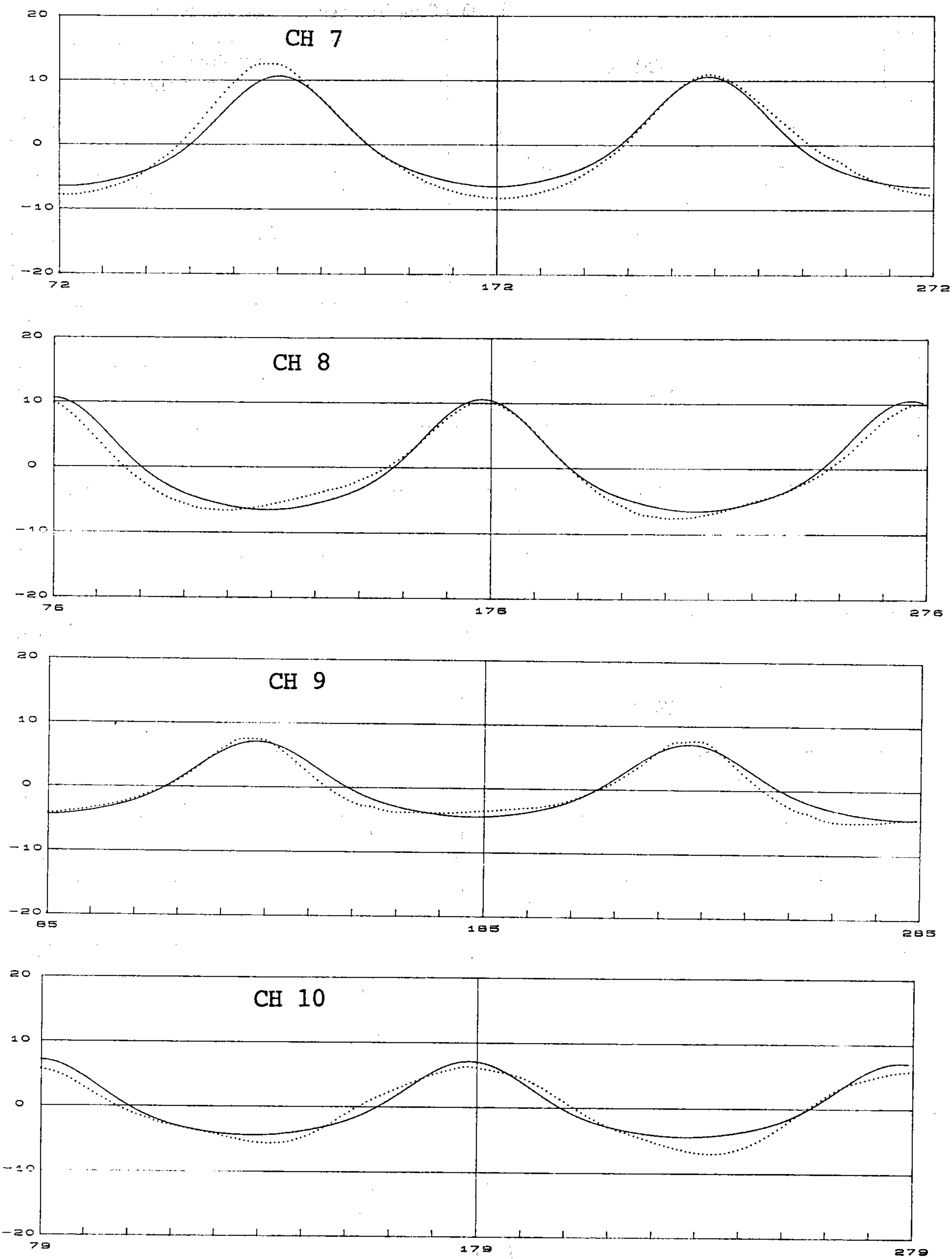

CH 7
20
10
0
-10
-20
72
172
272
CH 8
20
10
0
-10
-20
76
176
276
CH 9
20
10
0
-10
-20
85
185
285
CH 10
20
10
0
-10
-20
79
179
279

圖10-11

θ=45° (intersection angle), T_1=T_2=2.40sec(incident wave period)
d =60cm(water depth), H_1=H_2=13.07cm(incident wave height)

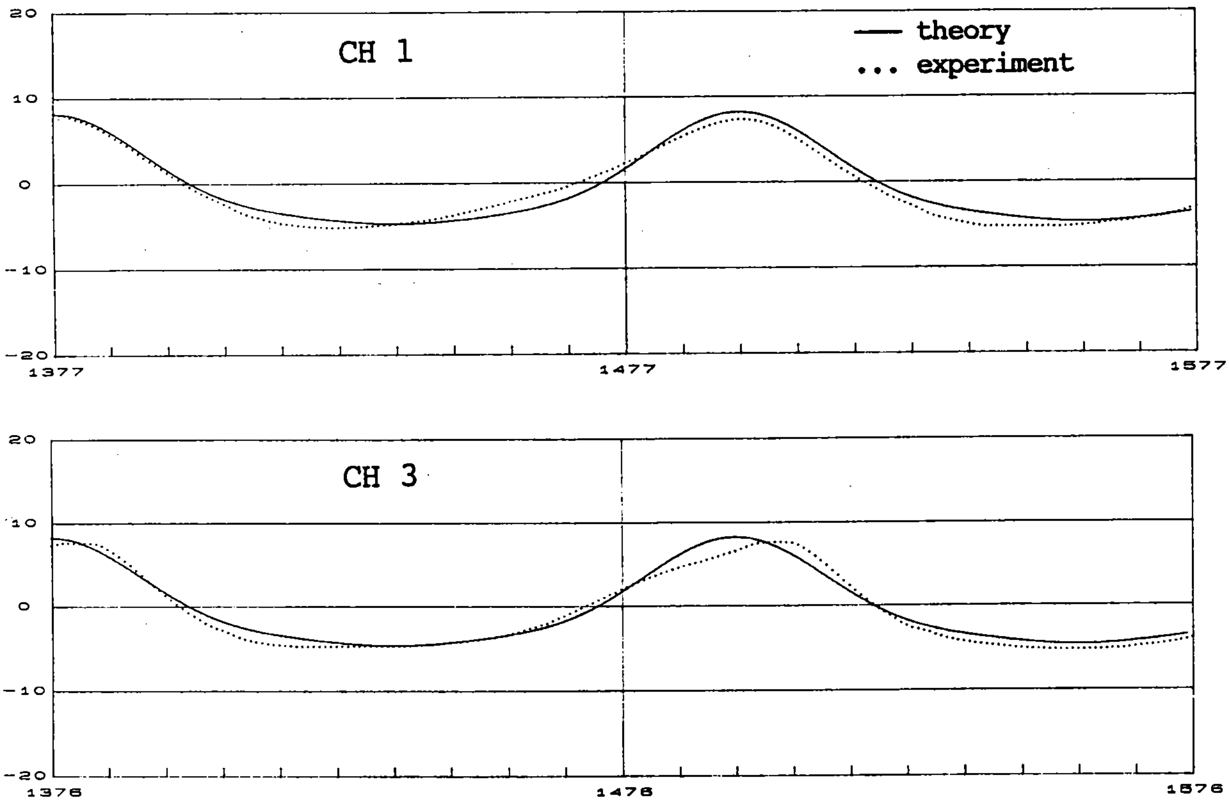
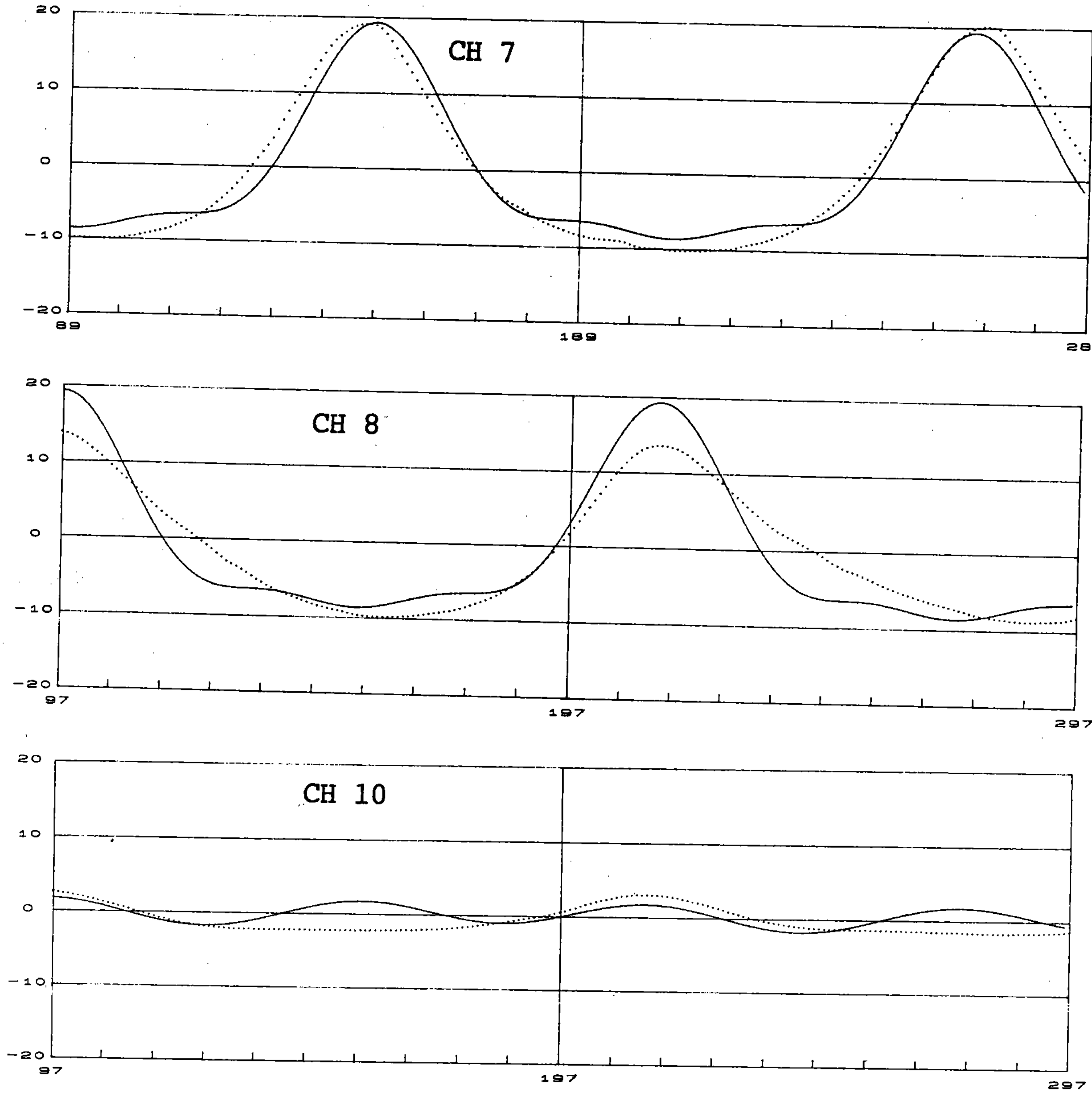


圖10-12



CH 7
20
10
0
-10
-20
89
189
289
CH 8
20
10
0
-10
-20
97
197
297
CH 10
20
10
0
-10
-20
97
197
297

圖10-12

θ=45° (intersection angle), T_1=T_2=2.40sec(incident wave period 1 & 2)
d =60cm(water depth), H_1=6.12cm(incident wave height 1)
H_2=3.06cm(incident wave height 2)

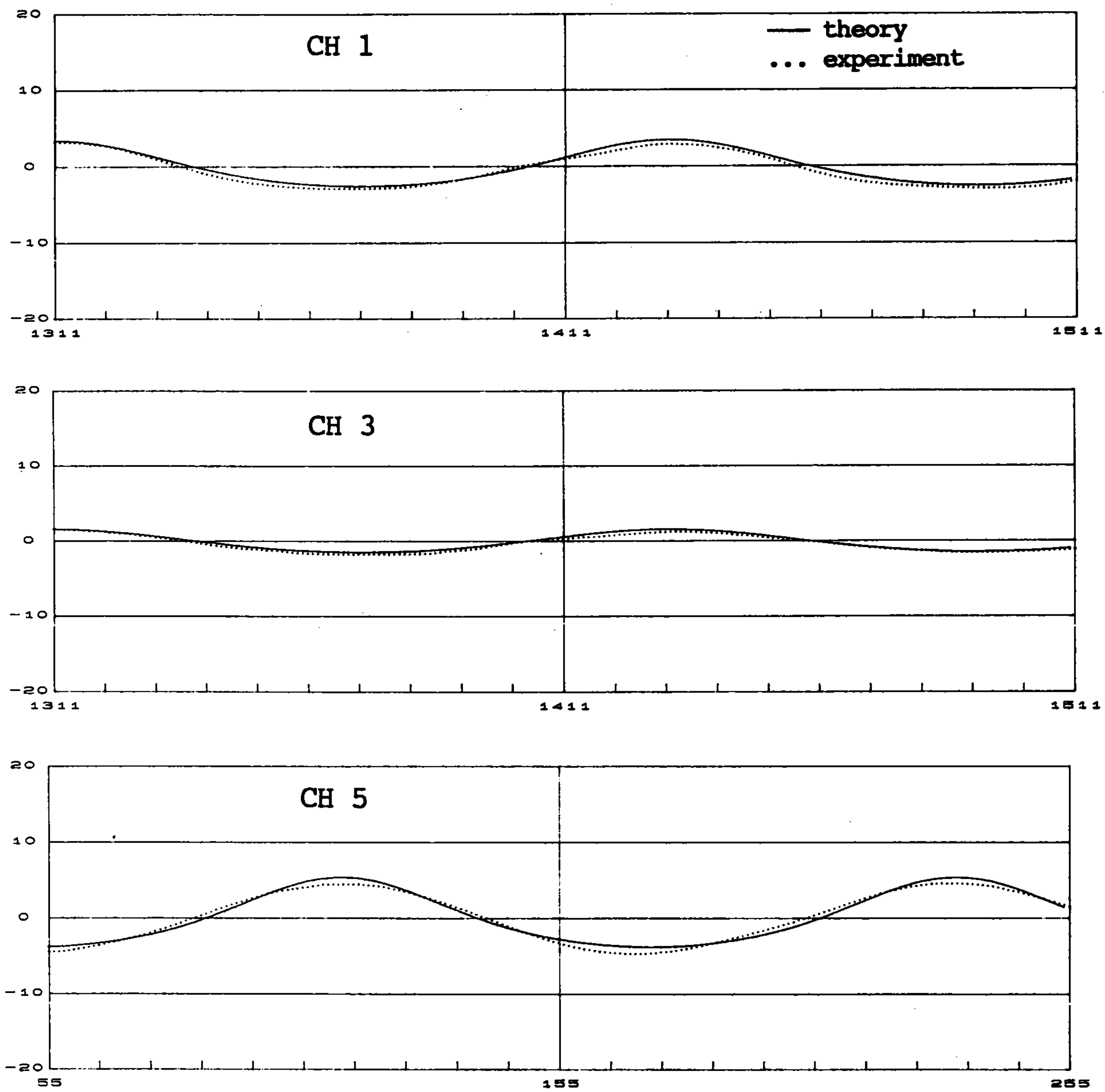


圖10-13

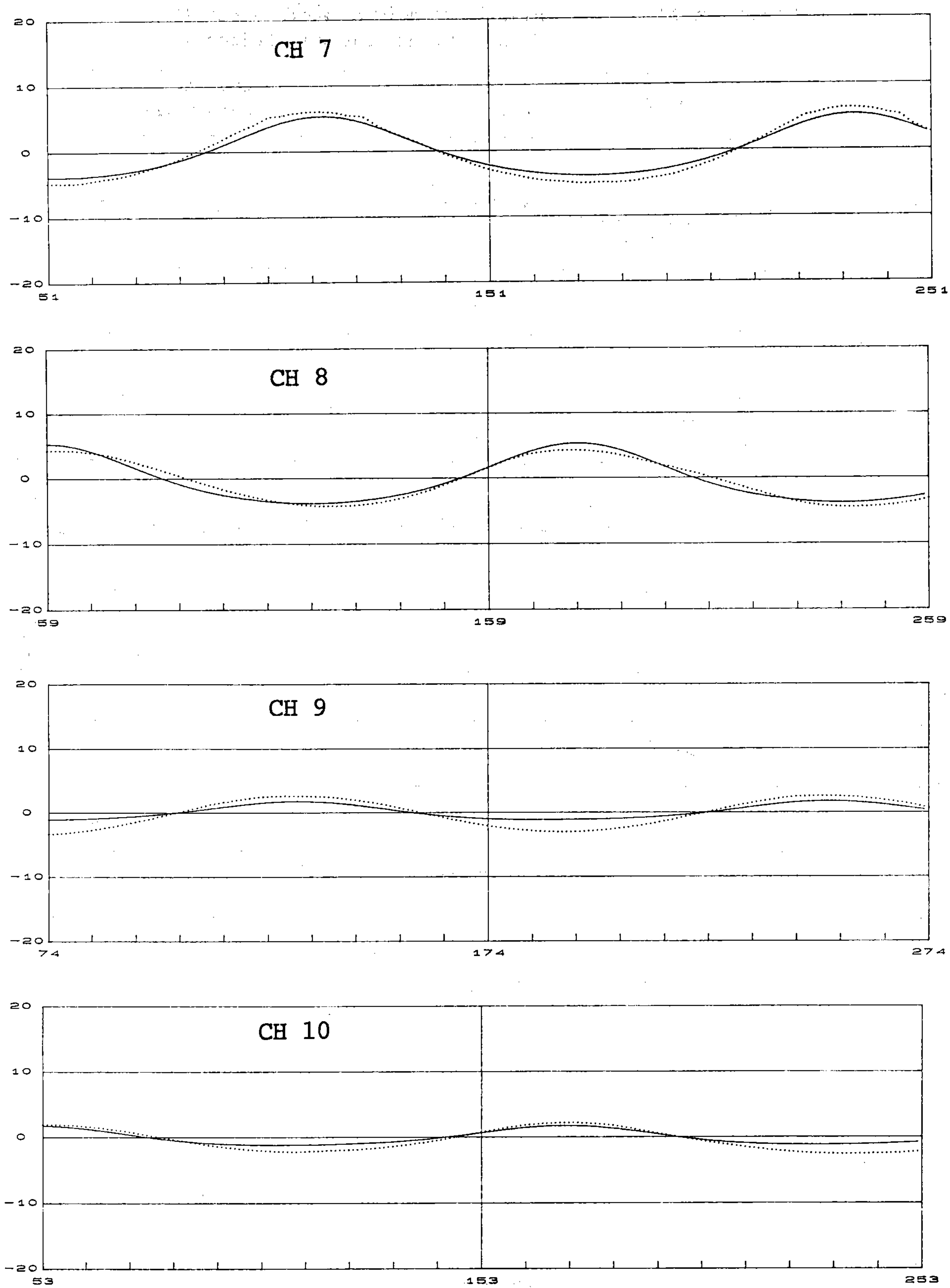
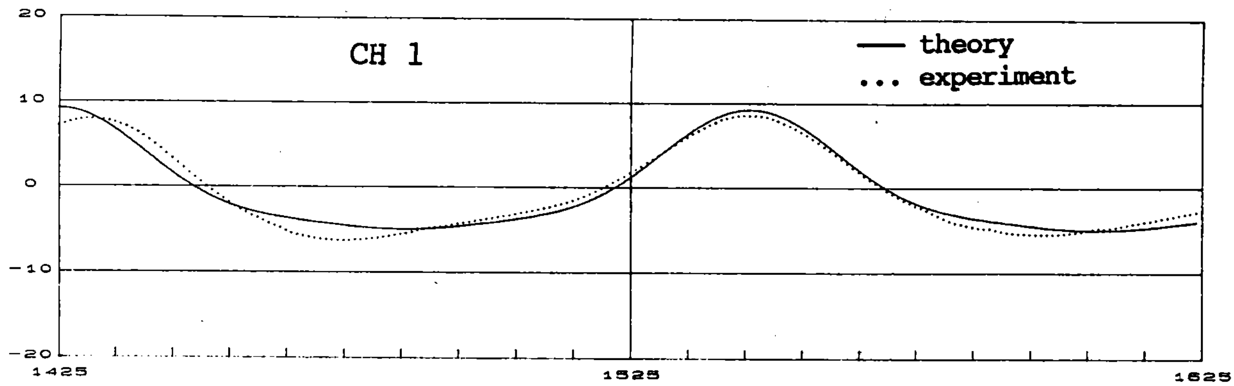

CH 7
20
10
0
-10
-20
51
151
251
CH 8
20
10
0
-10
-20
59
159
259
CH 9
20
10
0
-10
-20
74
174
274
CH 10
20
10
0
-10
-20
53
153
253

圖10-13

θ =45° (intersection angle), T_1=T_2=2.40sec(incident wave period 1 & 2)
d =60cm(water depth), H_1=14.22cm(incident wave height 1)
H_2=3.60cm(incident wave height 2)



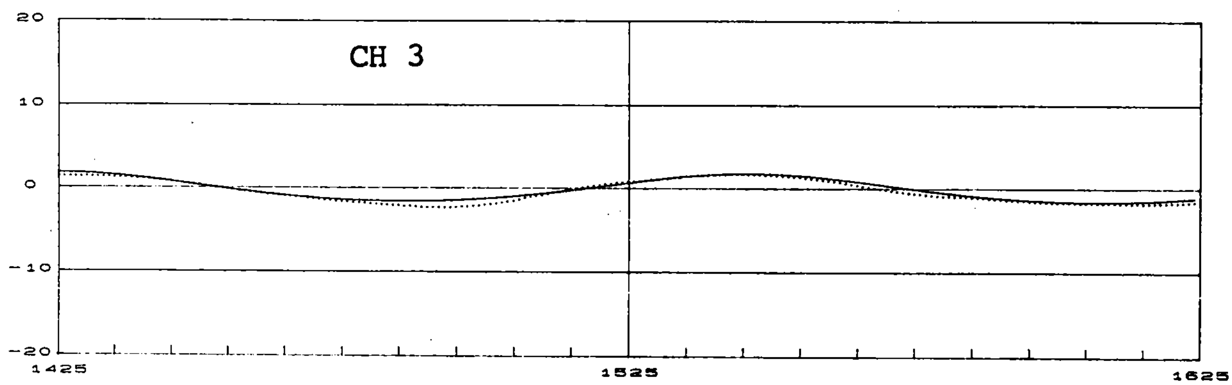


圖10-14

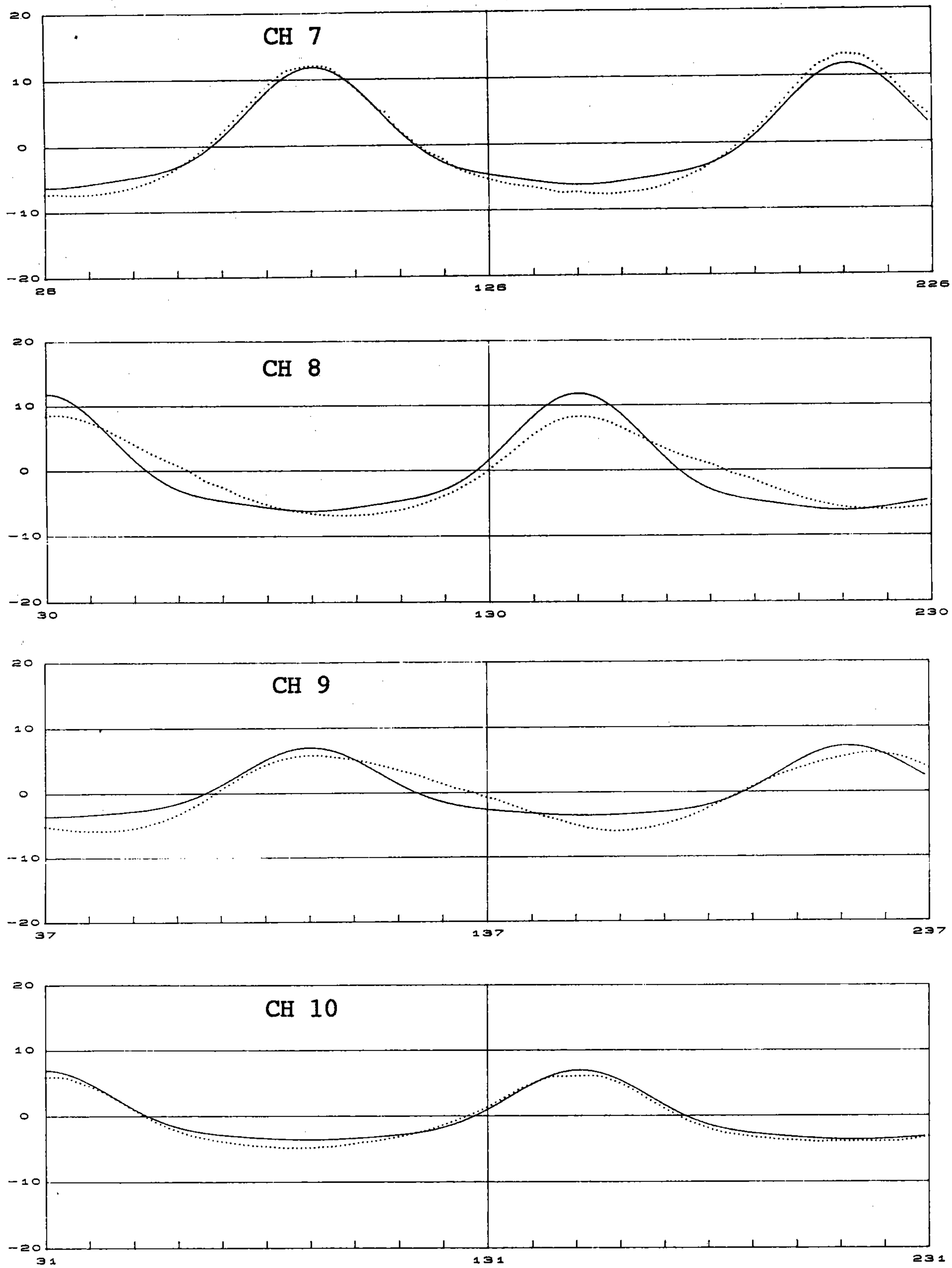

CH 7
20
10
0
-10
-20
26
126
226
CH 8
20
10
0
-10
-20
30
130
230
CH 9
20
10
0
-10
-20
37
137
237
CH 10
20
10
0
-10
-20
31
131
231

圖10-14

θ =45° (intersection angle), T_1=T_2=0.8sec(incident wave period)
d =50cm(water depth), H_1=H_2=7.05cm(incident wave height)

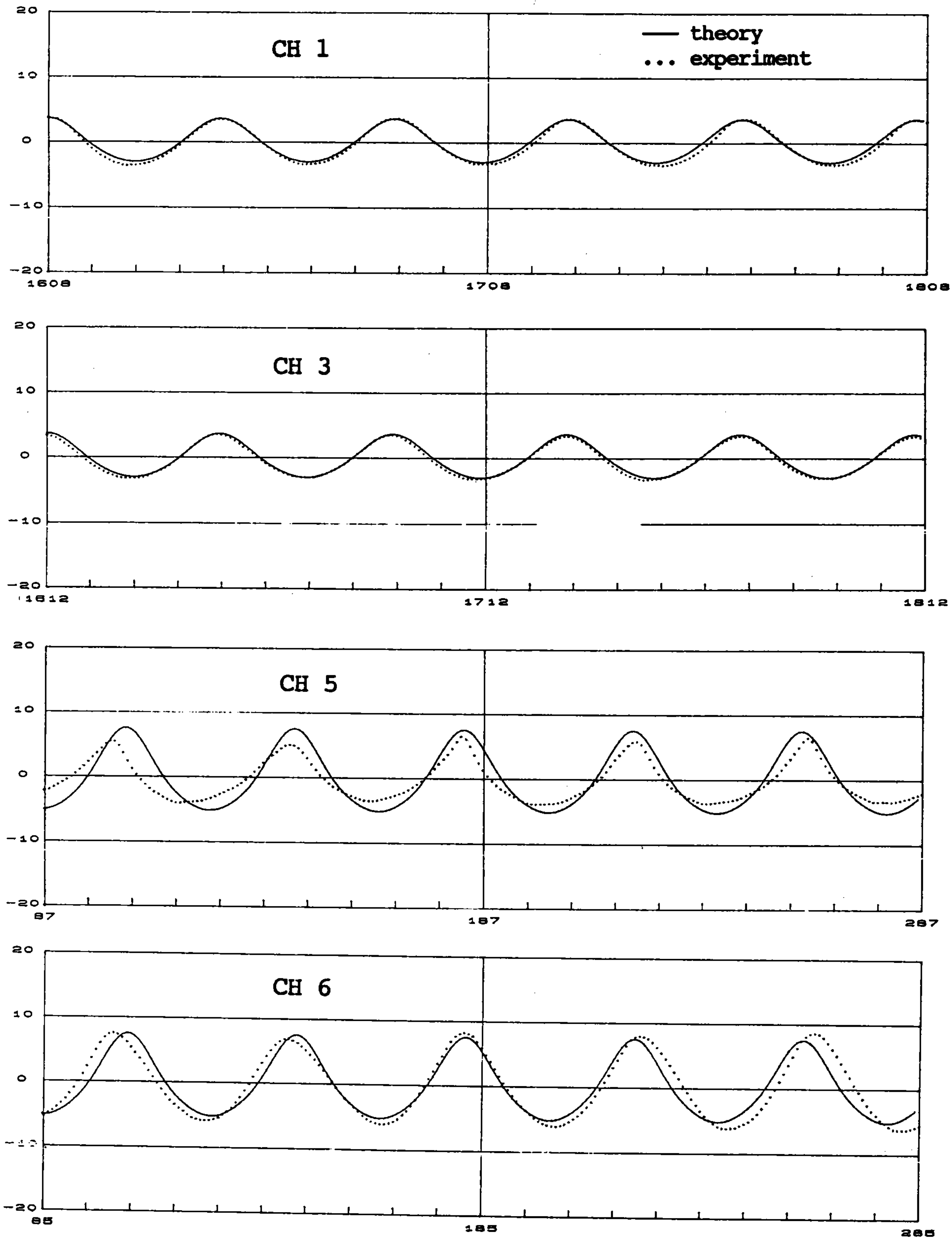


圖10-15

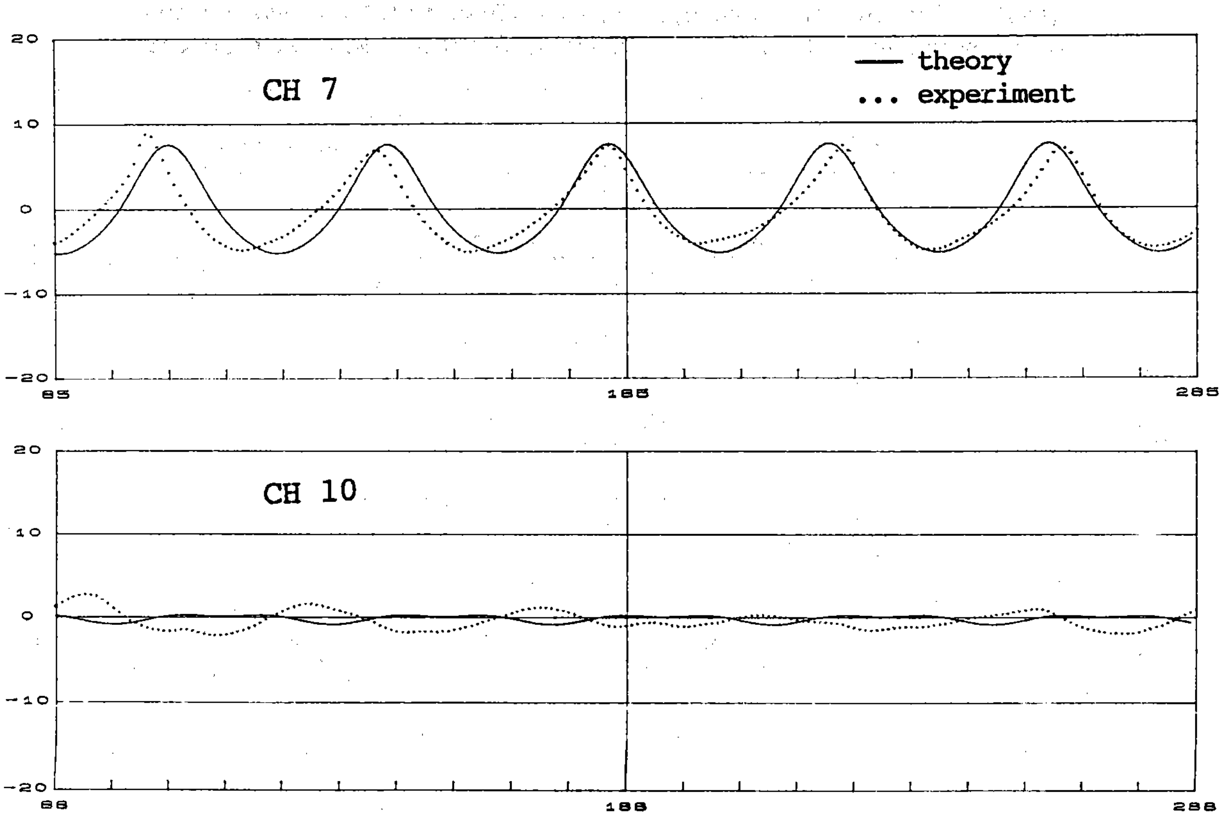
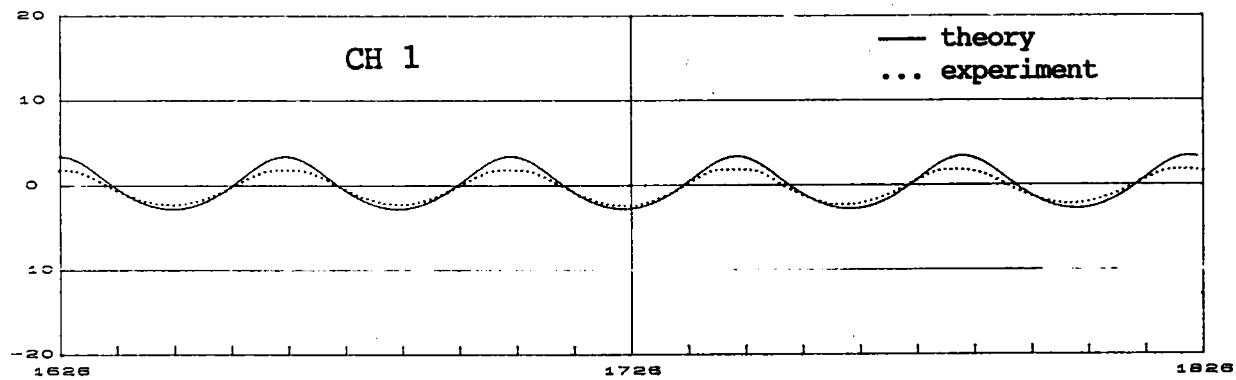
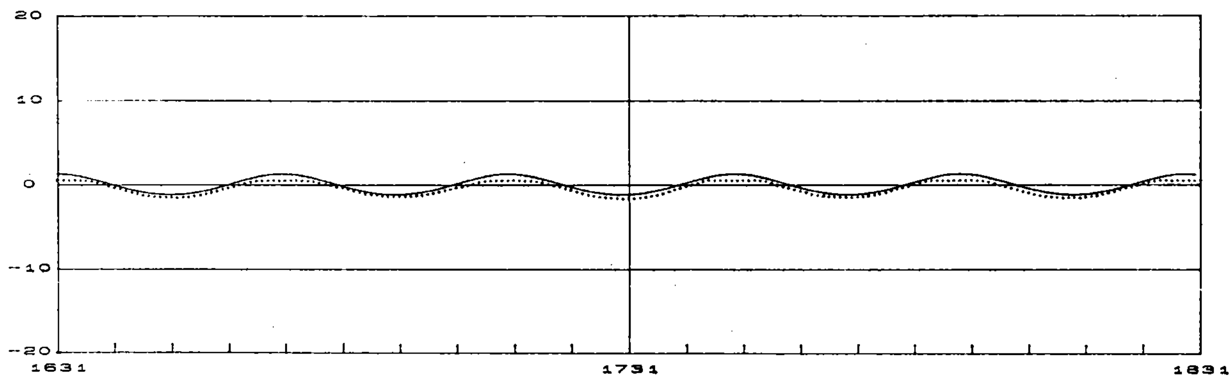


圖10-15

θ =45° (intersection angle), T_1=T_2=0.8sec(incident wave period 1 & 2)
d =50cm(water depth), H_1=4.14cm(incident wave height 1)
H_2=2.07cm(incident wave height 2)





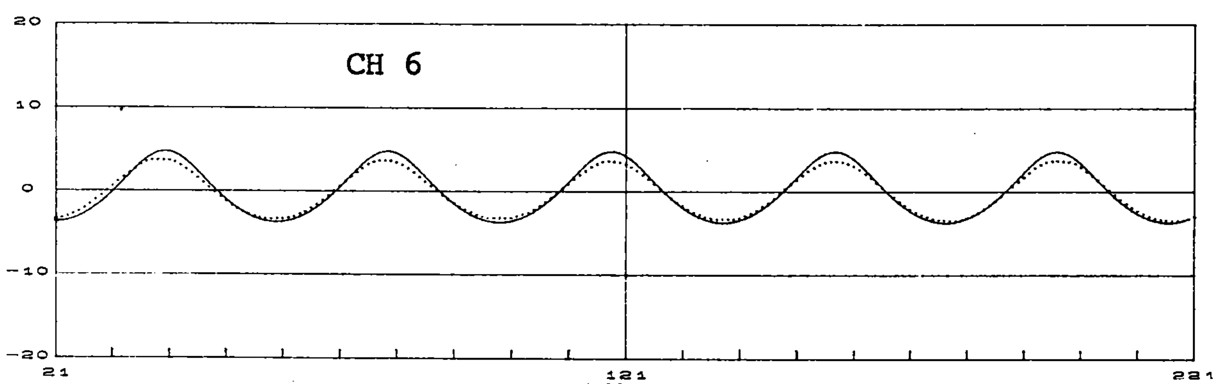


圖10-16

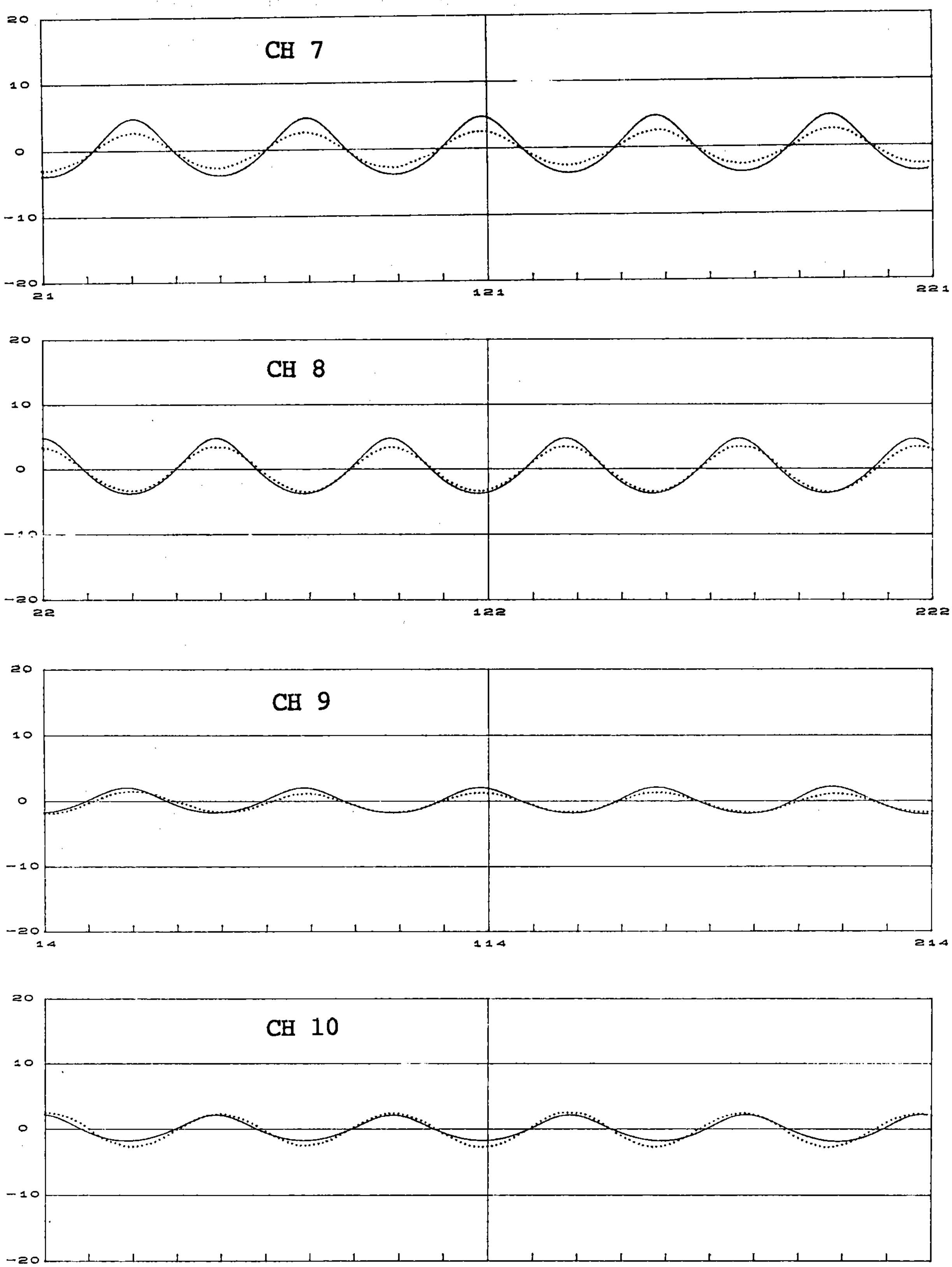

CH 7
20
10
0
-10
-20
21
121
221
CH 8
20
10
0
-20
22
122
222
CH 9
20
10
0
-10
-20
14
114
214
CH 10
20
10
0
-10
-20
14
114
214

圖10-16

θ =45° (intersection angle), $T_1=T_2=1.20$sec(incident wave period)
d =50cm(water depth), $H_1=H_2=9.82$cm(incident wave height)

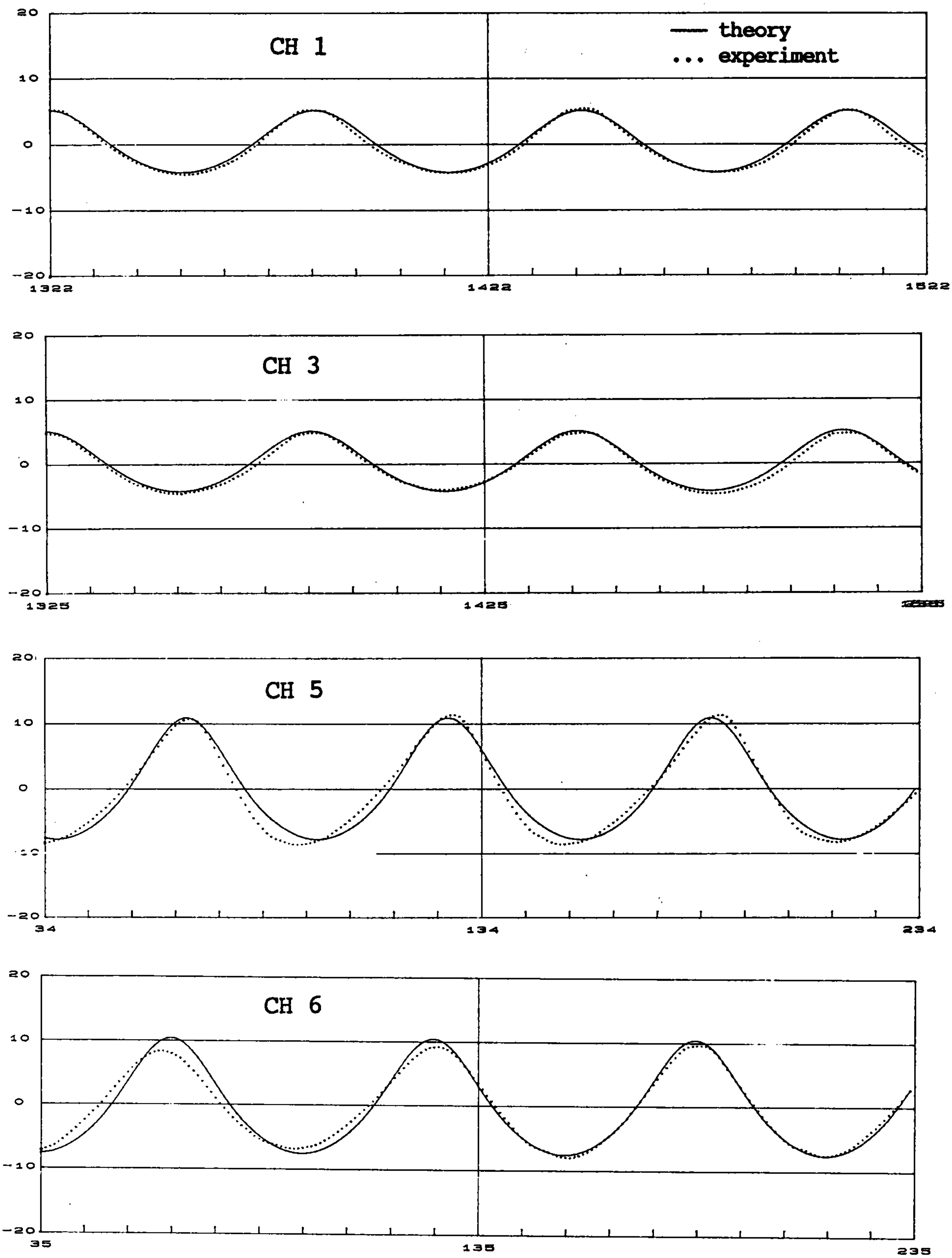


圖10-17

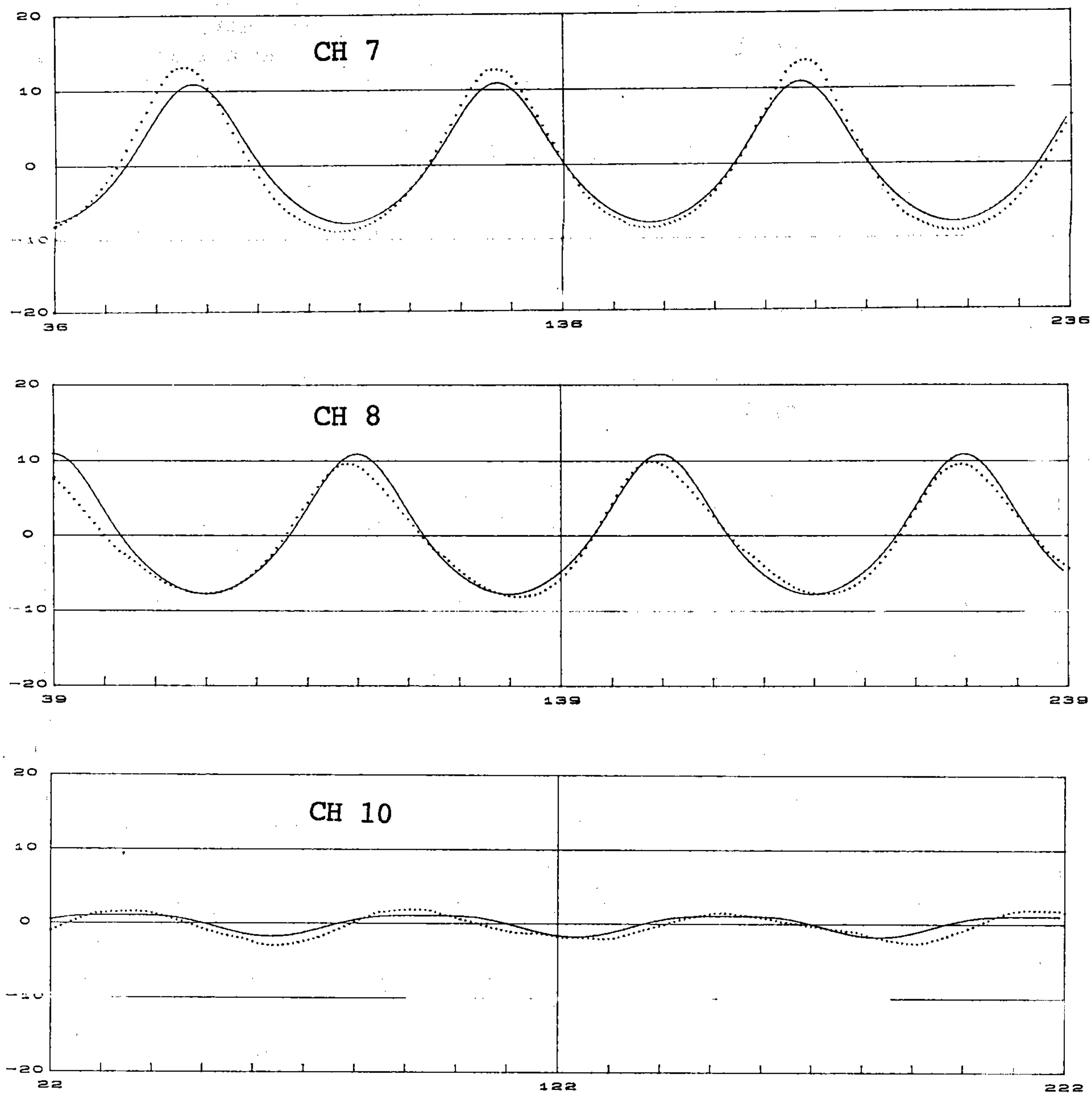

CH 7
20
10
0
-10
-20
36
136
236
CH 8
20
10
0
-10
-20
39
139
239
CH 10
20
10
0
-20
22
122
222

圖10-17

θ=45° (intersection angle), $T_1=T_2$=1.20sec(incident wave period 1 & 2)
d =50cm(water depth), H_1=4.87cm(incident wave height 1)
H_2=2.41cm(incident wave height 2)

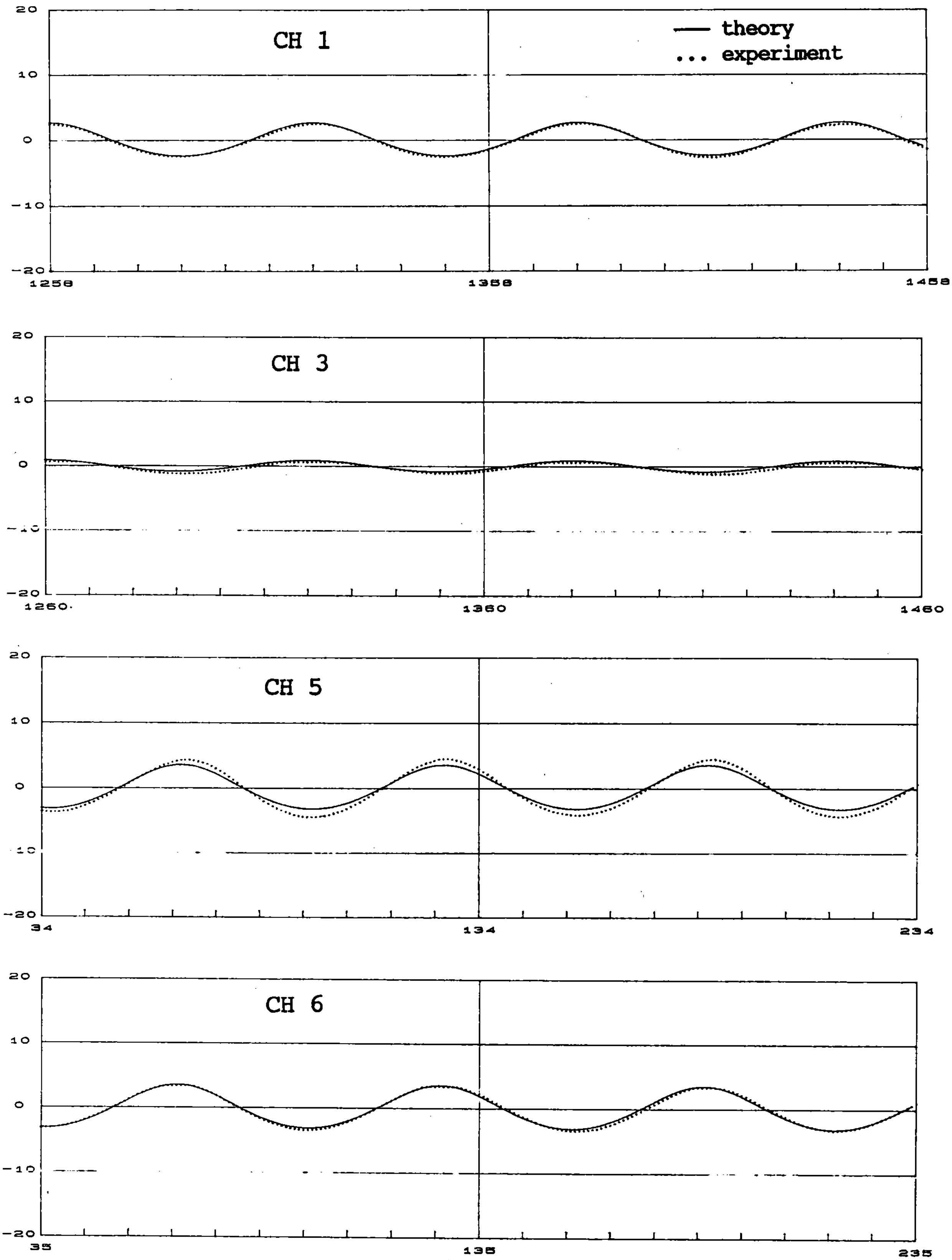


圖10-18

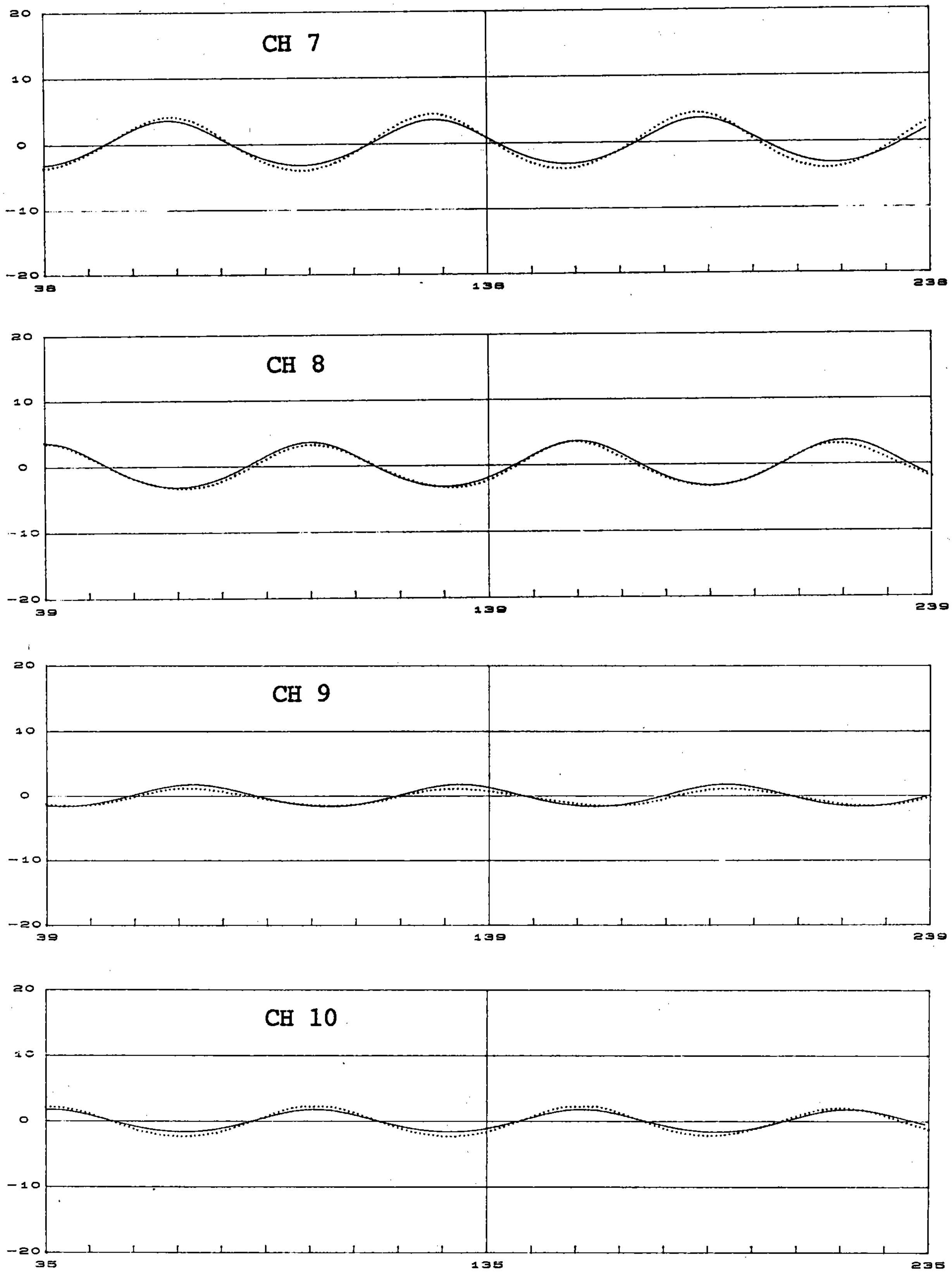

CH 7
20
10
0
-10
-20
38
138
238
CH 8
20
10
0
-10
-20
39
139
239
CH 9
20
10
0
-10
-20
39
139
239
CH 10
20
10
0
-10
-20
35
135
235

圖10-18

θ =45° (intersection angle), T_1=T_2=1.20sec(incident wave period 1 & 2)
d =50cm(water depth), H_1=9.57cm(incident wave height 1)
H_2=2.84cm(incident wave height 2)

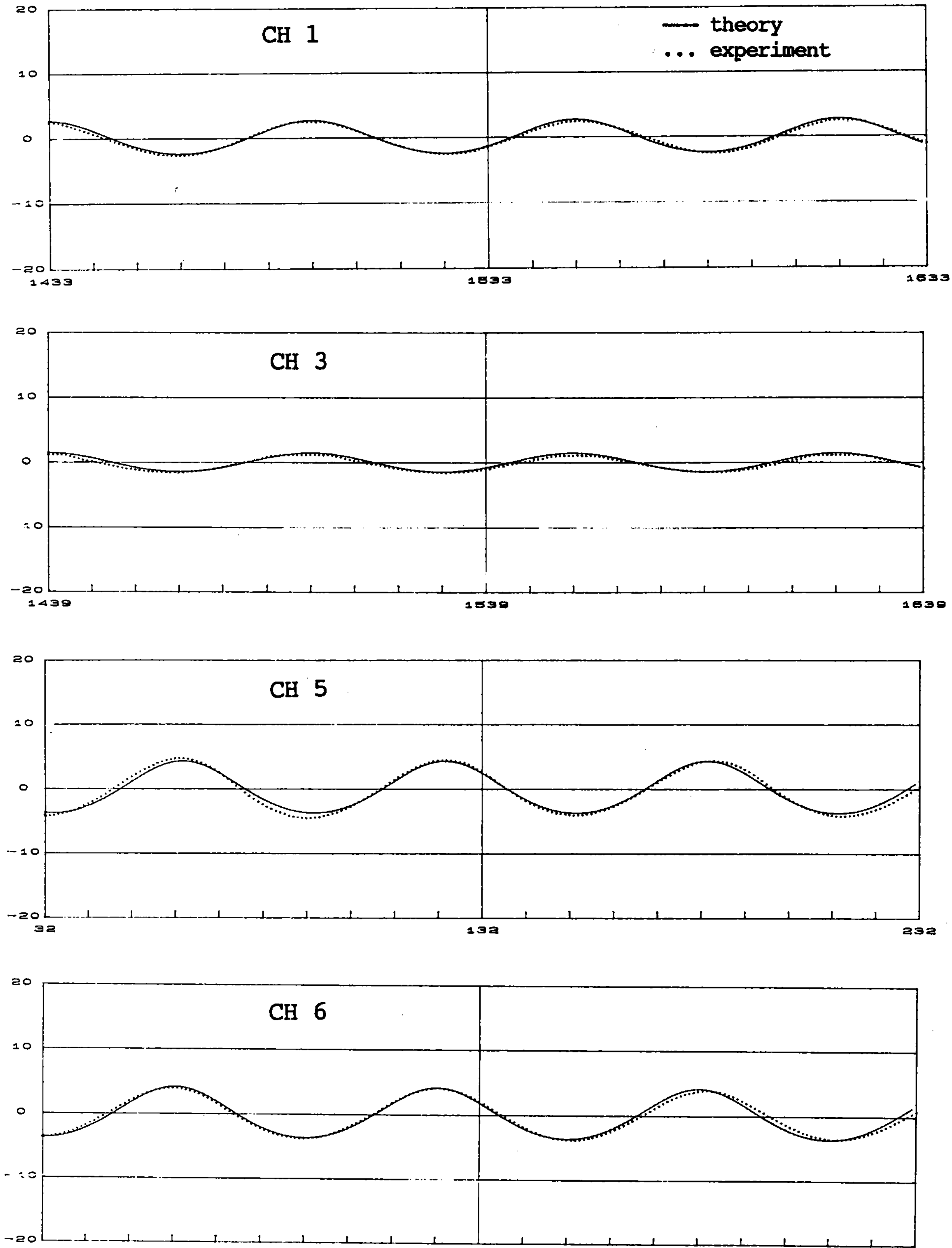


圖10-19

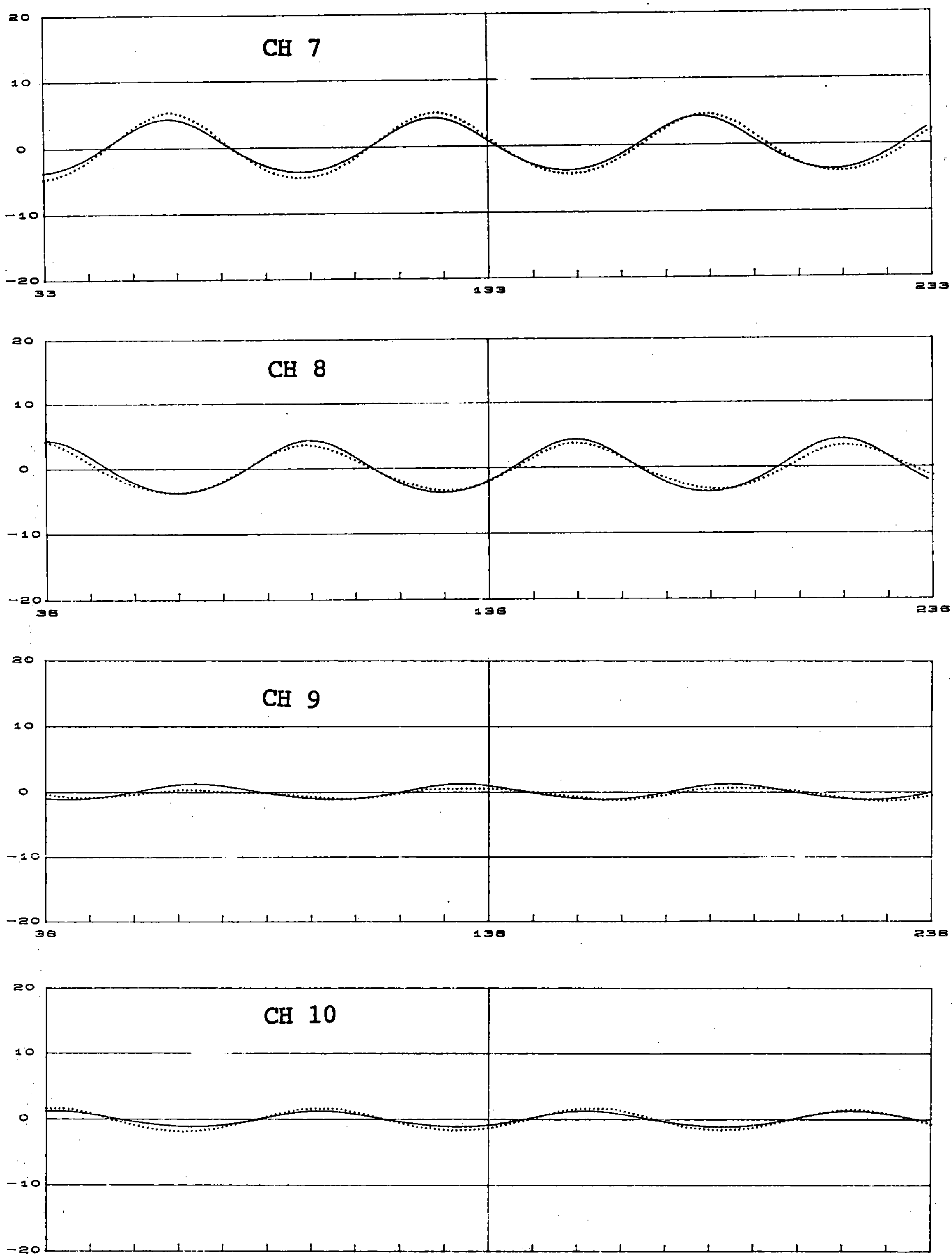

CH 7
20
10
0
−10
−20
33
133
233
CH 8
20
10
0
−10
−20
36
136
236
CH 9
20
10
0
−10
−20
38
138
238
CH 10
20
10
0
−10
−20
28
128
228

圖10-19

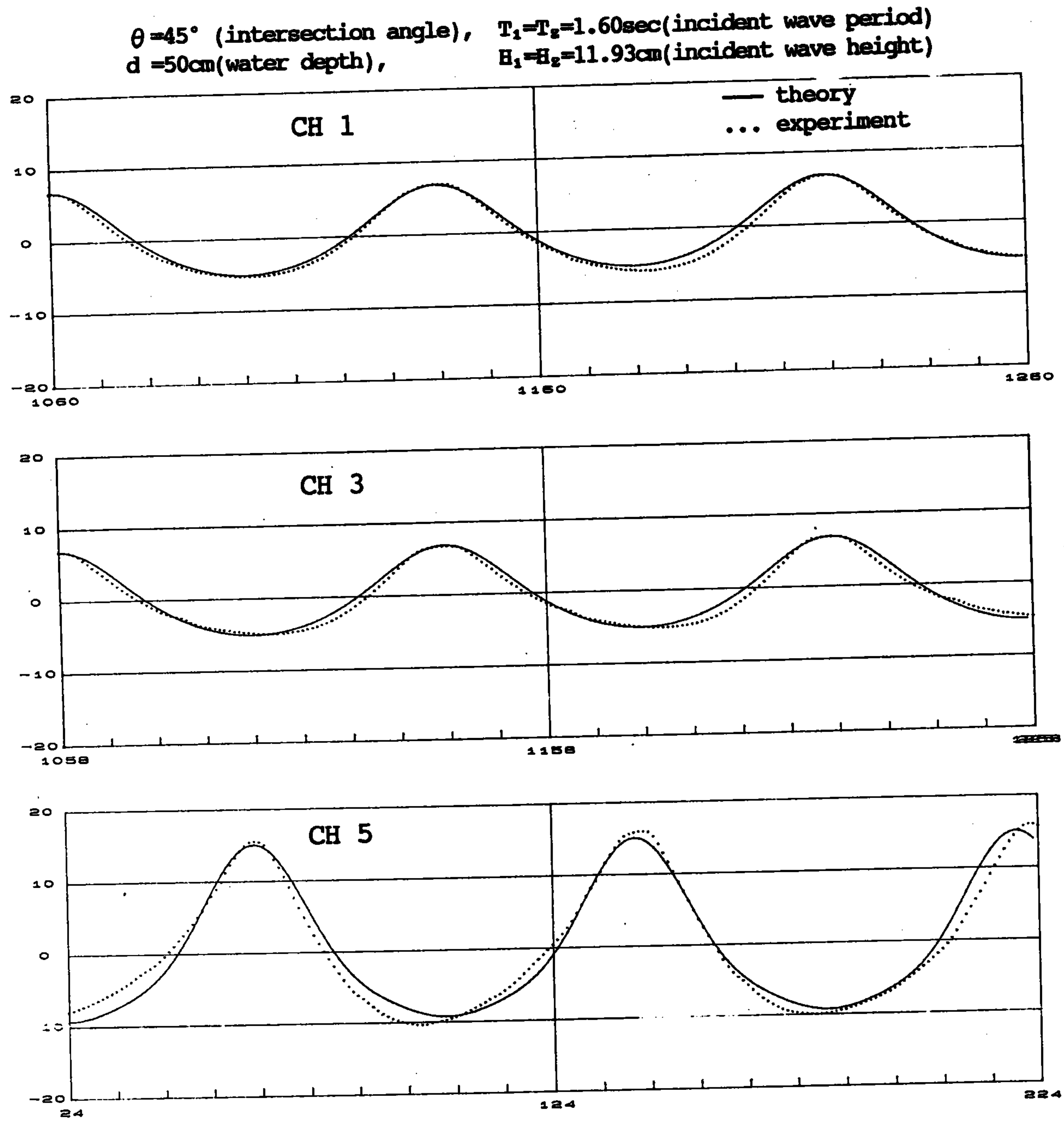
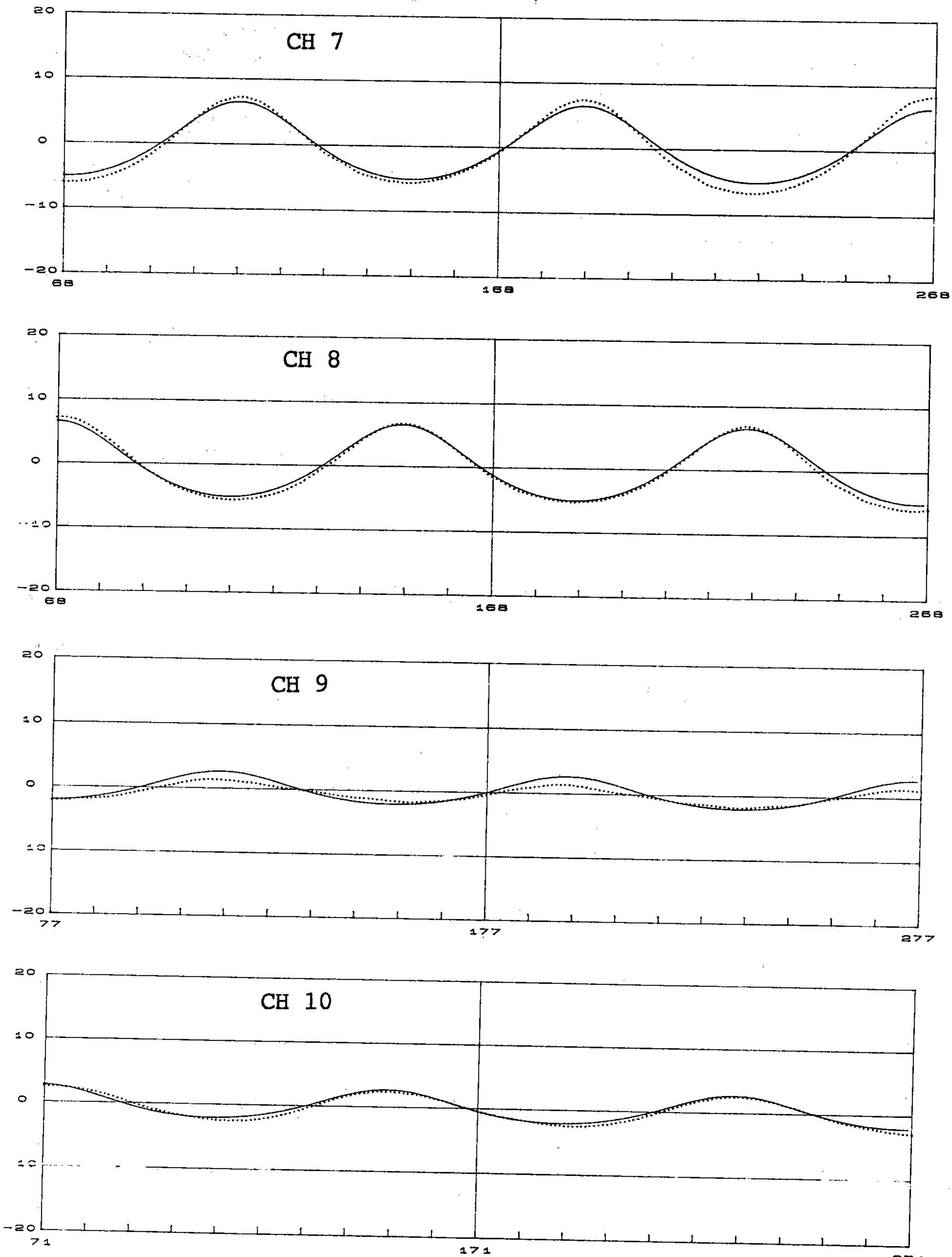

θ=45° (intersection angle), T₁=T₂=1.60sec(incident wave period)
d =50cm(water depth), H₁=H₂=11.93cm(incident wave height)
— theory
... experiment
CH 1
20
10
0
-10
-20
1060
1150
1260
CH 3
20
10
0
-10
-20
1058
1158
CH 5
20
10
0
-20
24
124
224

圖10-20



CH 7
20
10
0
−10
−20
68
168
268
CH 8
20
10
0
−10
−20
68
168
268
CH 9
20
10
0
10
−20
77
177
277
CH 10
20
10
0
10
−20
71
171
271

圖10-21

θ=45° (intersection angle), T_1=T_2=1.60sec(incident wave period 1 & 2)
d =50cm(water depth), H_1=13.40cm(incident wave height 1)
H_2=4.02cm(incident wave height 2)

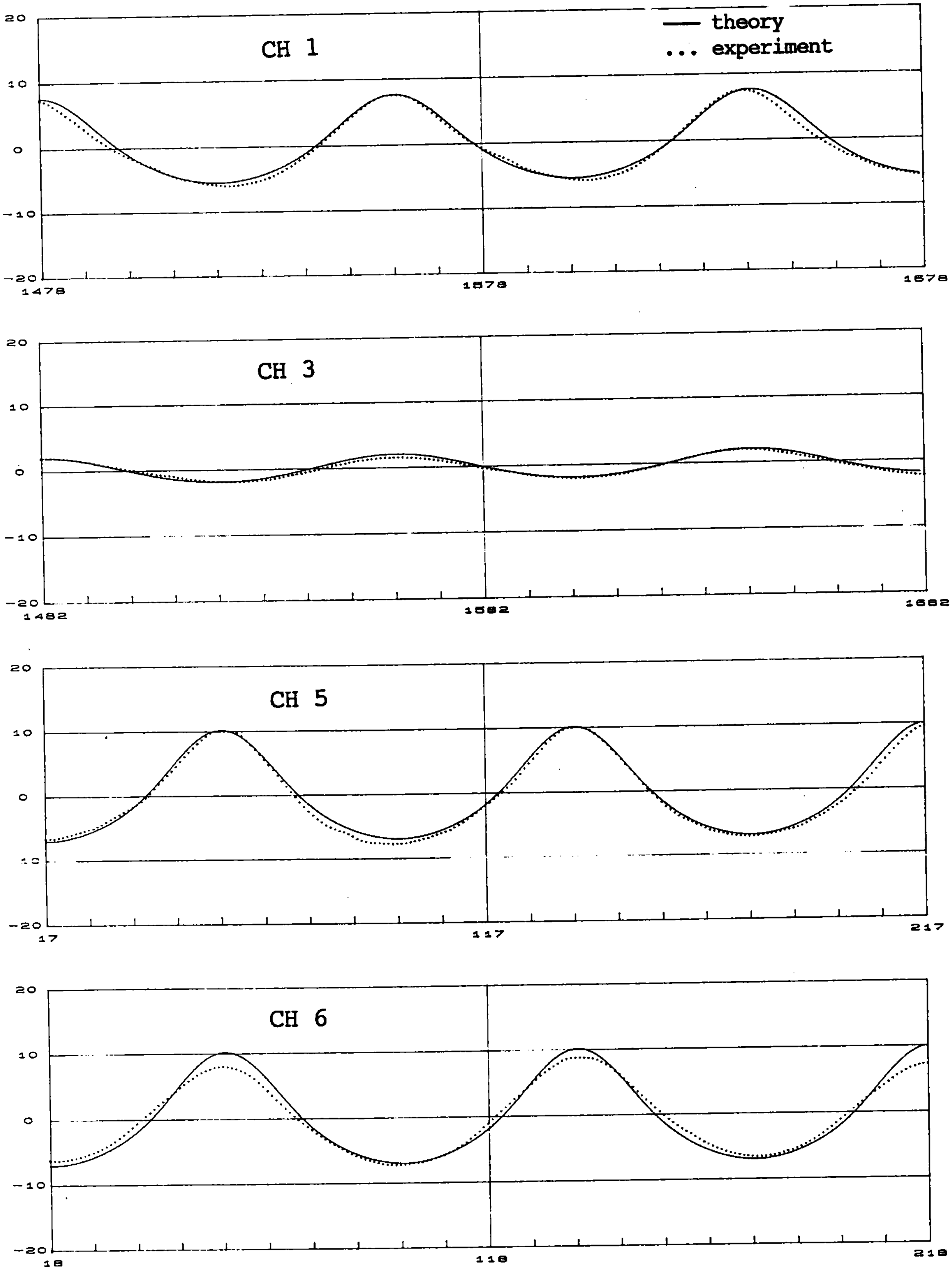


圖10-22

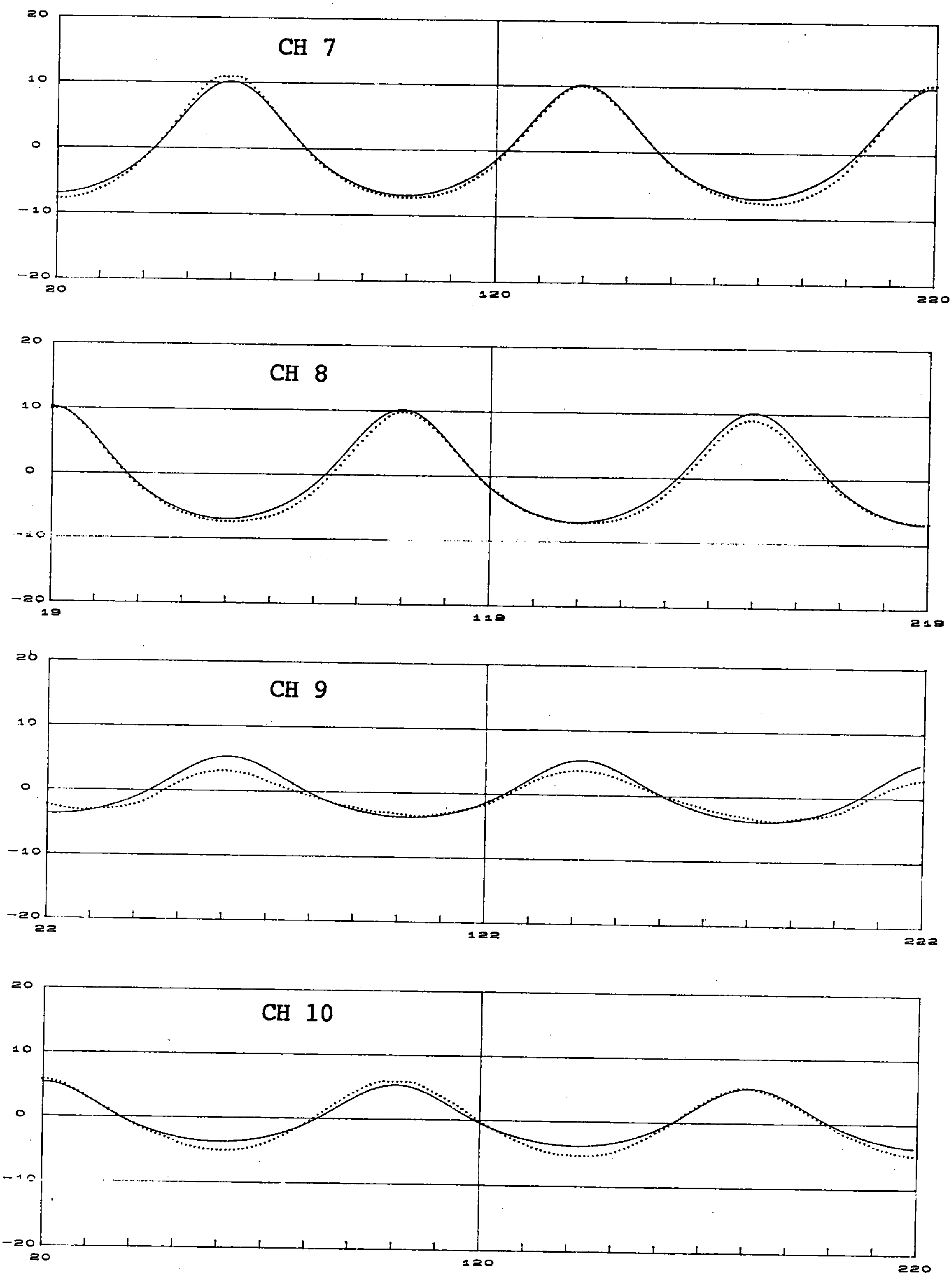

20
CH 7
10
0
-10
-20
20
120
220
20
CH 8
10
0
-10
-20
19
119
219
20
CH 9
10
0
-10
-20
22
122
222
20
CH 10
10
0
-10
-20
20
120
220

圖10-22

θ=45° (intersection angle), $T_1=T_2$=2.00sec(incident wave period)
d =50cm(water depth), $H_1=H_2$=11.26cm(incident wave height)

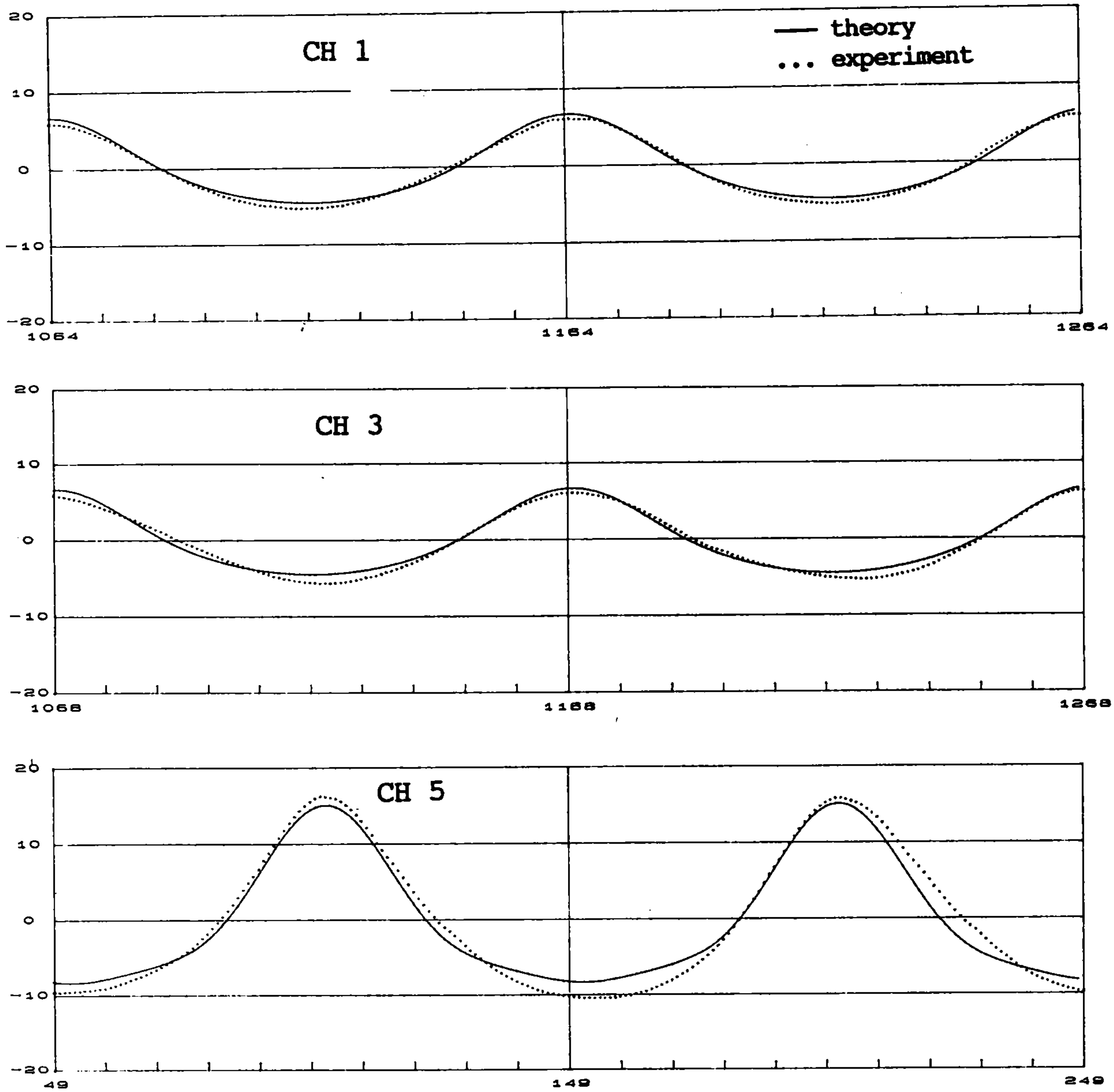


圖10-23

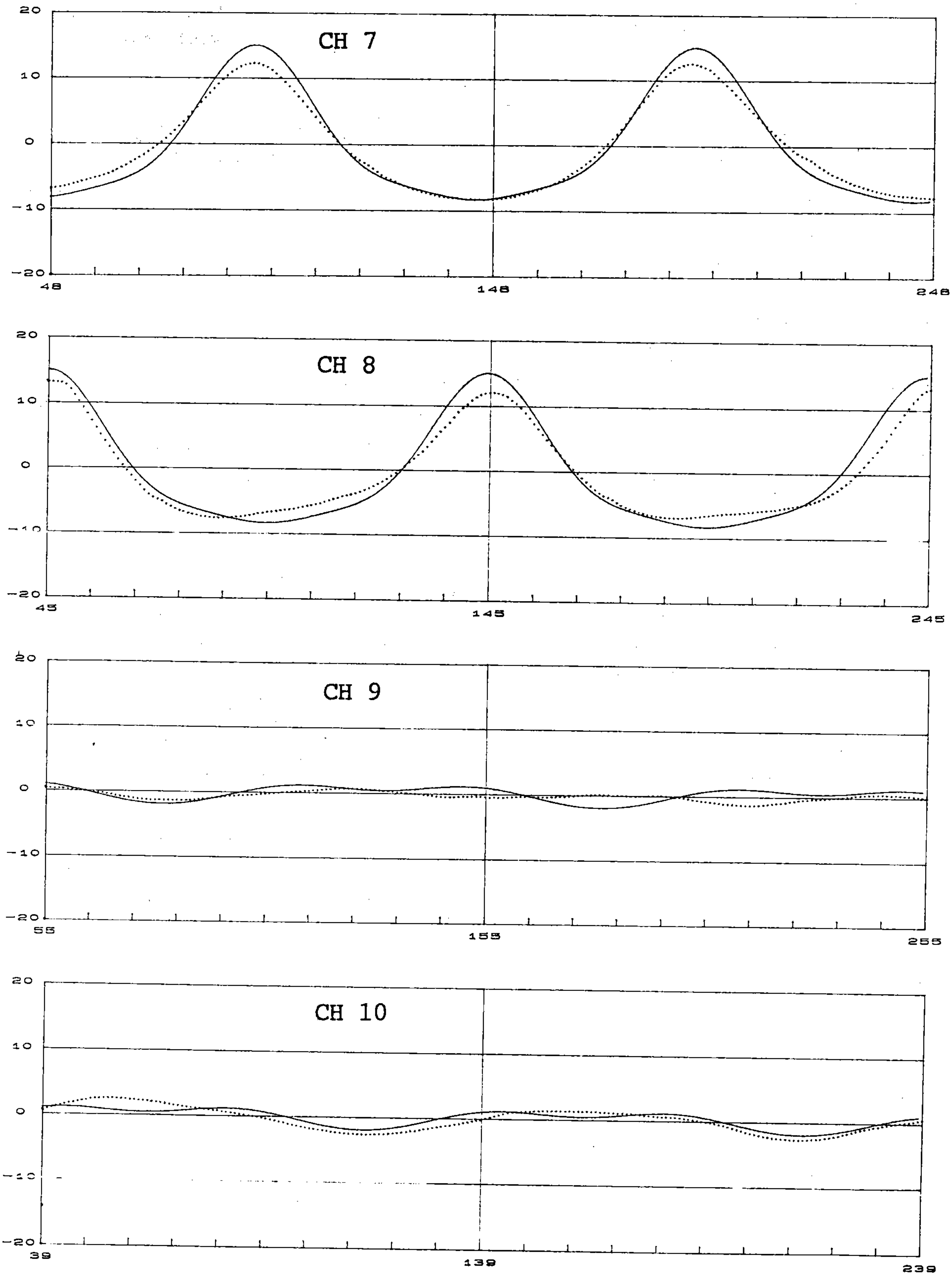

CH 7
20
10
0
−10
−20
48
148
248
CH 8
20
10
0
−20
45
145
245
CH 9
20
10
0
−10
−20
55
155
255
CH 10
20
10
0
−20
39
139
239

圖10-23

θ=45° (intersection angle), T_1=T_2=2.00sec(incident wave period 1 & 2)
d =50cm(water depth), H_1=6.16cm(incident wave height 1)
H_2=3.08cm(incident wave height 2)

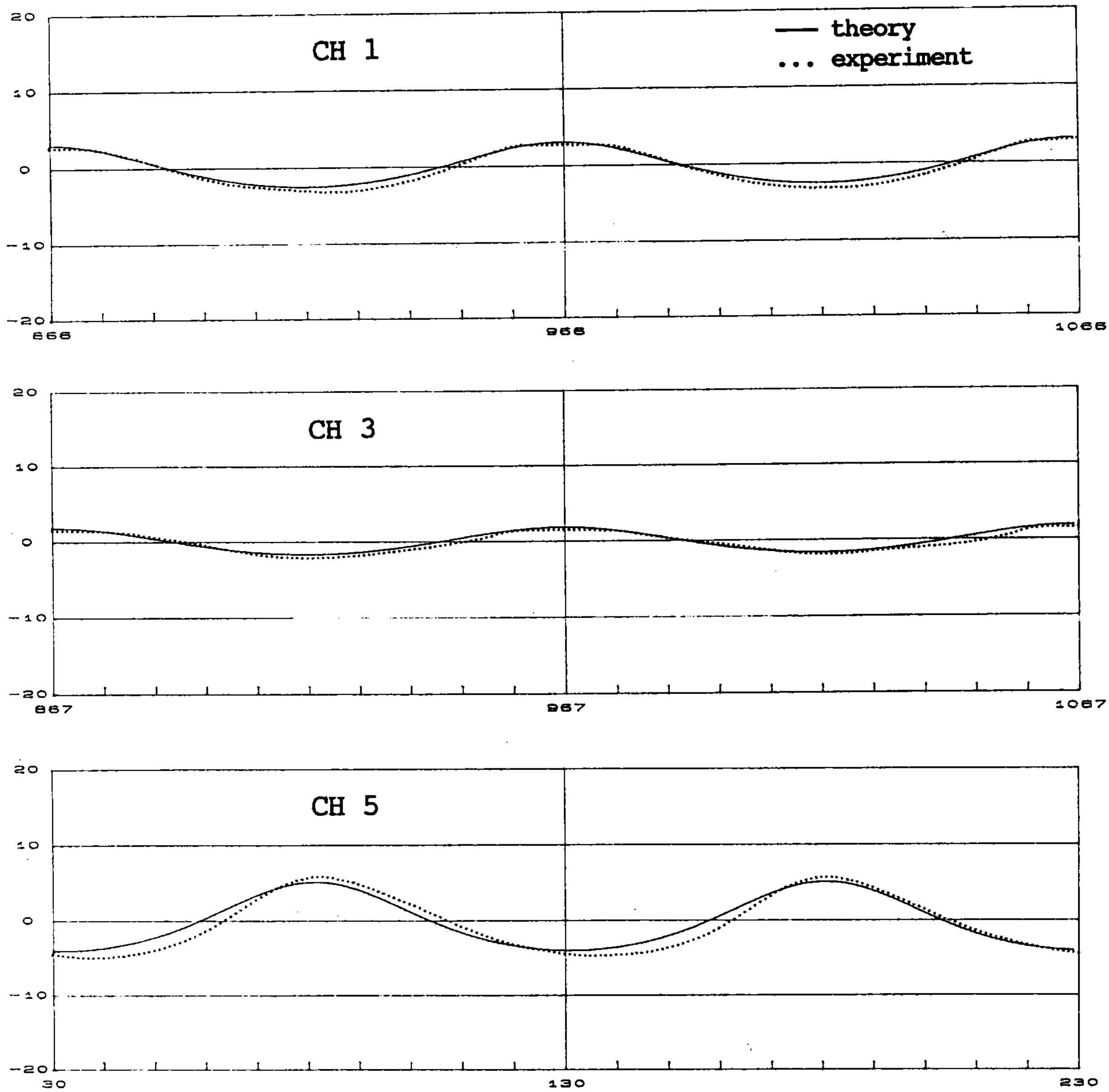


圖10-24

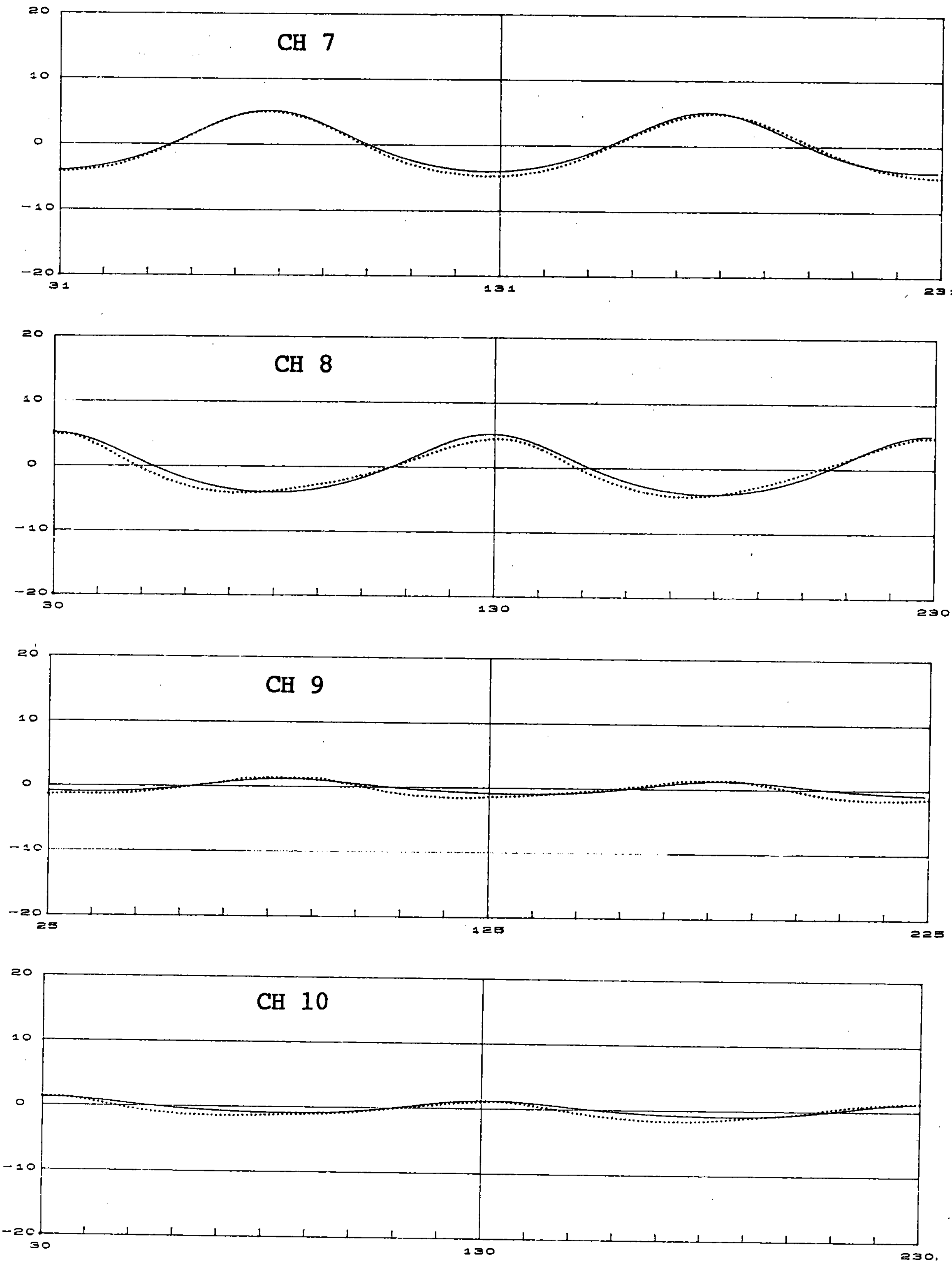

CH 7
20
10
0
-10
-20
31
131
231
CH 8
20
10
0
-10
-20
30
130
230
CH 9
20
10
0
-10
-20
25
125
225
CH 10
20
10
0
-10
-20
30
130
230

圖10-24

θ=45° (intersection angle), T_1=T_2=2.00sec(incident wave period 1 & 2)
d =50cm(water depth), H_1=12.14cm(incident wave height 1)
H_2=3.23cm(incident wave height 2)

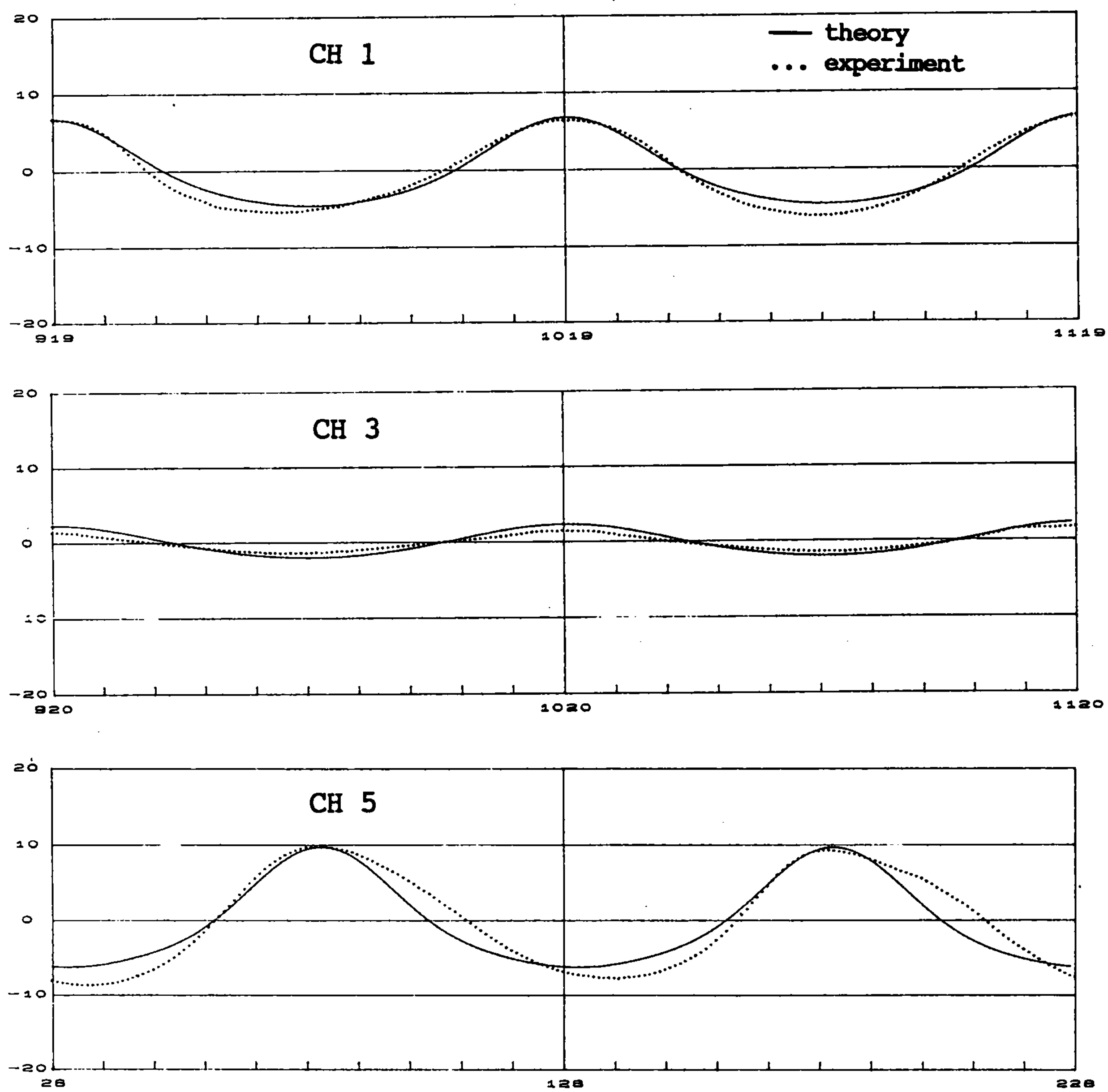


圖10-25

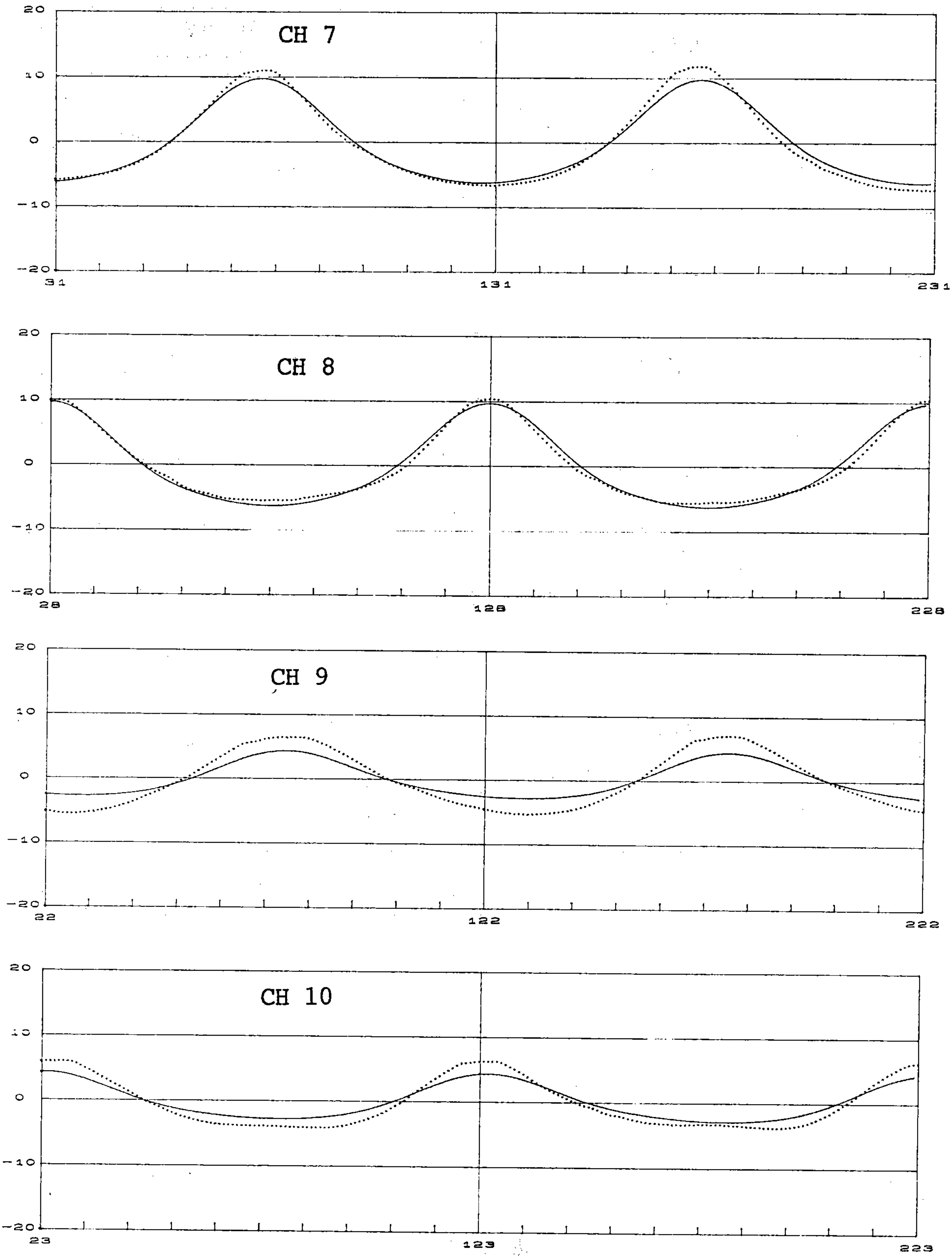

CH 7
20
10
0
-10
-20
31
131
231
CH 8
20
10
0
-10
-20
28
128
228
CH 9
20
10
0
-10
-20
22
122
222
CH 10
20
10
0
-10
-20
23
123
223

圖10-25

θ=45° (intersection angle), T_1=T_2=2.40sec(incident wave period)
d =50cm(water depth), H_1=H_2=14.74cm(incident wave height)

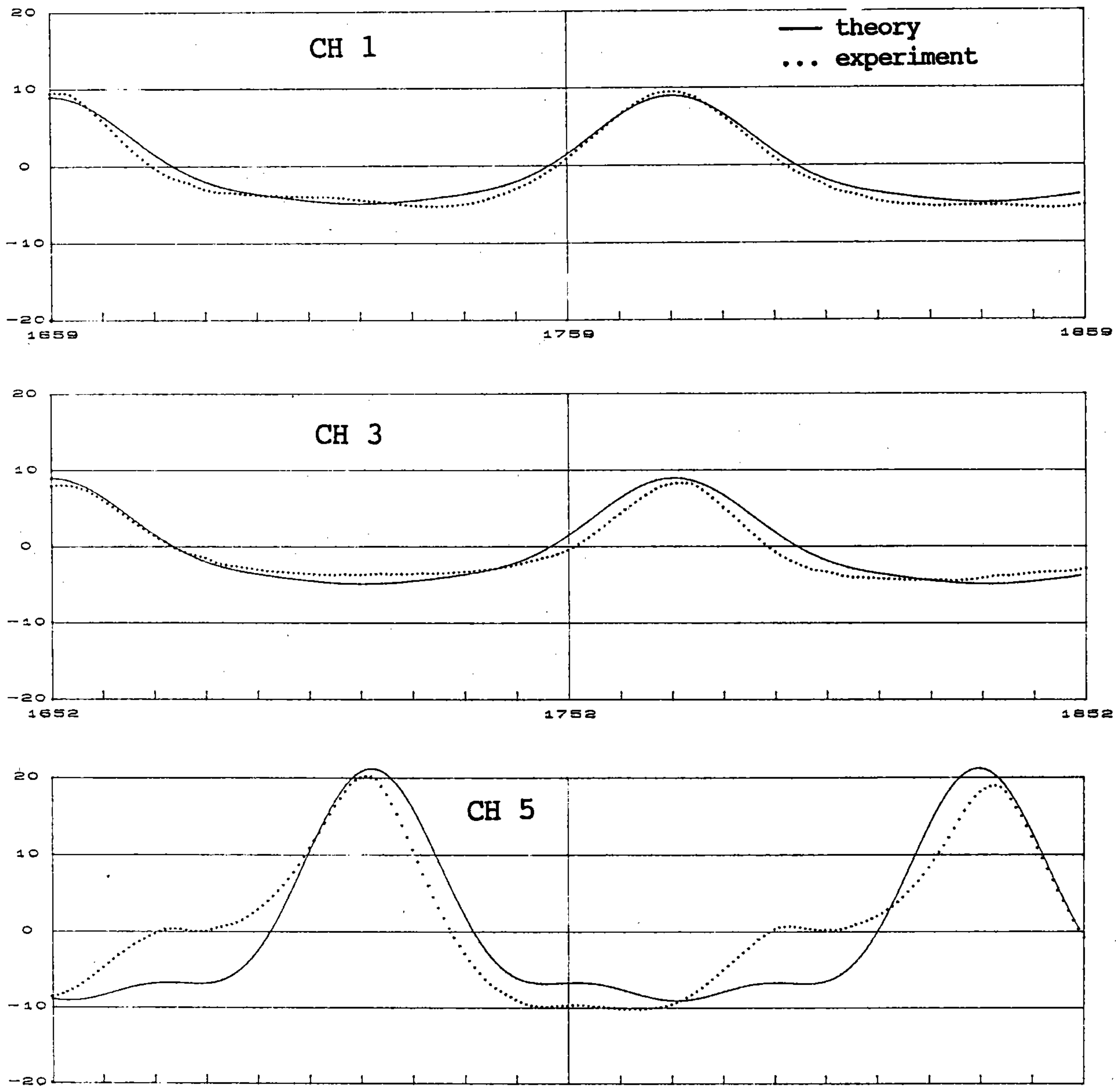


圖10-26

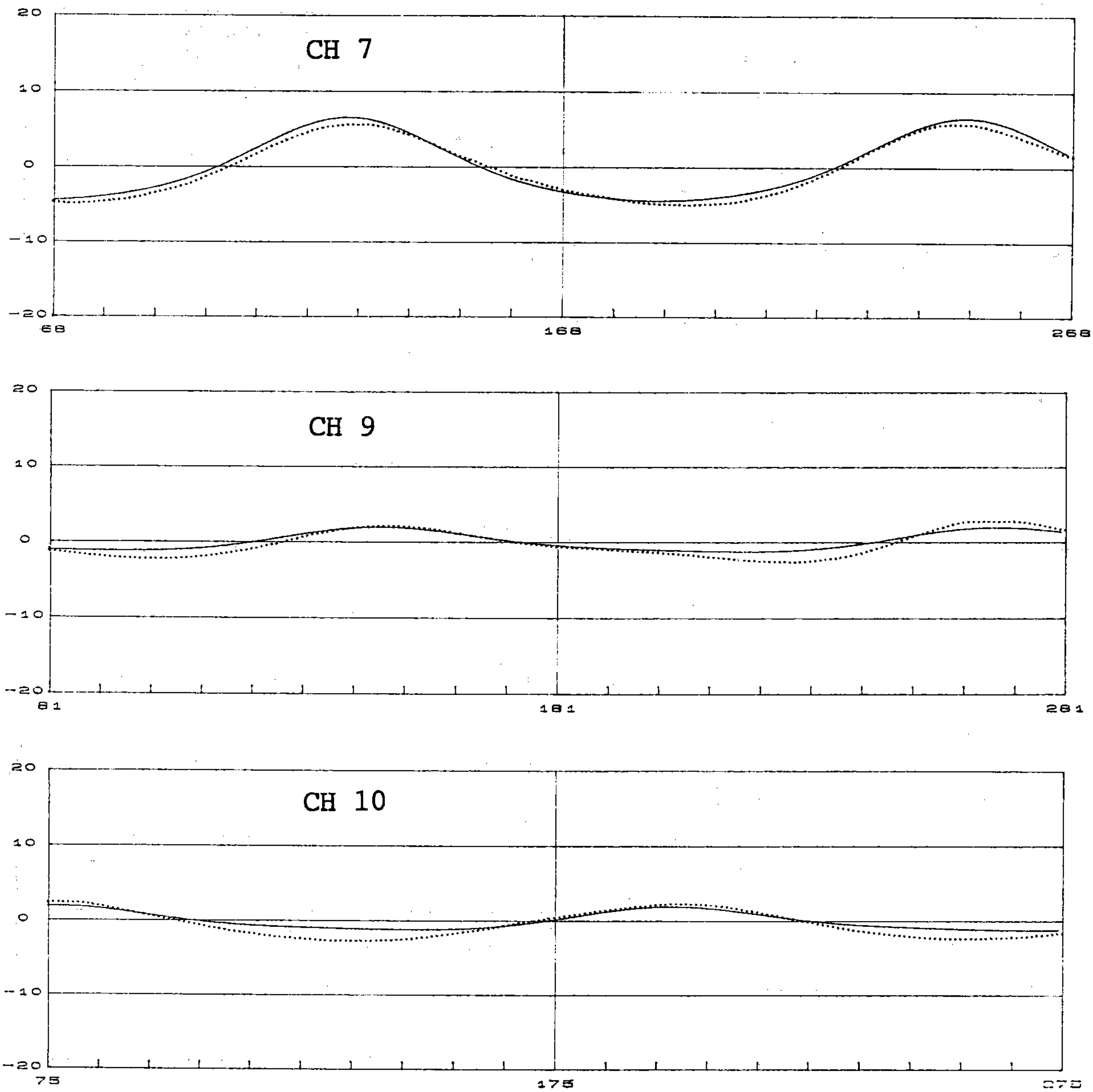

CH 7
20
10
0
-10
-20
68
168
268
CH 9
20
10
0
-10
-20
81
181
281
CH 10
20
10
0
-10
-20
75
175

圖10-27

θ=45° (intersection angle), T_1=T_2=2.40sec(incident wave period 1 & 2)
d =50cm(water depth), H_1=13.27cm(incident wave height 1)
H_2=4.93cm(incident wave height 2)

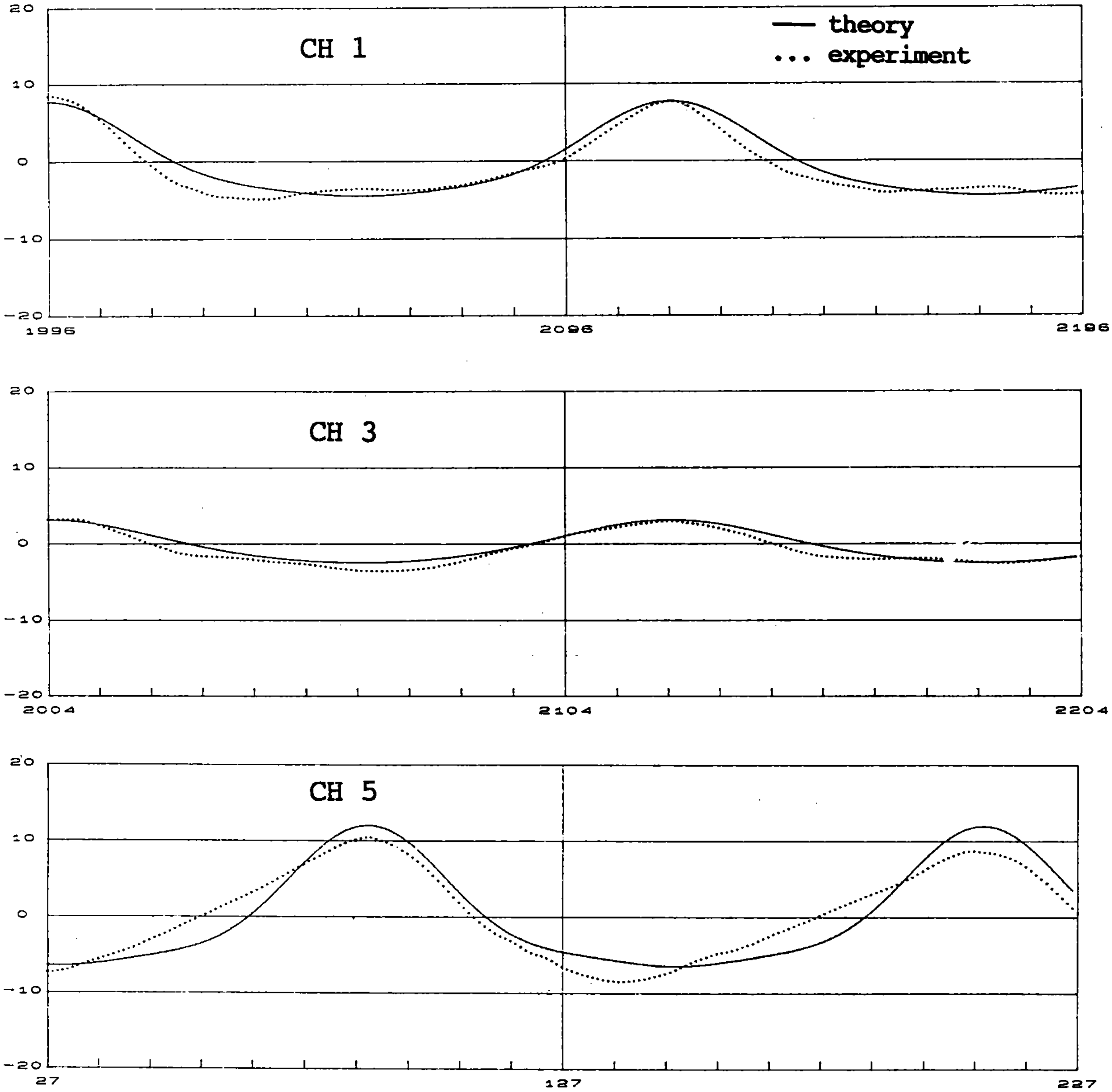


圖10-28

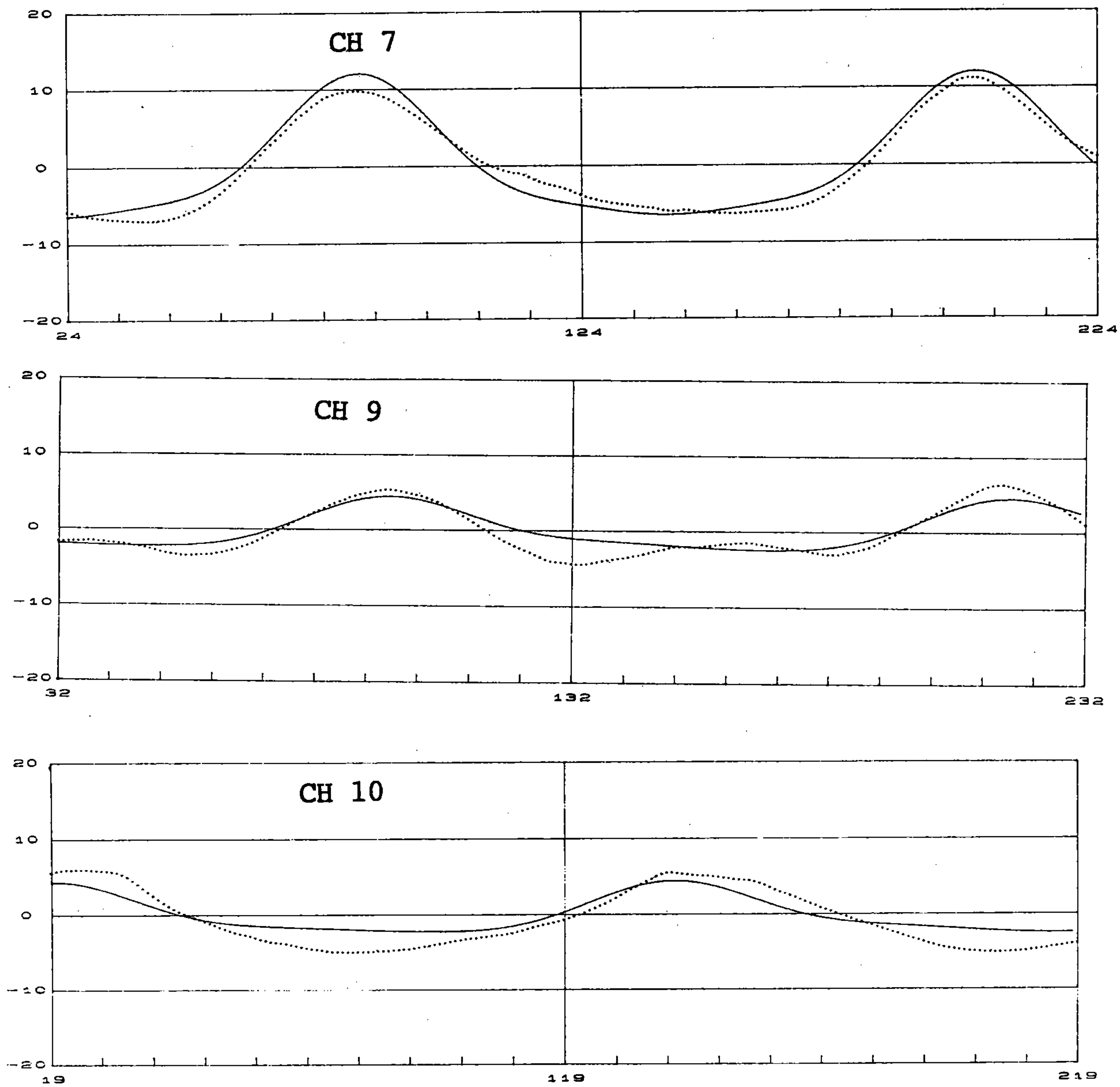

CH 7
20
10
0
−10
−20
24
124
224
CH 9
20
10
0
−10
−20
32
132
232
CH 10
20
10
0
−10
−20
19
119
219

圖10-28

θ=45° (intersection angle), T_1=T_2=0.8sec(incident wave period)
d =40cm(water depth), H_1=H_2=6.13cm(incident wave height)

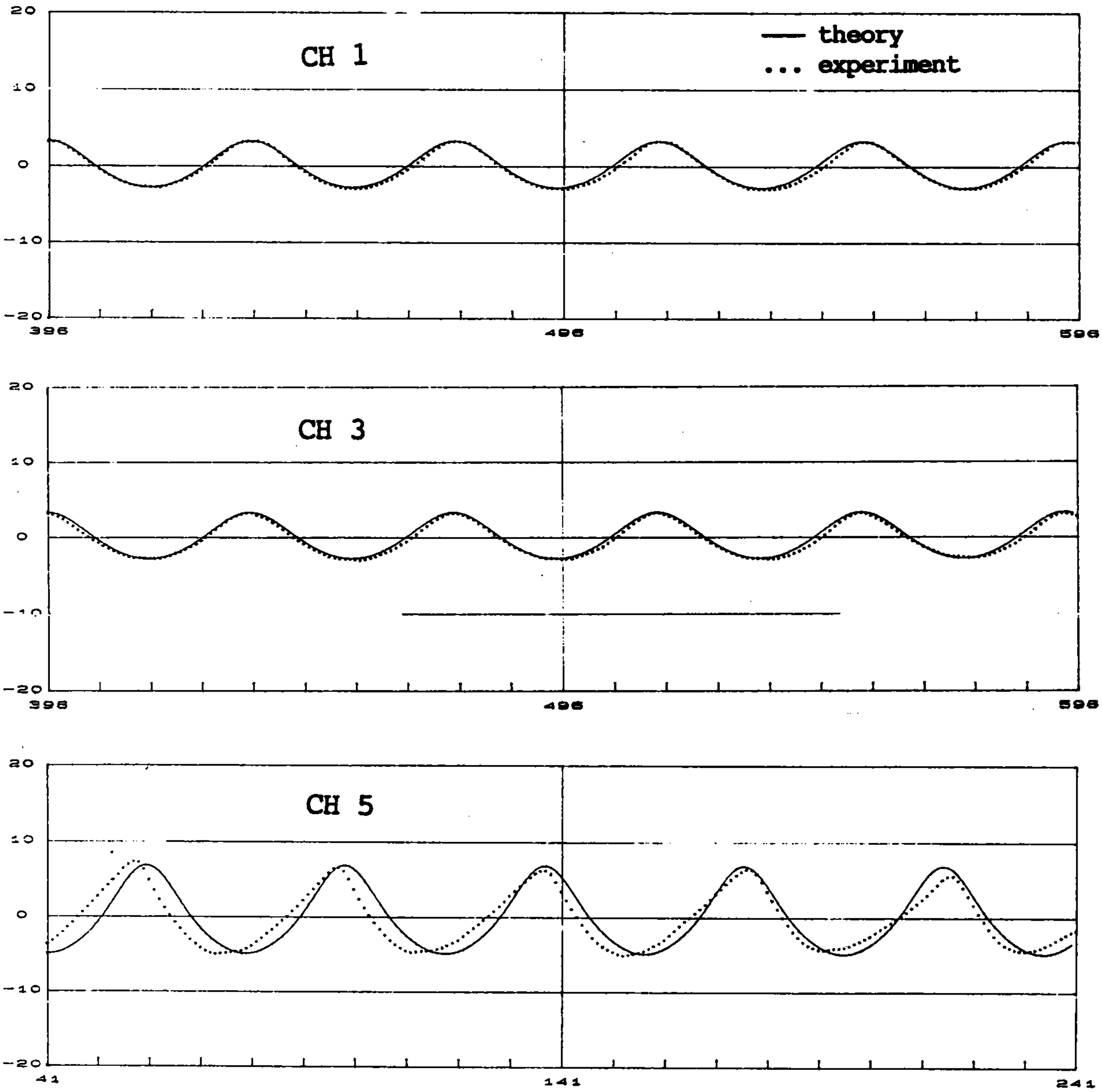


圖10-29

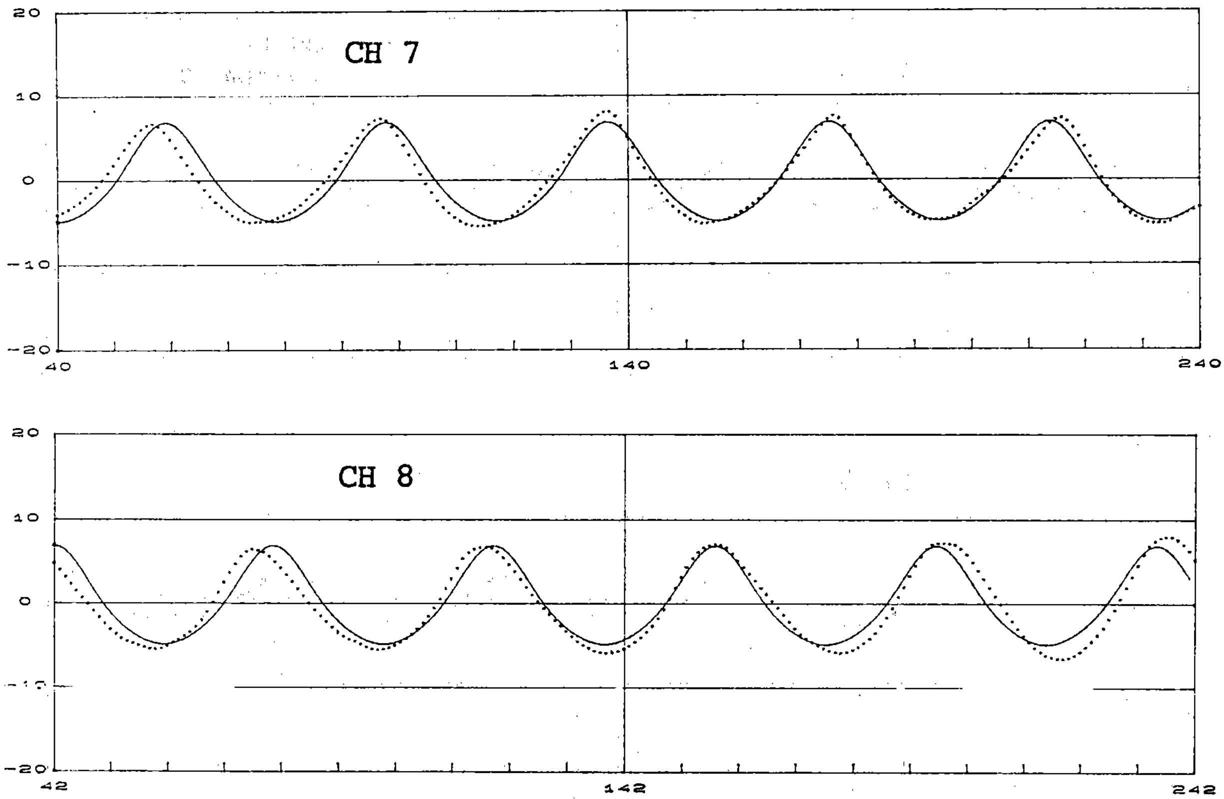


圖10-29

θ =45° (intersection angle), T_1=T_2=0.8sec(incident wave period 1 & 2)
d =40cm(water depth), H_1=6.68cm(incident wave height 1)
H_2=2.15cm(incident wave height 2)

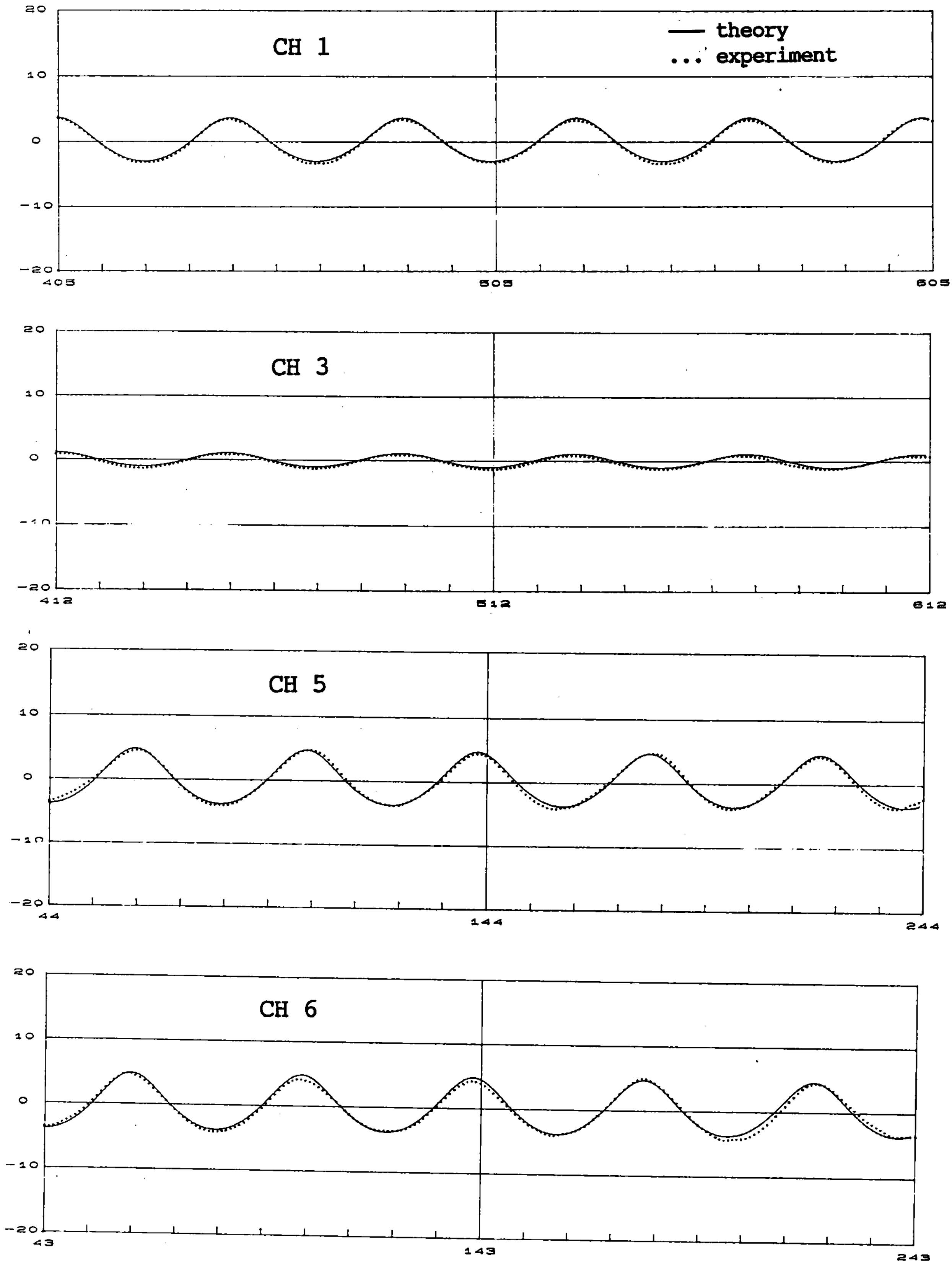


圖10-30

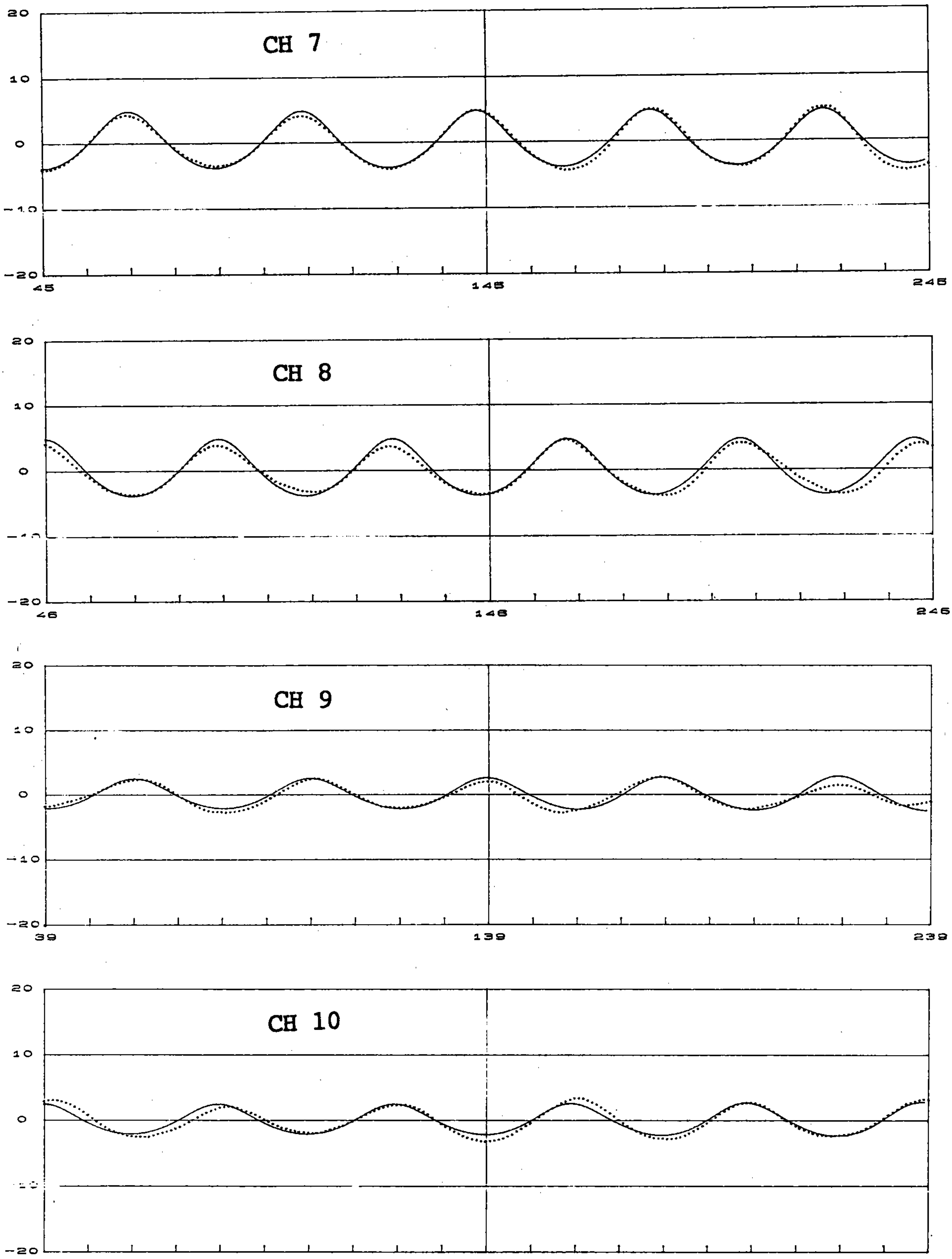

CH 7
20
10
0
-10
-20
45
145
245
CH 8
20
10
0
-20
46
146
246
CH 9
20
10
0
-10
-20
39
139
239
CH 10
20
10
0
-20
39
139
239

圖10-30

θ=45° (intersection angle), T_1=T_2=1.20sec(incident wave period)
d =40cm(water depth), H_1=H_2=9.45cm(incident wave height)

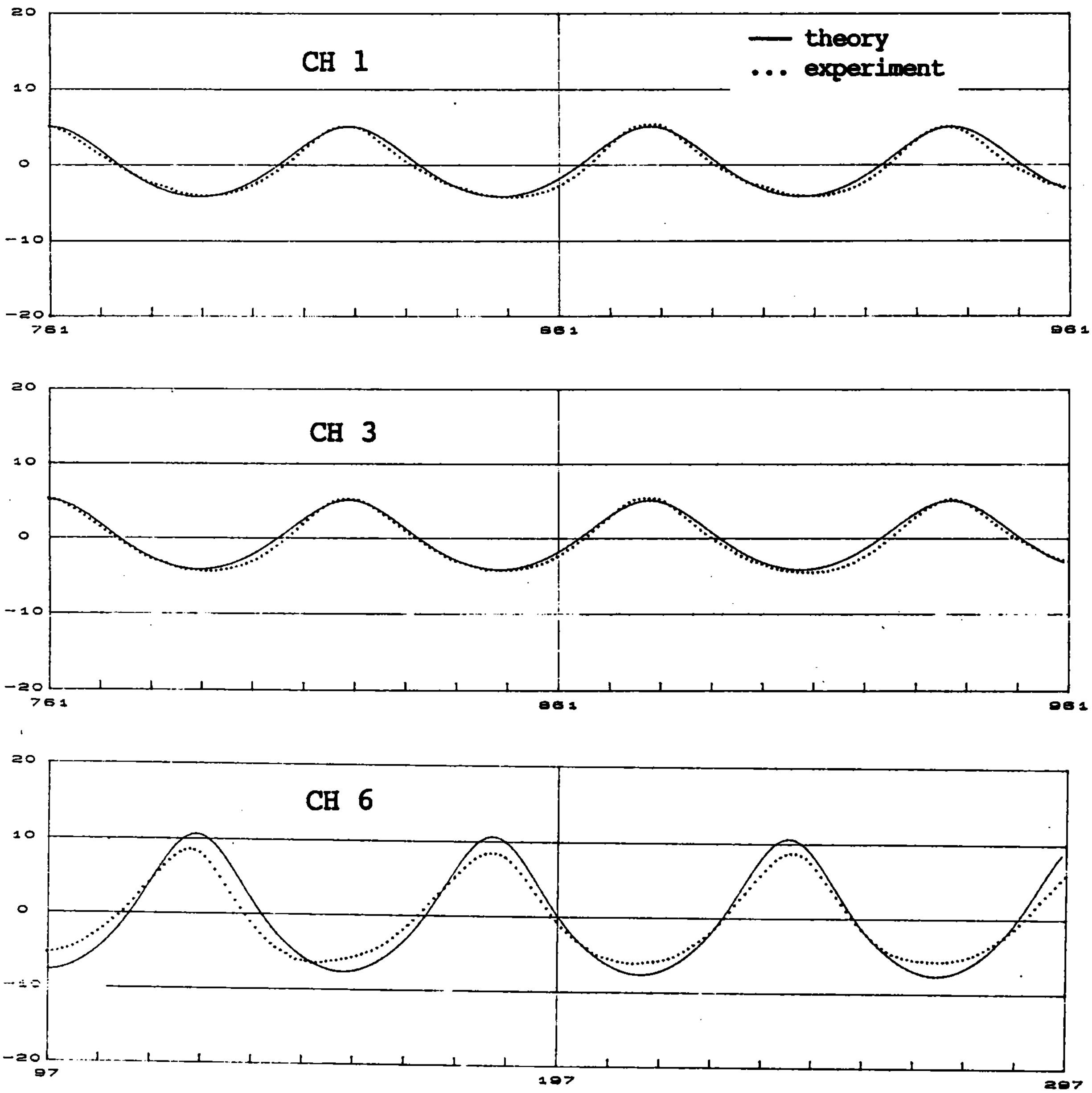
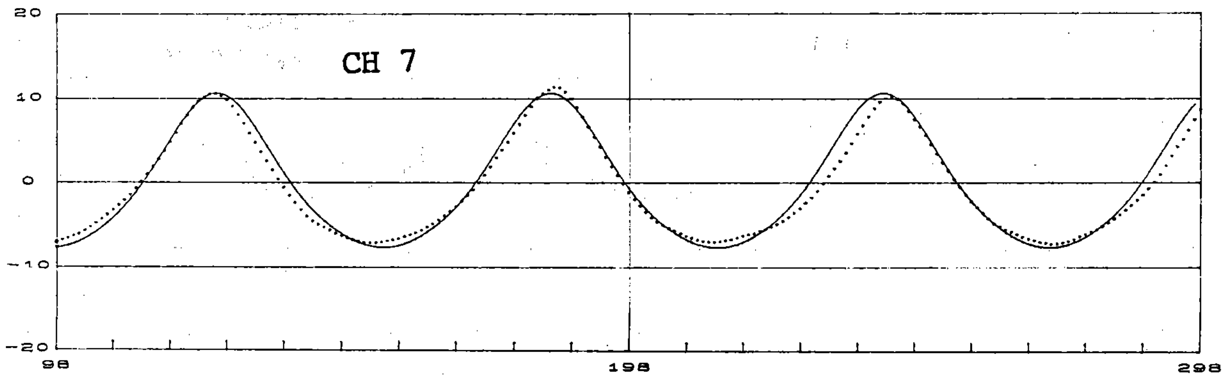


圖10-31



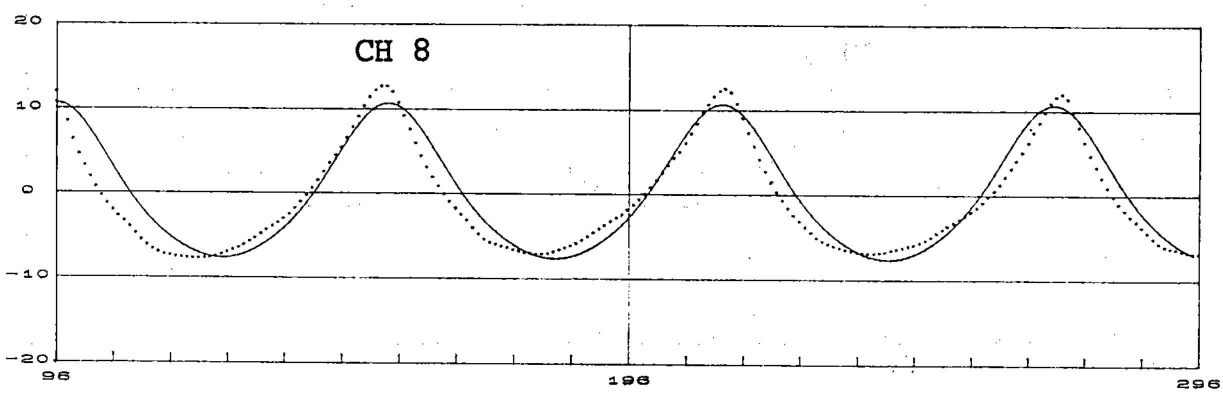


圖10-31

θ=45° (intersection angle), T_1=T_2=1.20sec(incident wave period 1 & 2)
d =40cm(water depth), H_1=10.18cm(incident wave height 1)
H_2=2.85cm(incident wave height 2)

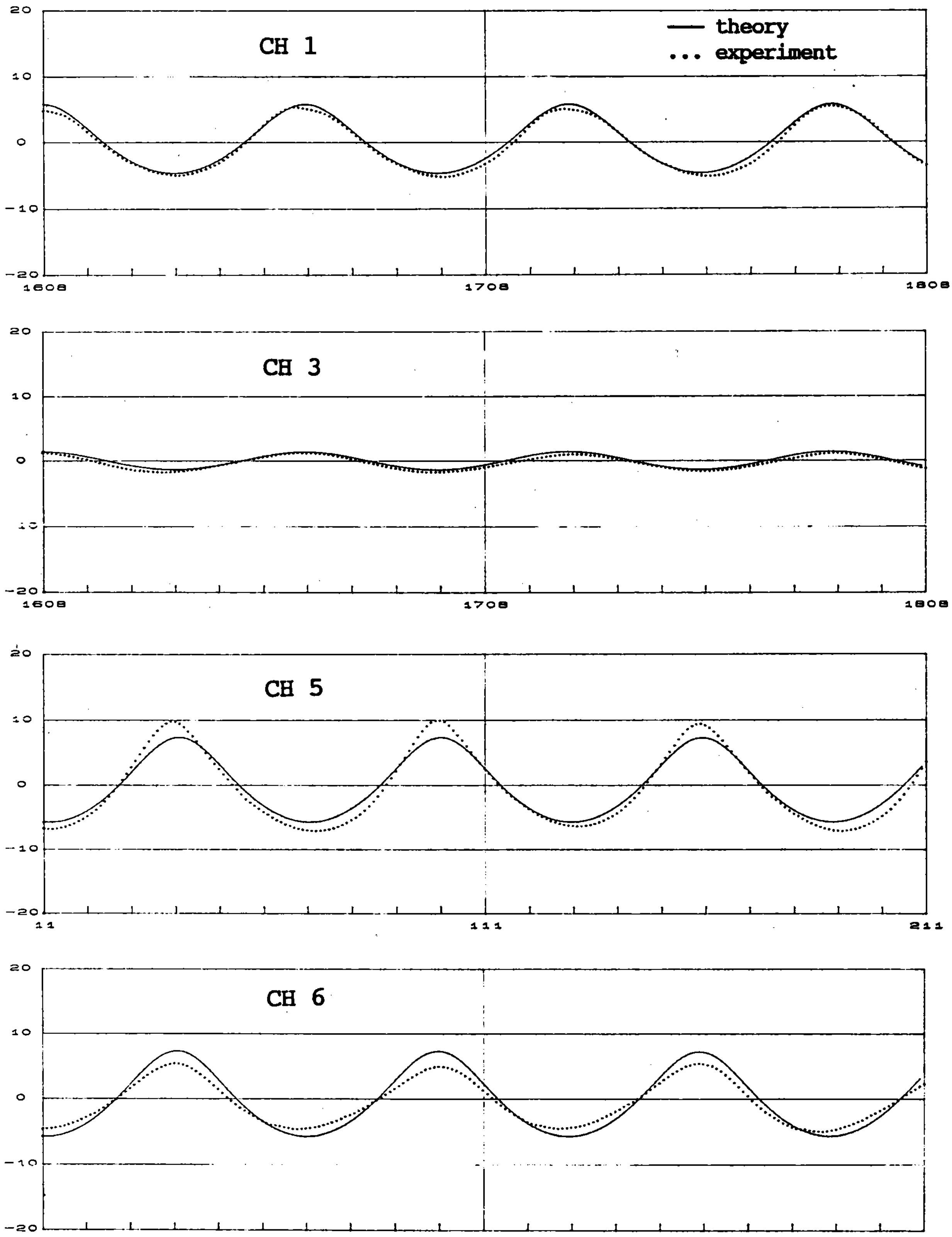


圖10-32

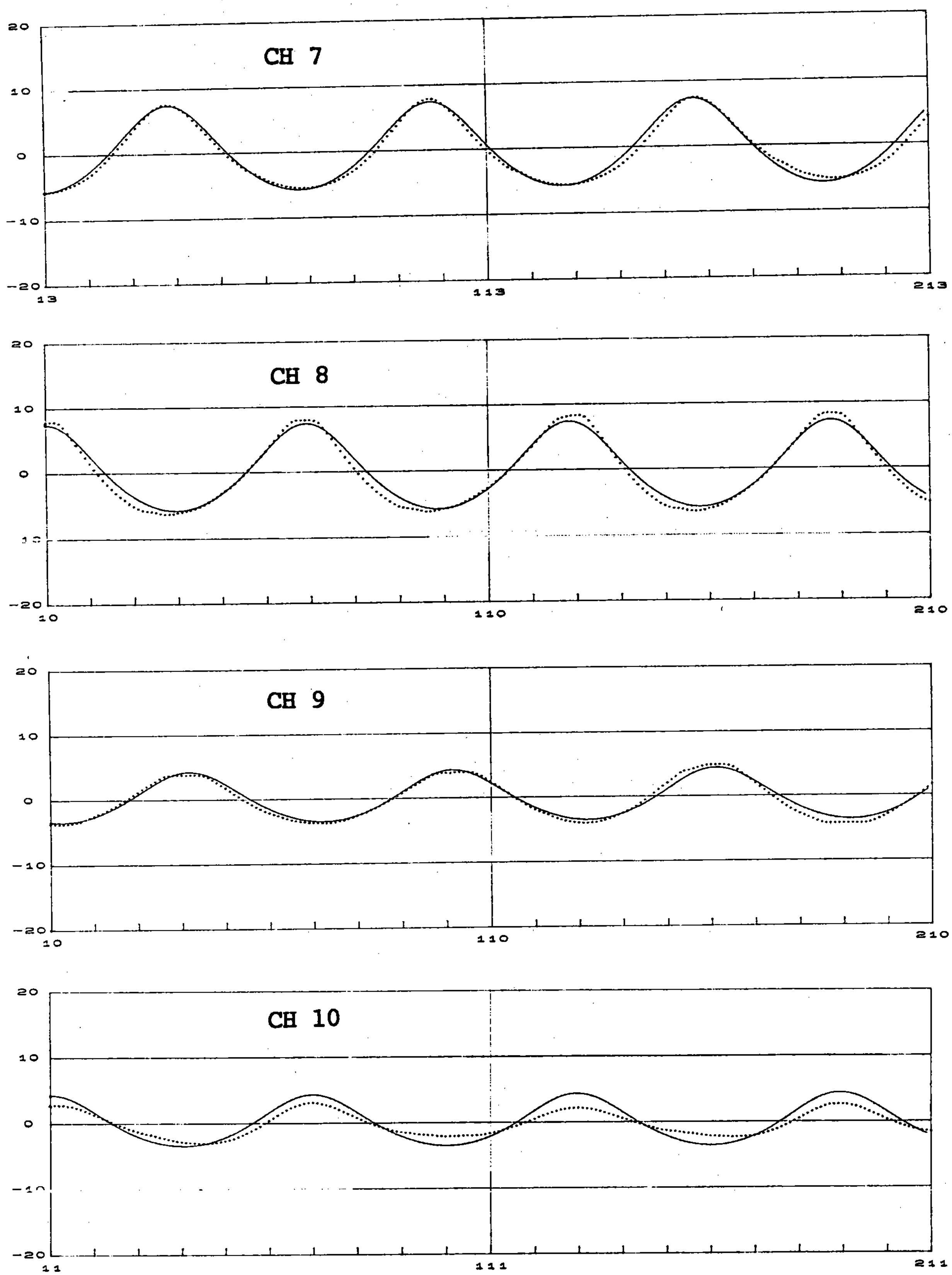

CH 7
20
10
0
-10
-20
13
113
213
CH 8
20
10
0
-20
10
110
210
CH 9
20
10
0
-10
-20
10
110
210
CH 10
20
10
0
-20
11
111
211

圖10-32

θ=45° (intersection angle), T_1=T_2=1.60sec(incident wave period)
d =40cm(water depth), H_1=H_2=11.38cm(incident wave height)

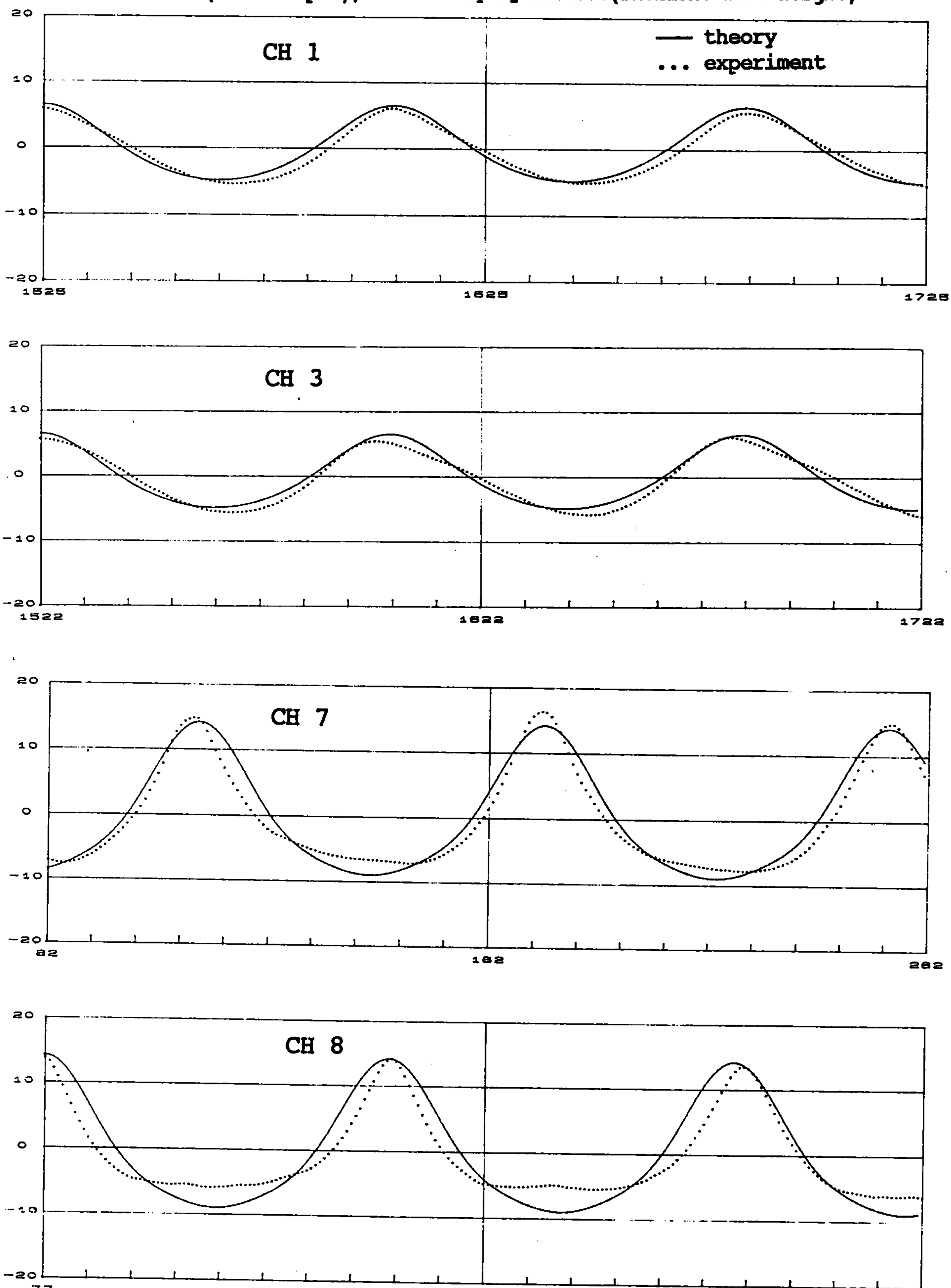


圖10-33

θ =45° (intersection angle), T_1=T_2=1.60sec(incident wave period 1 & 2)
d =40cm(water depth), H_1=6.40cm(incident wave height 1)
H_2=3.15cm(incident wave height 2)

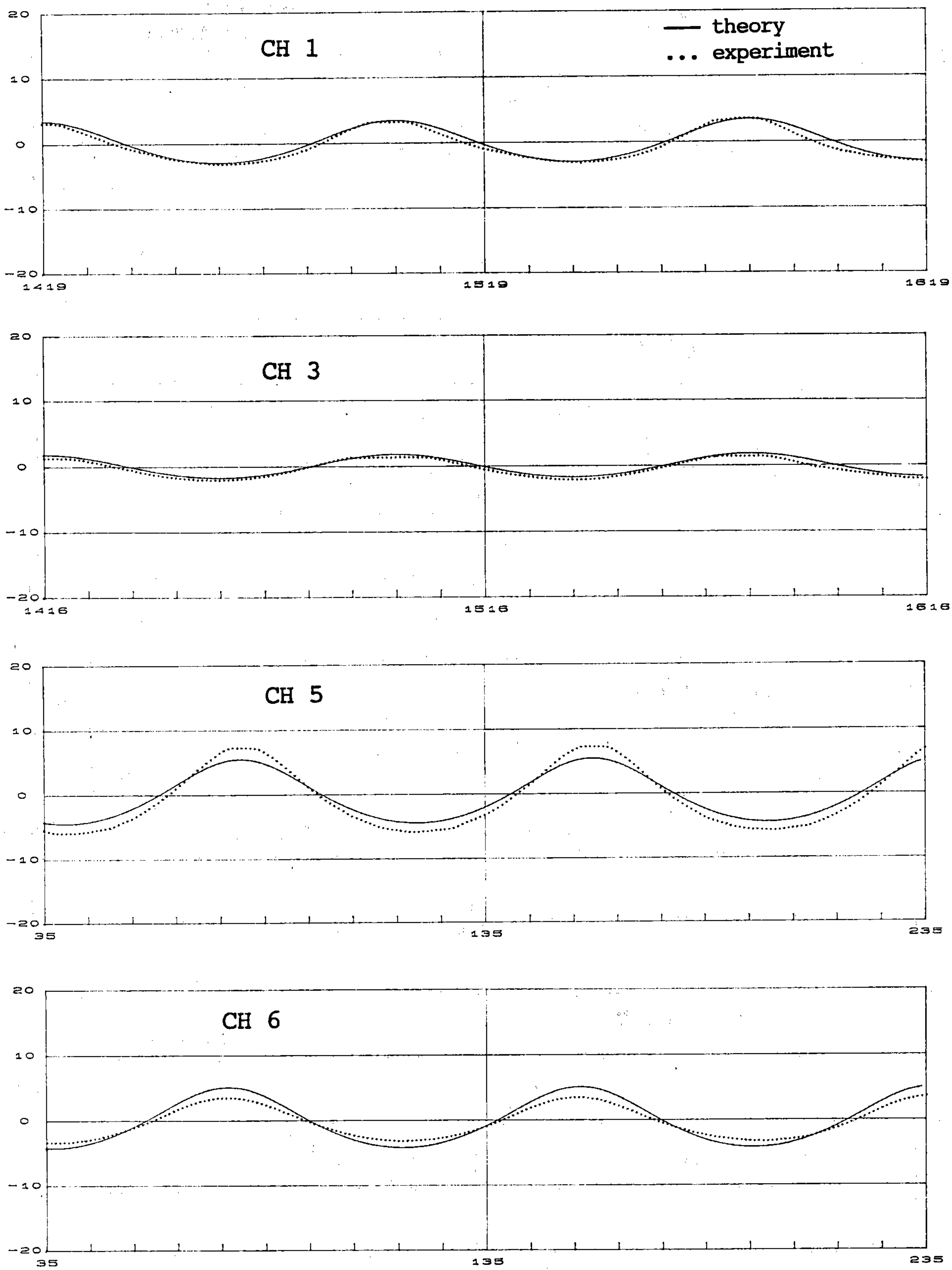
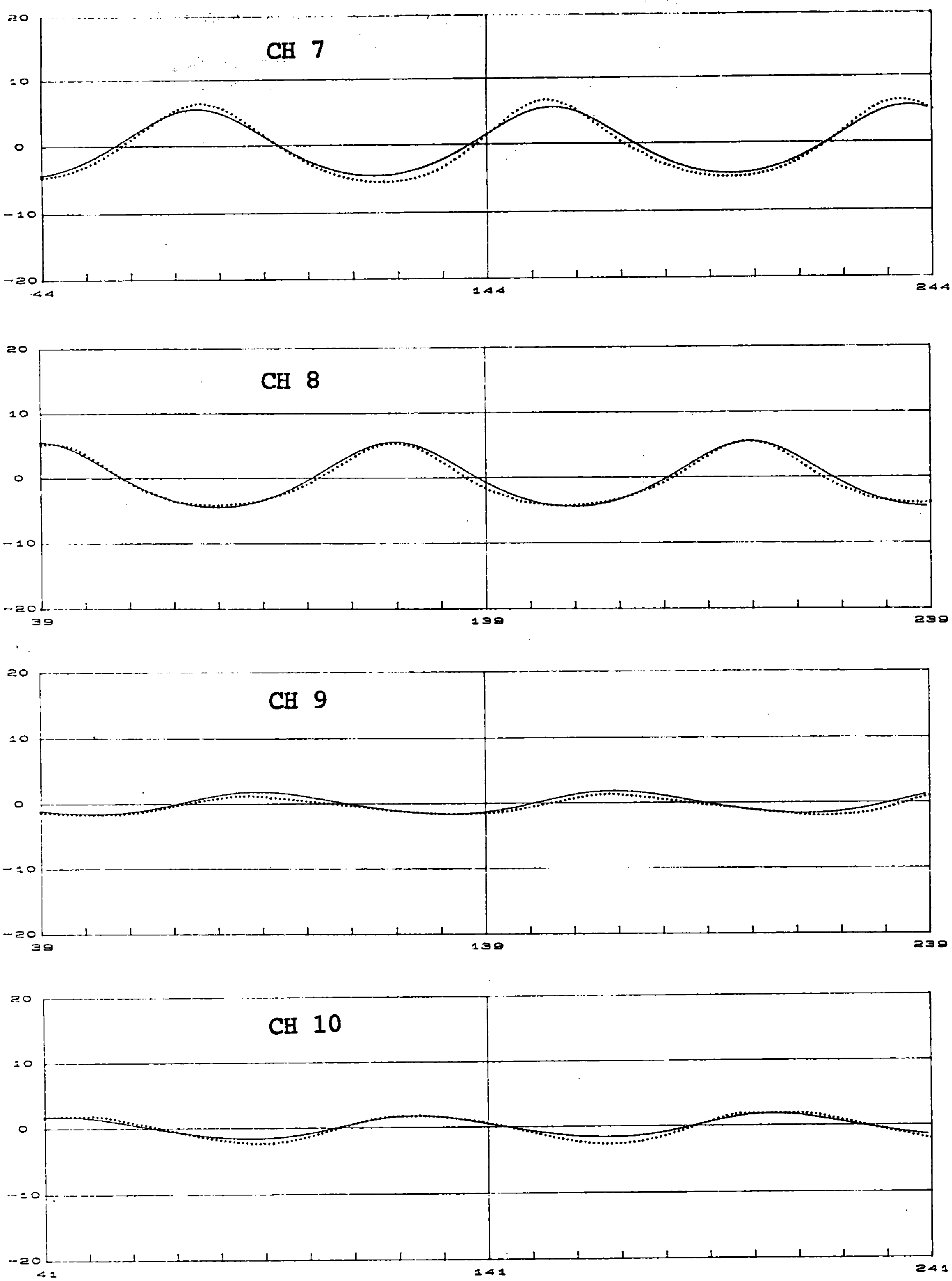


圖10-34



CH 7
20
10
0
-10
-20
44
144
244
CH 8
20
10
0
-10
-20
39
139
239
CH 9
20
10
0
-10
-20
39
139
239
CH 10
20
10
0
-10
-20
41
141
241

圖10-34

θ=45° (intersection angle), T_1=T_2=1.60sec(incident wave period 1 & 2)
d =40cm(water depth), H_1=11.80cm(incident wave height 1)
H_2=3.52cm(incident wave height 2)

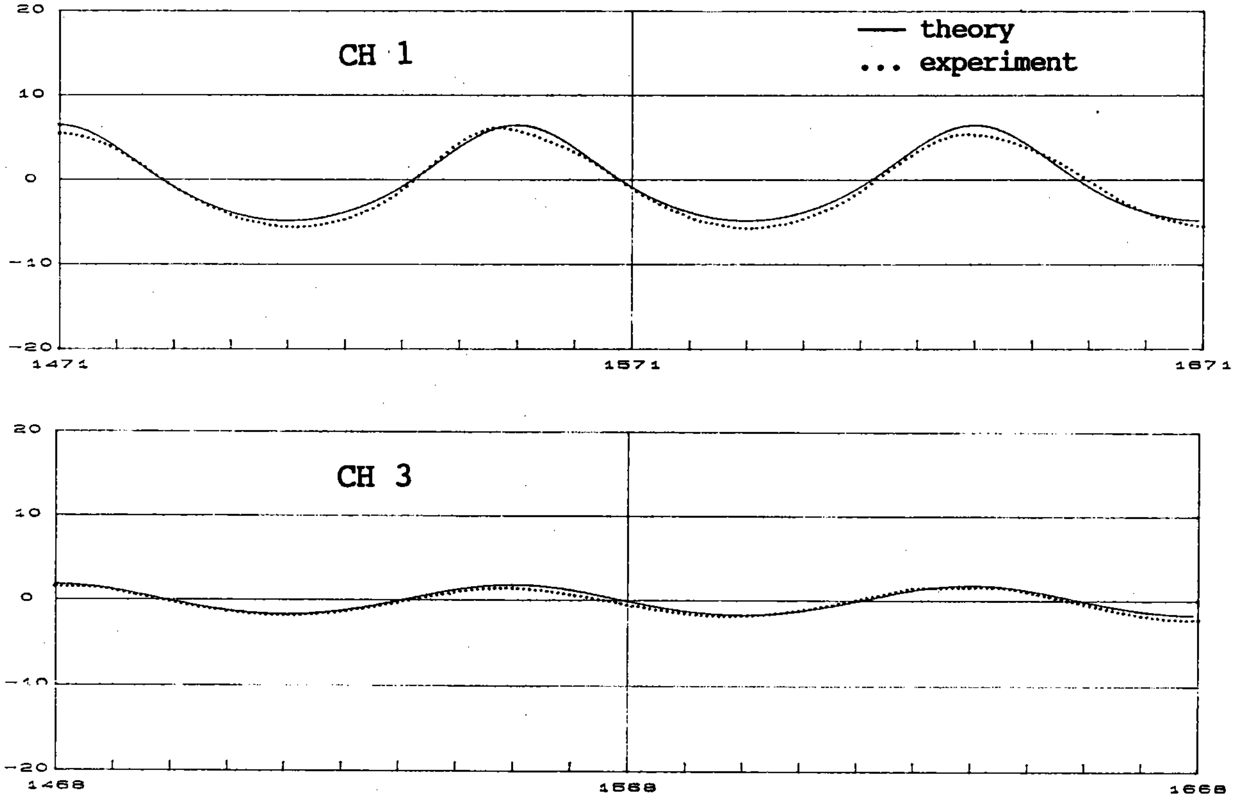


圖10-35

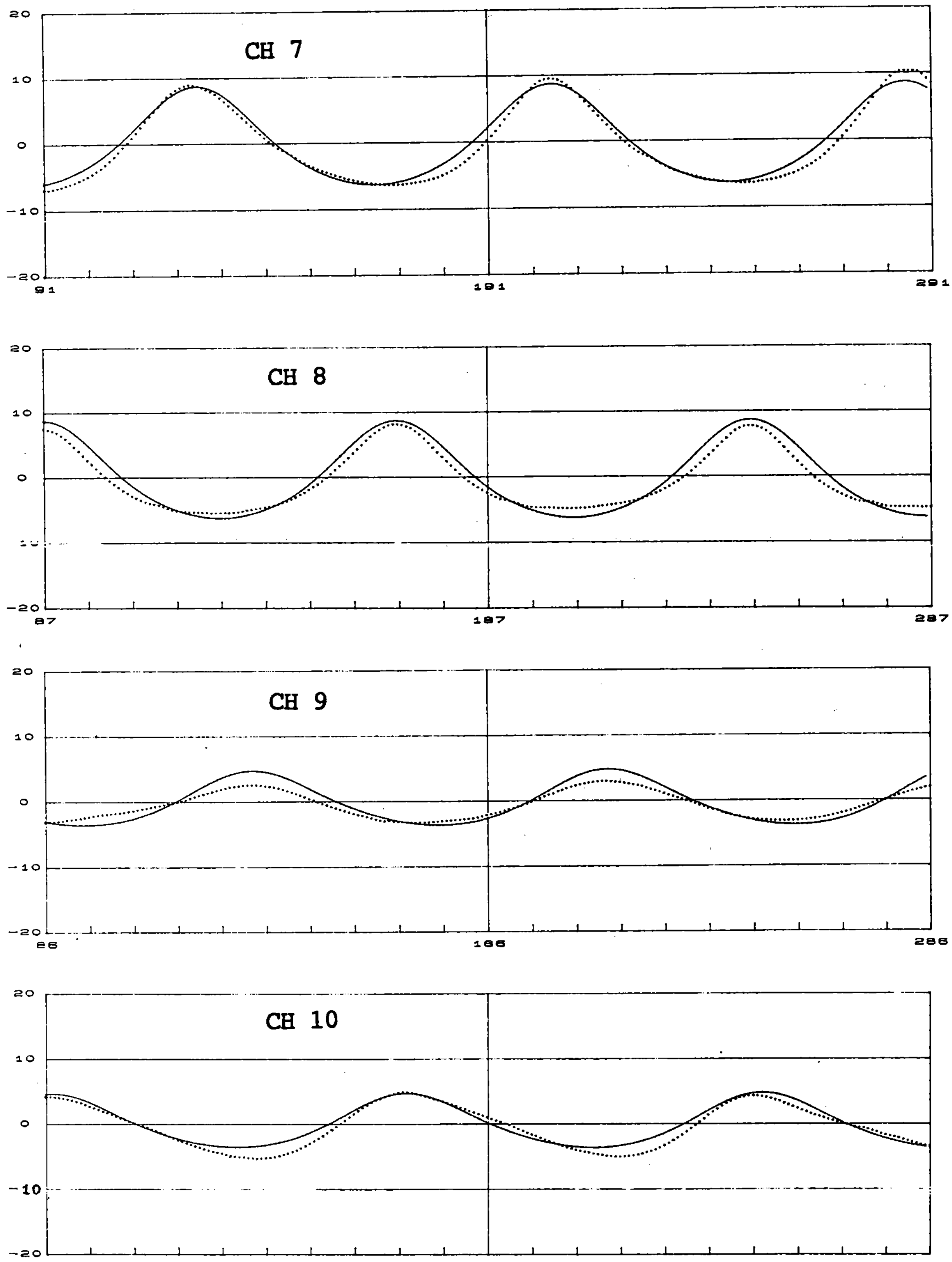

CH 7
20
10
0
-10
-20
91
191
291
CH 8
20
10
0
-20
87
187
287
CH 9
20
10
0
-10
-20
86
186
286
CH 10
20
10
0
-10
-20
88
188
288

圖10-35

θ =45° (intersection angle), T_1=T_2=2.00sec(incident wave period)
d =40cm(water depth), H_1=H_2=11.93cm(incident wave height)

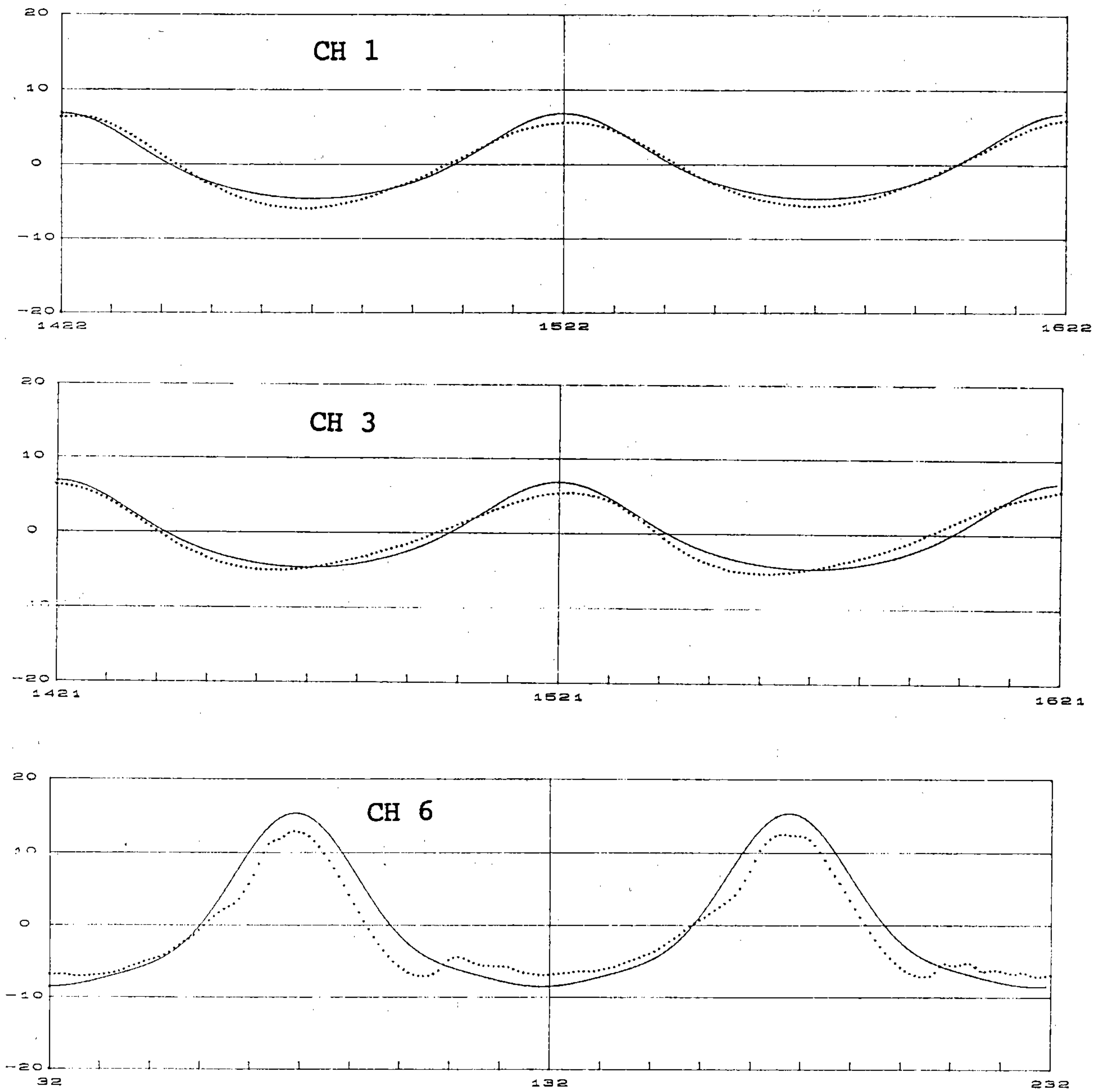


圖10-36

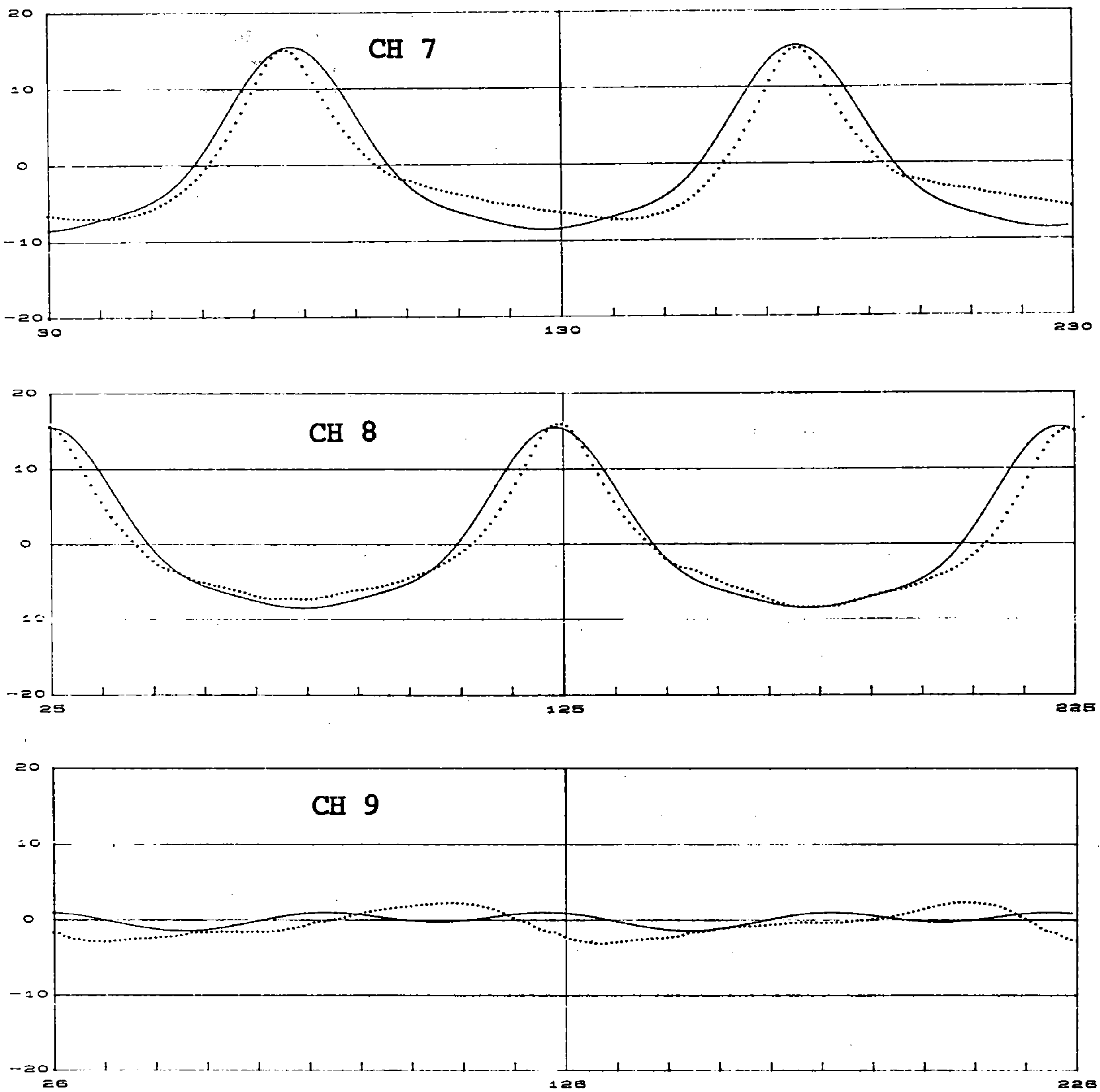

CH 7
20
10
0
-10
-20
30
130
230
CH 8
20
10
0
-20
25
125
225
CH 9
20
10
0
-10
-20
26
126
226

圖10-36

θ=45° (intersection angle), T_1=T_2=2.00sec(incident wave period 1 & 2)
d =40cm(water depth), H_1=6.24cm(incident wave height 1)
H_2=2.92cm(incident wave height 2)

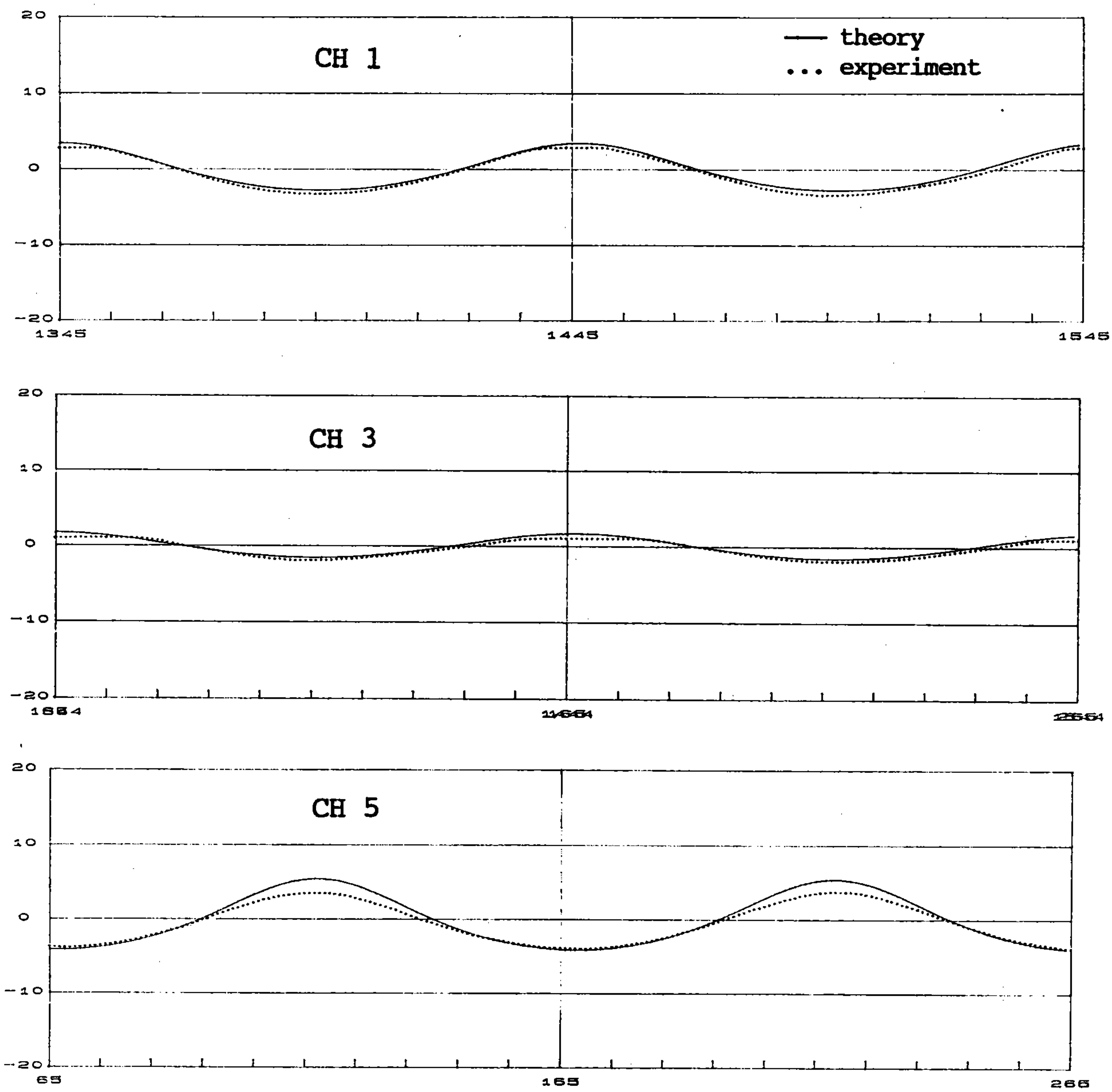


圖10-37

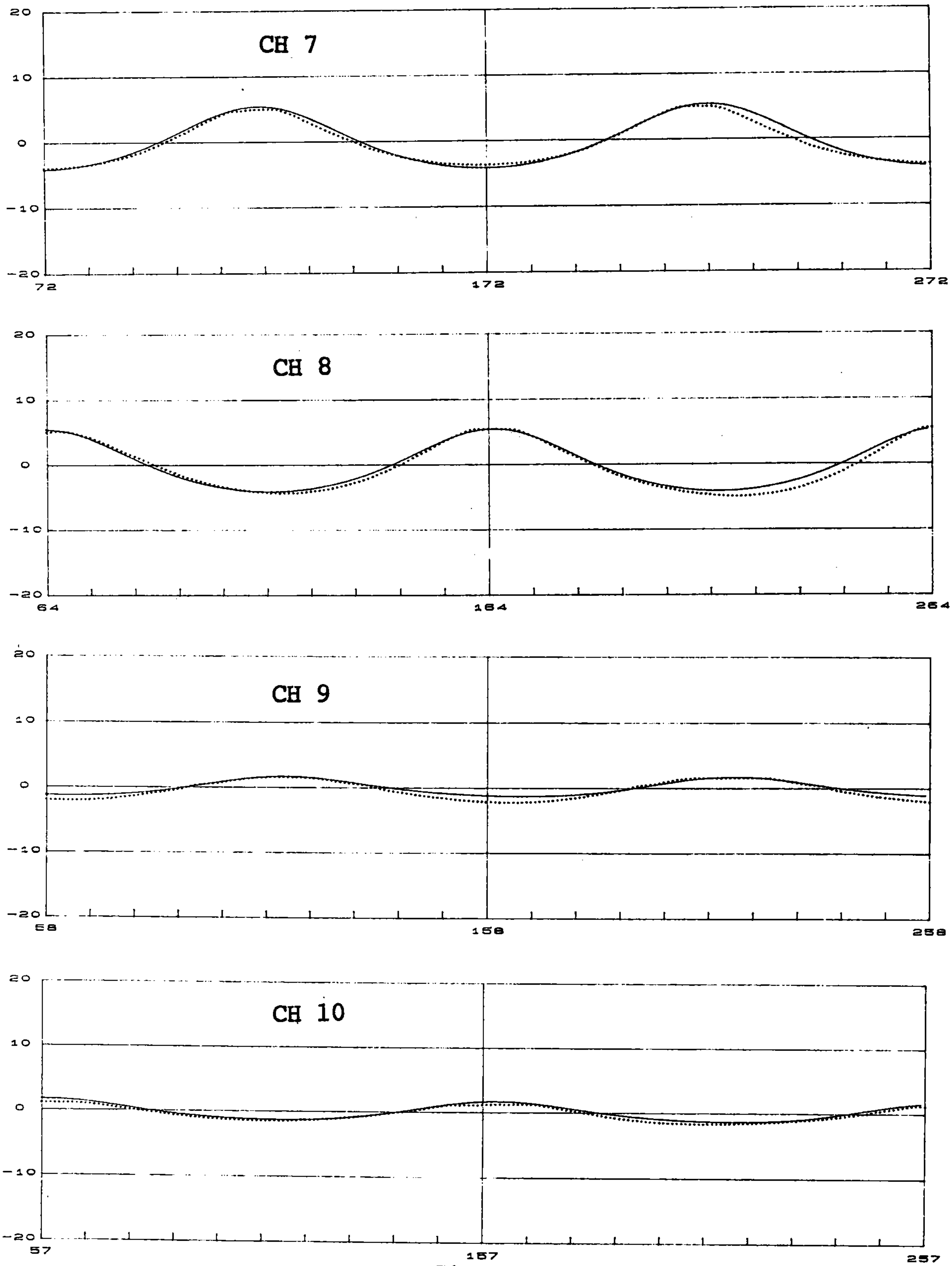

CH 7
20
10
0
−10
−20
72
172
272
CH 8
20
10
0
−10
−20
64
164
264
CH 9
20
10
0
−10
−20
58
158
258
CH 10
20
10
0
−10
−20
57
157
257

圖10-37

θ =45° (intersection angle), T_1=T_2=2.00sec(incident wave period 1 & 2)
d =40cm(water depth), H_1=12.38cm(incident wave height 1)
H_2=3.41cm(incident wave height 2)

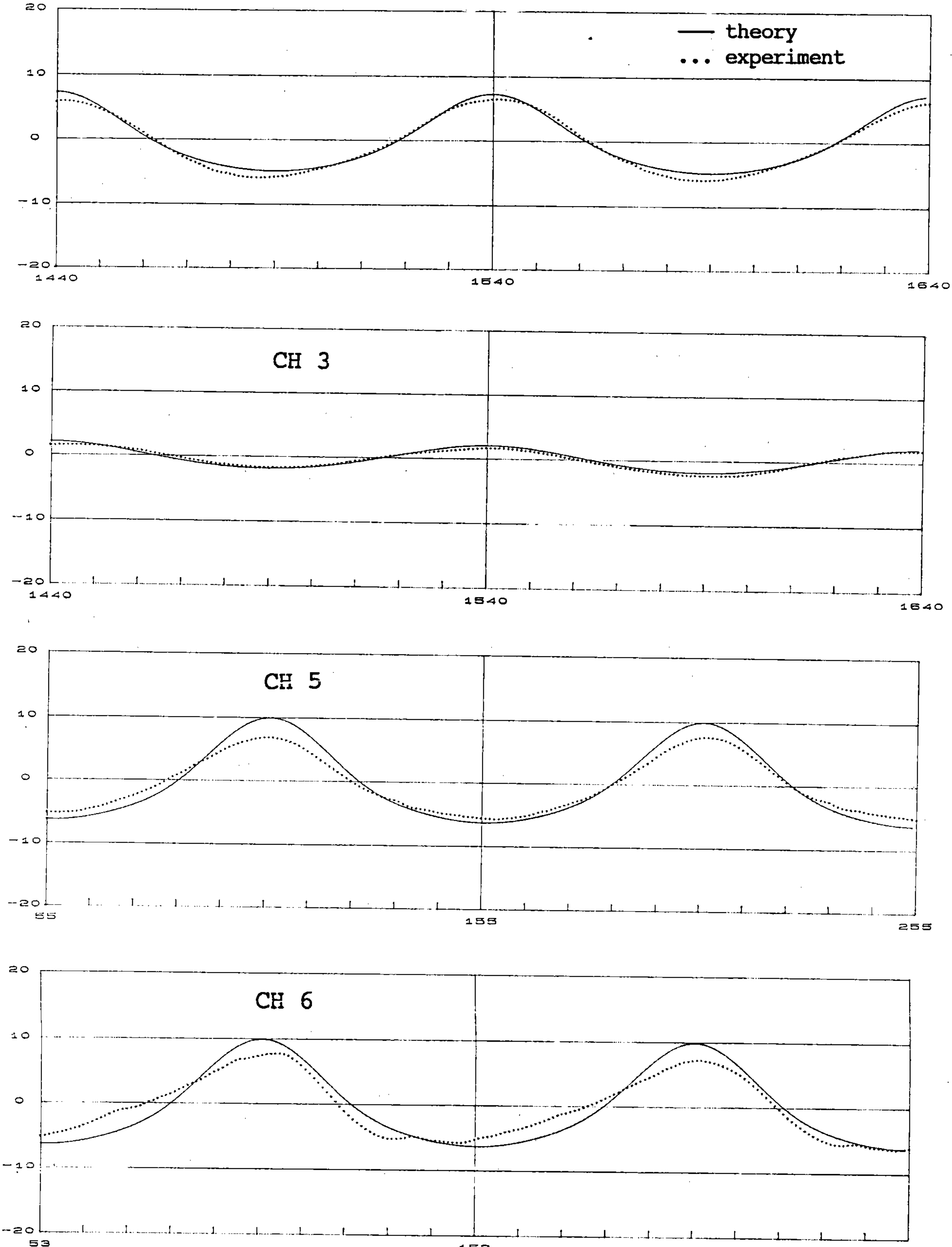


圖10-38

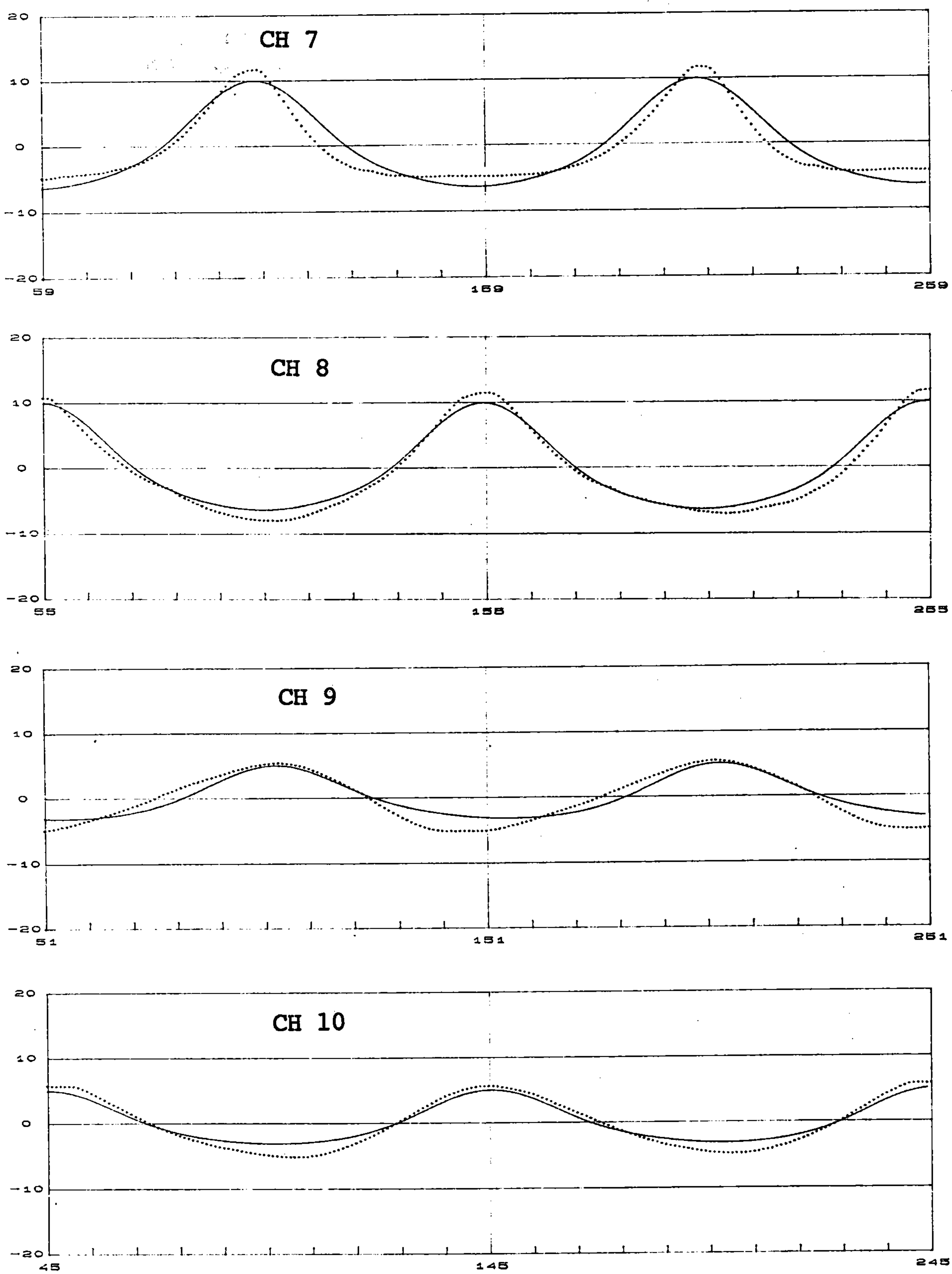

CH 7
20
10
0
-10
-20
59
159
259
CH 8
20
10
0
-10
-20
55
155
255
CH 9
20
10
0
-10
-20
51
151
251
CH 10
20
10
0
-10
-20
45
145
245

圖10-38

θ =45° (intersection angle), T_1=T_2=2.40sec(incident wave period)
d =40cm(water depth), H_1=H_2=9.62cm(incident wave height)

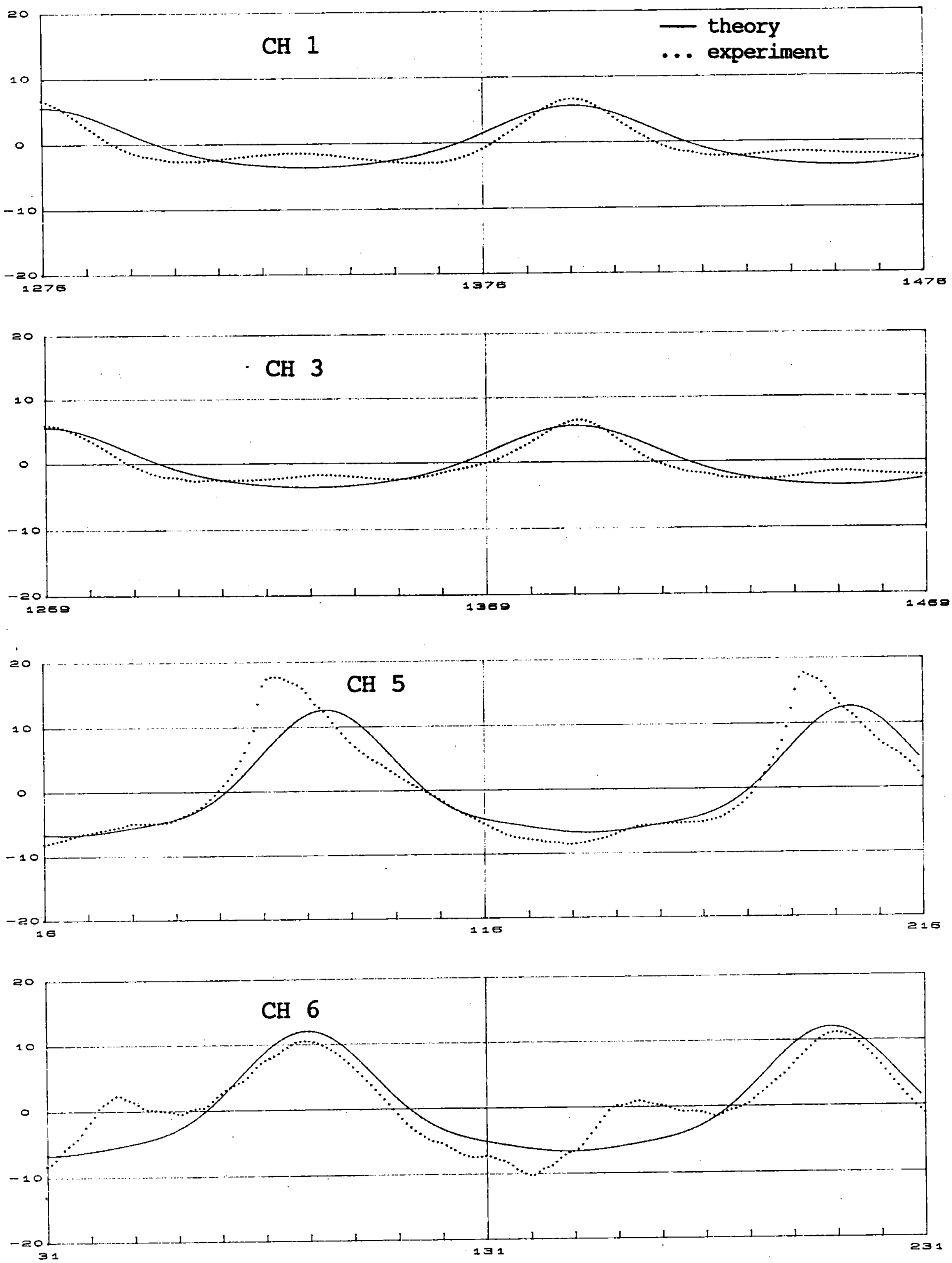
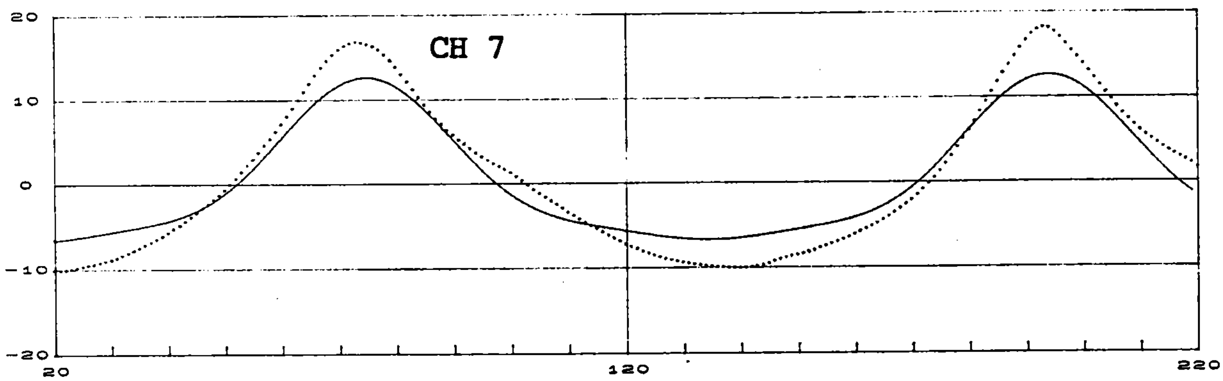


圖10-39



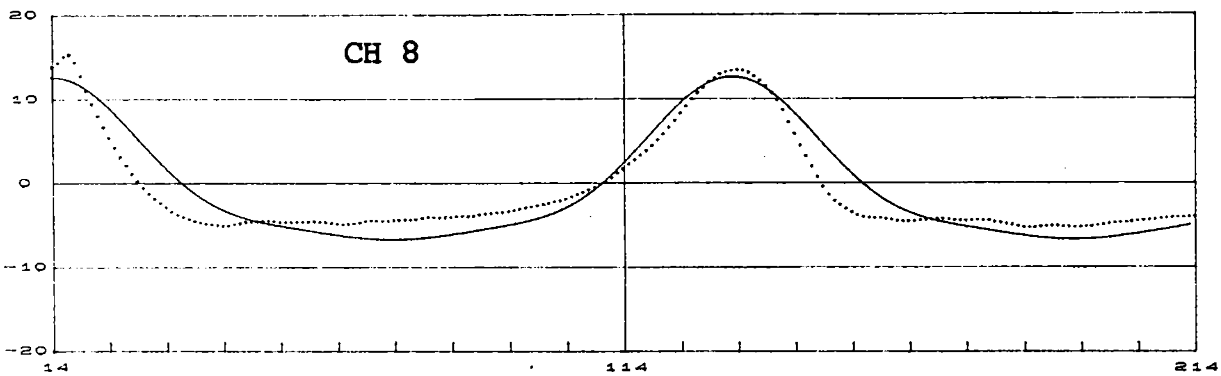


圖10-39

θ=45° (intersection angle), d =40cm(water depth),

T_1=T_2=2.40sec(incident wave period 1 & 2)
H_1=5.34cm(incident wave height 1)
H_2=2.71cm(incident wave height 2)

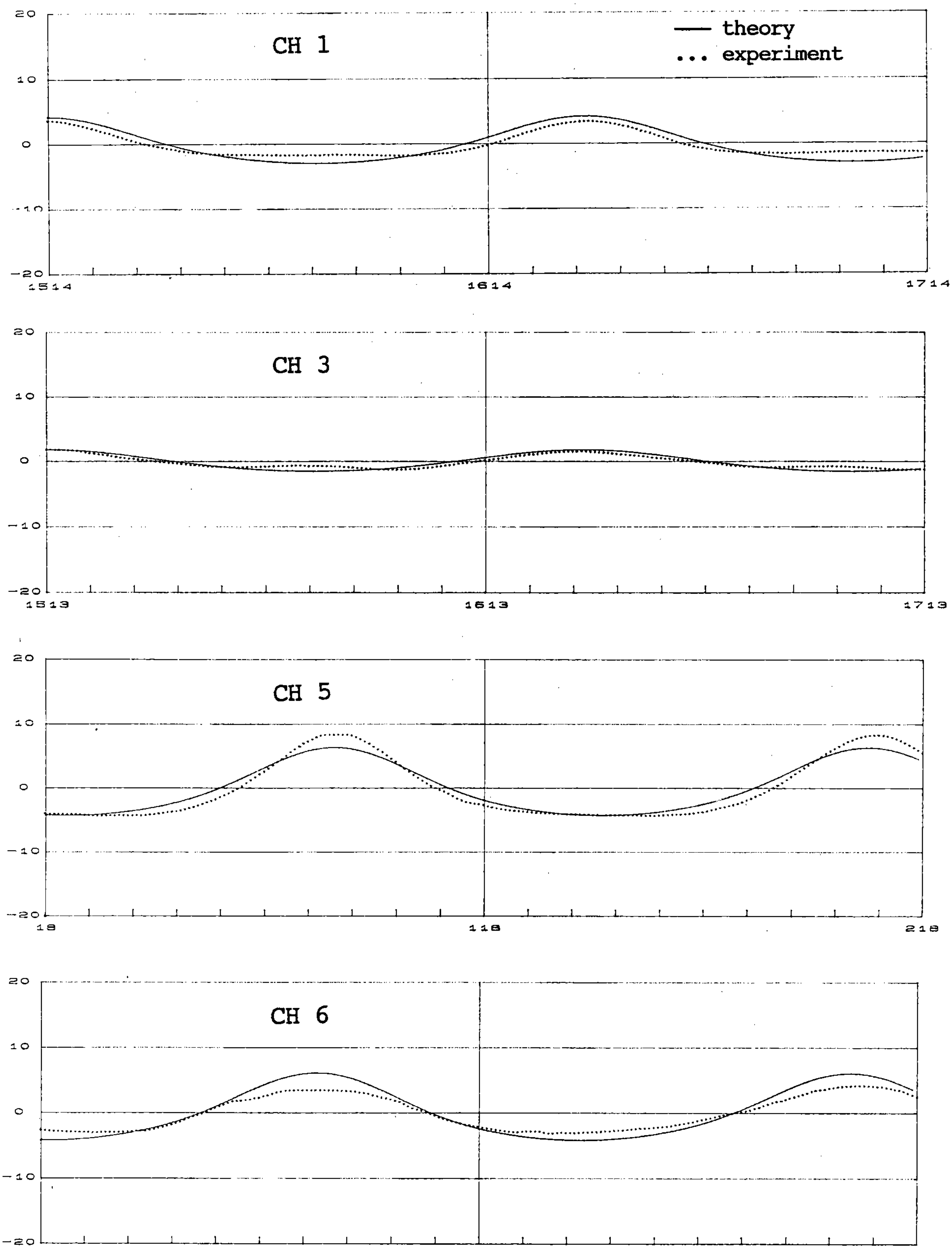


圖10-40

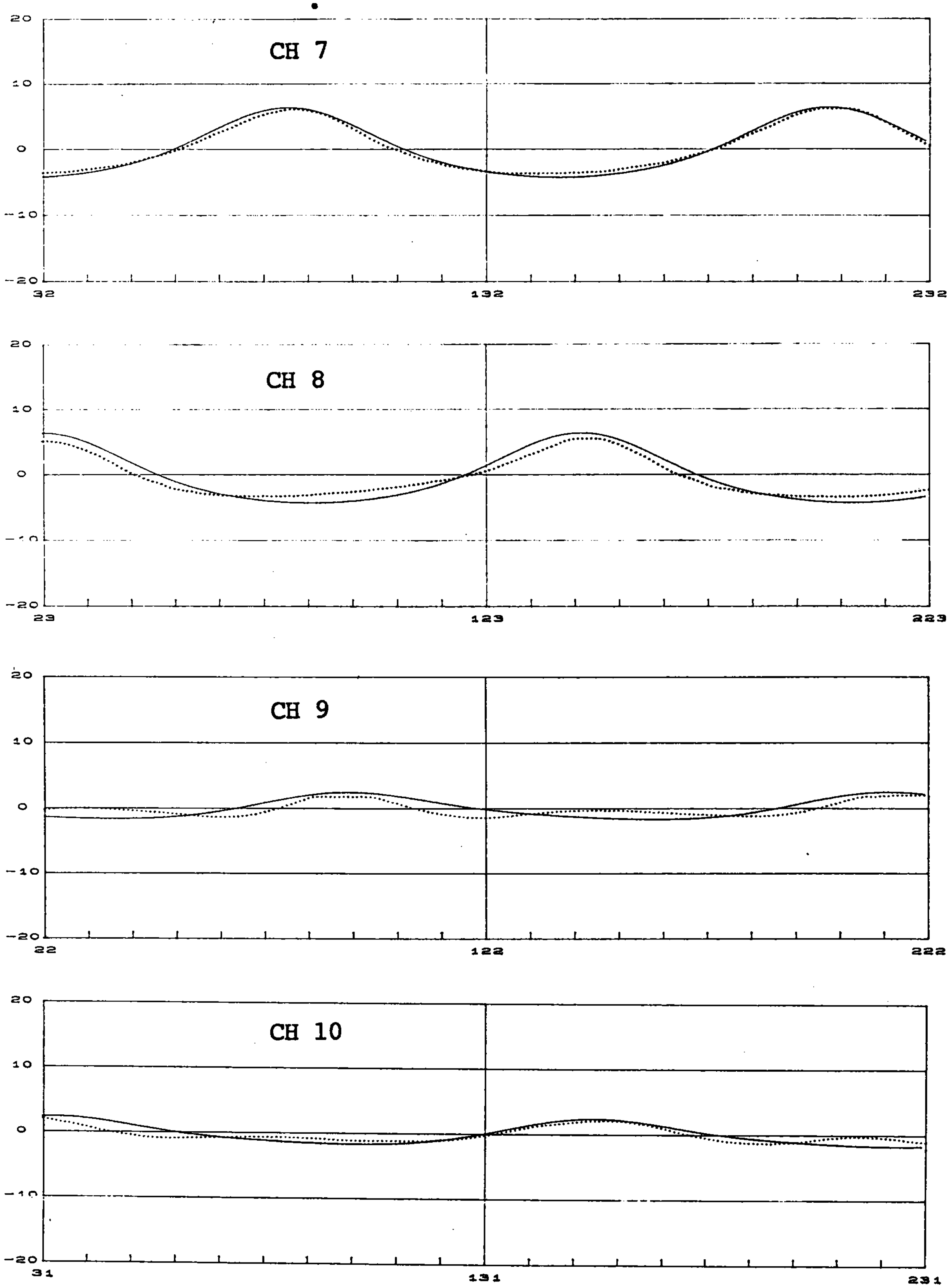

CH 7
20
10
0
-10
-20
32
132
232
CH 8
20
10
0
-20
23
123
223
CH 9
20
10
0
-10
-20
22
122
222
CH 10
20
10
0
-20
31
131
231

圖10-40

θ=45° (intersection angle), T_1=T_2=2.40sec(incident wave period 1 & 2)
d =40cm(water depth), H_1=12.57cm(incident wave height 1)
H_2=3.21cm(incident wave height 2)

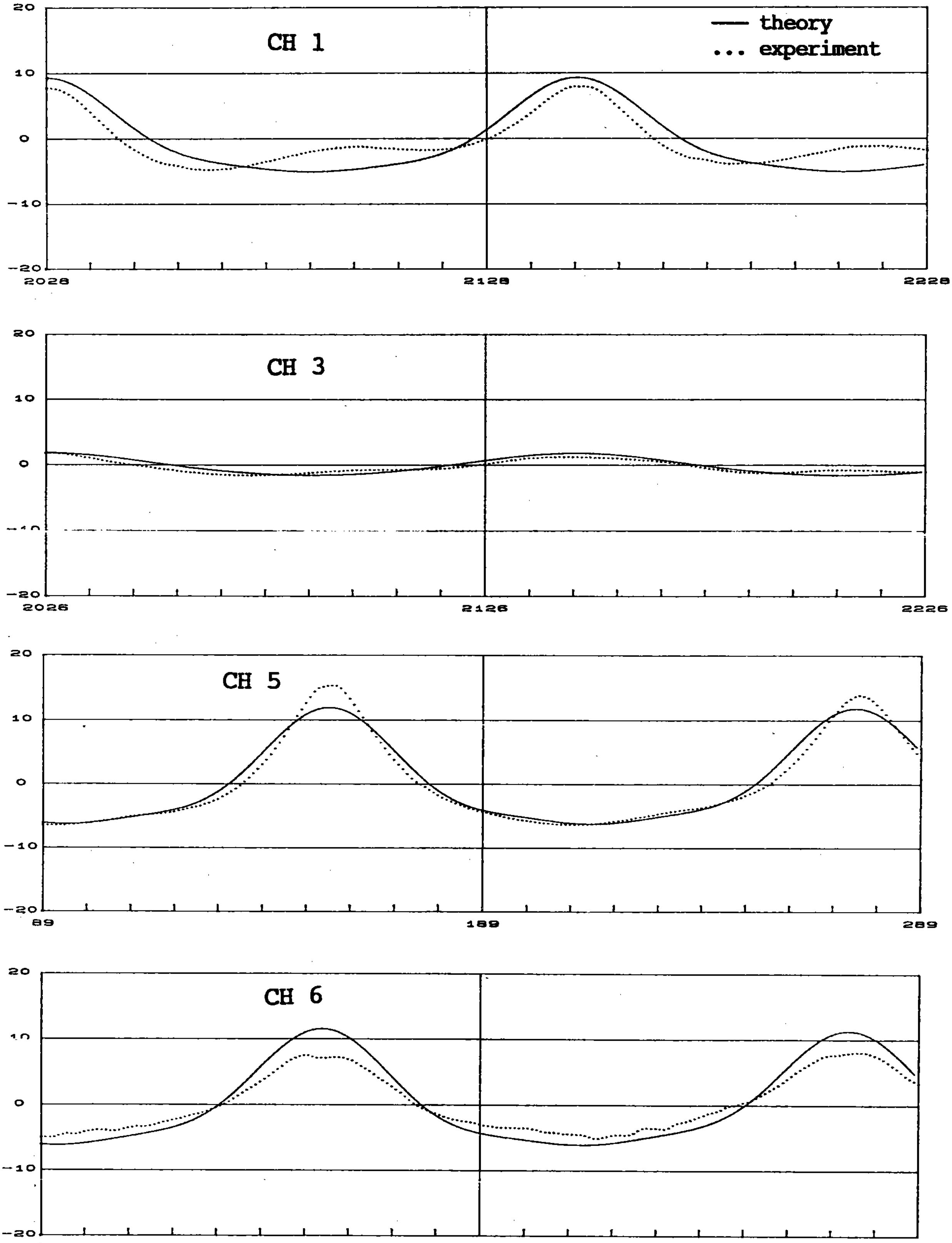


圖10-41

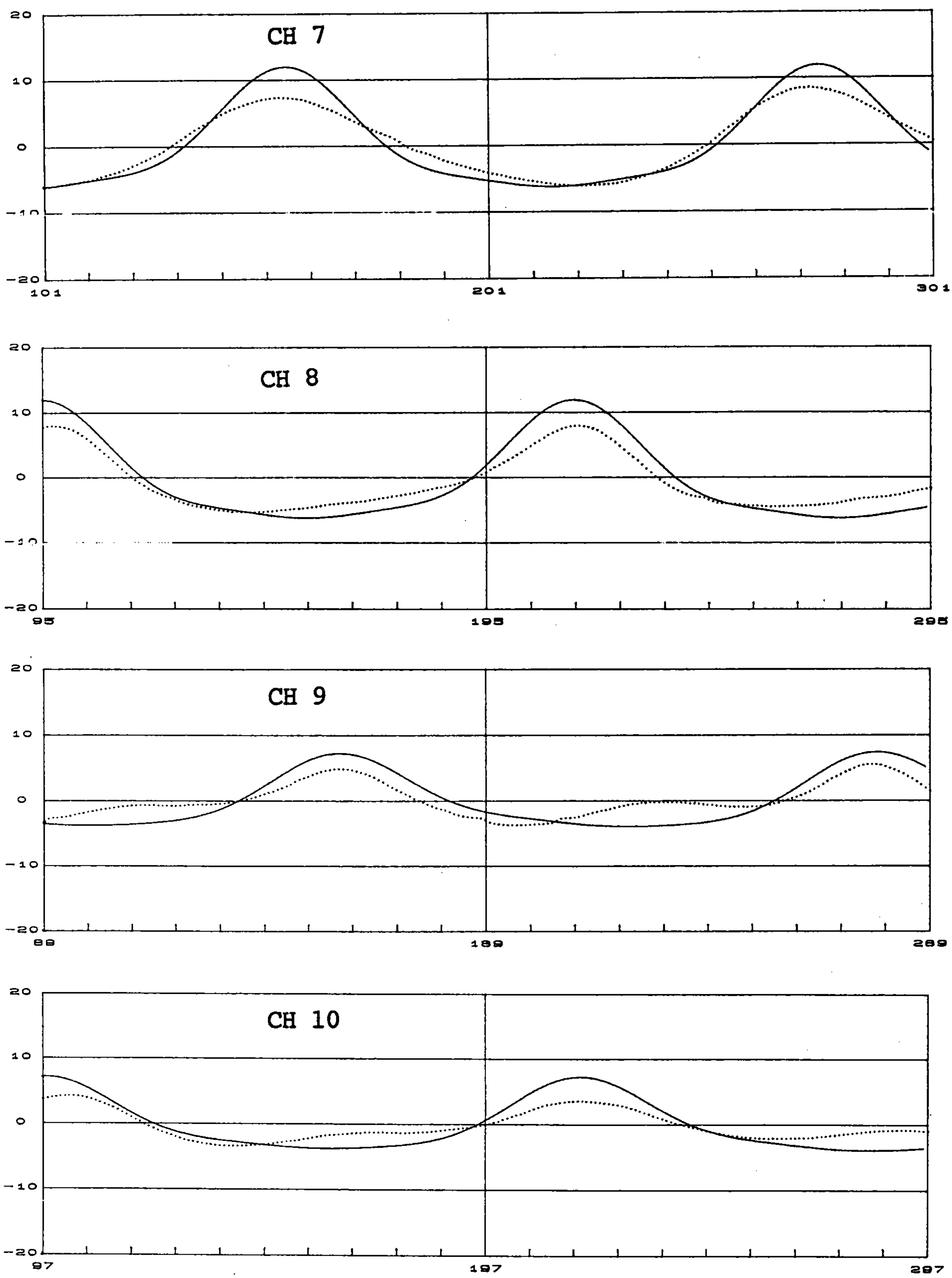

CH 7
20
10
0
-10
-20
101
201
301
CH 8
20
10
0
-10
-20
95
195
295
CH 9
20
10
0
-10
-20
89
189
289
CH 10
20
10
0
-10
-20
97
197
297

圖10-41

θ=45° (intersection angle), T_1=T_2=0.8sec(incident wave period 1 & 2)
d =60cm(water depth), H_1=7.71cm(incident wave height 1)
H_2=3.12cm(incident wave height 2)

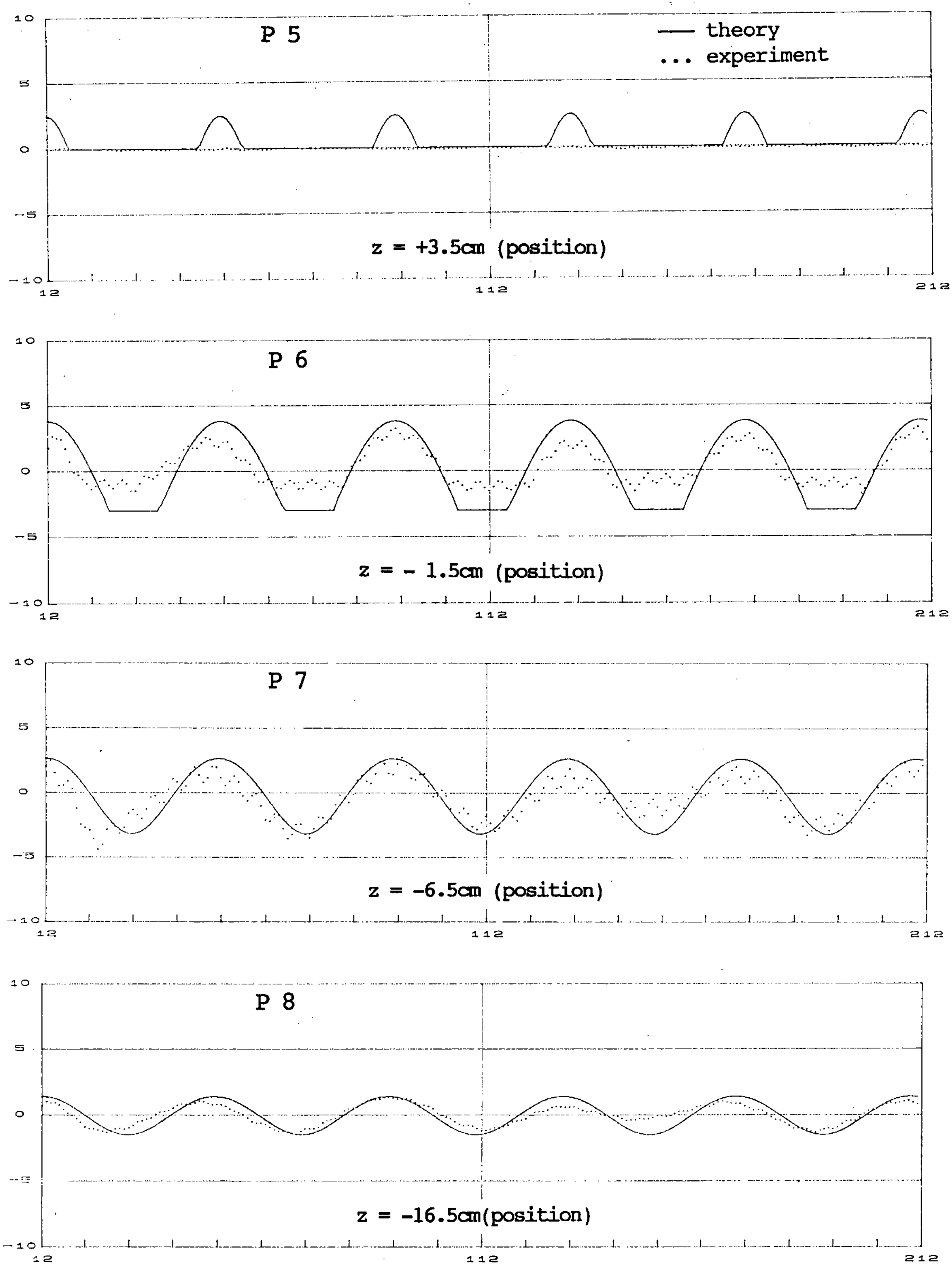


圖11-1

圖11 兩波交會相互作用所形成之波壓試驗值與理論值比較

Fig 11 The comparison of theoretical and experimental wave-pressure resulted from the interaction between two progressive gravity waves trains.

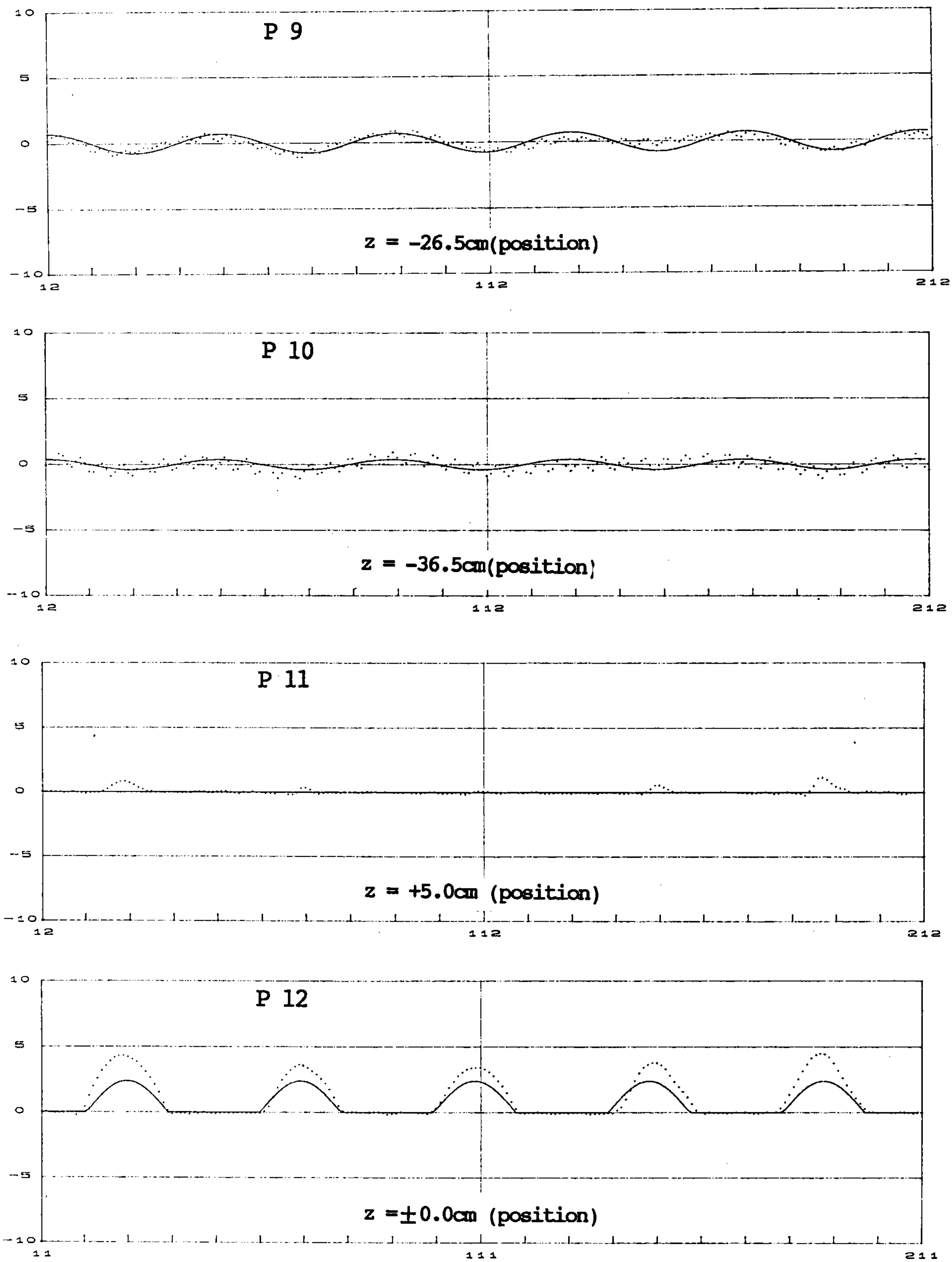

P 9
z = -26.5cm(position)
P 10
z = -36.5cm(position)
P 11
z = +5.0cm (position)
P 12
z =±0.0cm (position)

圖11-1

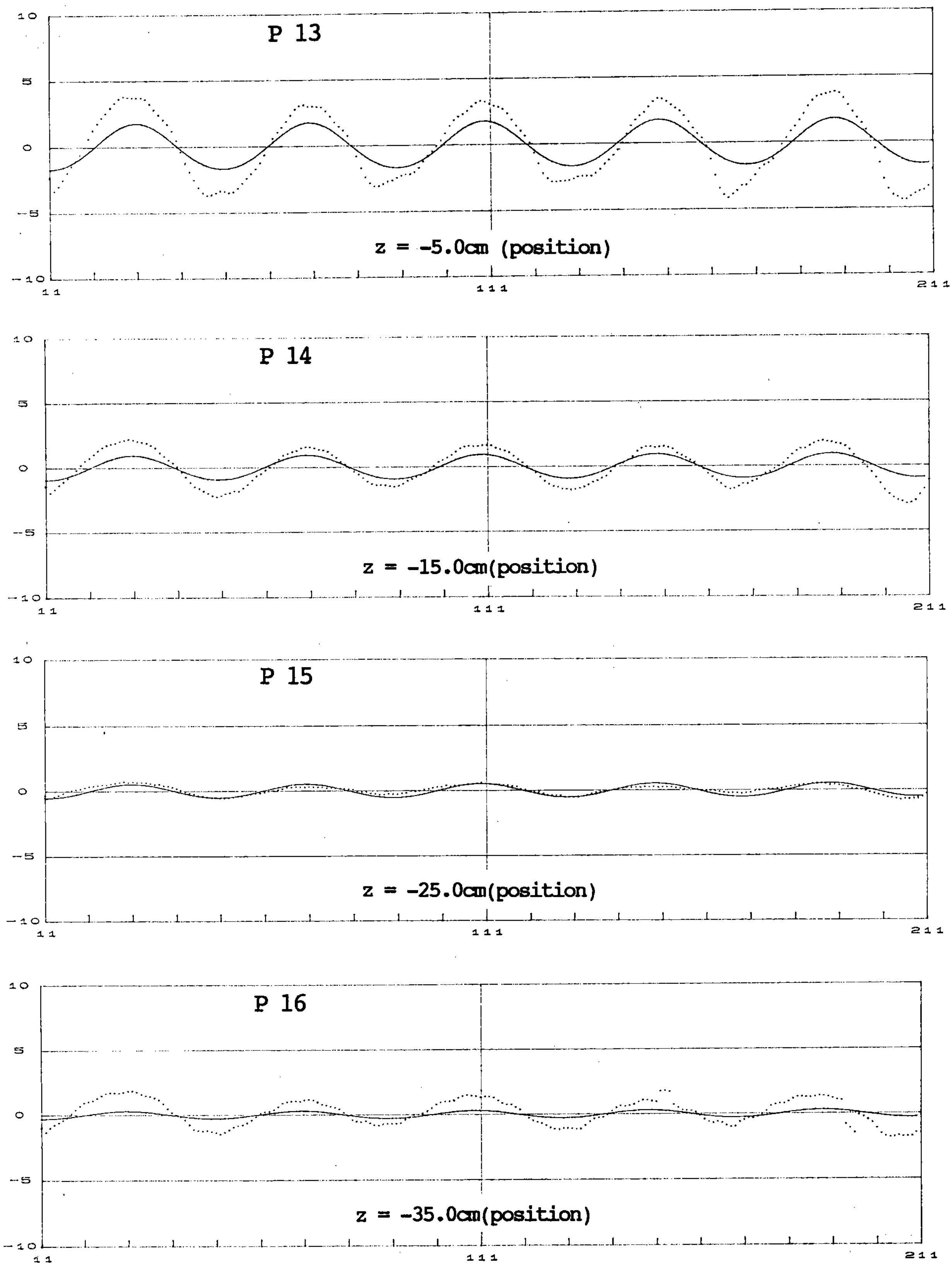

P 13
z = -5.0cm (position)
P 14
z = -15.0cm(position)
P 15
z = -25.0cm(position)
P 16
z = -35.0cm(position)
10
5
0
-5
-10
11
111
211

圖11-1

θ=45° (intersection angle), T_1=T_2=1.20sec(incident wave period 1 & 2)
d =60cm(water depth), H_1=6.39cm(incident wave height 1)
H_2=3.66cm(incident wave height 2)

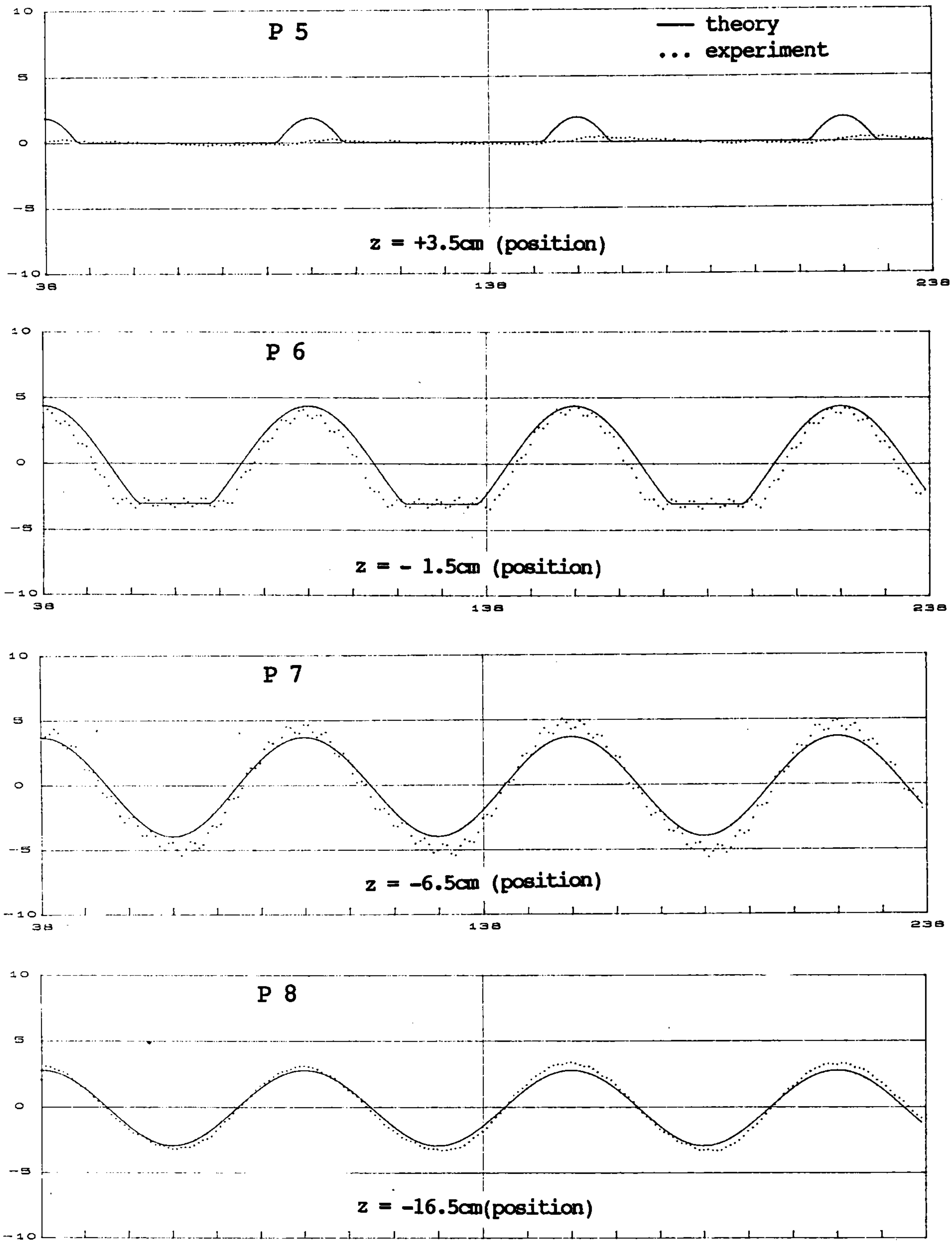


圖11-2

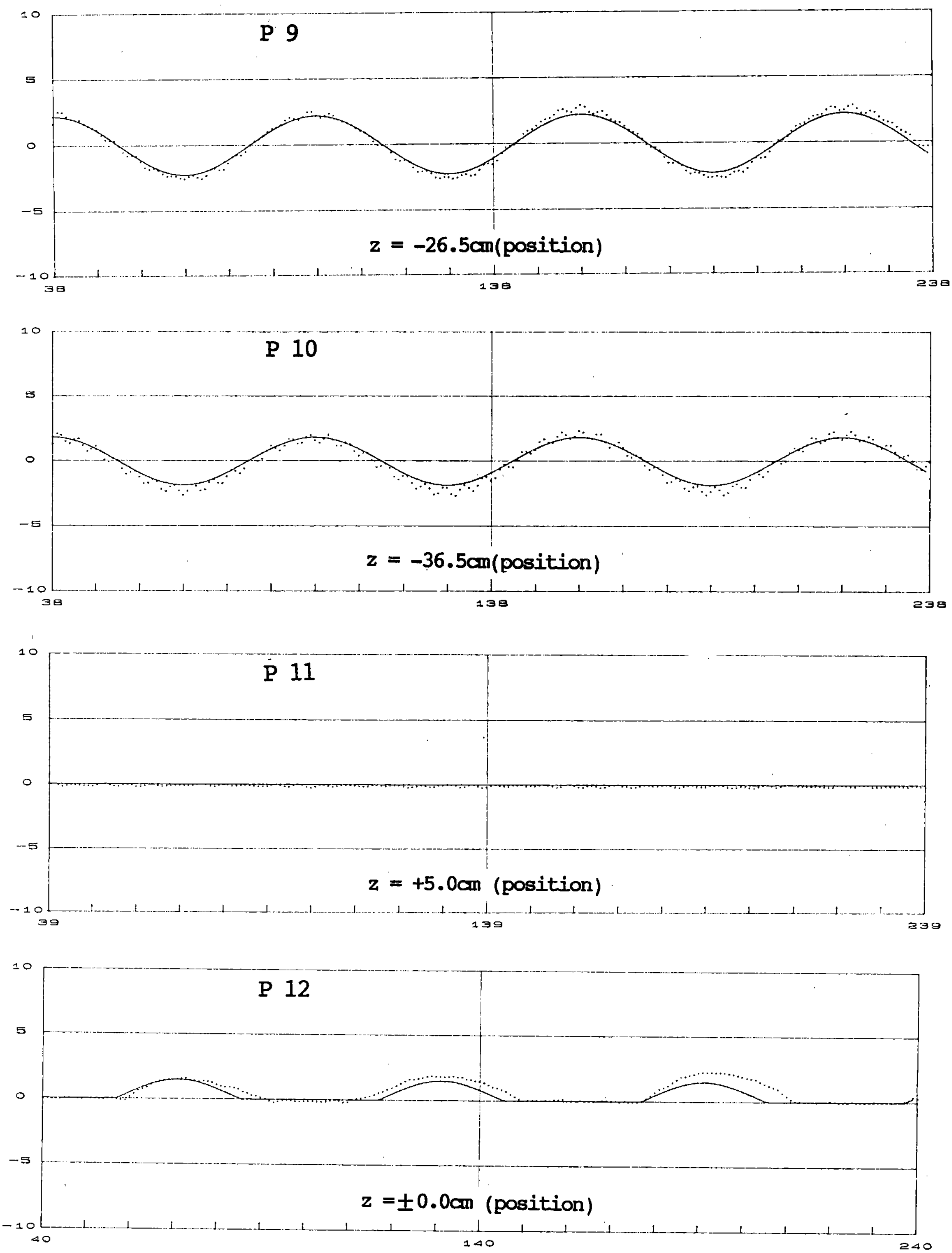

P 9
z = -26.5cm(position)
10
5
0
-5
-10
38
138
238
P 10
z = -36.5cm(position)
10
5
0
-5
-10
38
138
238
P 11
z = +5.0cm (position)
10
5
0
-5
-10
39
139
239
P 12
z =±0.0cm (position)
10
5
0
-5
-10
40
140
240

圖11-2

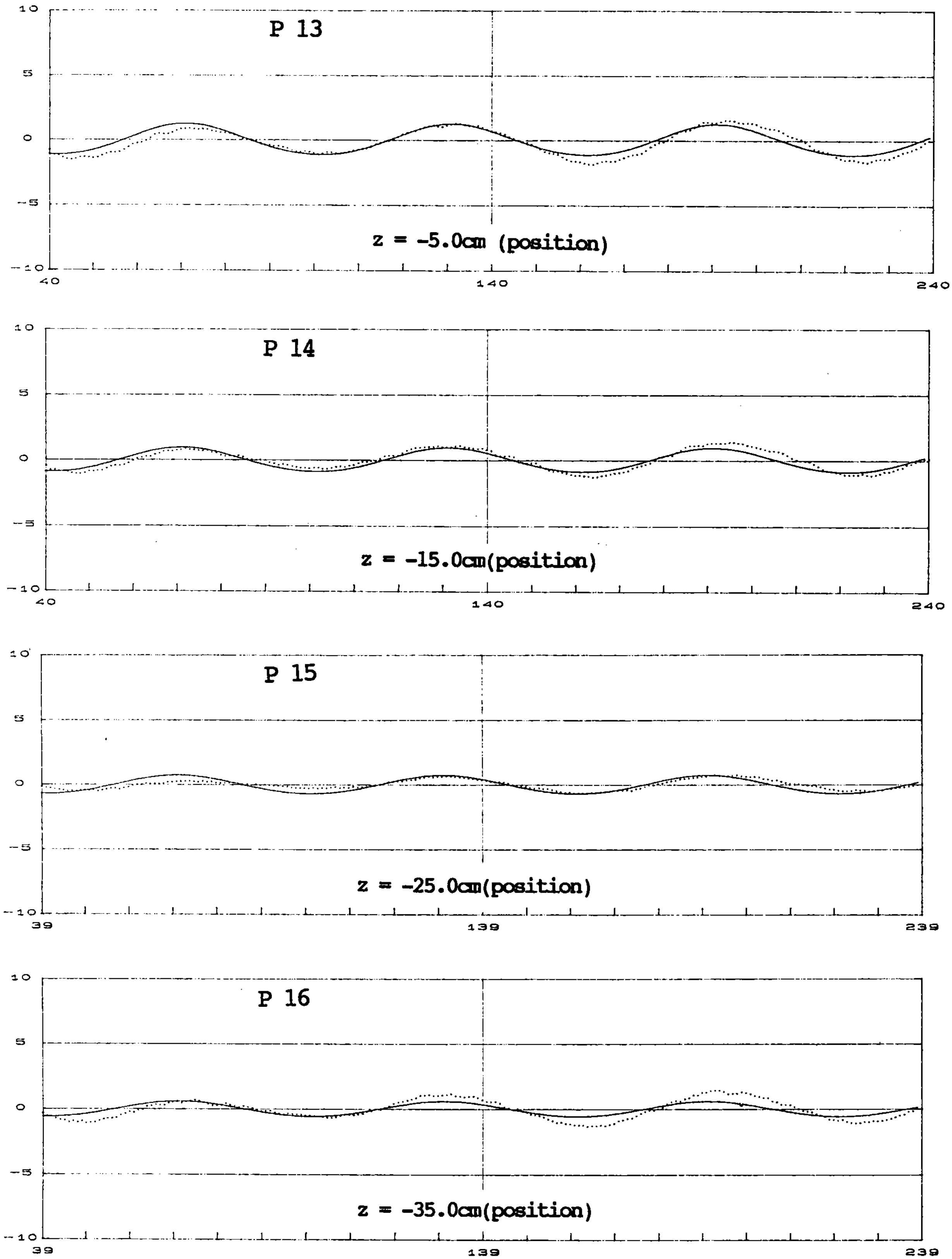

P 13
z = -5.0cm (position)
10
5
0
-5
-10
40
140
240
P 14
z = -15.0cm(position)
10
5
0
-5
-10
40
140
240
P 15
z = -25.0cm(position)
10
5
0
-5
-10
39
139
239
P 16
z = -35.0cm(position)
10
5
0
-5
-10
39
139
239

圖11-2

θ=45° (intersection angle), T_1=T_2=1.20sec(incident wave period 1 & 2)
d =60cm(water depth), H_1=10.22cm(incident wave height 1)
H_2=3.52cm(incident wave height 2)

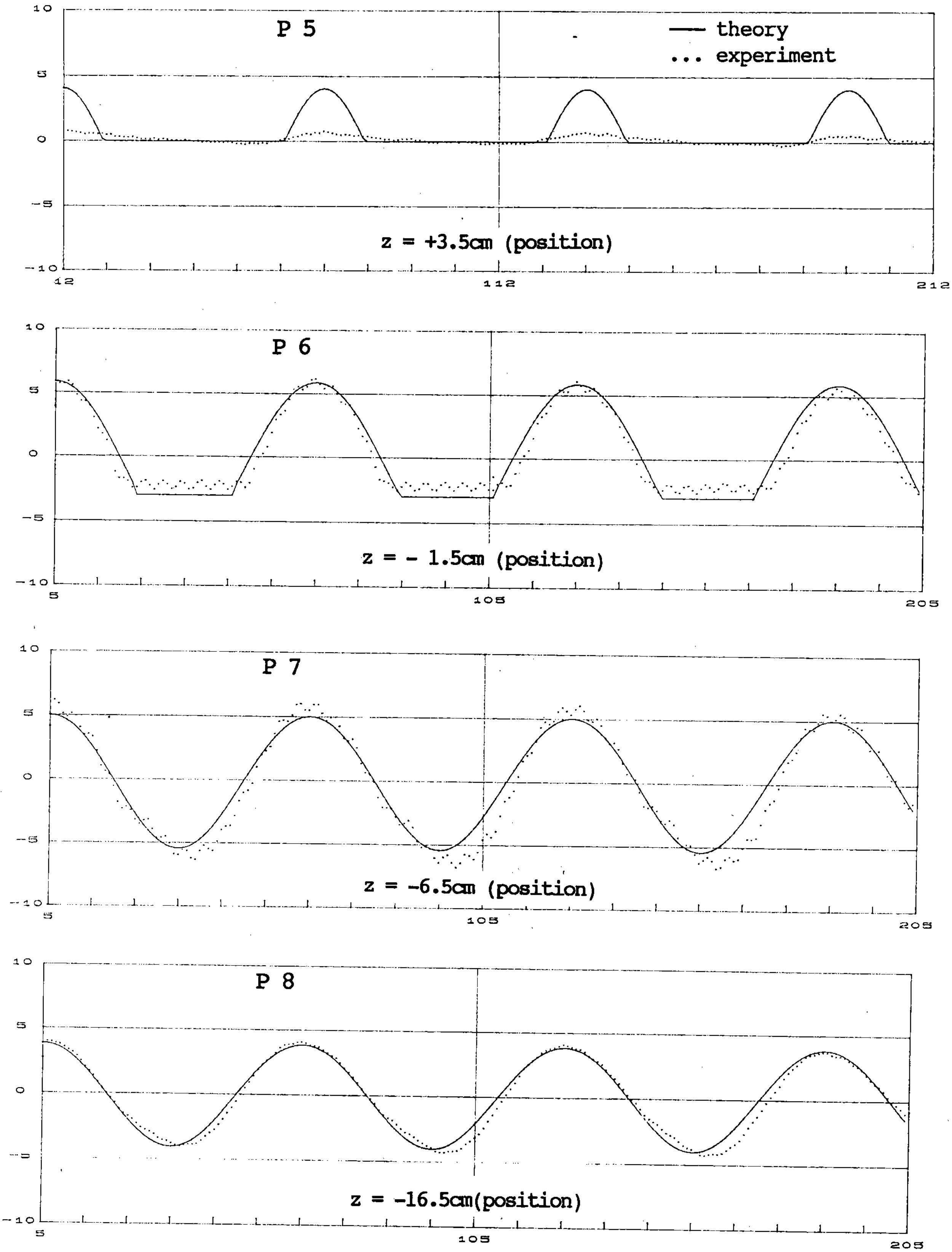
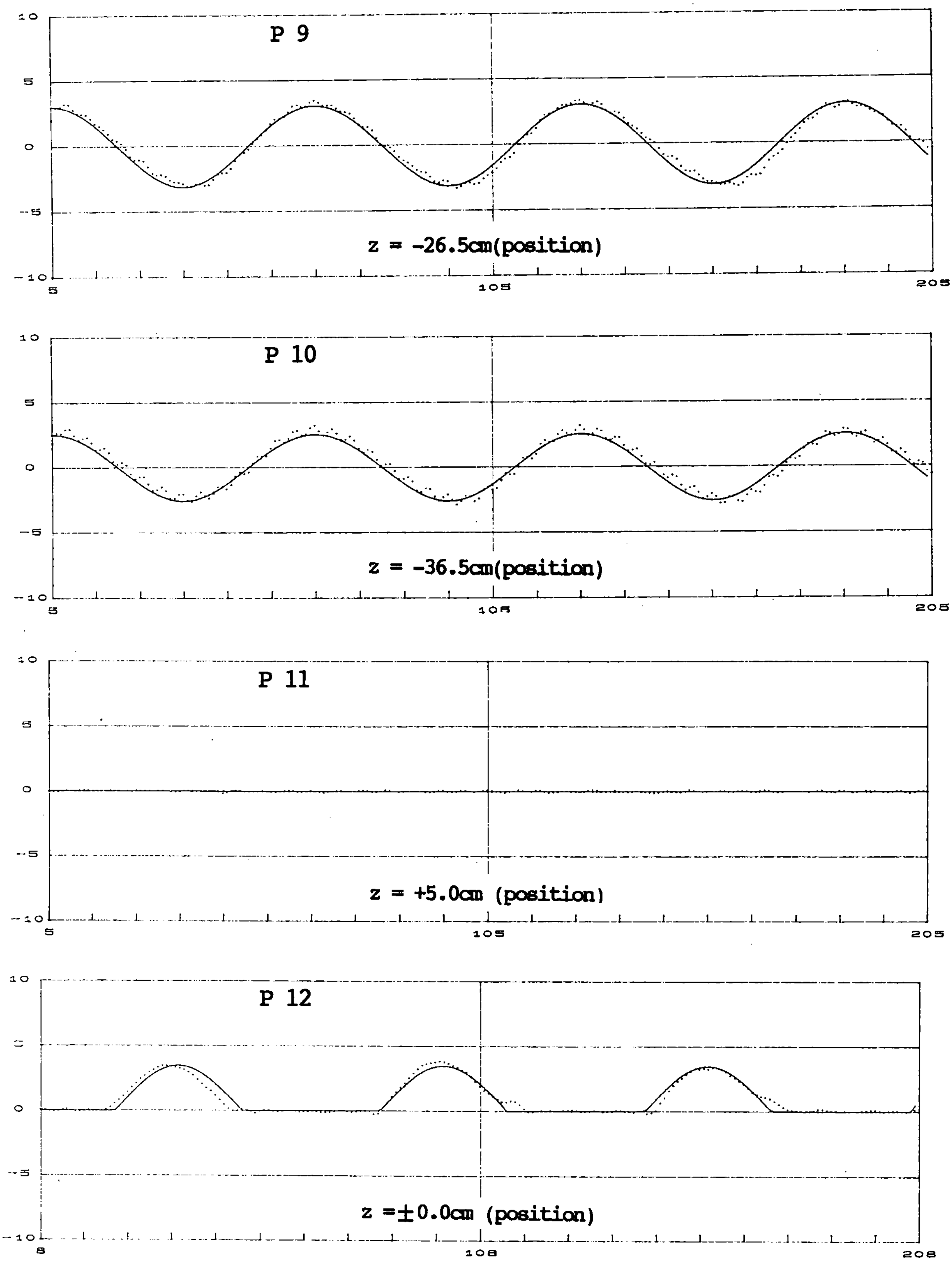


圖11-3



P 9
z = -26.5cm(position)
P 10
z = -36.5cm(position)
P 11
z = +5.0cm (position)
P 12
z =±0.0cm (position)

圖11-3

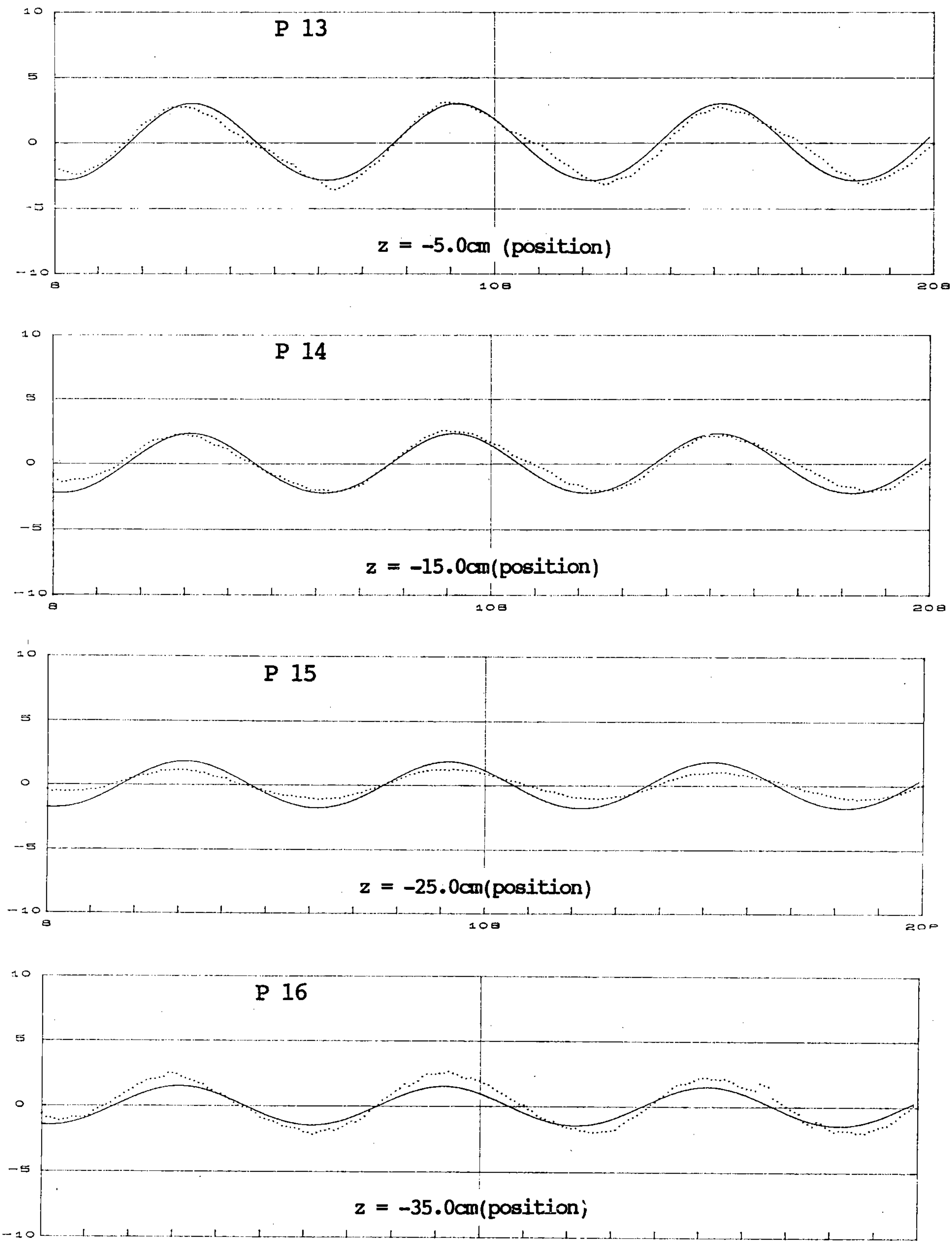

P 13
z = -5.0cm (position)
P 14
z = -15.0cm(position)
P 15
z = -25.0cm(position)
P 16
z = -35.0cm(position)

圖11-3

θ =45° (intersection angle), T_1=T_2=1.60sec(incident wave period 1 & 2)
d =60cm(water depth), H_1=7.31cm(incident wave height 1)
H_2=3.47cm(incident wave height 2)

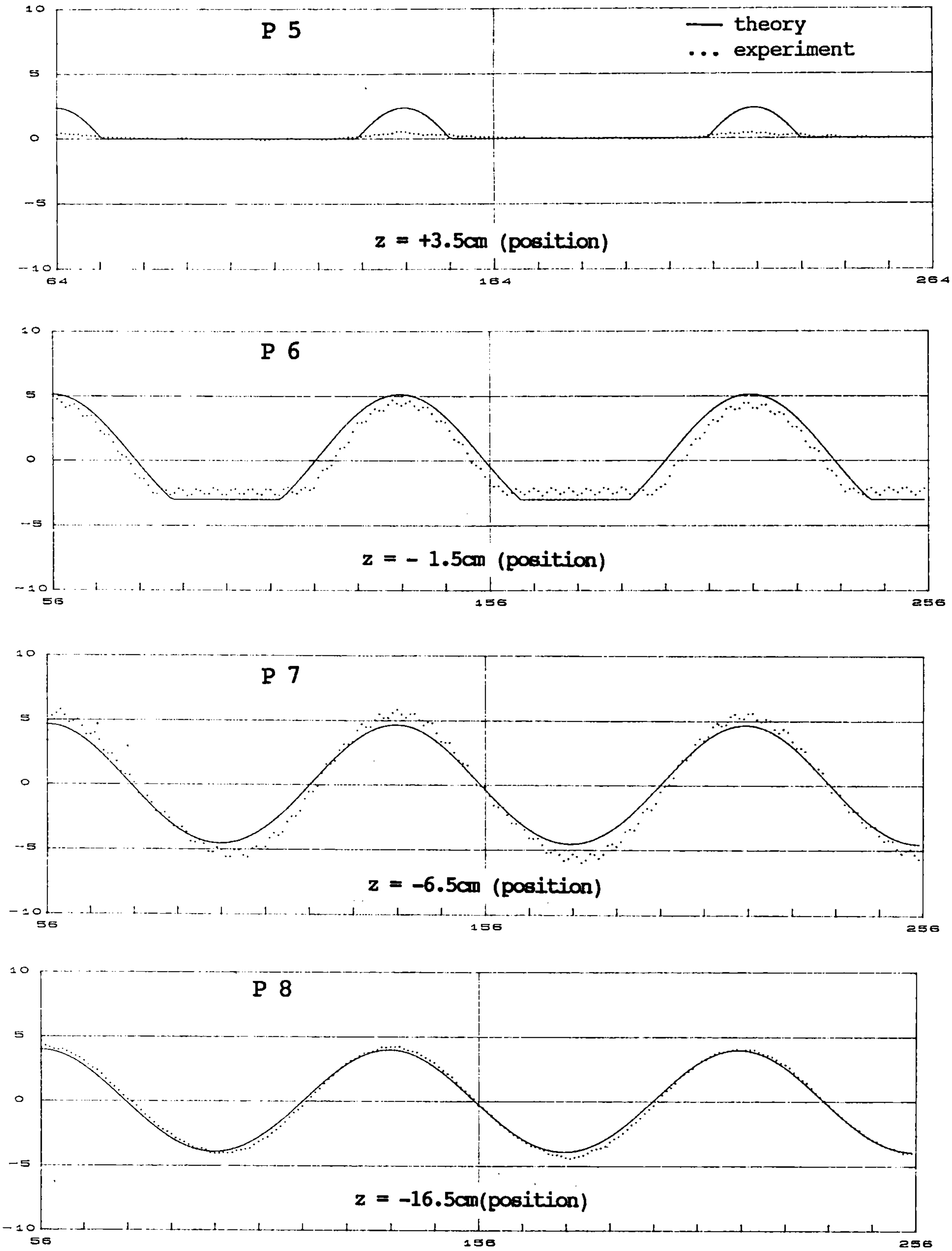


圖11-4

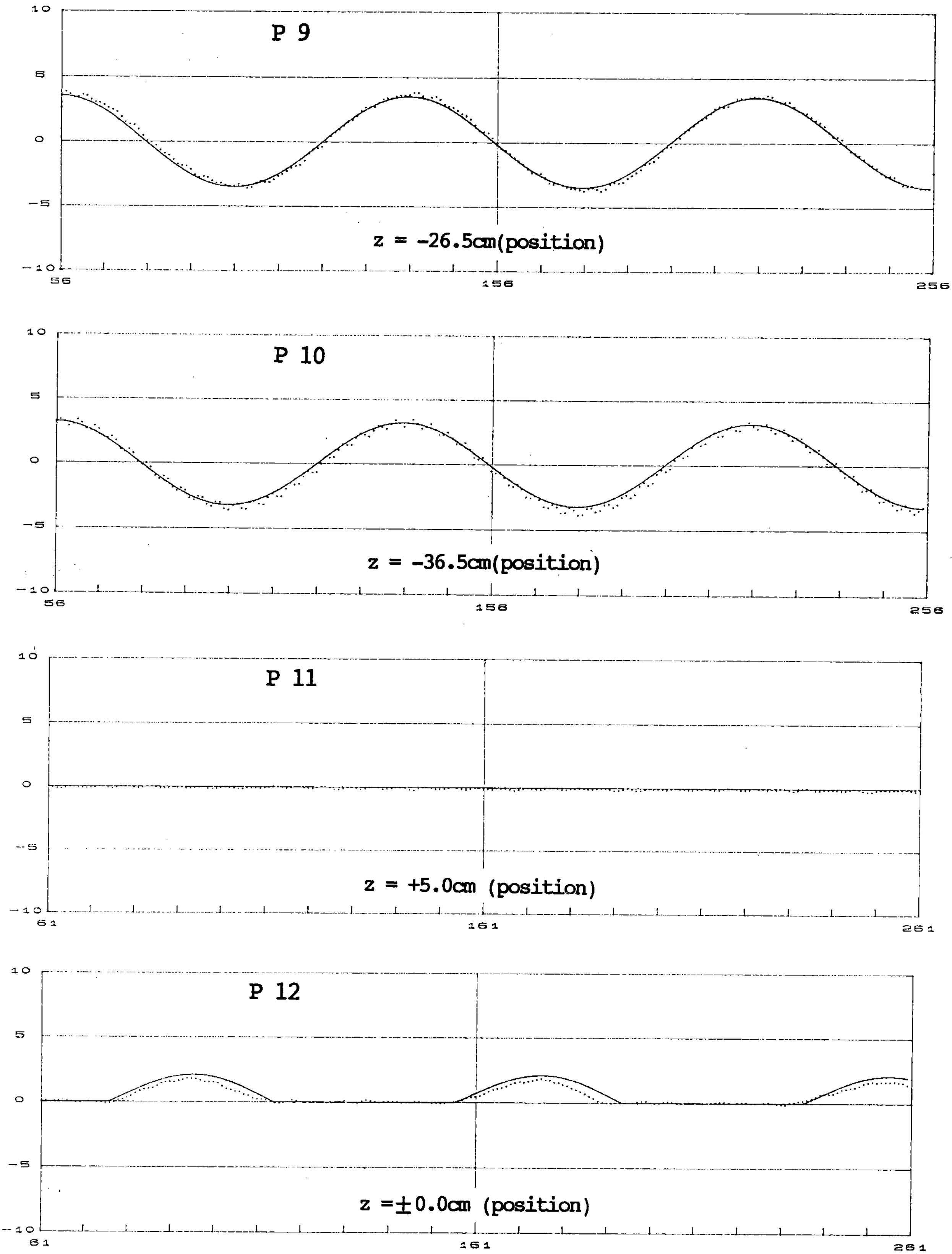

P 9
z = -26.5cm(position)
P 10
z = -36.5cm(position)
P 11
z = +5.0cm (position)
P 12
z =±0.0cm (position)
10
5
0
-5
-10
56
156
256
61
161
261

圖11-4

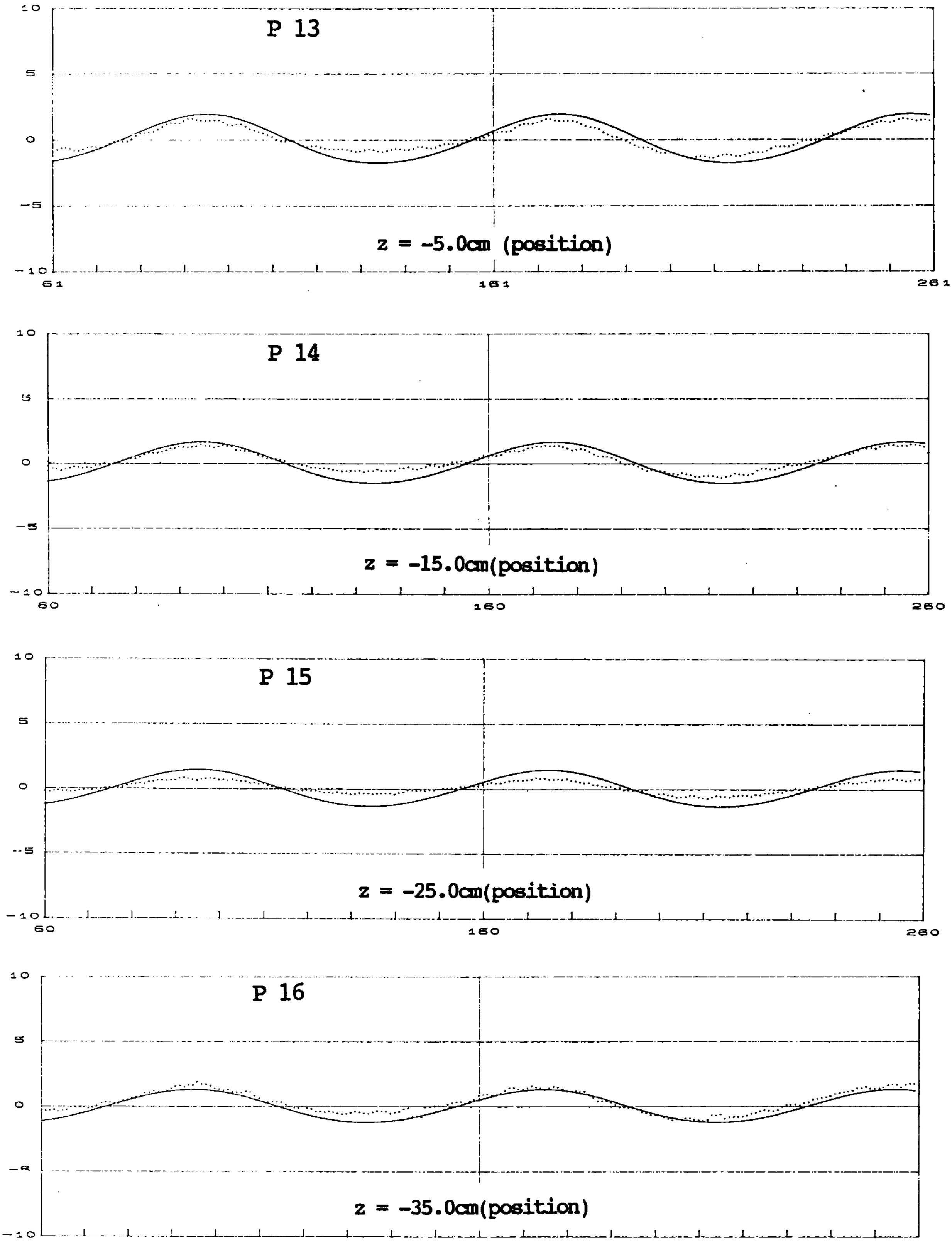

P 13
z = -5.0cm (position)
10
5
0
-5
-10
61
161
261
P 14
z = -15.0cm(position)
10
5
0
-5
-10
60
160
260
P 15
z = -25.0cm(position)
10
5
0
-5
-10
60
160
260
P 16
z = -35.0cm(position)
10
5
0
-10
60
160
260

圖11-4

θ=45° (intersection angle), T_1=T_2=1.60sec(incident wave period 1 & 2)
d =60cm(water depth), H_1=12.50cm(incident wave height 1)
H_2=3.99cm(incident wave height 2)

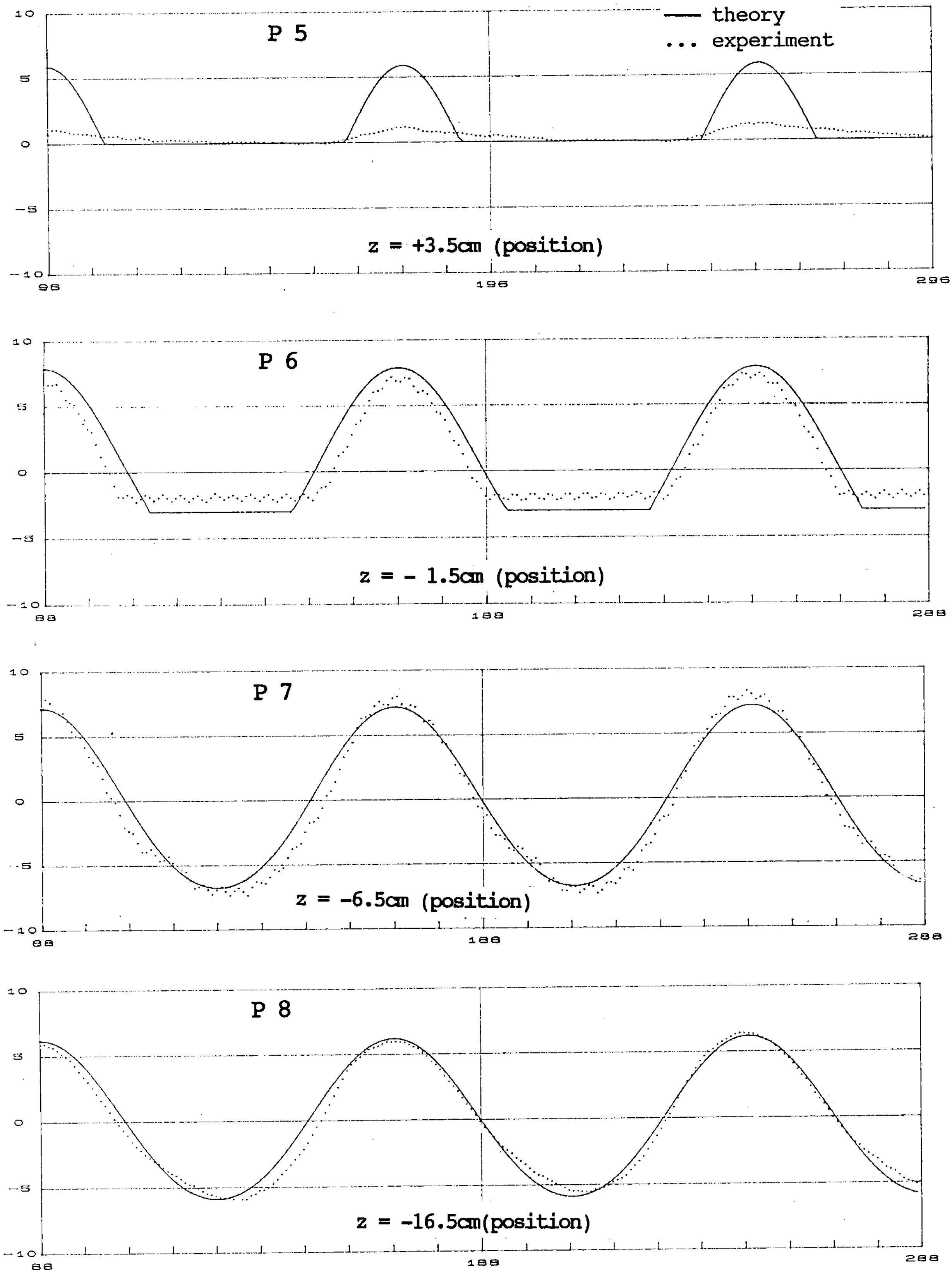


圖11-5

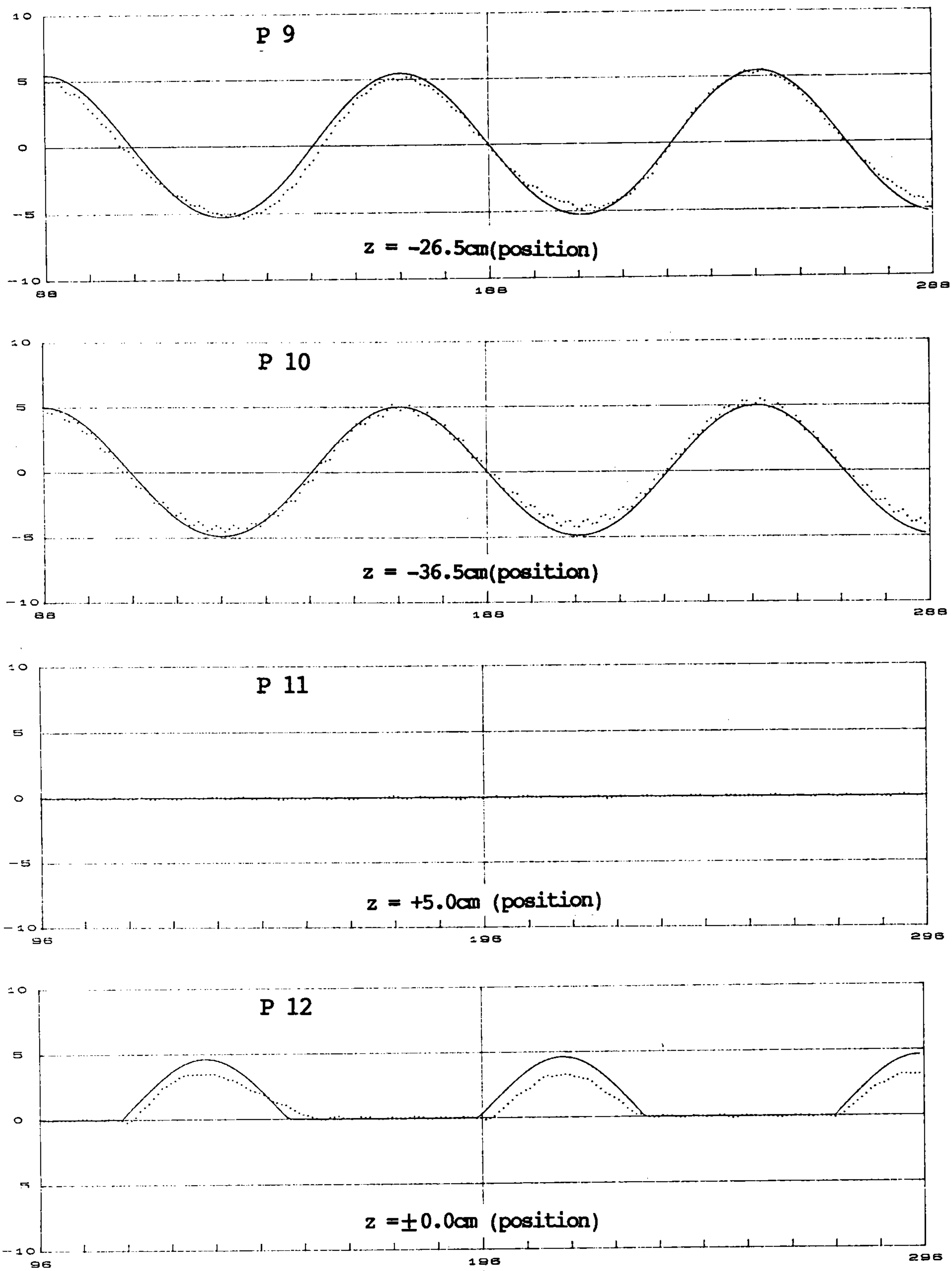

P 9
z = -26.5cm(position)
P 10
z = -36.5cm(position)
P 11
z = +5.0cm (position)
P 12
z =±0.0cm (position)
10
5
0
-5
-10
88
188
288
96
196
296

圖11-5

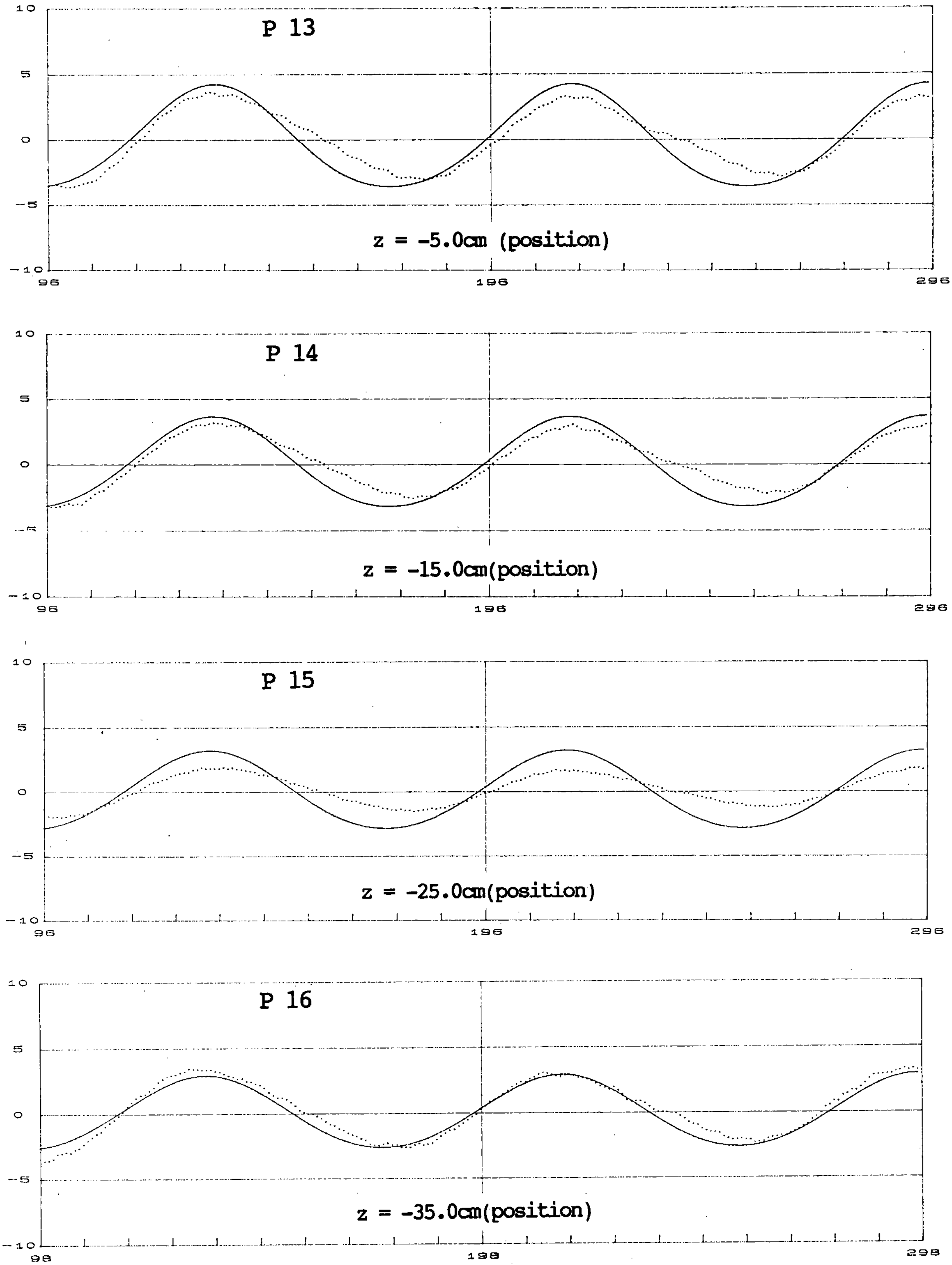

P 13
z = -5.0cm (position)
10
5
0
-5
-10
96
196
296
P 14
z = -15.0cm(position)
10
5
0
-10
96
196
296
P 15
z = -25.0cm(position)
10
5
0
-5
-10
96
196
296
P 16
z = -35.0cm(position)
10
5
0
-5
-10
98
198
298

圖11-5

θ=45° (intersection angle), T_1=T_2=2.00sec(incident wave period 1 & 2)
d =60cm(water depth), H_1=7.25cm(incident wave height 1)
H_2=3.28cm(incident wave height 2)

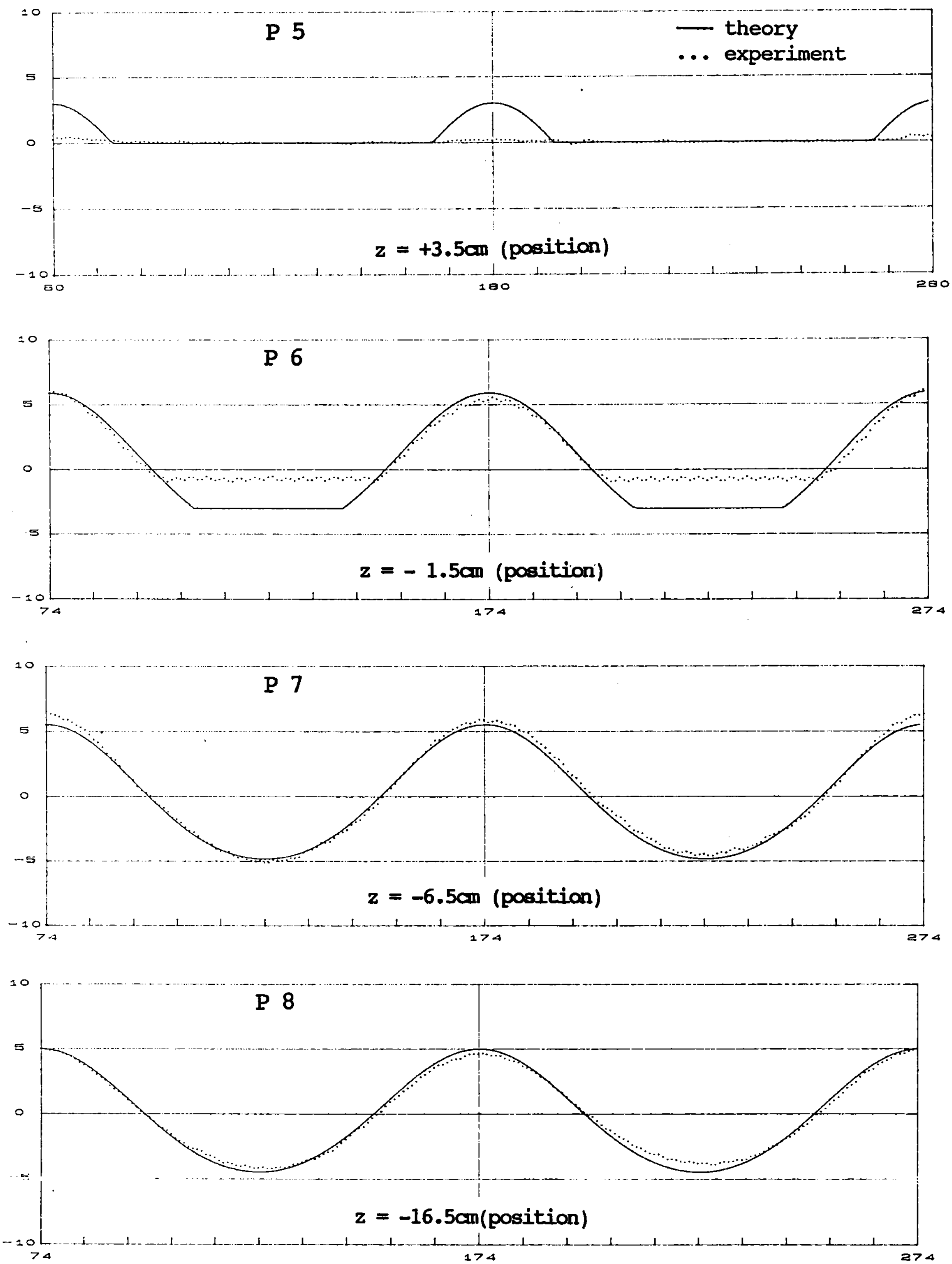


圖11-6

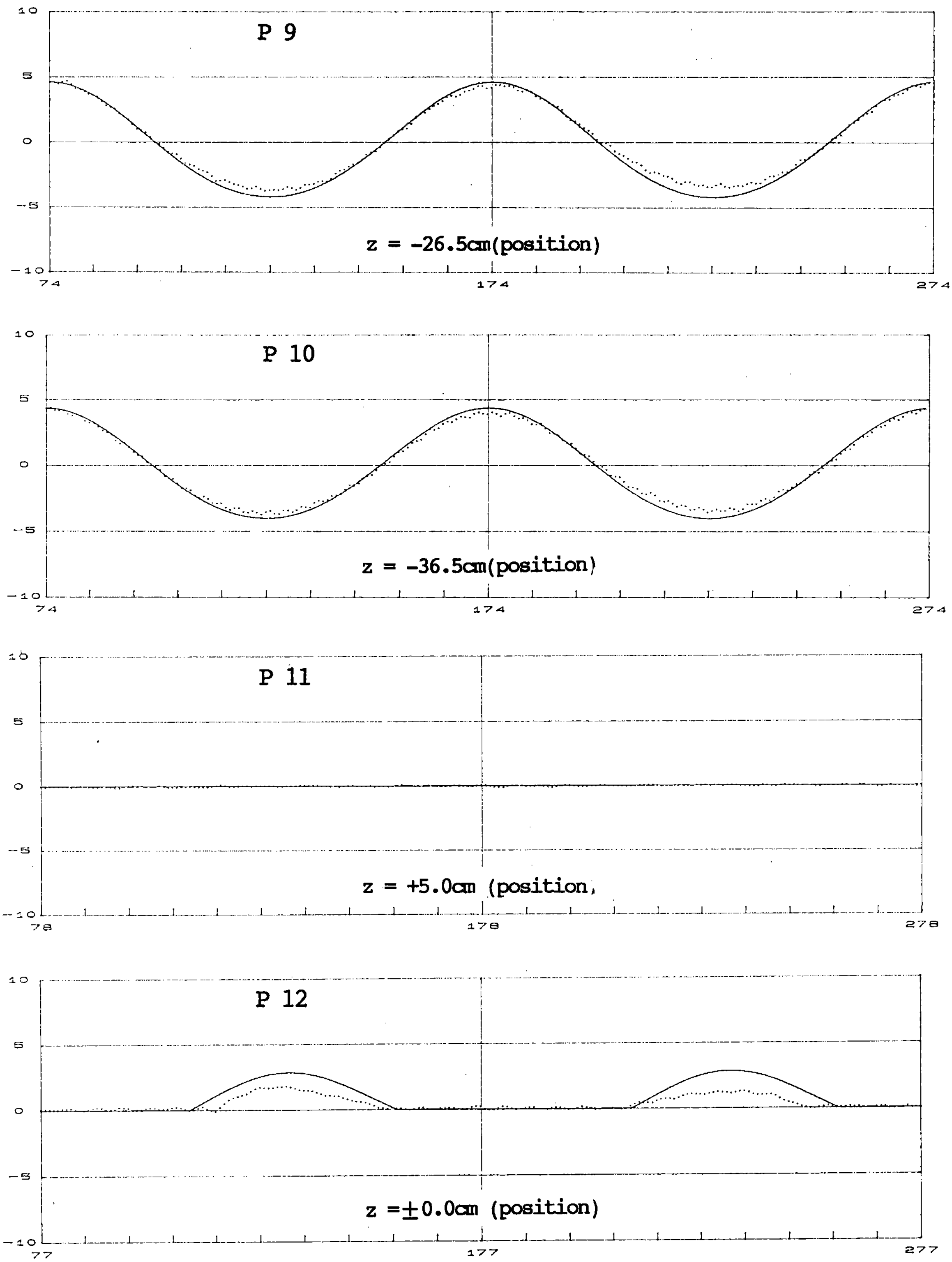

P 9
z = -26.5cm(position)
10
5
0
-5
-10
74
174
274
P 10
z = -36.5cm(position)
10
5
0
-5
-10
74
174
274
P 11
z = +5.0cm (position)
10
5
0
-5
-10
78
178
278
P 12
z =±0.0cm (position)
10
5
0
-5
-10
77
177
277

圖11-6

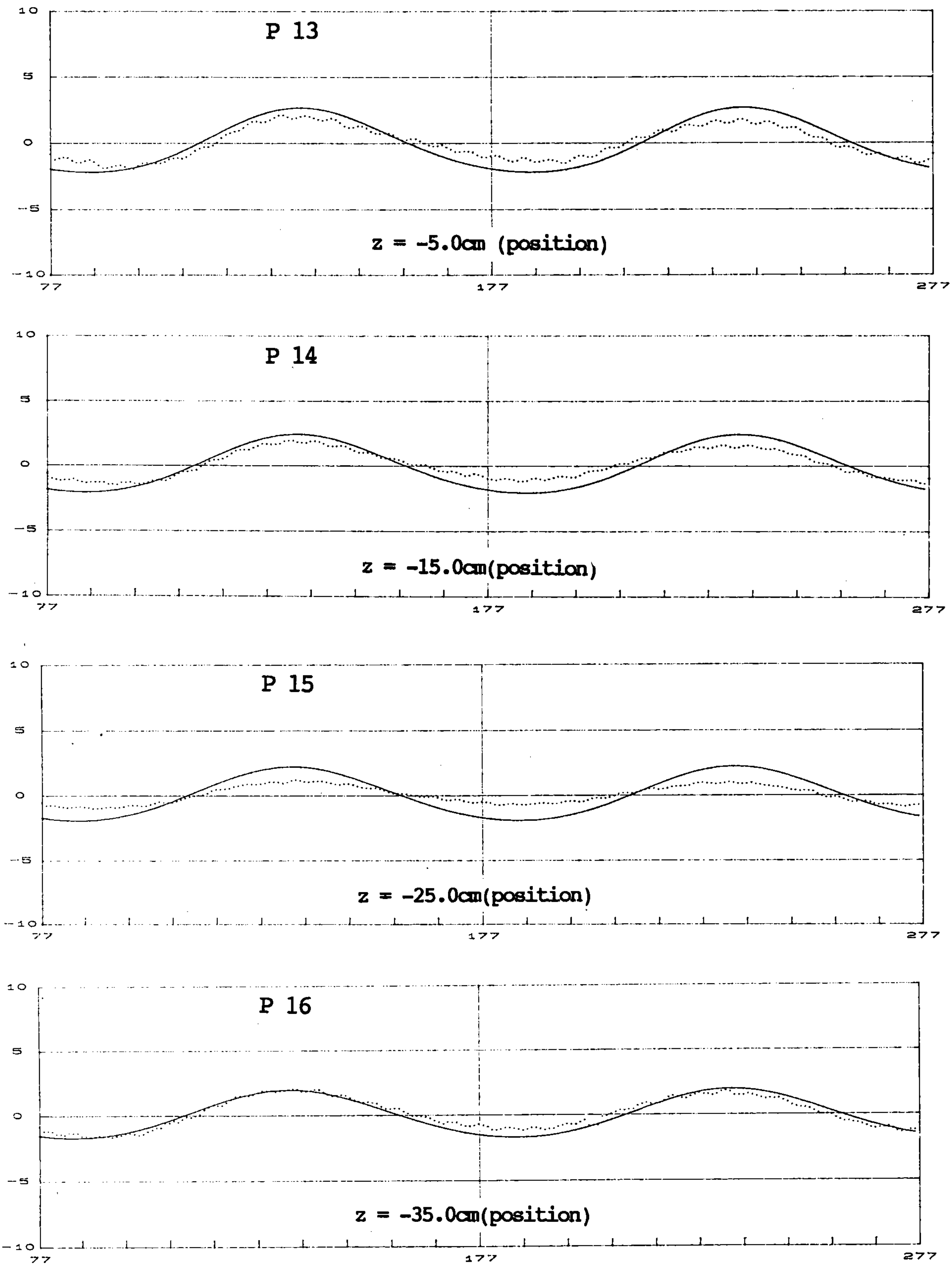

P 13
z = -5.0cm (position)
P 14
z = -15.0cm(position)
P 15
z = -25.0cm(position)
P 16
z = -35.0cm(position)
10
5
0
-5
-10
77
177
277

圖11-6

θ =45° (intersection angle), T_1=T_2=2.00sec(incident wave period 1 & 2)
d =60cm(water depth), H_1=14.86cm(incident wave height 1)
H_2=3.89cm(incident wave height 2)

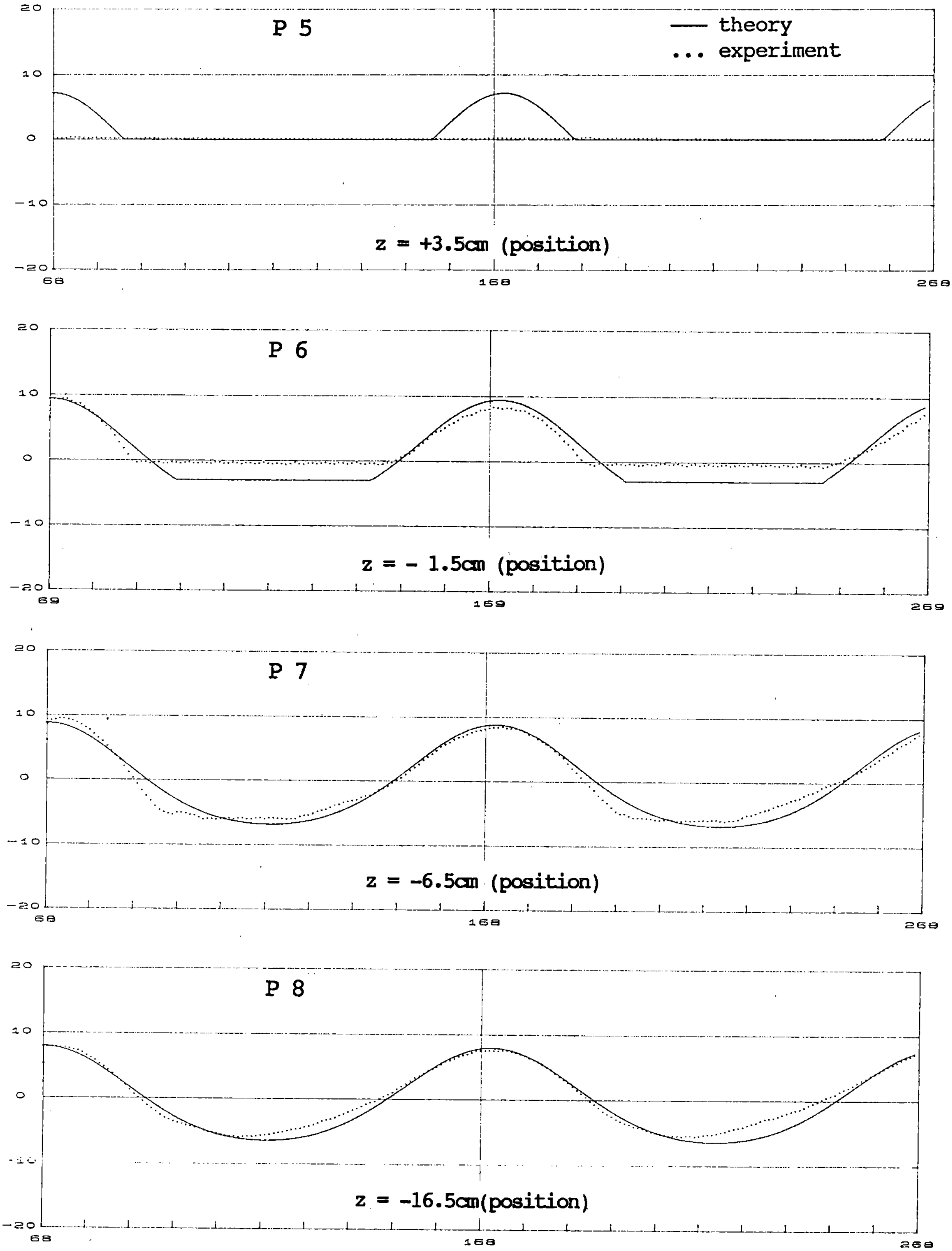


圖11-7

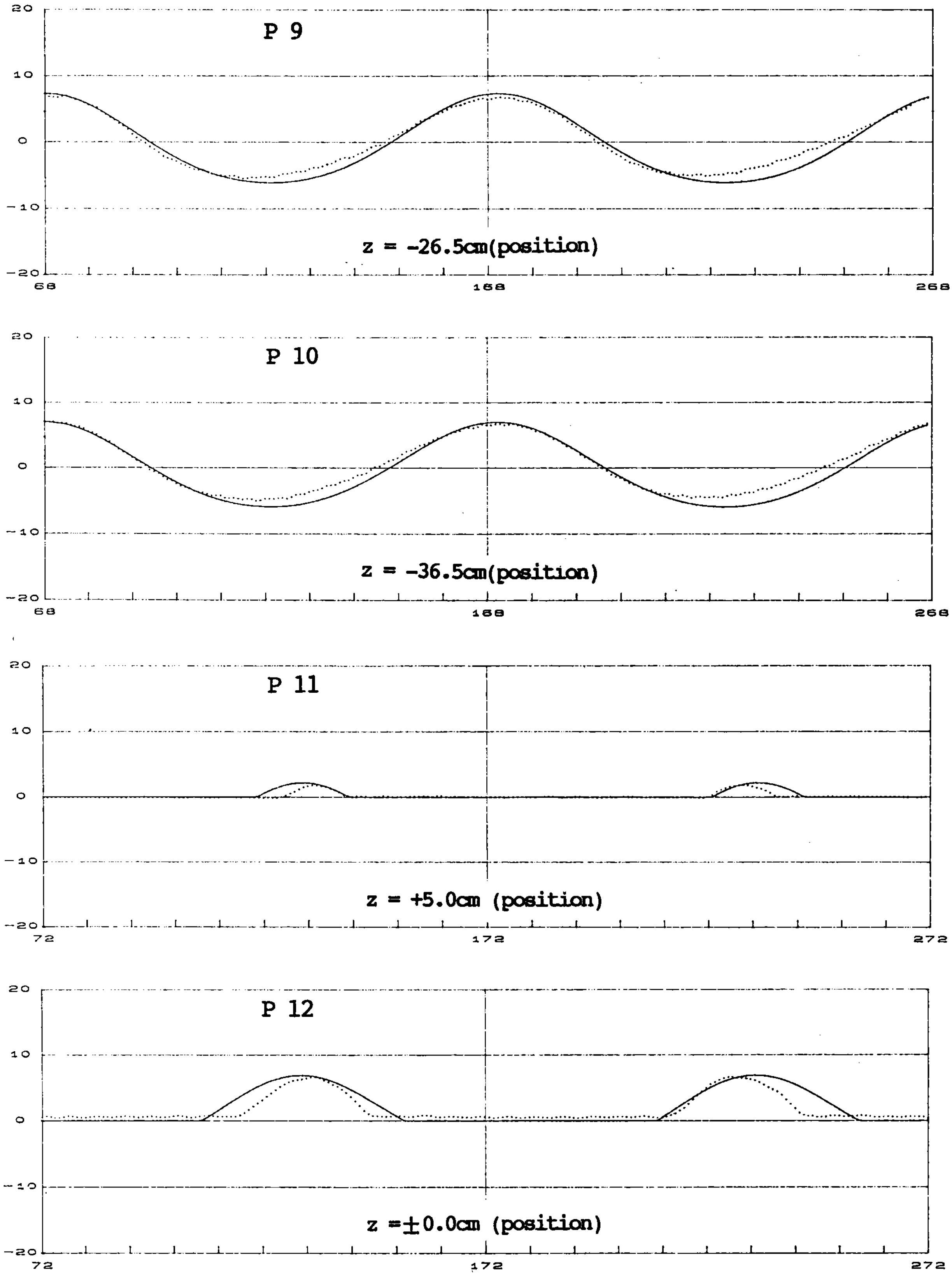
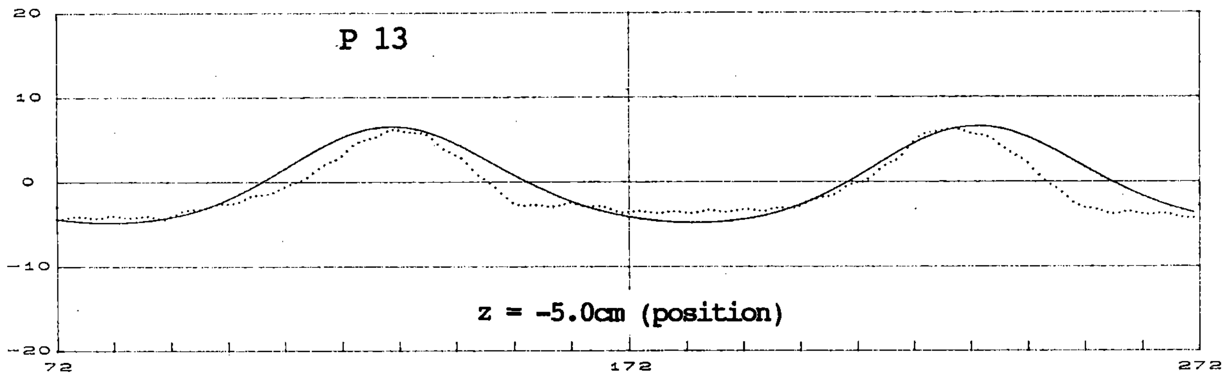
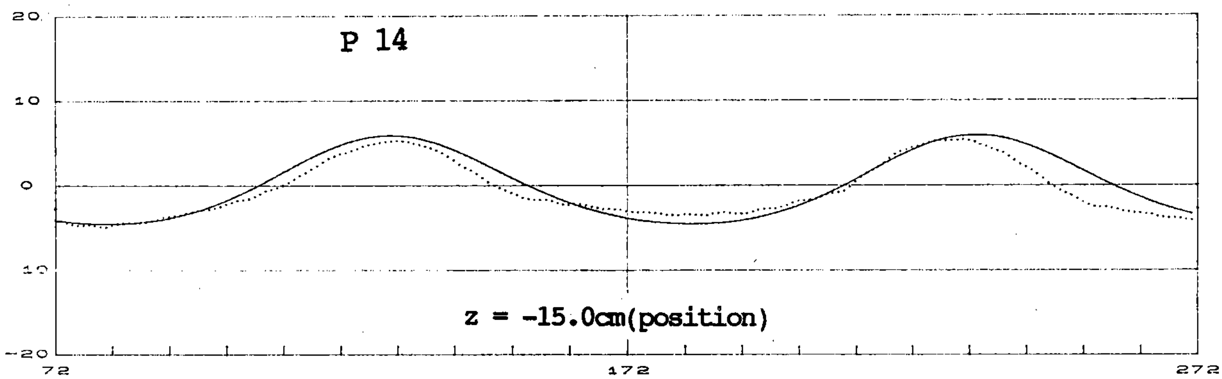
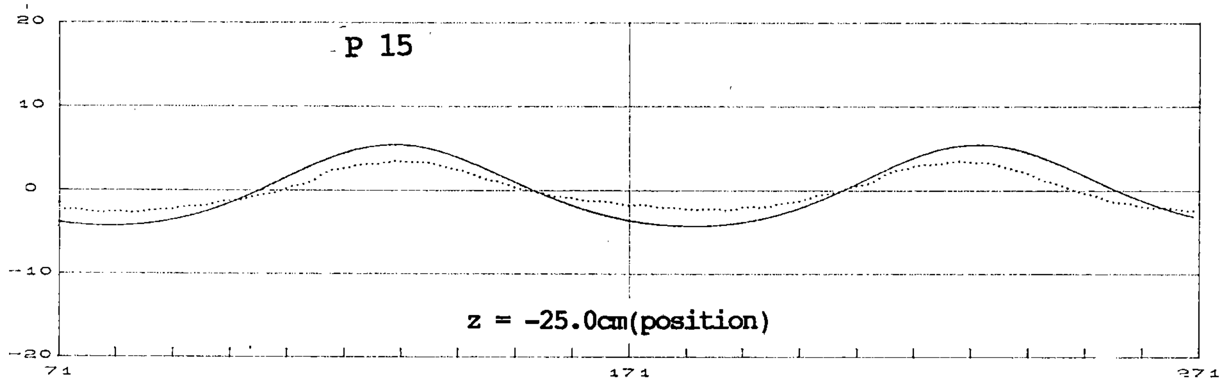

P 9
z = -26.5cm(position)
20
10
0
−10
−20
68
168
268
P 10
z = -36.5cm(position)
20
10
0
−10
−20
68
168
268
P 11
z = +5.0cm (position)
20
10
0
−10
−20
72
172
272
P 12
z =±0.0cm (position)
20
10
0
−10
−20
72
172
272

圖11-7


P 13
z = -5.0cm (position)
20
10
0
-10
-20
72
172
272


P 14
z = -15.0cm(position)
20
10
0
-20
72
172
272


P 15
z = -25.0cm(position)
20
10
0
-10
-20
71
171
271

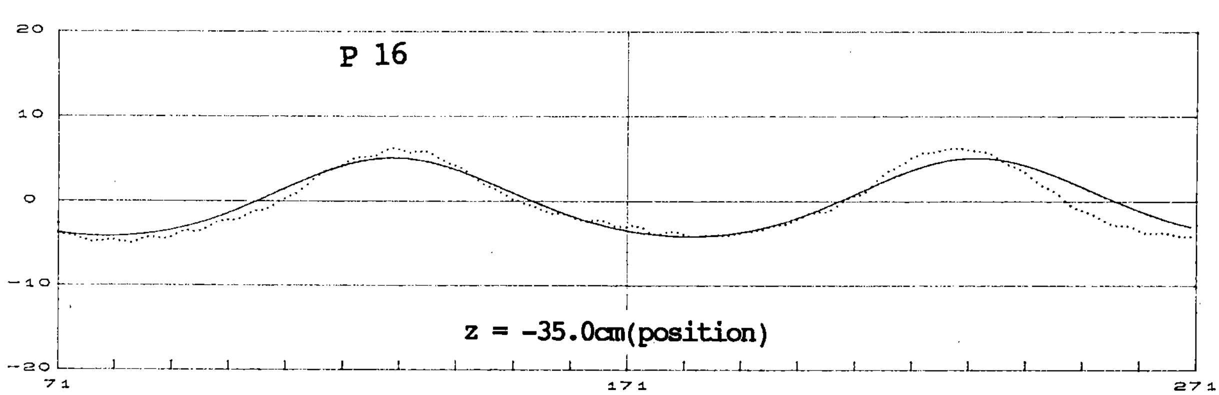

P 16
z = -35.0cm(position)
20
10
0
-10
-20
71
171
271

圖11-7

θ =45° (intersection angle), d =60cm(water depth),

T_1=T_2=2.40sec(incident wave period 1 & 2)
H_1=7.47cm(incident wave height 1)
H_2=3.52cm(incident wave height 2)

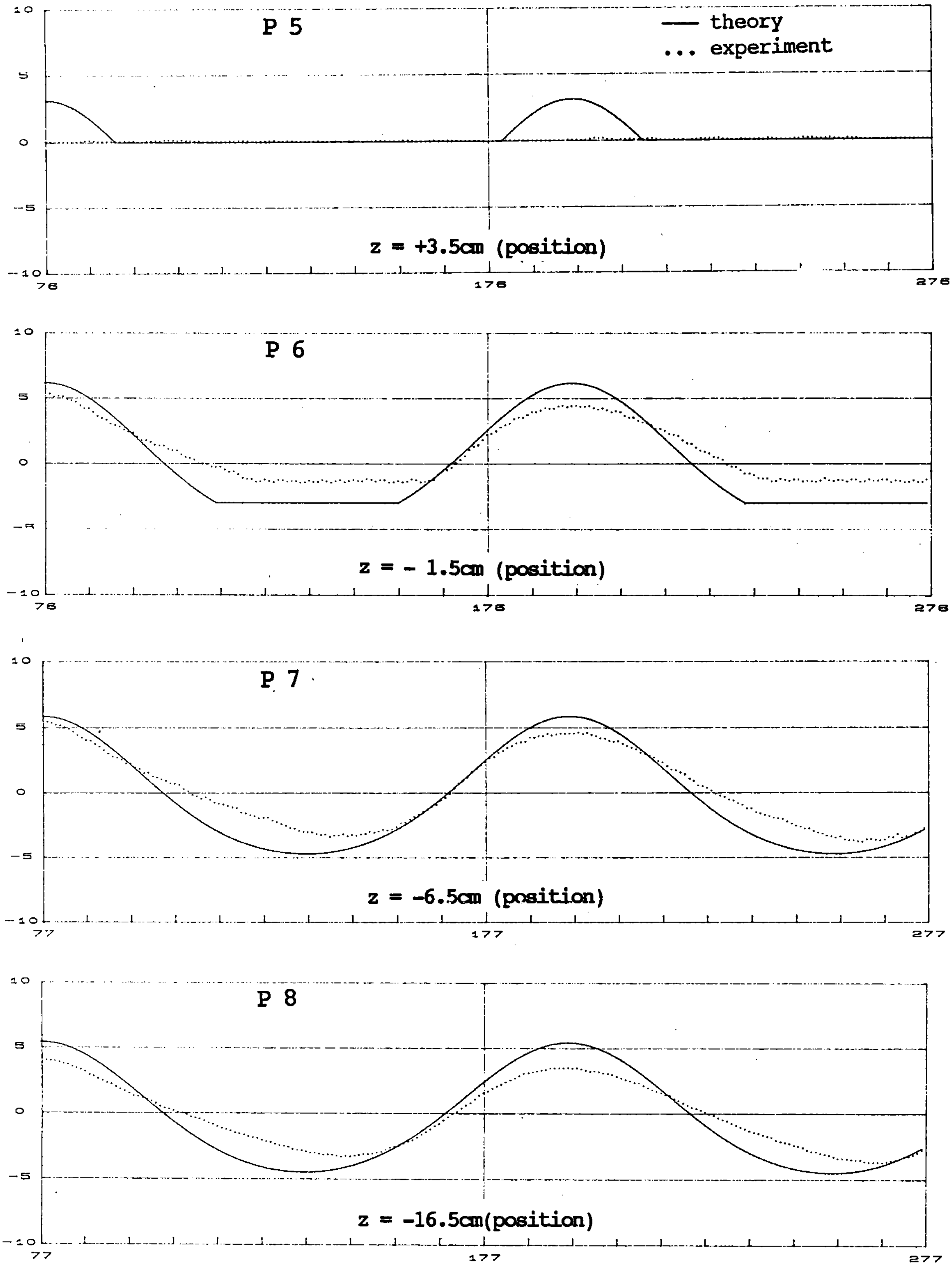


圖11-8

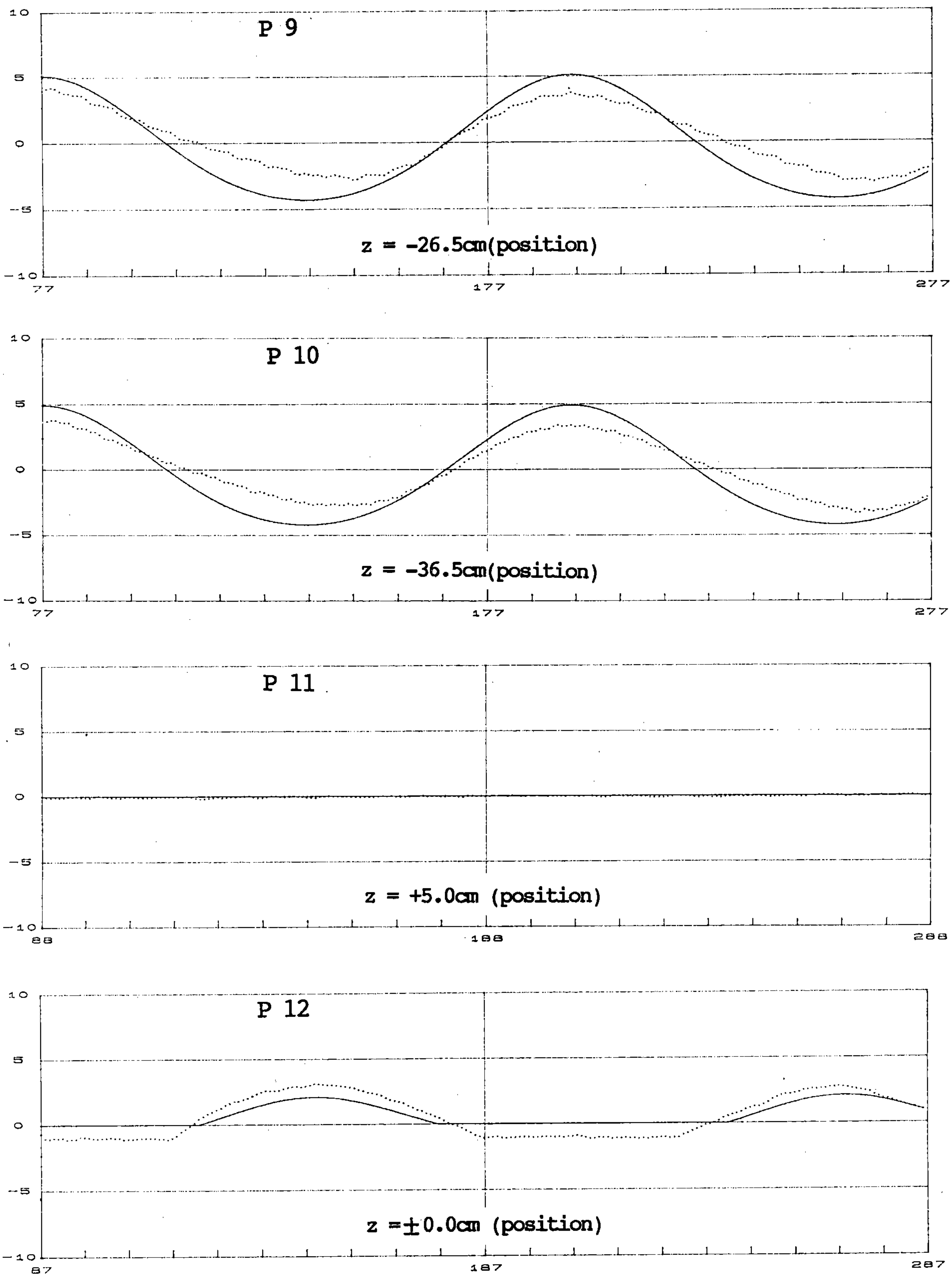
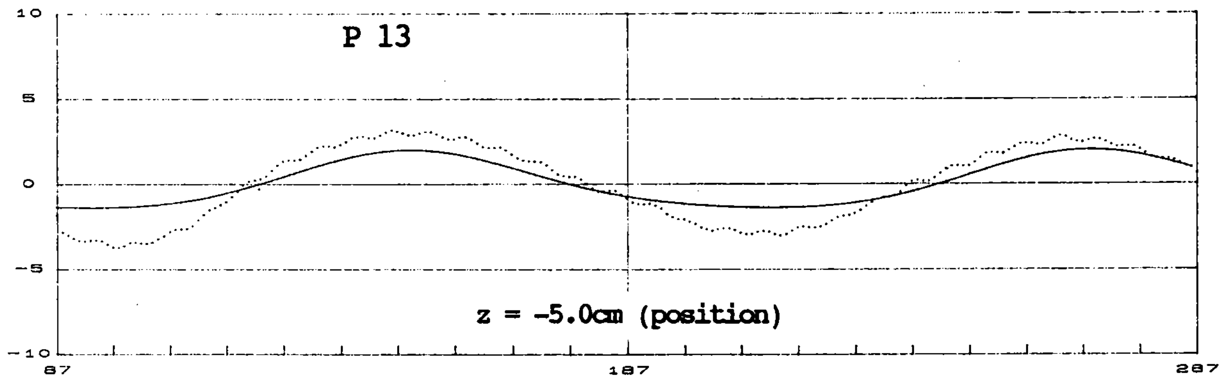
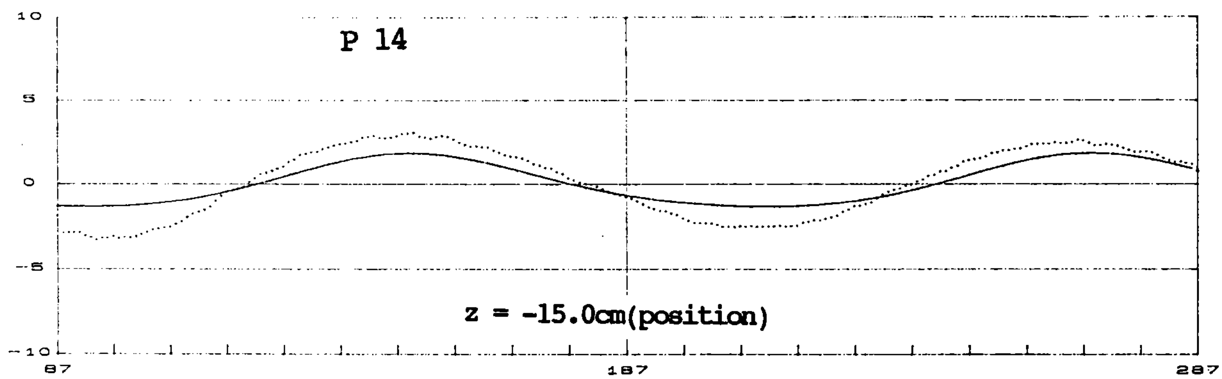

P 9
z = -26.5cm(position)
10
5
0
-5
-10
77
177
277
P 10
z = -36.5cm(position)
10
5
0
-5
-10
77
177
277
P 11
z = +5.0cm (position)
10
5
0
-5
-10
88
188
288
P 12
z =±0.0cm (position)
10
5
0
-5
-10
87
187
287

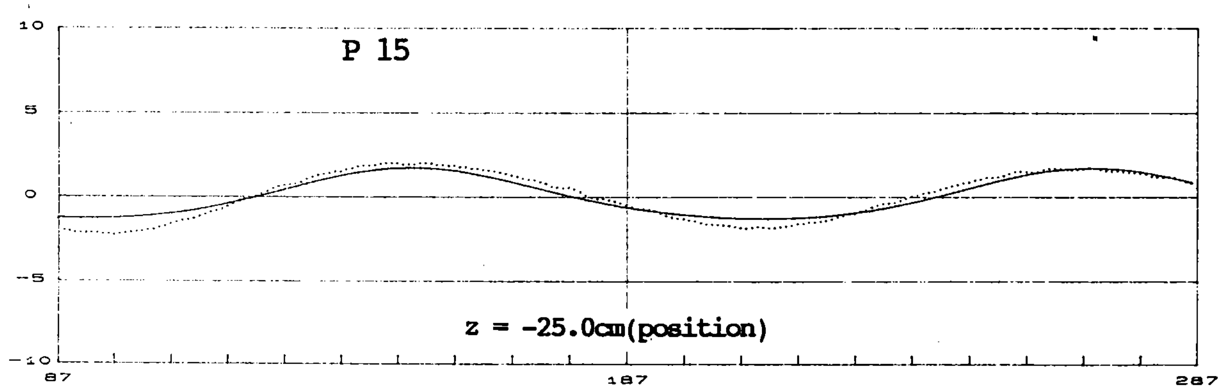
圖11-8



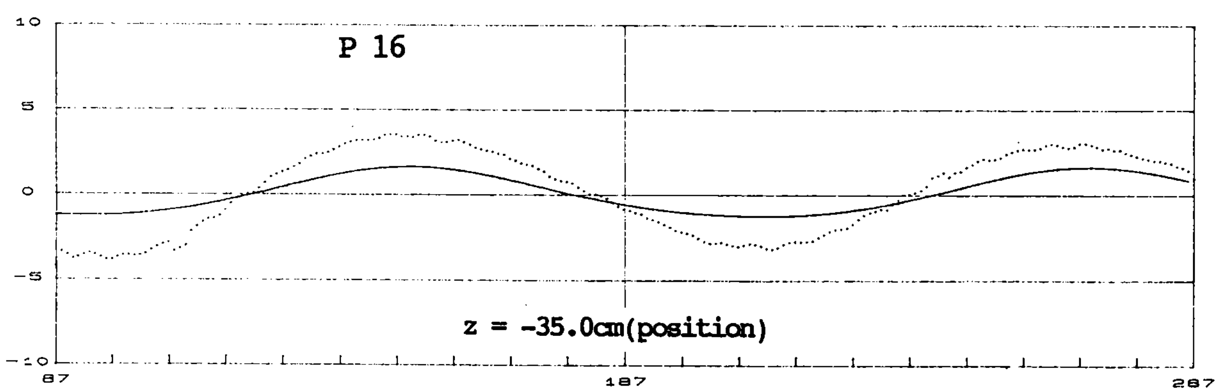
10
5
0
-5
-10
P 13
z = -5.0cm (position)
87
187
287



10
5
0
-5
-10
P 14
z = -15.0cm(position)
87
187
287



10
5
0
-5
-10
P 15
z = -25.0cm(position)
87
187
287



10
5
0
-5
-10
P 16
z = -35.0cm(position)
87
187
287

圖11-8

θ =45° (intersection angle), T_1=T_2=2.40sec(incident wave period 1 & 2)
d =60cm(water depth), H_1=13.39cm(incident wave height 1)
H_2=3.34cm(incident wave height 2)

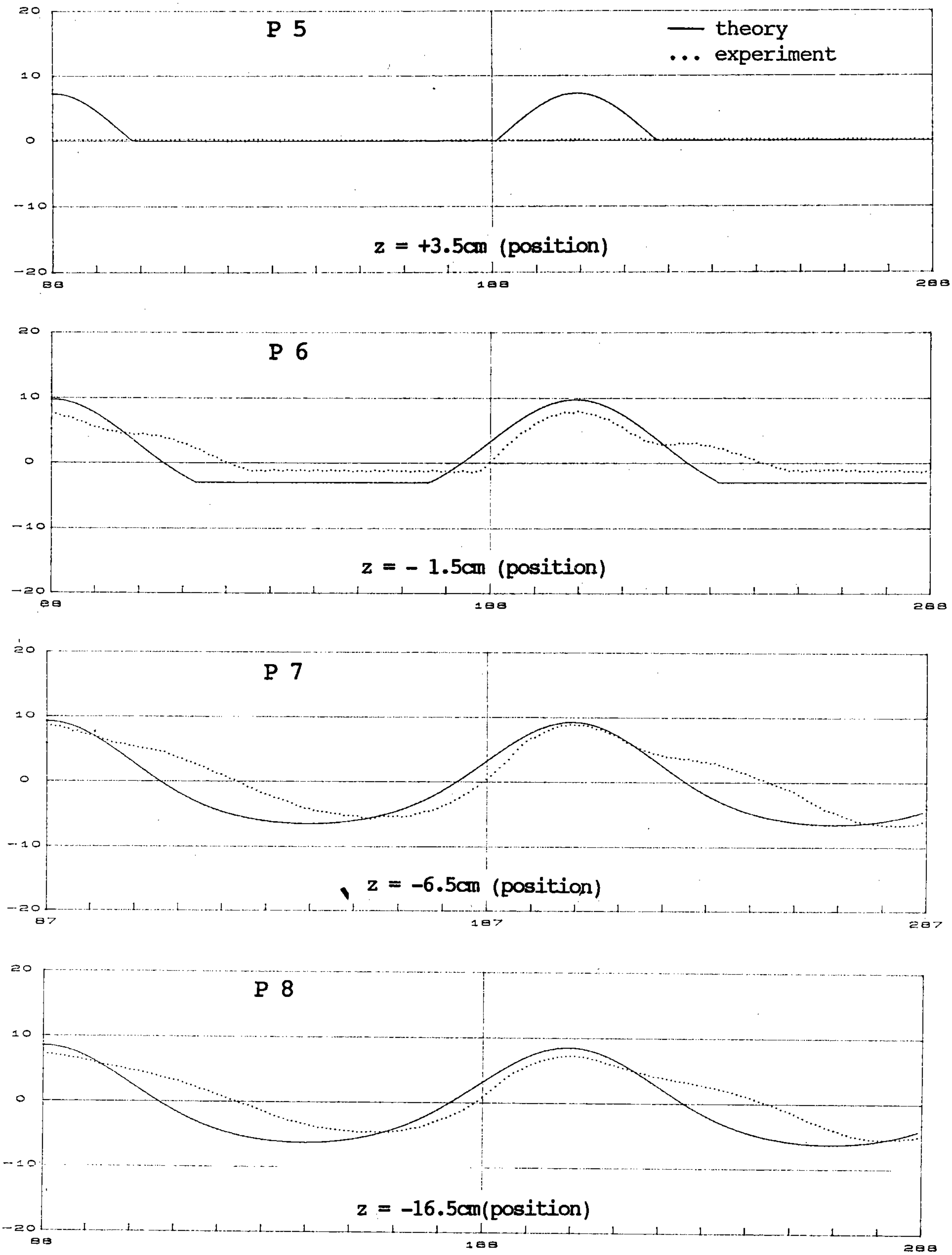


圖11-9

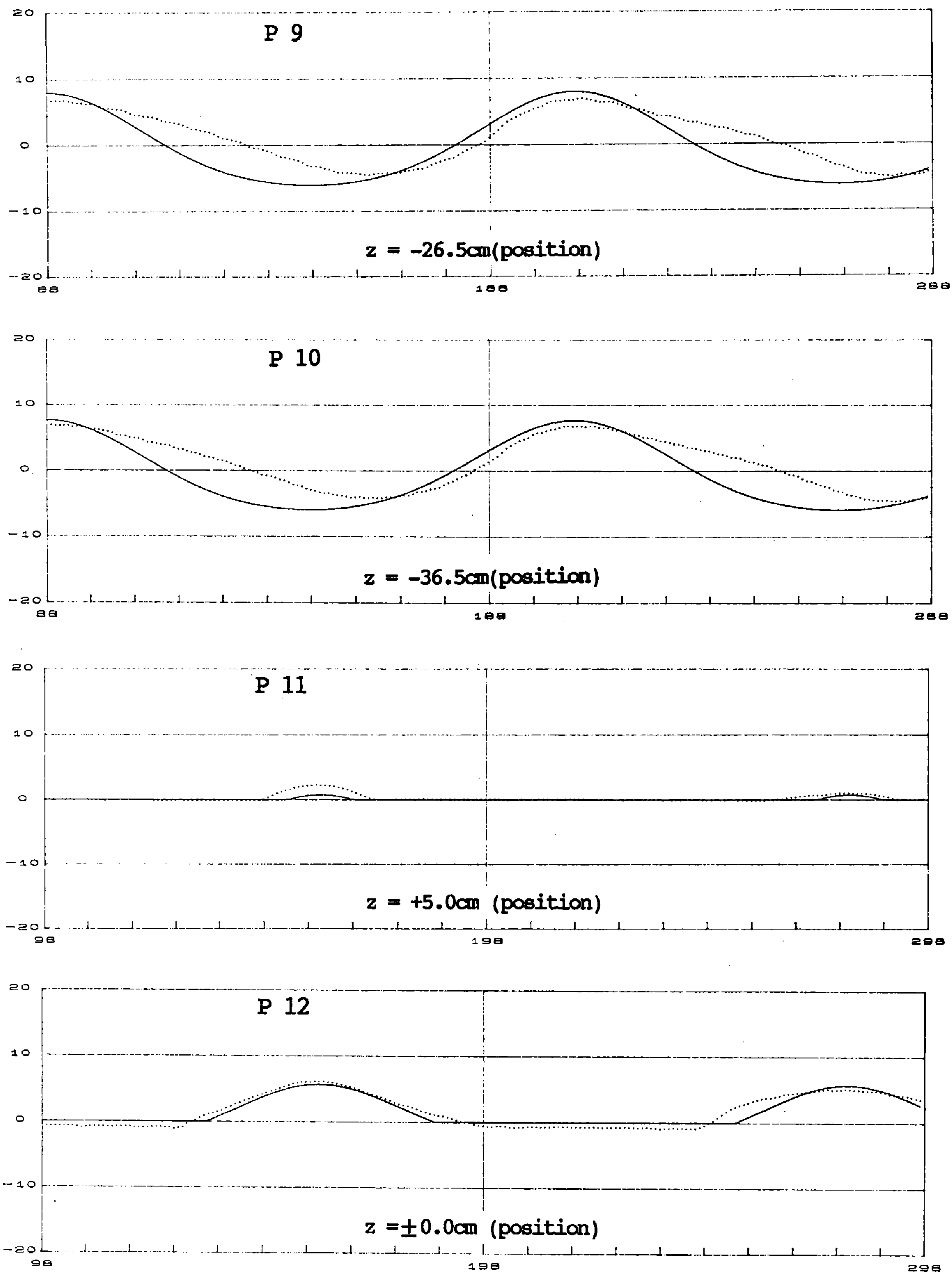

P 9
z = -26.5cm(position)
P 10
z = -36.5cm(position)
P 11
z = +5.0cm (position)
P 12
z =±0.0cm (position)

圖11-9

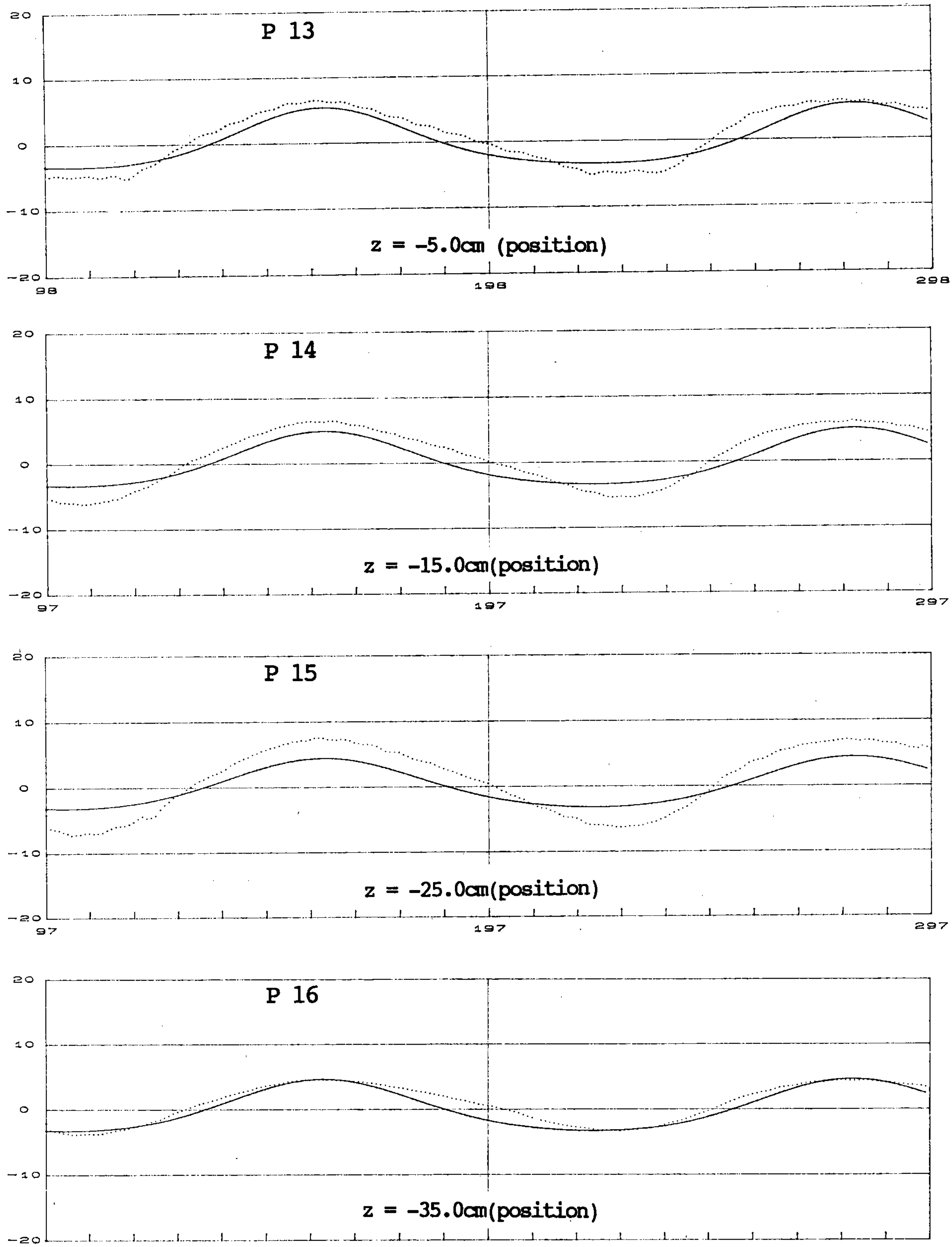

P 13
z = -5.0cm (position)
20
10
0
-10
-20
98
198
298
P 14
z = -15.0cm(position)
97
197
297
P 15
z = -25.0cm(position)
P 16
z = -35.0cm(position)

圖11-9

θ=45° (intersection angle), T_1=T_2=0.8sec(incident wave period 1 & 2)
d =50cm(water depth), H_1=6.95cm(incident wave height 1)
H_2=3.01cm(incident wave height 2)

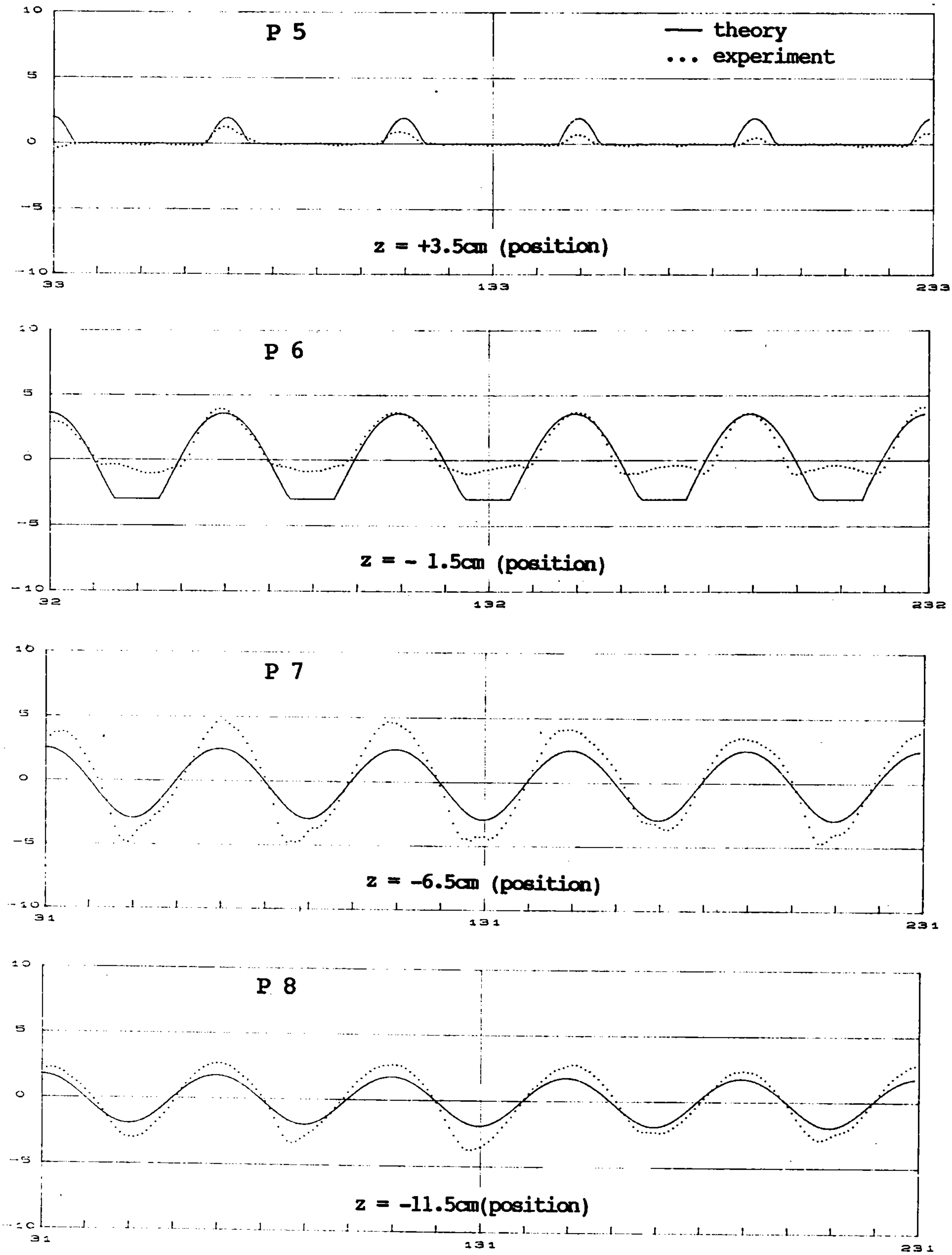


圖11-10

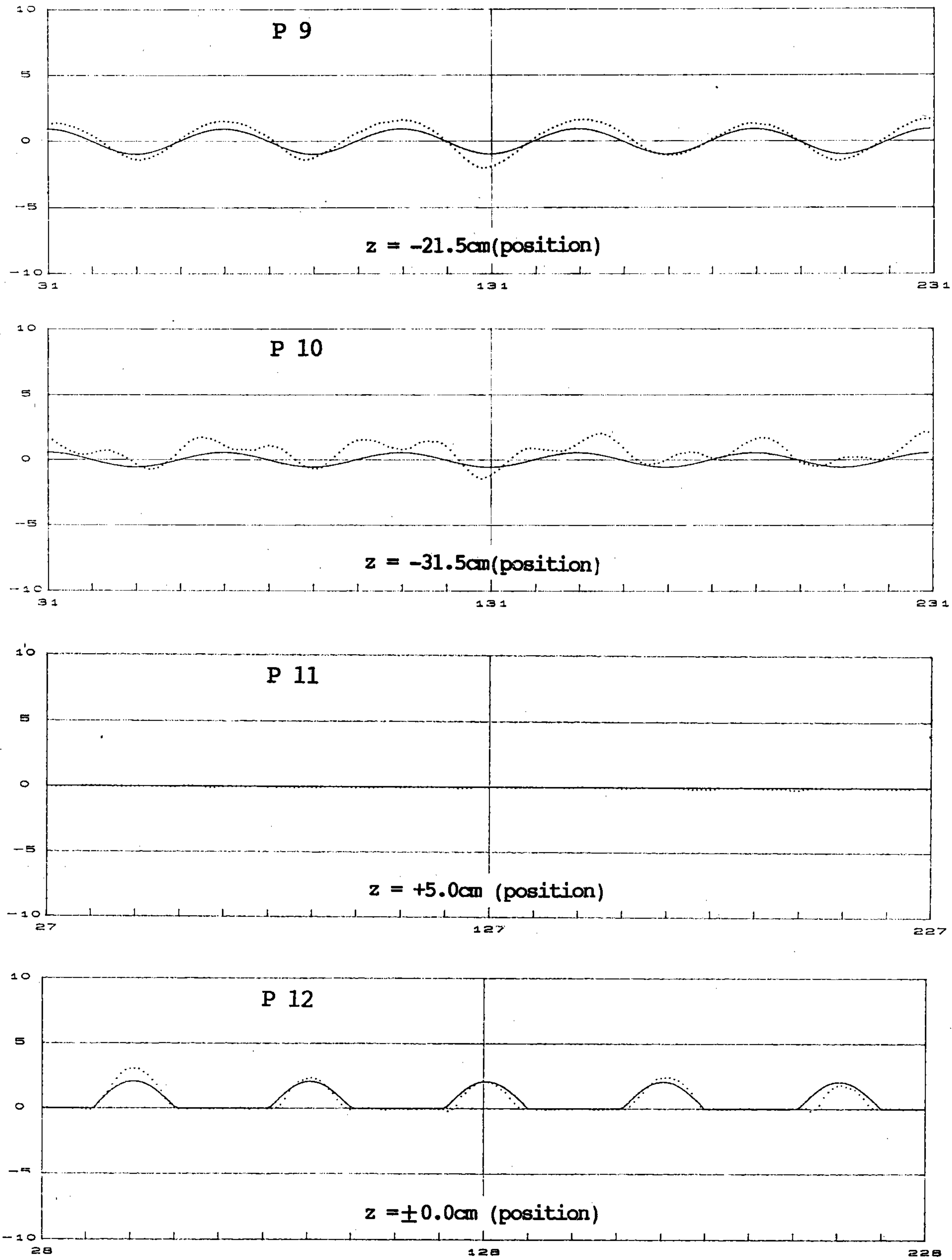

P 9
z = -21.5cm(position)
10
5
0
-5
-10
31
131
231
P 10
z = -31.5cm(position)
10
5
0
-5
-10
31
131
231
P 11
z = +5.0cm (position)
10
5
0
-5
-10
27
127
227
P 12
z =±0.0cm (position)
10
5
0
-5
-10
28
128
228

圖11-10

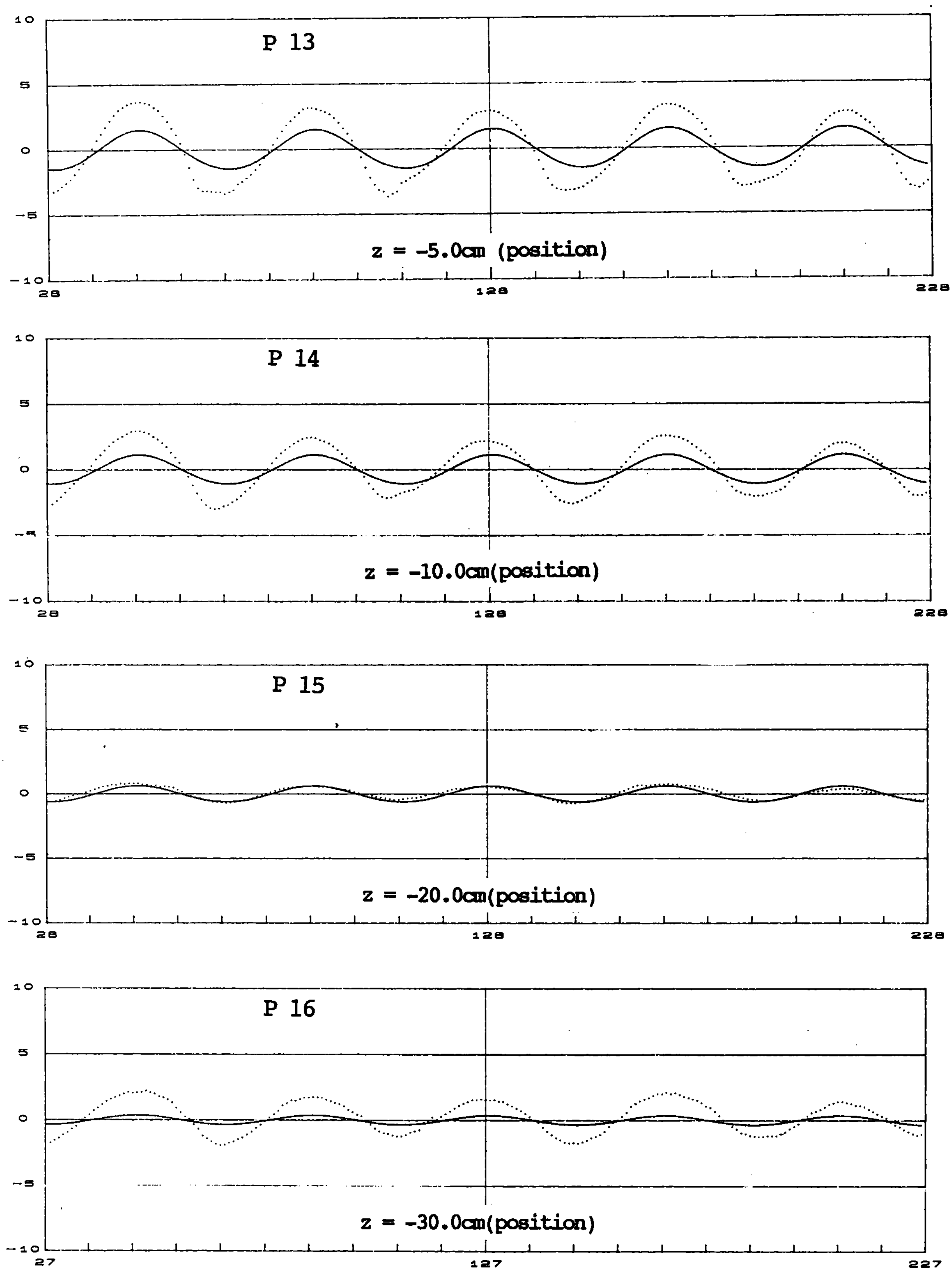

P 13
z = -5.0cm (position)
P 14
z = -10.0cm(position)
P 15
z = -20.0cm(position)
P 16
z = -30.0cm(position)
10
5
0
-5
-10
28
128
228
27
127
227

圖11-10

θ=45° (intersection angle), T_1=T_2=1.20sec(incident wave period 1 & 2)
d =50cm(water depth), H_1=6.28cm(incident wave height 1)
H_2=2.56cm(incident wave height 2)

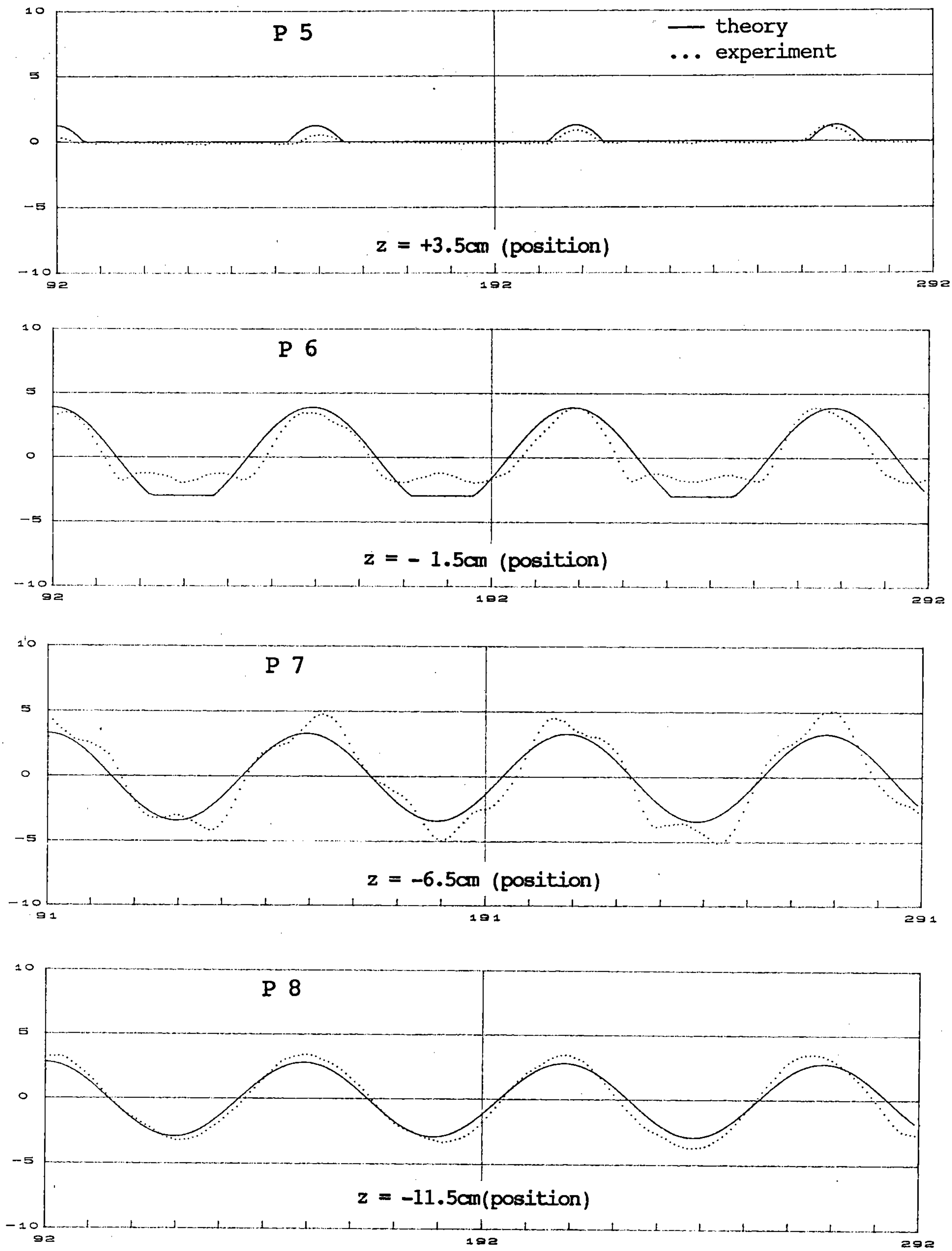


圖11-11

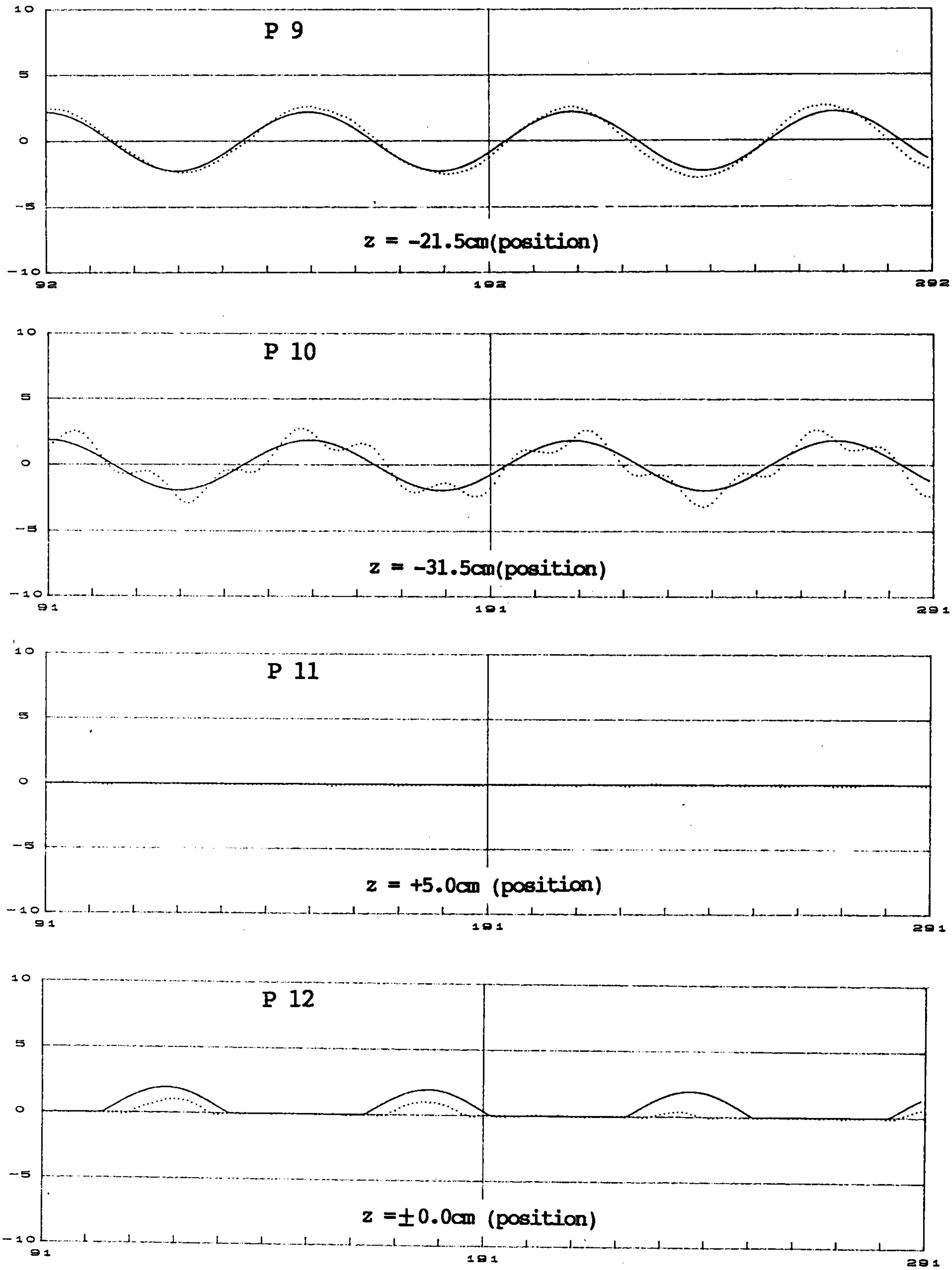
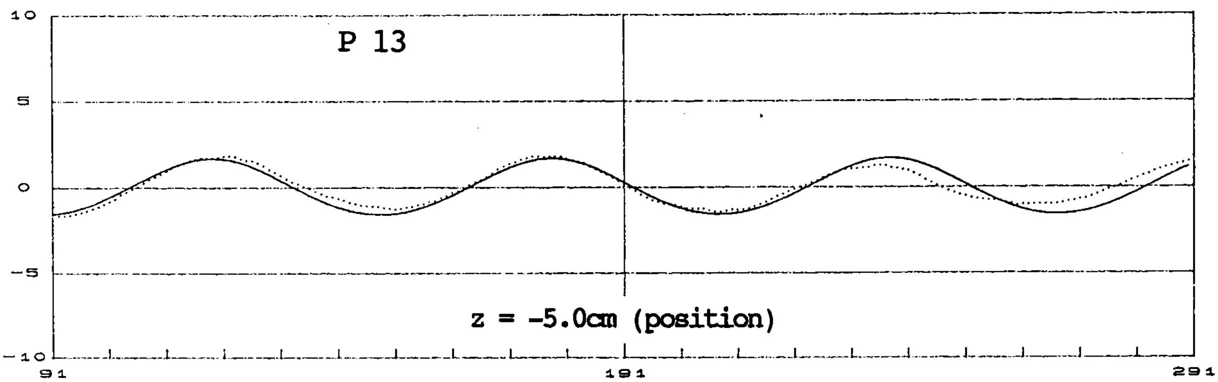
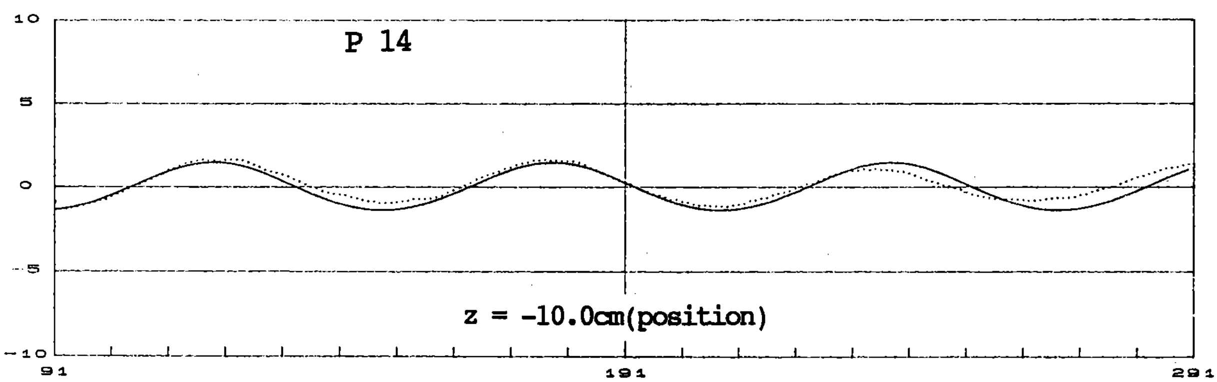

P 9
z = -21.5cm(position)
P 10
z = -31.5cm(position)
P 11
z = +5.0cm (position)
P 12
z =±0.0cm (position)
10
5
0
-5
-10
92
192
292
91
191
291

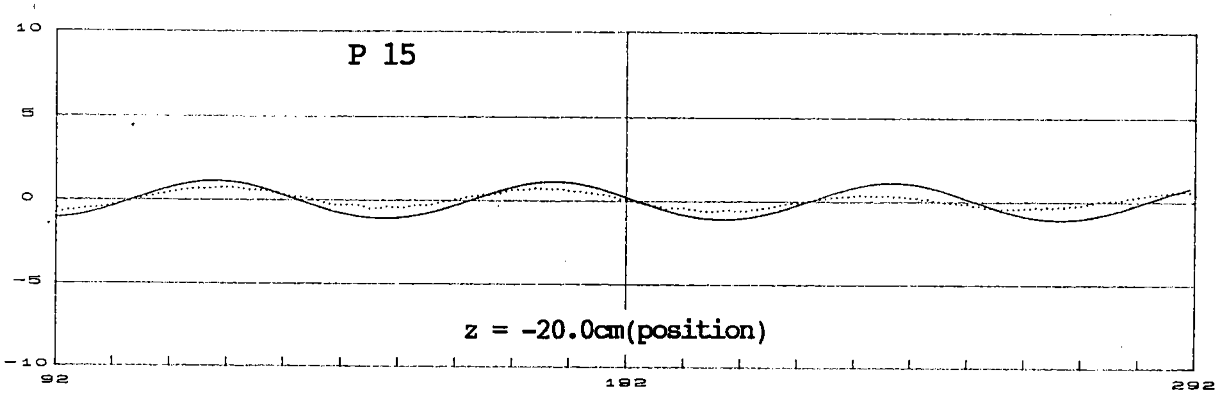
圖11-11



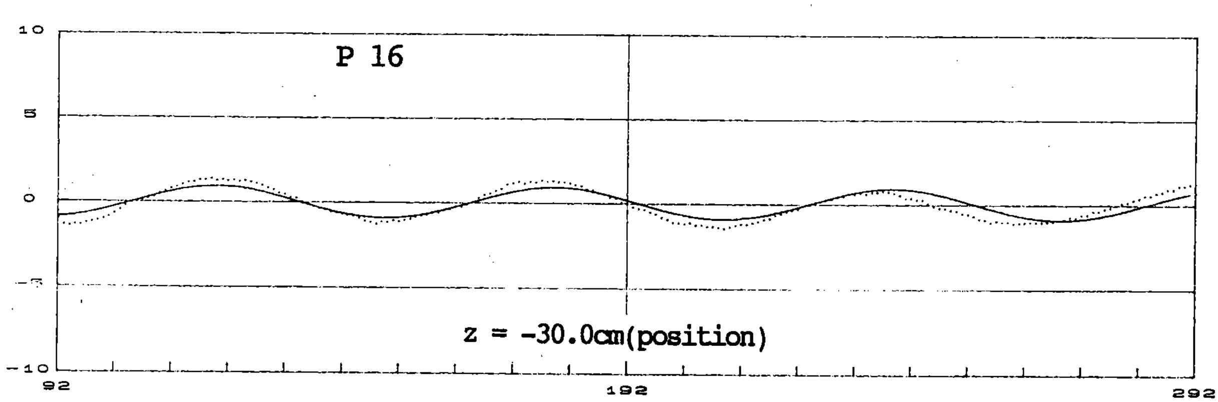
P 13
z = -5.0cm (position)



P 14
z = -10.0cm(position)



P 15
z = -20.0cm(position)



P 16
z = -30.0cm(position)

圖11-11

θ =45° (intersection angle), T_1=T_2=1.20sec(incident wave period 1 & 2)
d =50cm(water depth), H_1=9.88cm(incident wave height 1)
H_2=3.20cm(incident wave height 2)

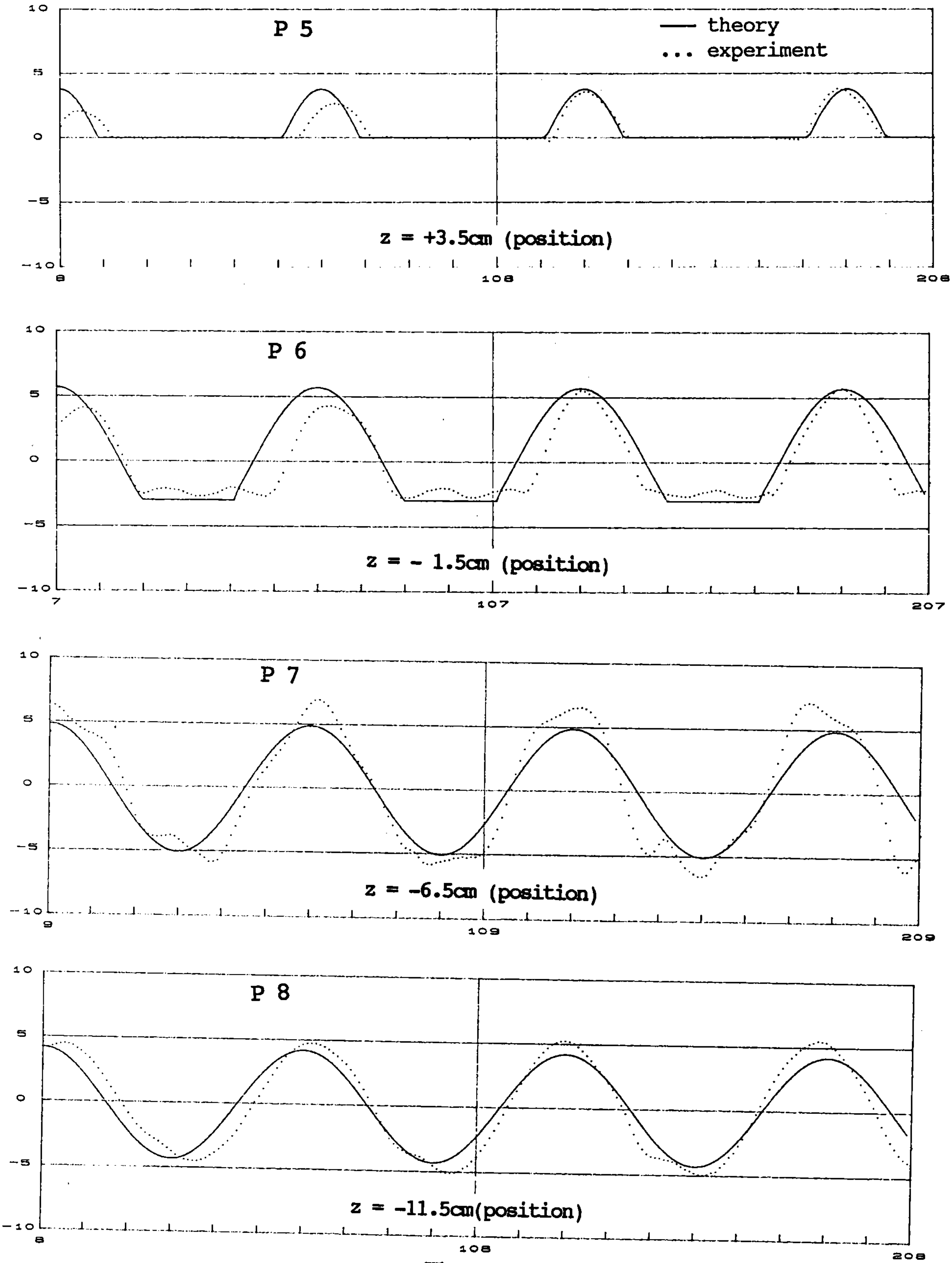


圖11-12

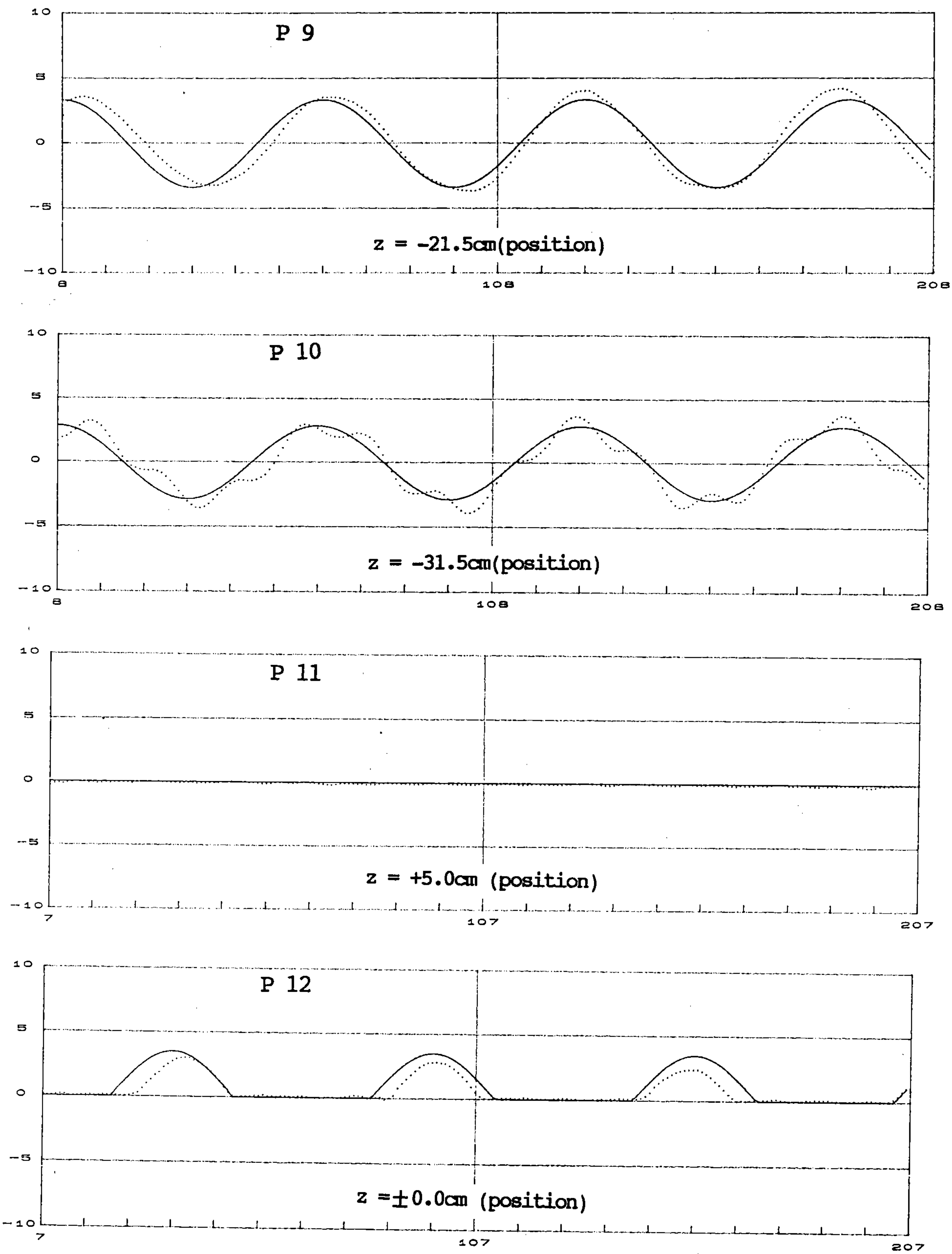

10
5
0
-5
-10
P 9
z = -21.5cm(position)
8
108
208
10
5
0
-5
-10
P 10
z = -31.5cm(position)
8
108
208
10
5
0
-5
-10
P 11
z = +5.0cm (position)
7
107
207
10
5
0
-5
-10
P 12
z =±0.0cm (position)
7
107
207

圖11-12

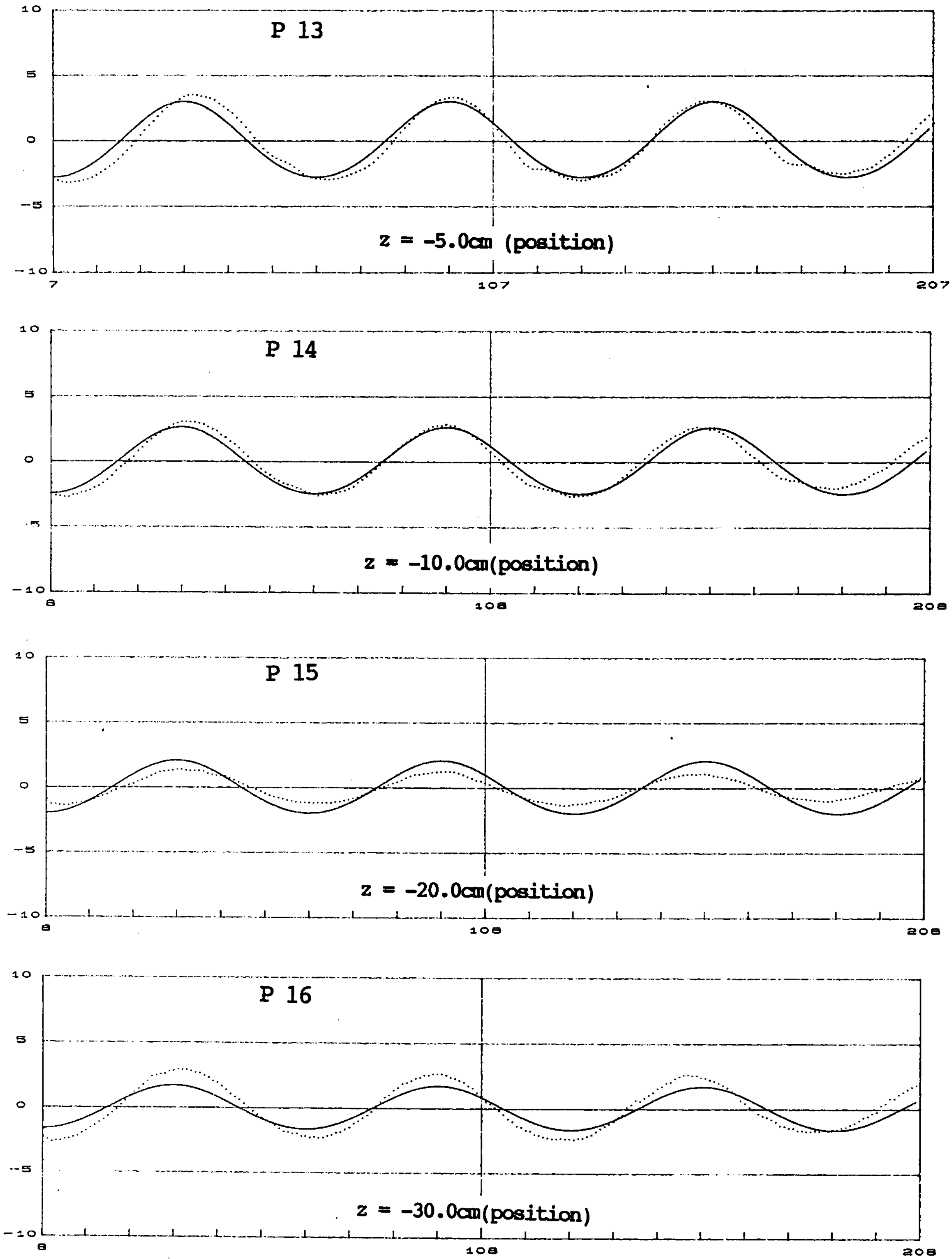

P 13
z = -5.0cm (position)
10
5
0
-5
-10
7
107
207
P 14
z = -10.0cm(position)
10
5
0
-5
-10
8
108
208
P 15
z = -20.0cm(position)
10
5
0
-5
-10
8
108
208
P 16
z = -30.0cm(position)
10
5
0
-5
-10
8
108
208

圖11-12

θ=45° (intersection angle), T_1=T_2=1.60sec(incident wave period 1 & 2)
d =50cm(water depth), H_1=7.63cm(incident wave height 1)
H_2=3.38cm(incident wave height 2)

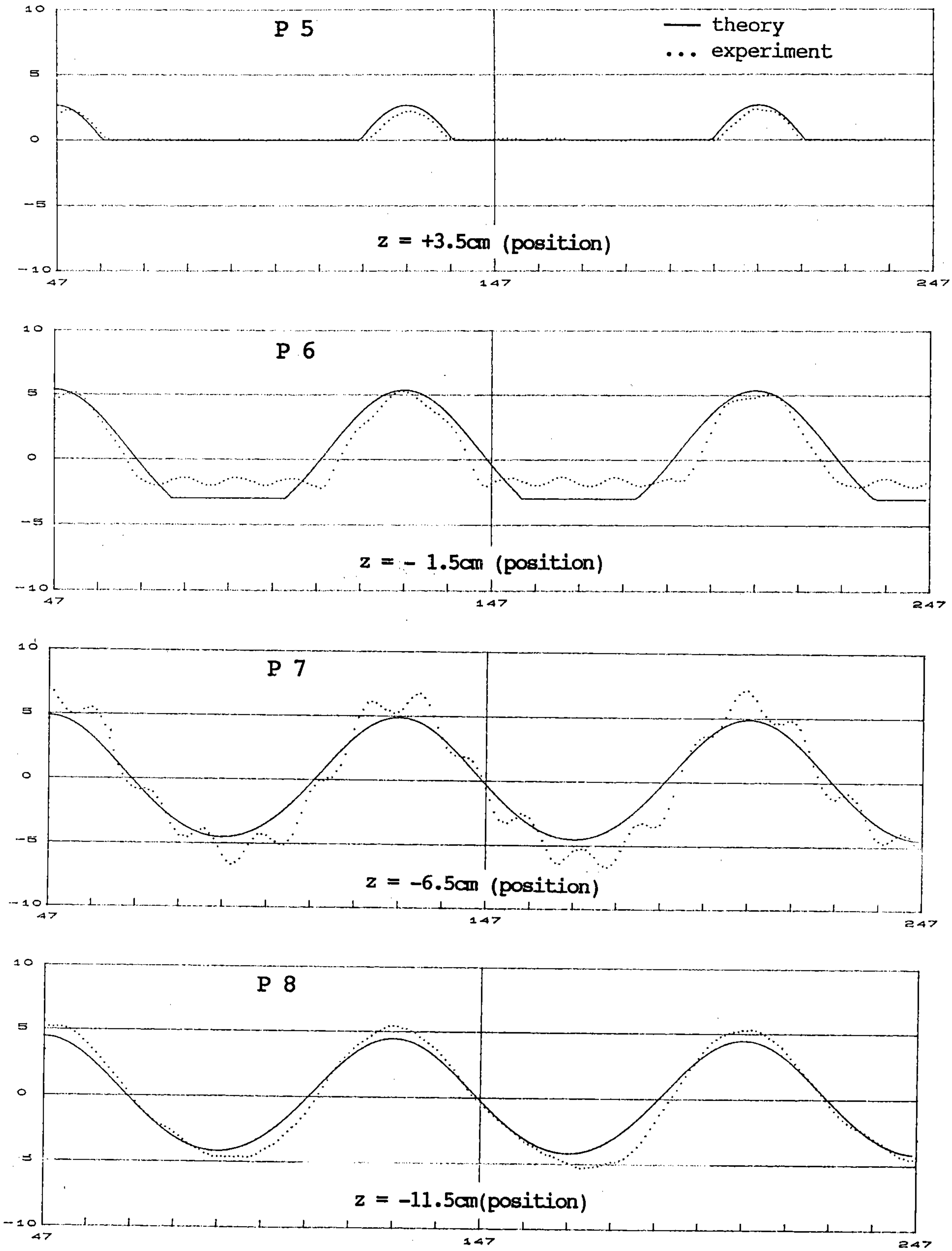


圖11-13

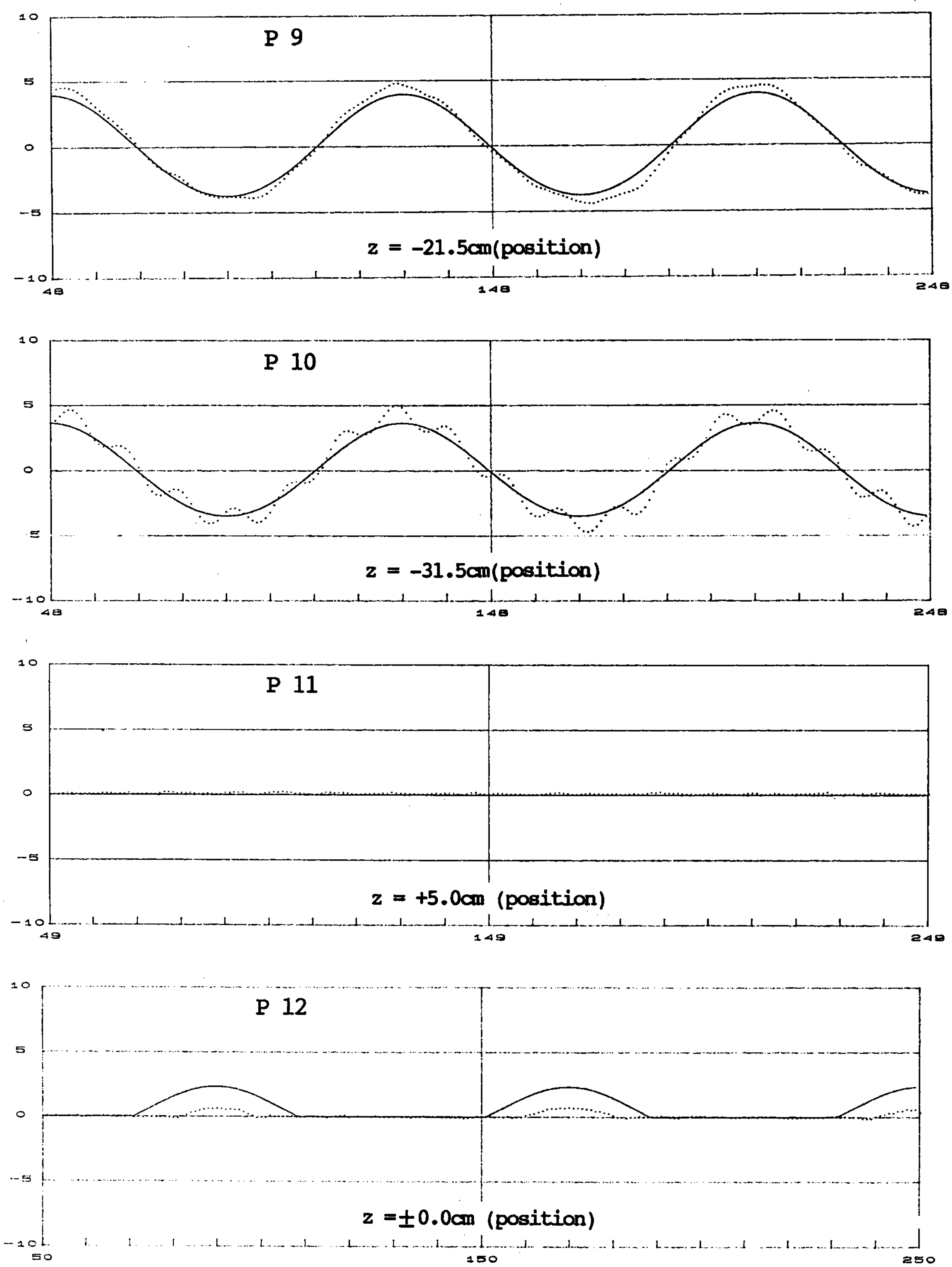
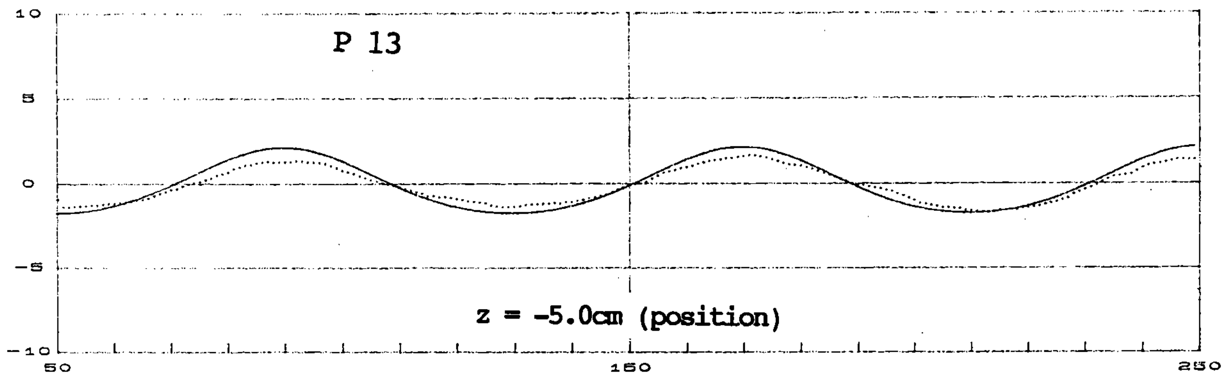
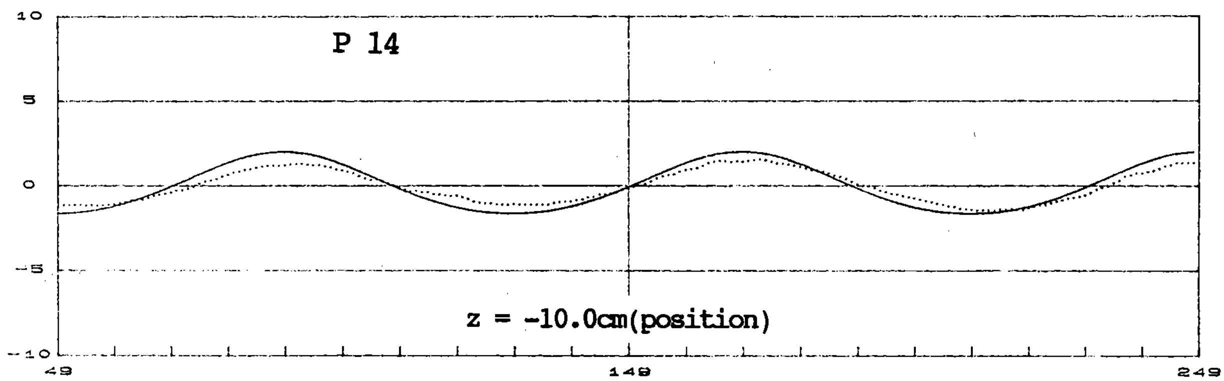
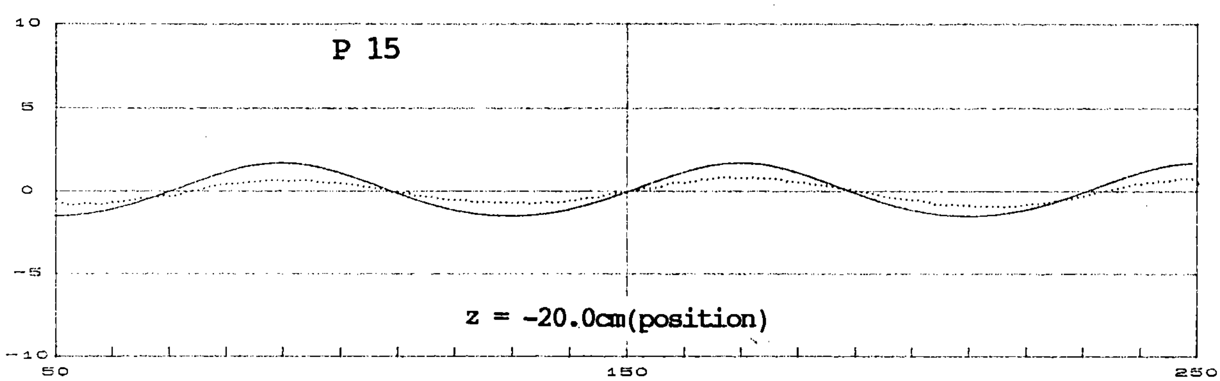

P 9
z = -21.5cm(position)
P 10
z = -31.5cm(position)
P 11
z = +5.0cm (position)
P 12
z =±0.0cm (position)

圖11-13


P 13
z = -5.0cm (position)
10
5
0
-5
-10
50
150
250


P 14
z = -10.0cm(position)
10
5
0
-5
-10
49
149
249


P 15
z = -20.0cm(position)
10
5
0
-5
-10
50
150
250

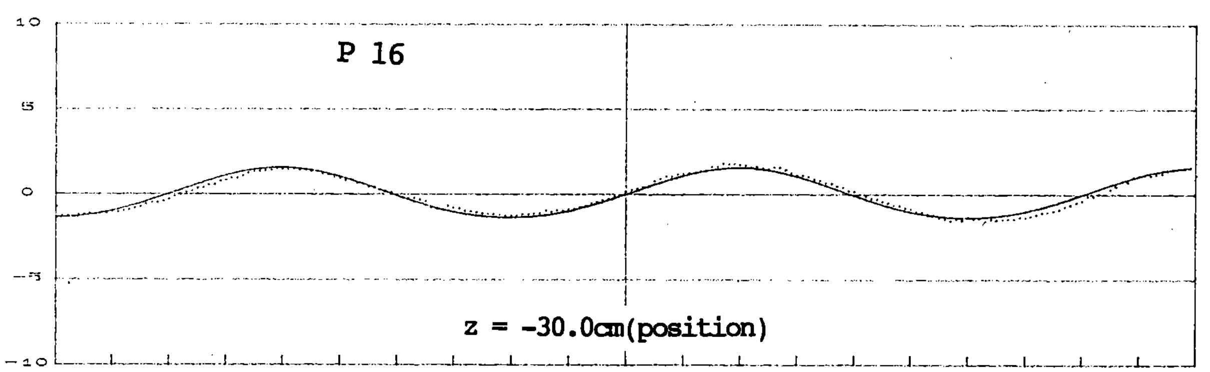

P 16
z = -30.0cm(position)
10
0
-10
50
150
250

圖11-13

θ=45° (intersection angle), T_1=T_2=1.60sec(incident wave period 1 & 2)
d =50cm(water depth), H_1=12.94cm(incident wave height 1)
H_2=3.23cm(incident wave height 2)

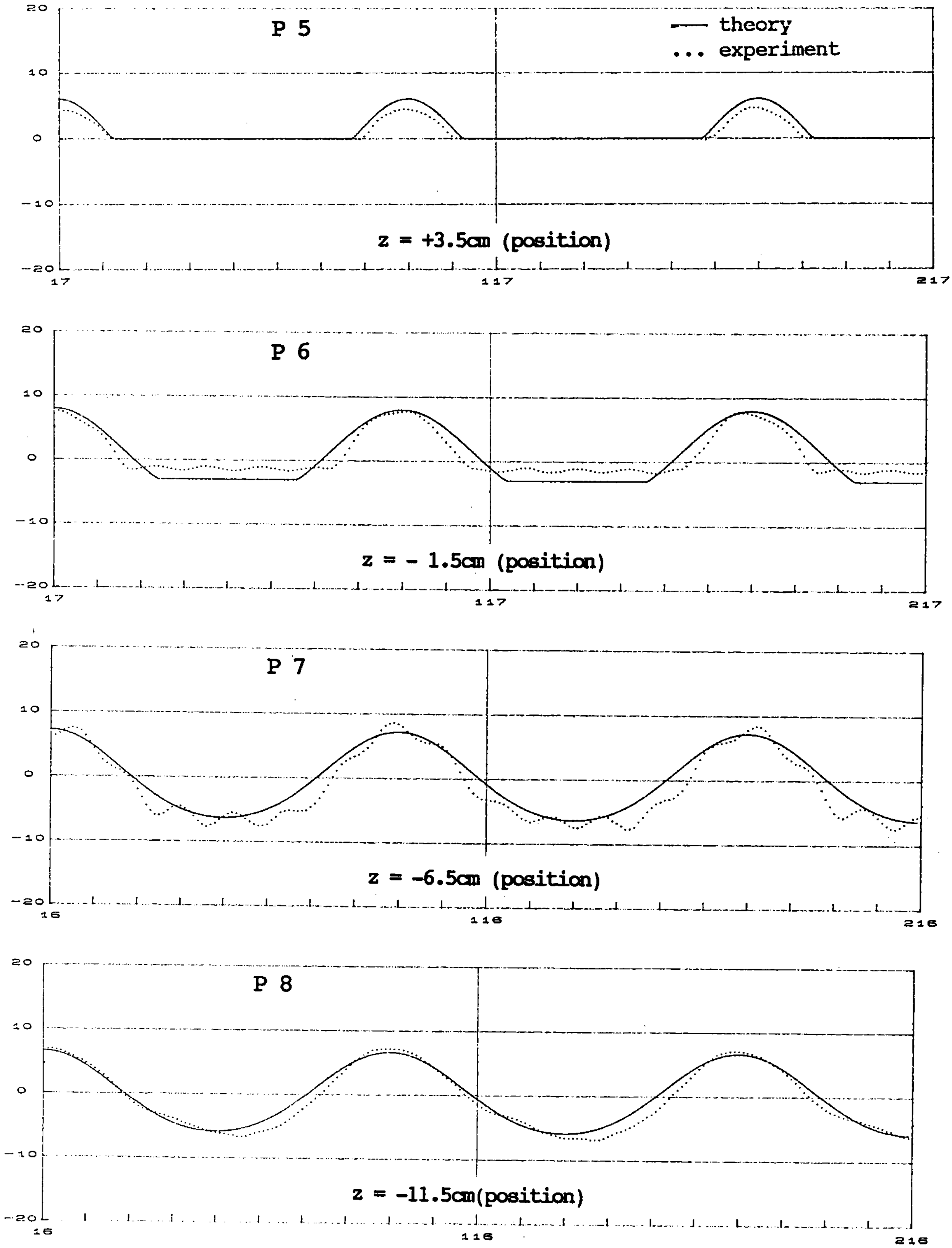


圖11-14

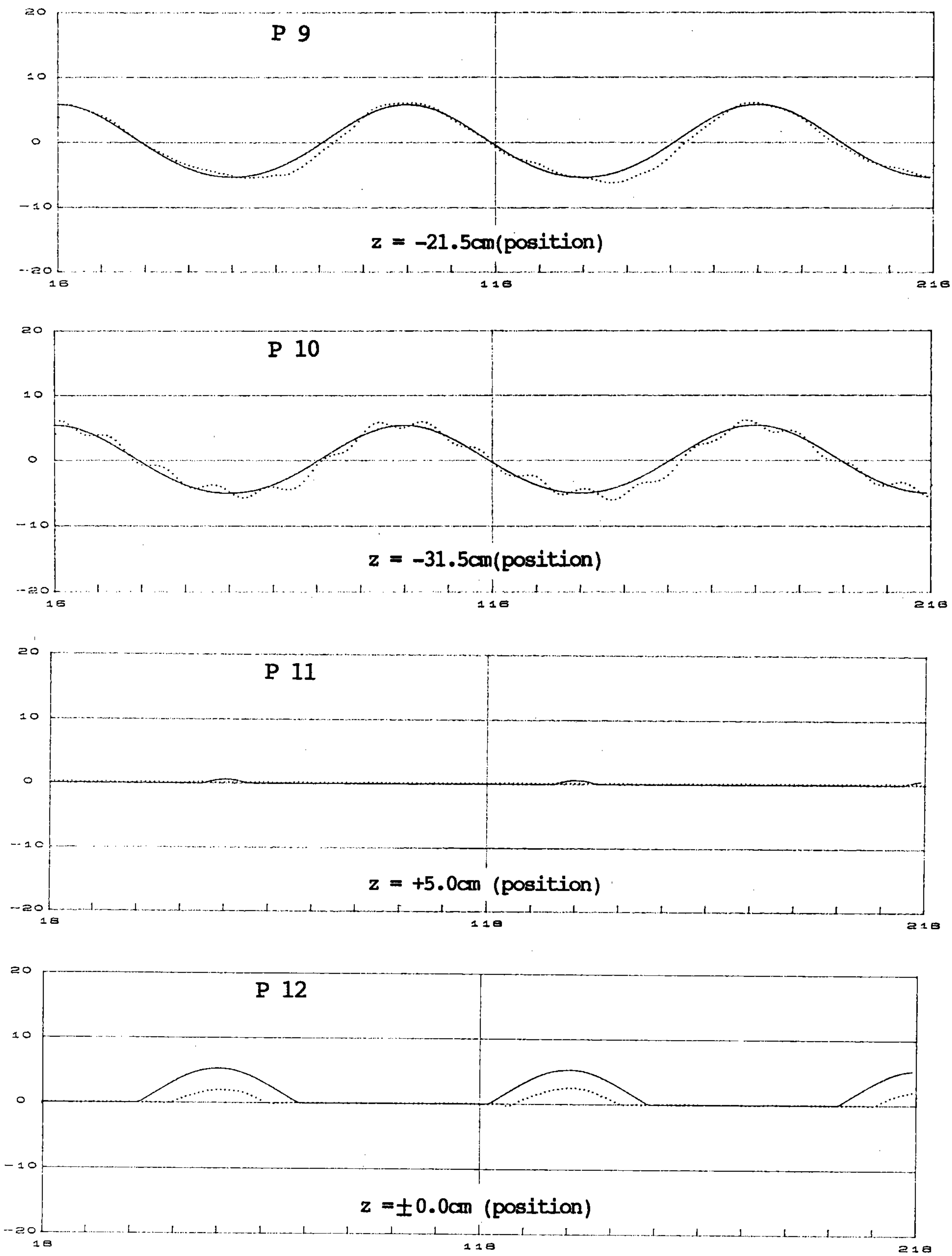

P 9
z = -21.5cm(position)
P 10
z = -31.5cm(position)
P 11
z = +5.0cm (position)
P 12
z =±0.0cm (position)
20
10
0
-10
-20
16
116
216

圖11-14

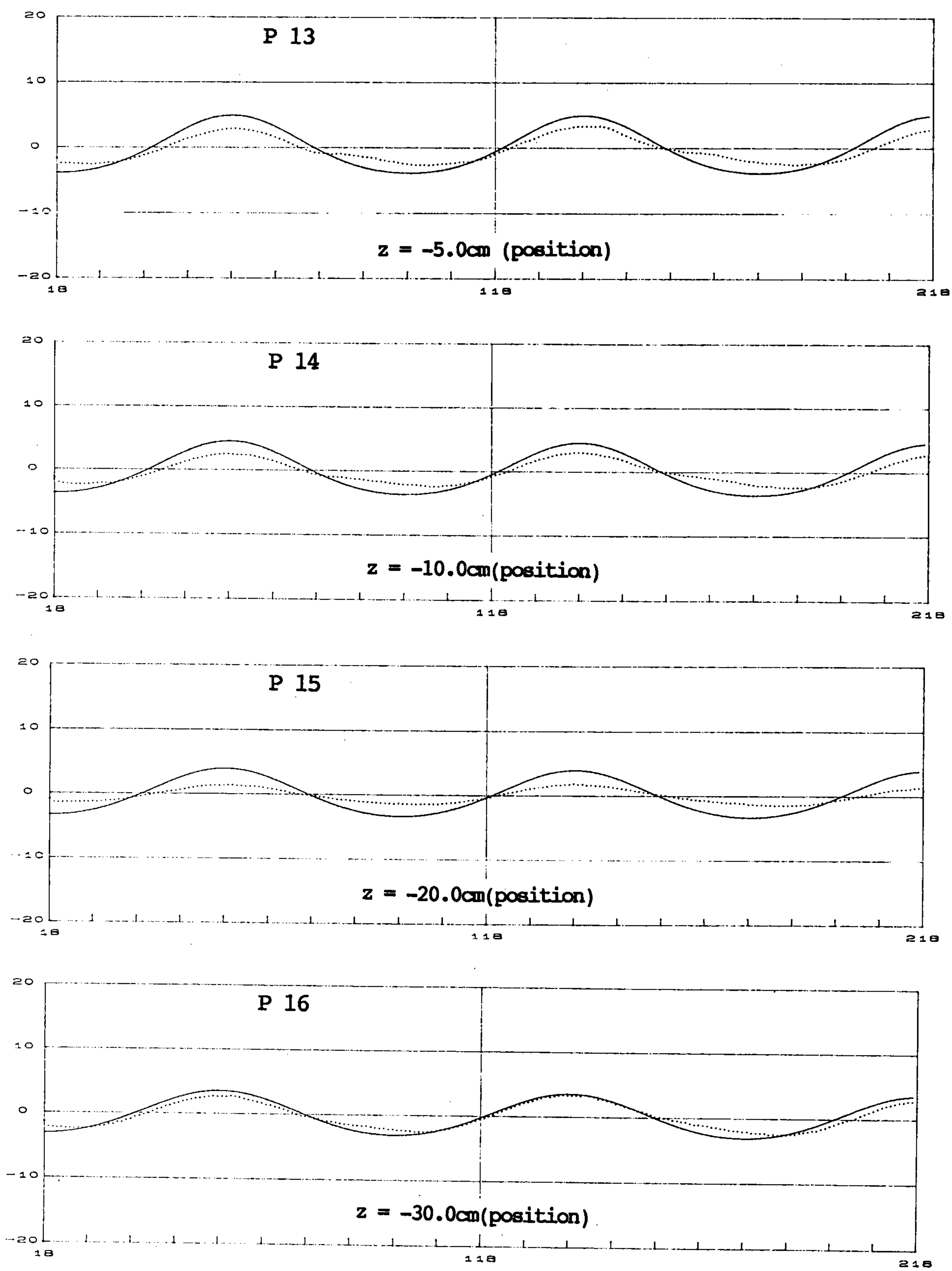

P 13
z = -5.0cm (position)
P 14
z = -10.0cm(position)
P 15
z = -20.0cm(position)
P 16
z = -30.0cm(position)
20
10
0
-10
-20
18
118
218

圖11-14

θ=45° (intersection angle), T_1=T_2=2.00sec(incident wave period 1 & 2)
d =50cm(water depth), H_1=6.83cm(incident wave height 1)
H_2=2.80cm(incident wave height 2)

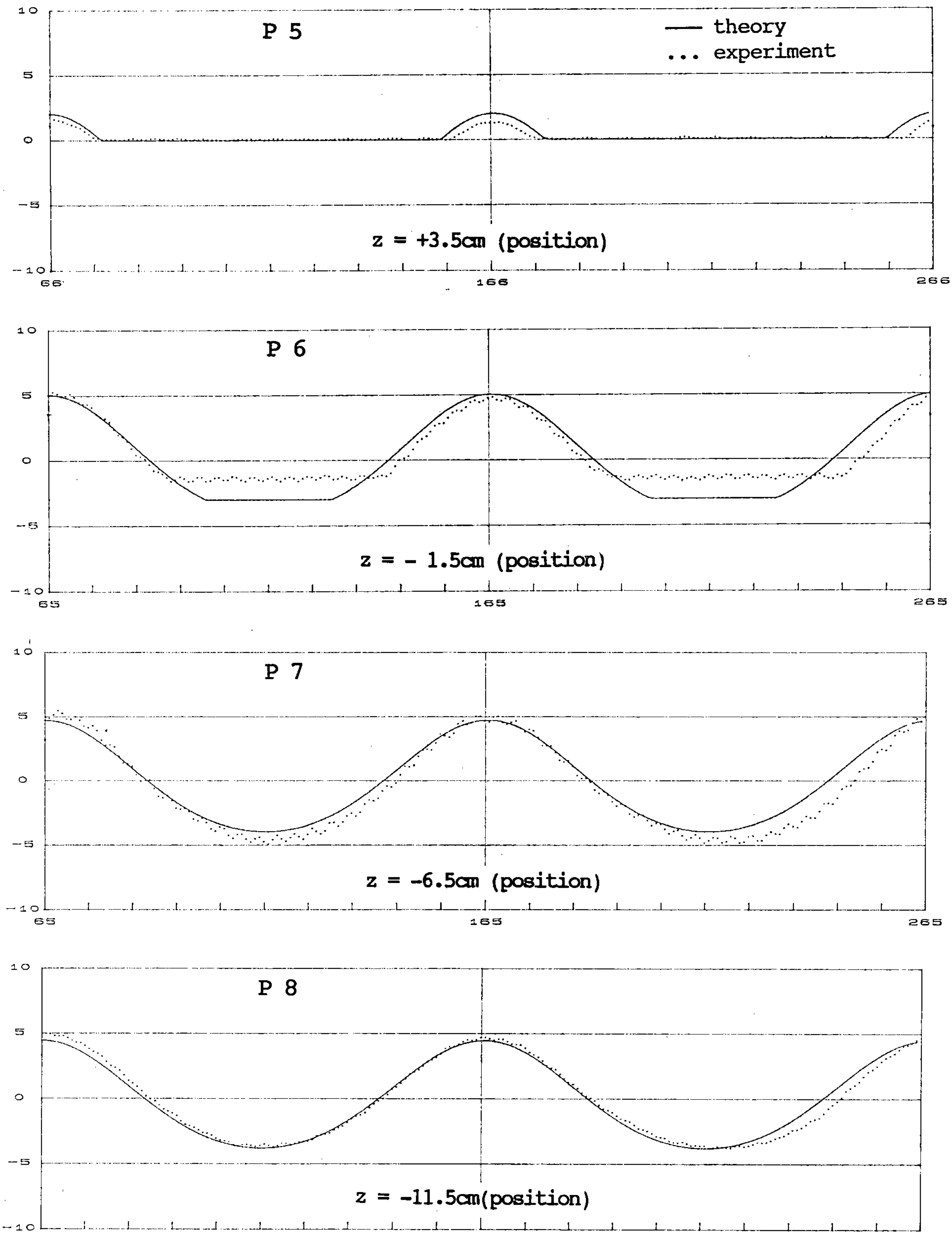


圖11-15

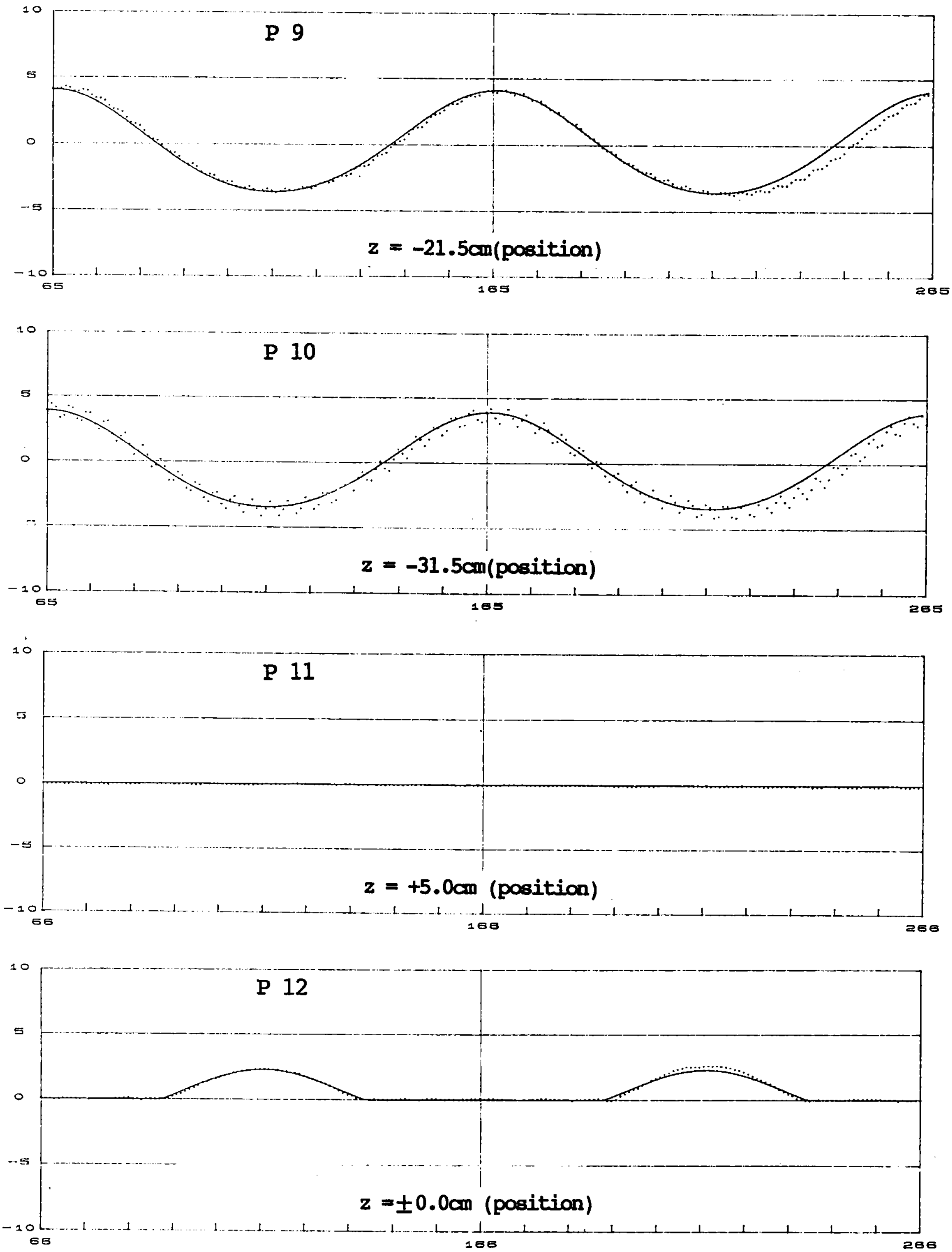
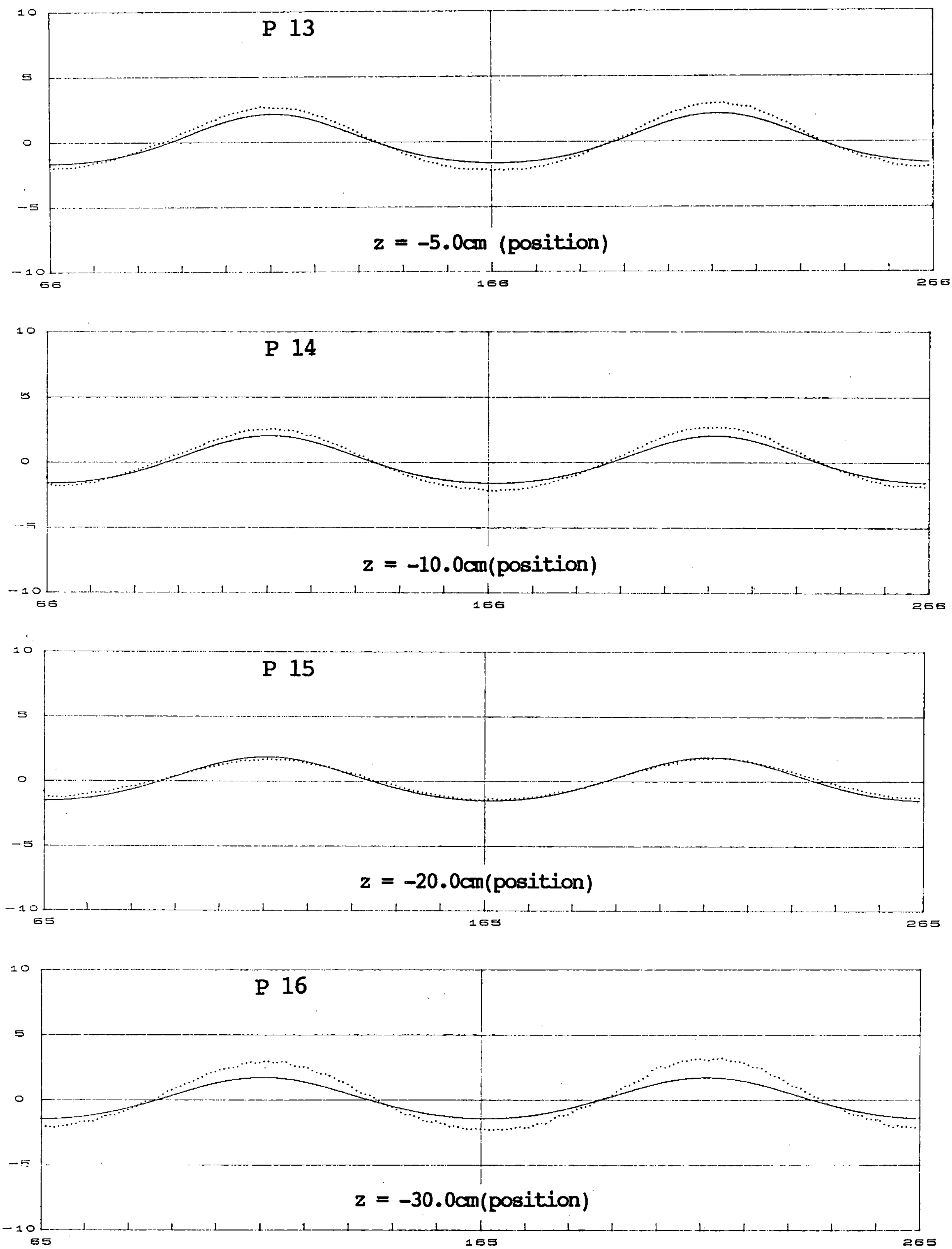

P 9
z = -21.5cm(position)
P 10
z = -31.5cm(position)
P 11
z = +5.0cm (position)
P 12
z =±0.0cm (position)
10
5
0
-5
-10
65
165
265
66
166
266

圖11-15



P 13
z = -5.0cm (position)
10
5
0
-5
-10
66
166
266
P 14
z = -10.0cm(position)
10
5
0
-5
-10
66
166
266
P 15
z = -20.0cm(position)
10
5
0
-5
-10
65
165
265
P 16
z = -30.0cm(position)
10
5
0
-5
-10
65
165
265

圖11-15

θ=45° (intersection angle), d =50cm(water depth), T_1=T_2=2.00sec(incident wave period 1 & 2)
H_1=12.88cm(incident wave height 1)
H_2=3.31cm(incident wave height 2)

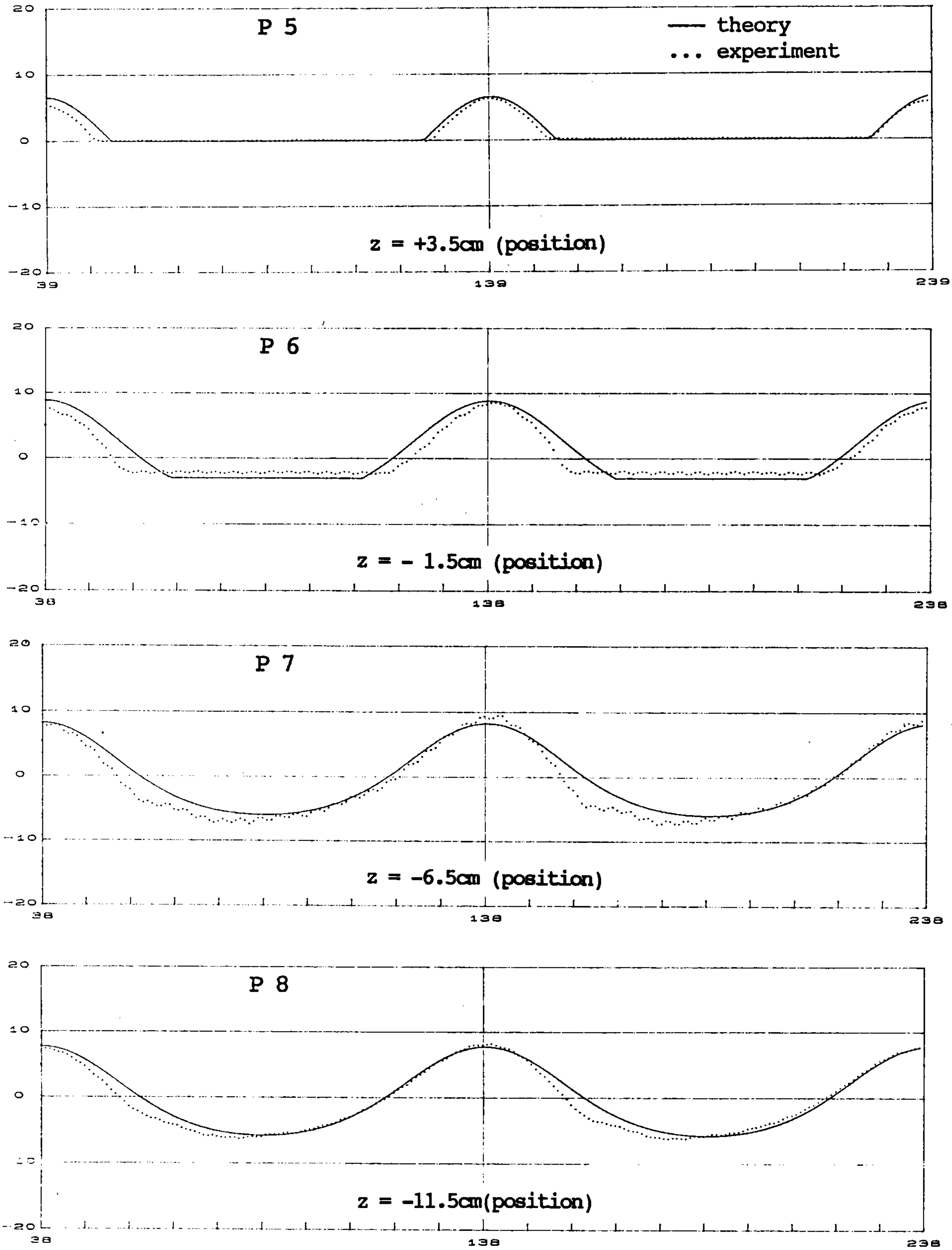
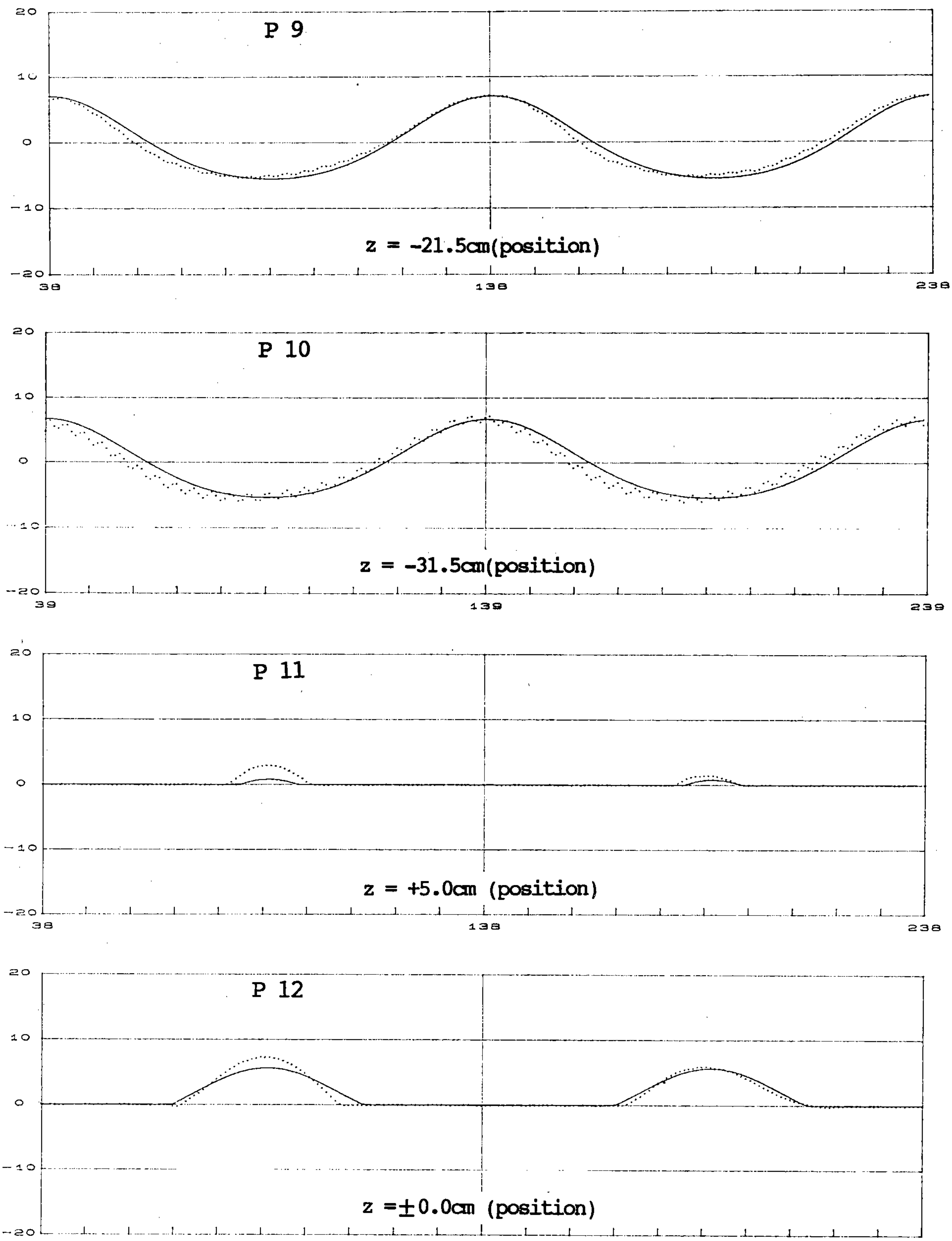


圖11-16


P 9
z = -21.5cm(position)
20
10
0
−10
−20
38
138
238
P 10
z = -31.5cm(position)
20
10
0
−10
−20
39
139
239
P 11
z = +5.0cm (position)
20
10
0
−10
−20
38
138
238
P 12
z =±0.0cm (position)
20
10
0
−10
−20
38
138
238
圖11-16

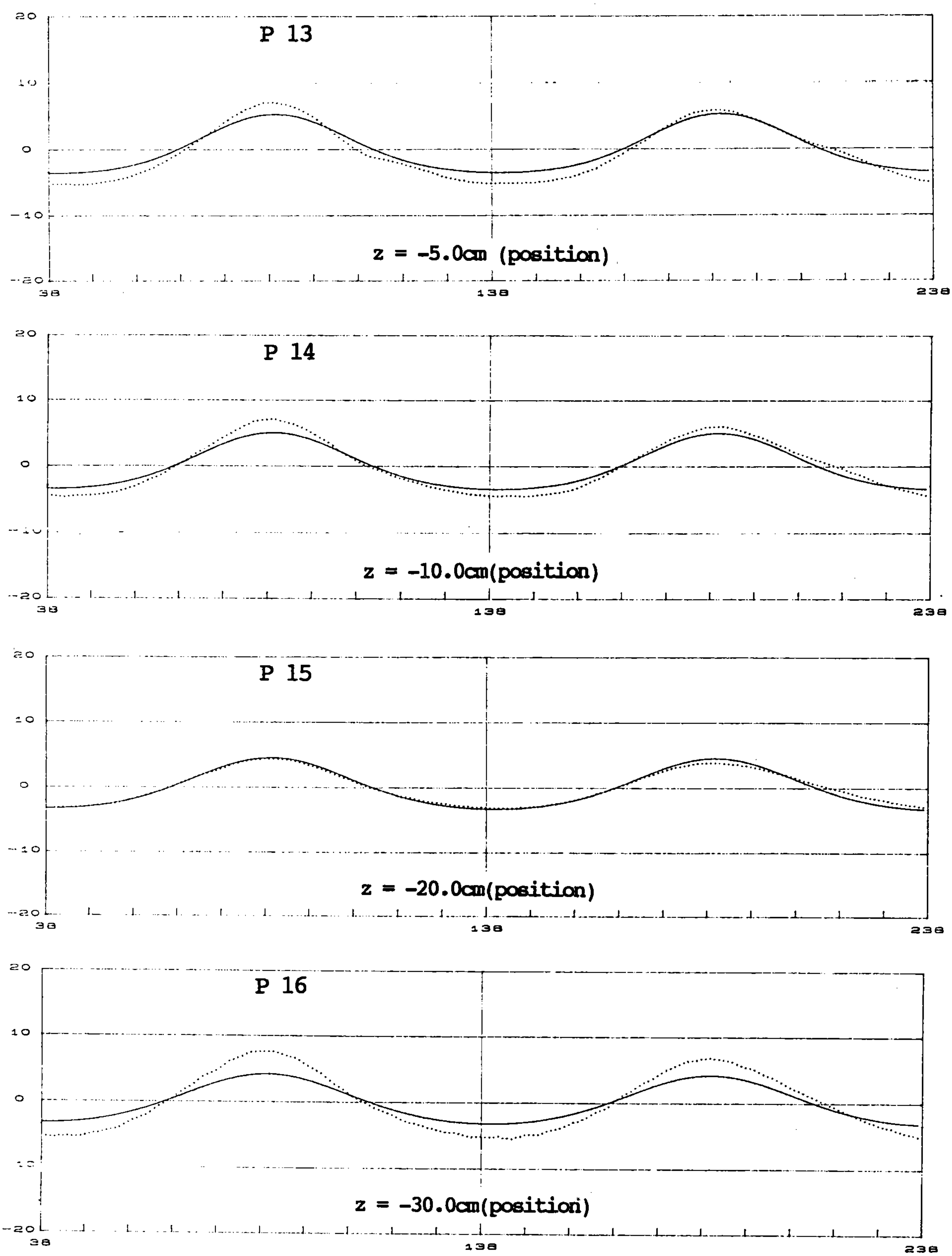

P 13
z = -5.0cm (position)
P 14
z = -10.0cm(position)
P 15
z = -20.0cm(position)
P 16
z = -30.0cm(position)
20
10
0
-10
-20
38
138
238

圖11-16

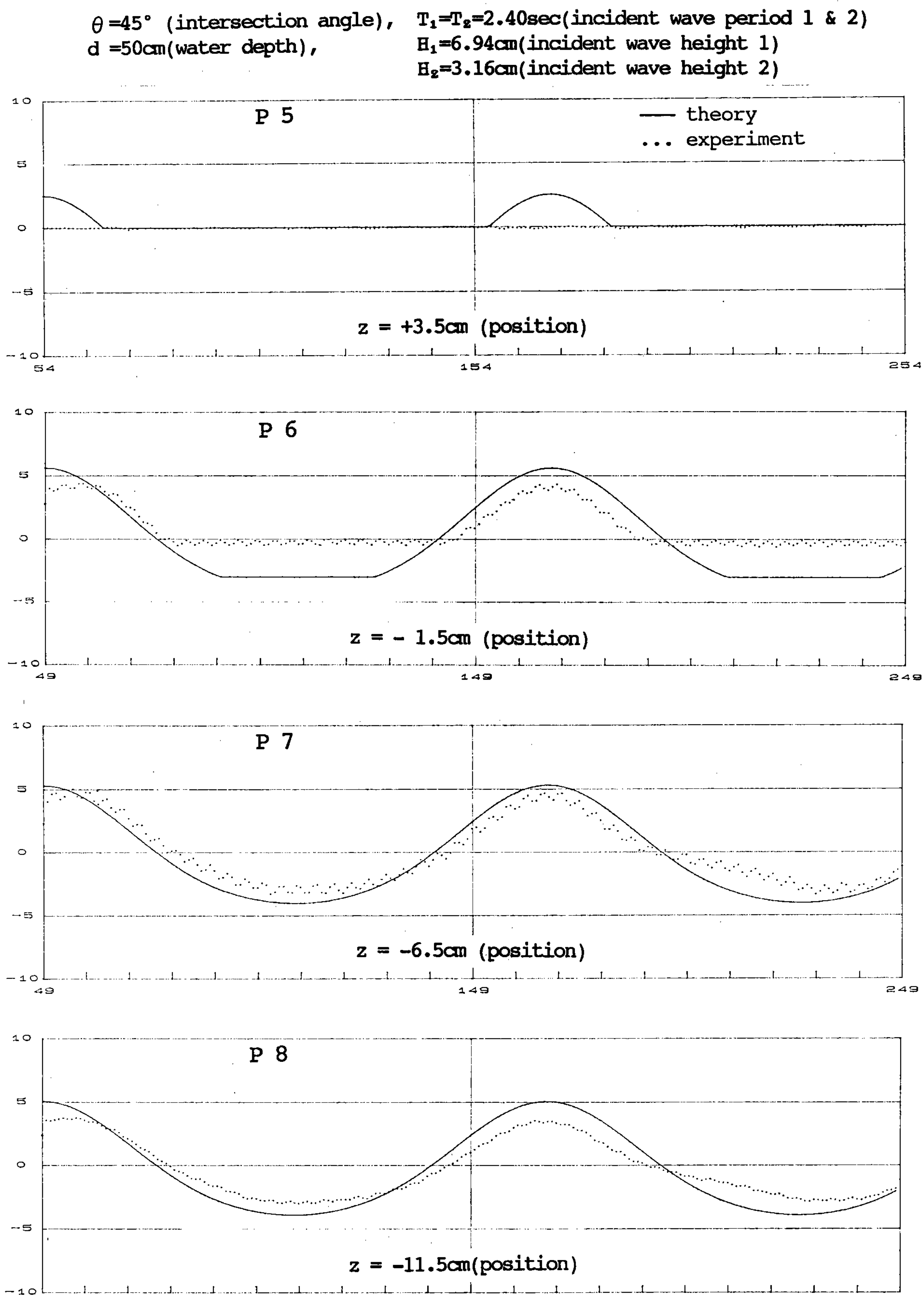
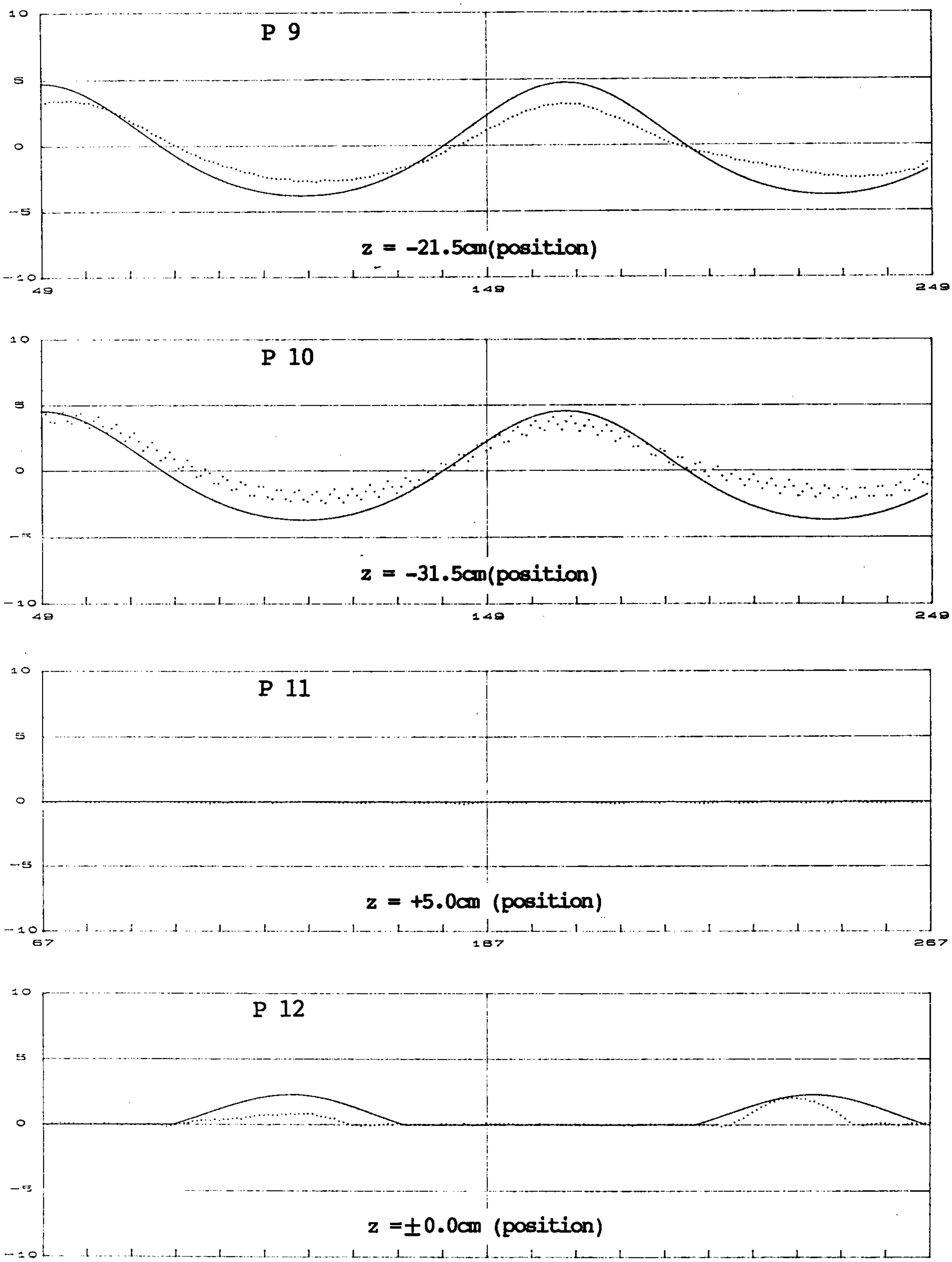

θ =45° (intersection angle), d =50cm(water depth),
T1=T2=2.40sec(incident wave period 1 & 2)
H1=6.94cm(incident wave height 1)
H2=3.16cm(incident wave height 2)
P 5
— theory
... experiment
z = +3.5cm (position)
P 6
z = - 1.5cm (position)
P 7
z = -6.5cm (position)
P 8
z = -11.5cm(position)

圖11-17



P 9
z = -21.5cm(position)
10
5
0
−5
−10
49
149
249
P 10
z = -31.5cm(position)
10
5
0
−5
−10
49
149
249
P 11
z = +5.0cm (position)
10
5
0
−5
−10
67
167
267
P 12
z =±0.0cm (position)
10
5
0
−5
−10
66
166
266

圖11-17

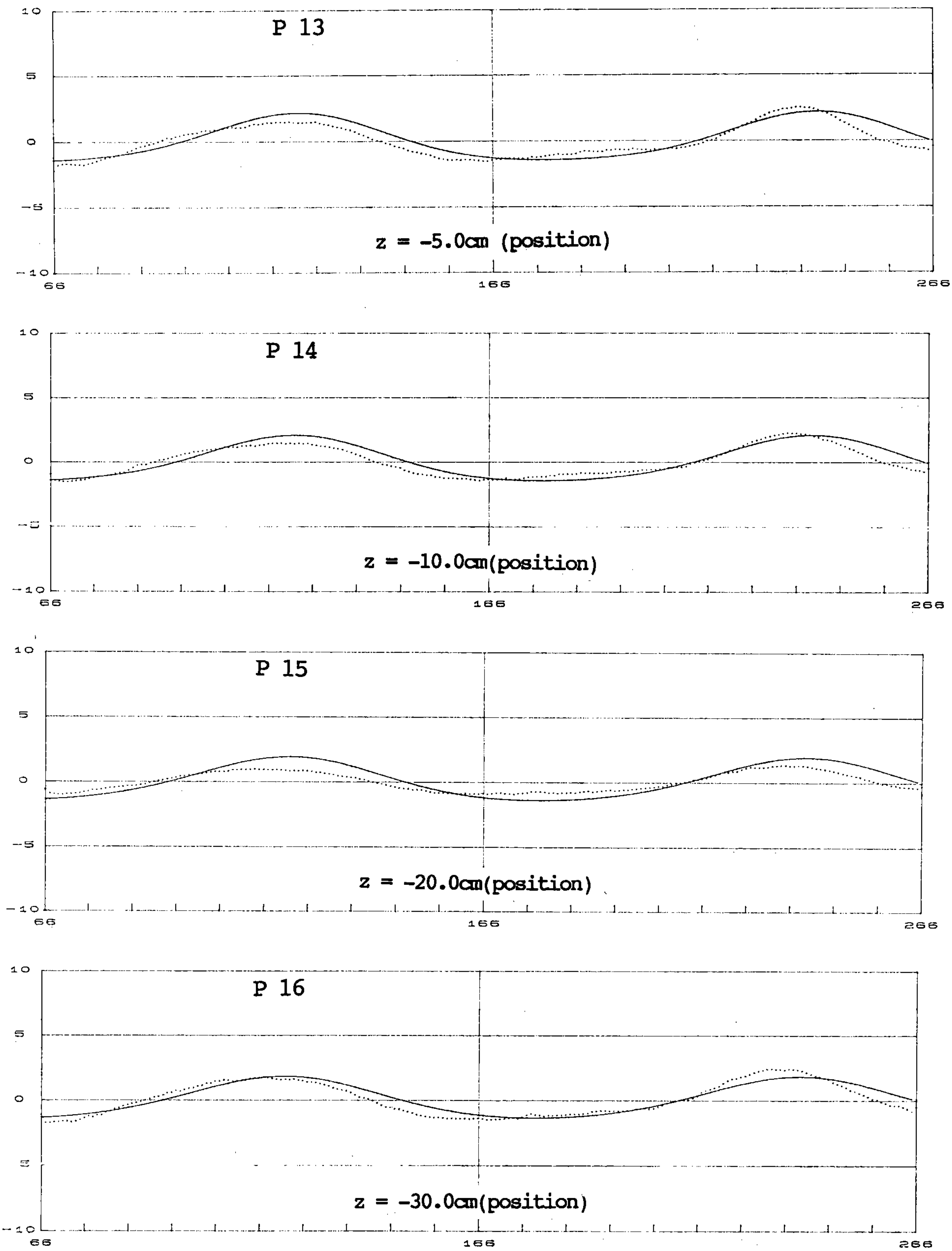

P 13
z = -5.0cm (position)
P 14
z = -10.0cm(position)
P 15
z = -20.0cm(position)
P 16
z = -30.0cm(position)
10
5
0
-5
-10
66
166
266

圖11-17

θ =45° (intersection angle), T_1=T_2=2.40sec(incident wave period 1 & 2)
d =50cm(water depth), H_1=11.18cm(incident wave height 1)
H_2=3.62cm(incident wave height 2)

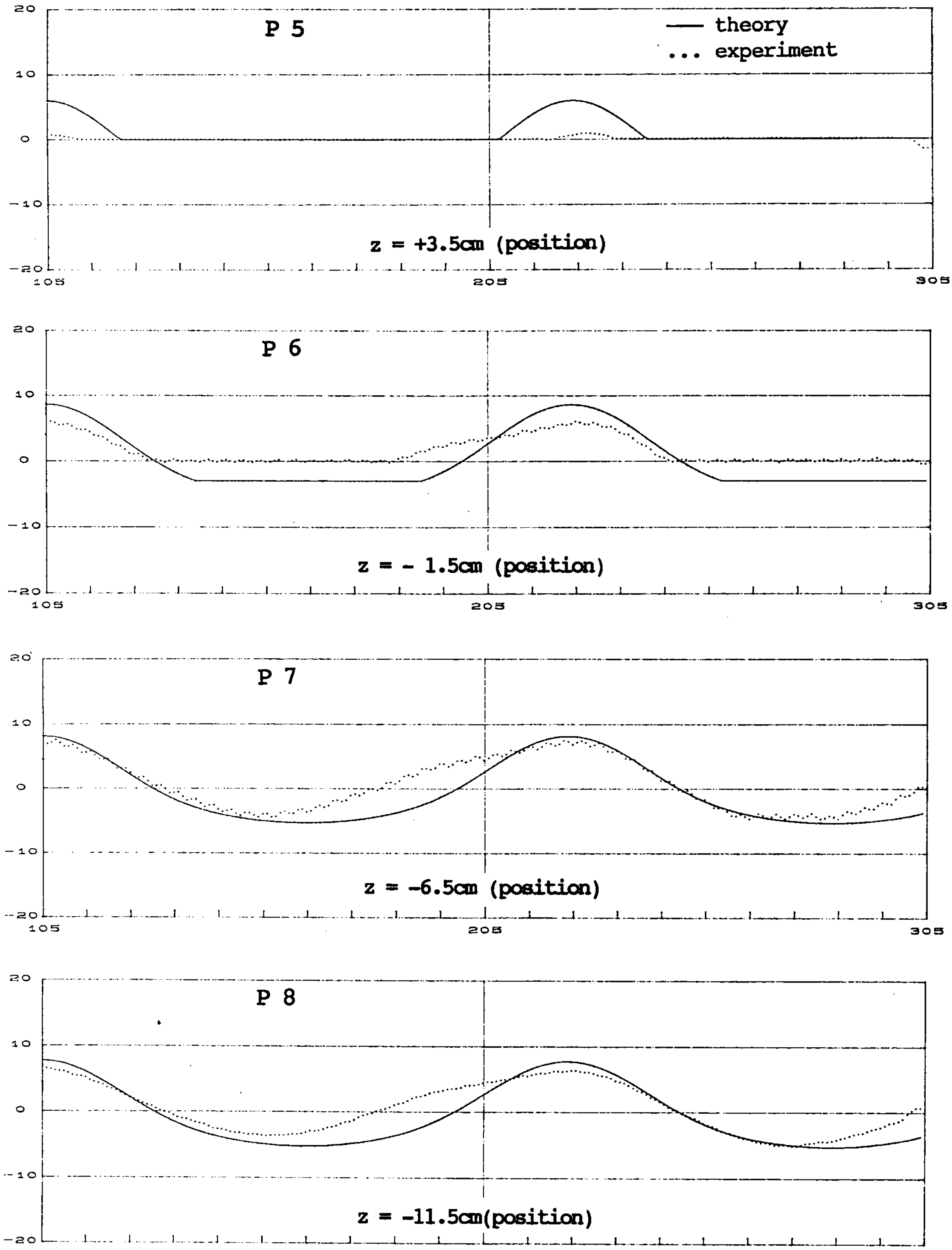


圖11-18

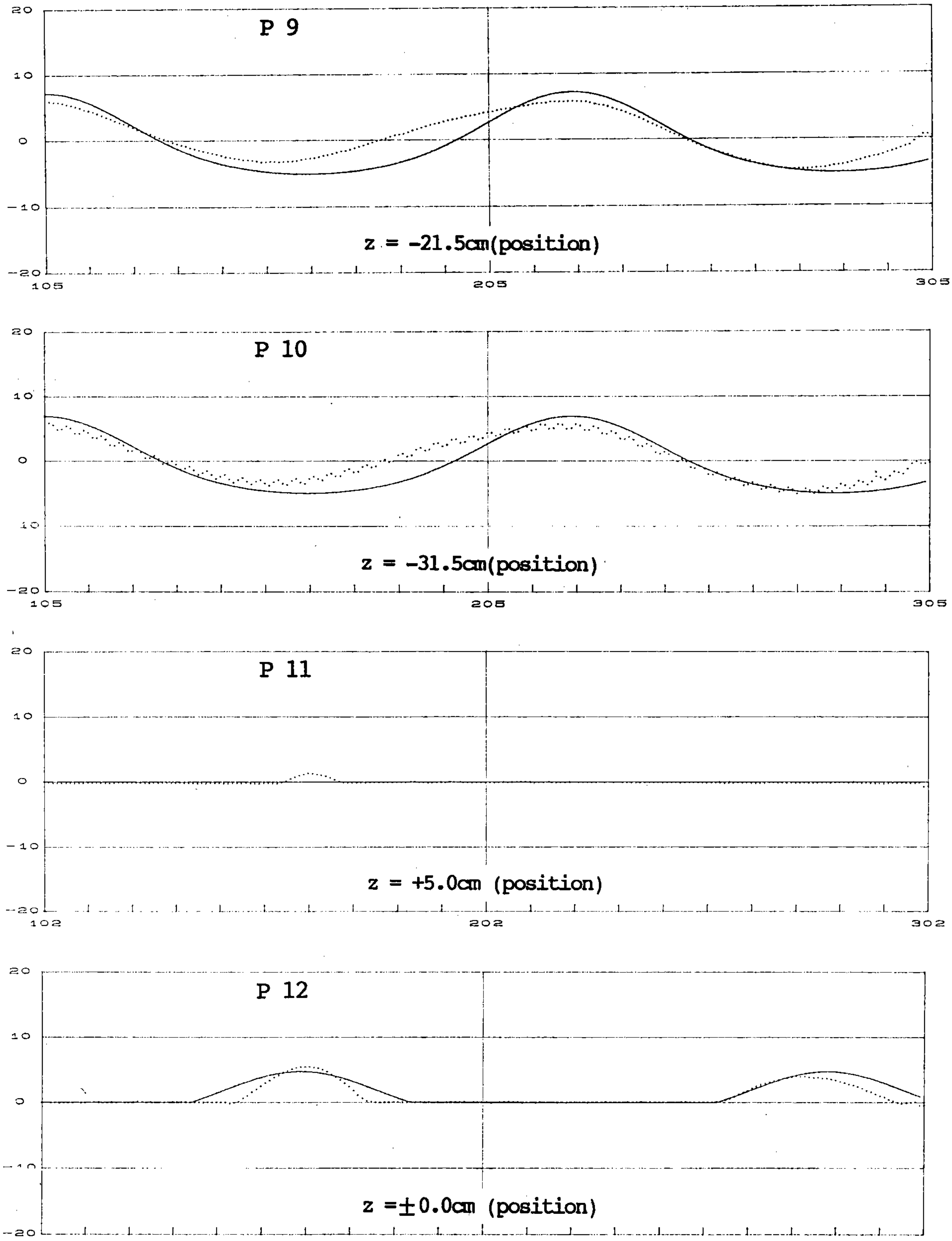
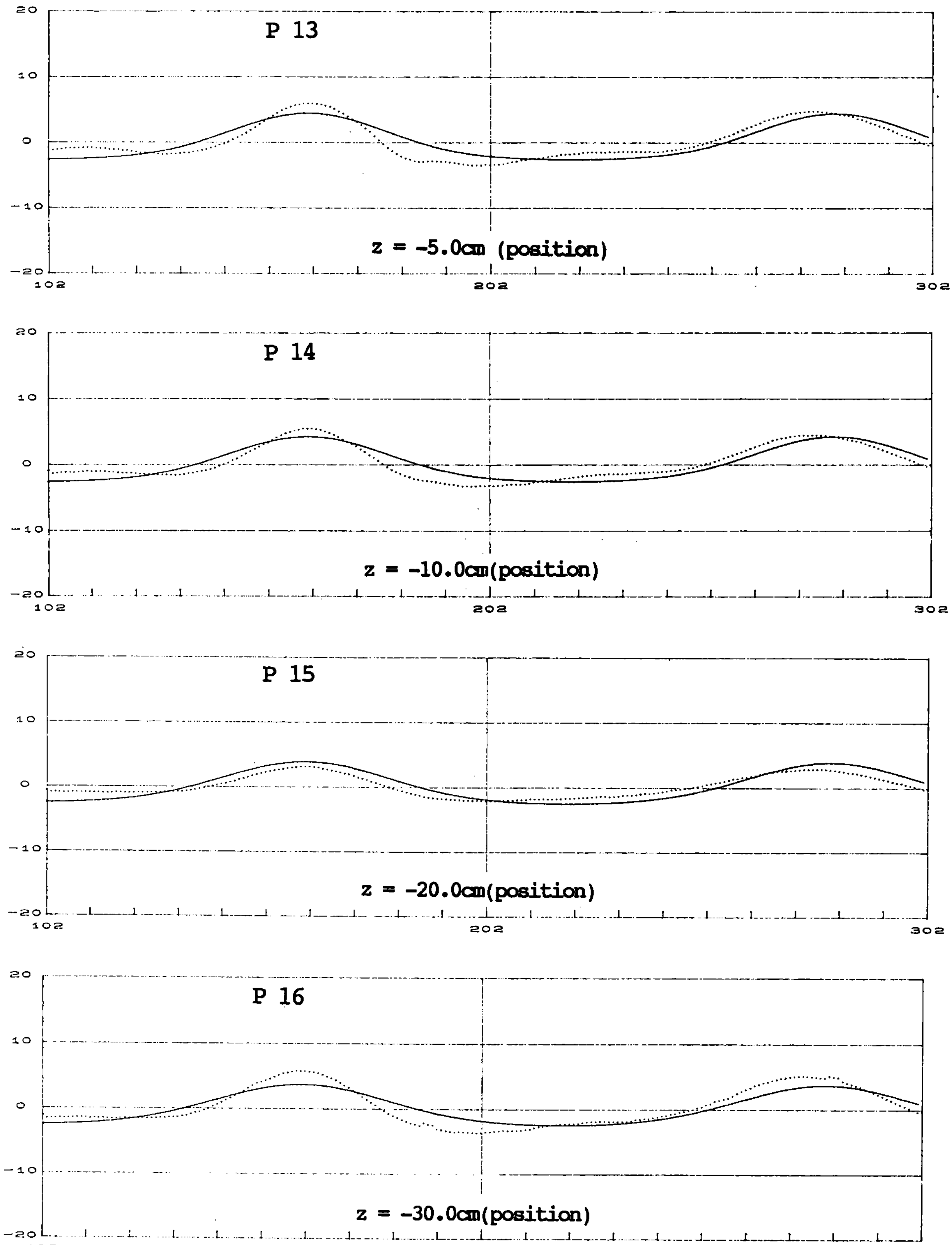

P 9
z = -21.5cm(position)
20
10
0
−10
−20
105
205
305
P 10
z = -31.5cm(position)
20
10
0
−20
105
205
305
P 11
z = +5.0cm (position)
20
10
0
−10
−20
102
202
302
P 12
z =±0.0cm (position)
20
10
0
−20
102
202
302

圖11-18



P 13
z = -5.0cm (position)
P 14
z = -10.0cm(position)
P 15
z = -20.0cm(position)
P 16
z = -30.0cm(position)
20
10
0
-10
-20
102
202
302

圖11-18

θ =45° (intersection angle), T_1=T_2=1.20sec(incident wave period 1 & 2)
d =40cm(water depth), H_1=7.36cm(incident wave height 1)
H_2=3.20cm(incident wave height 2)

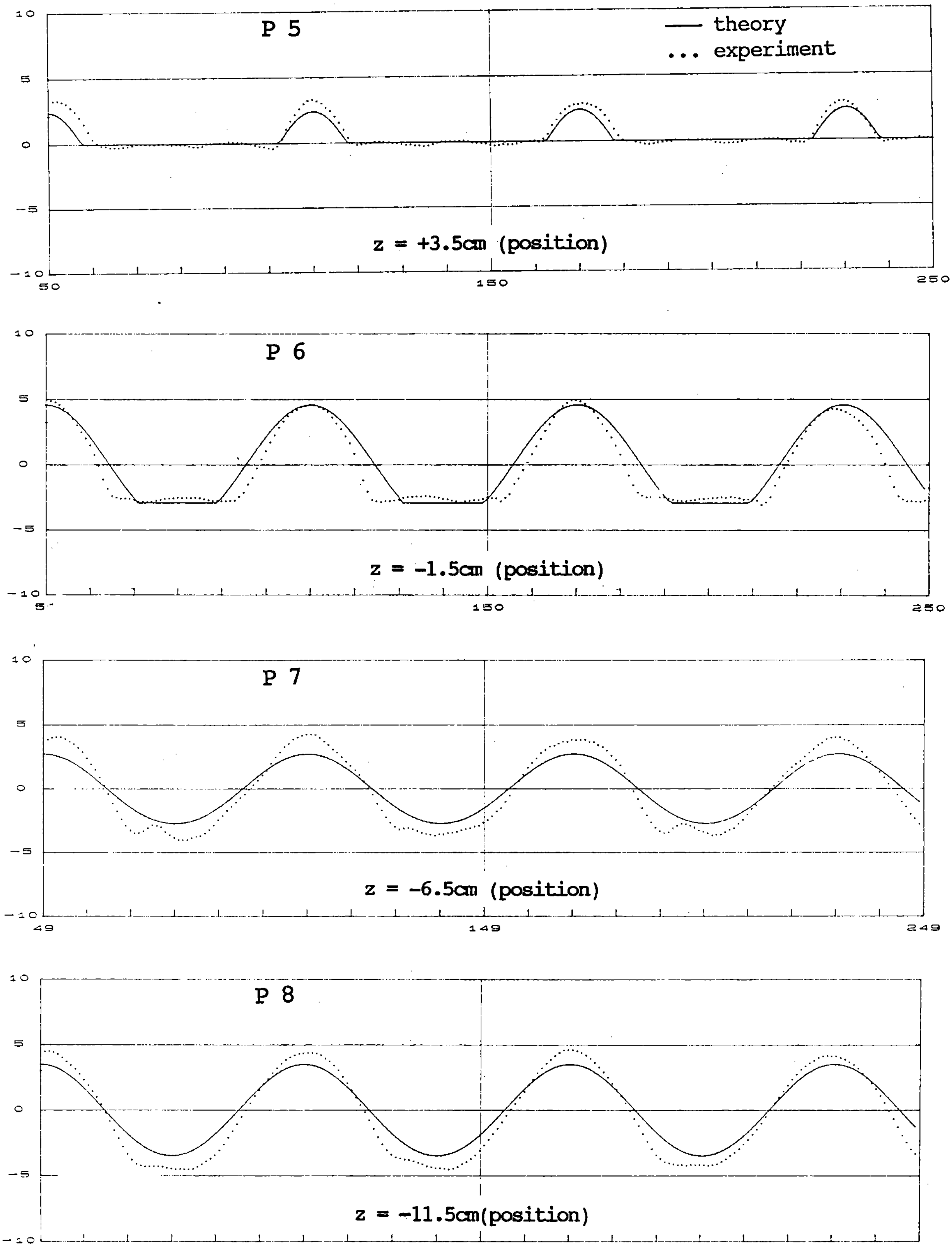
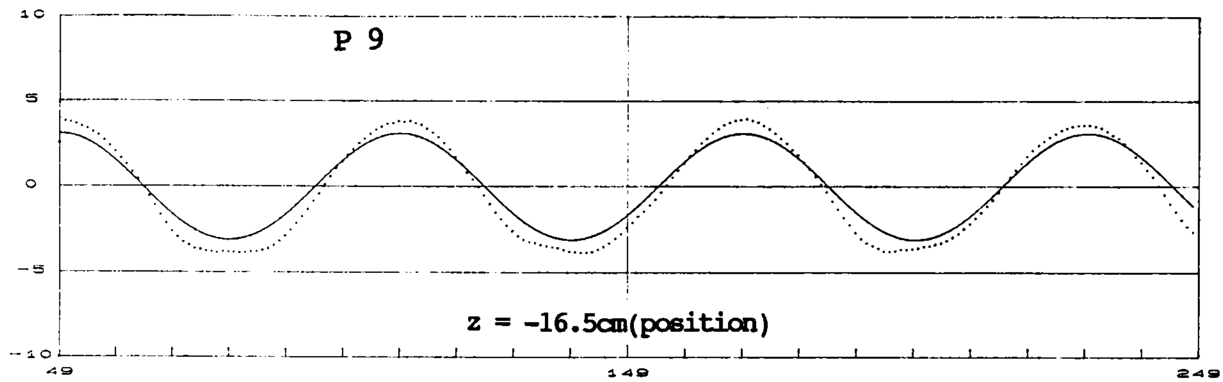
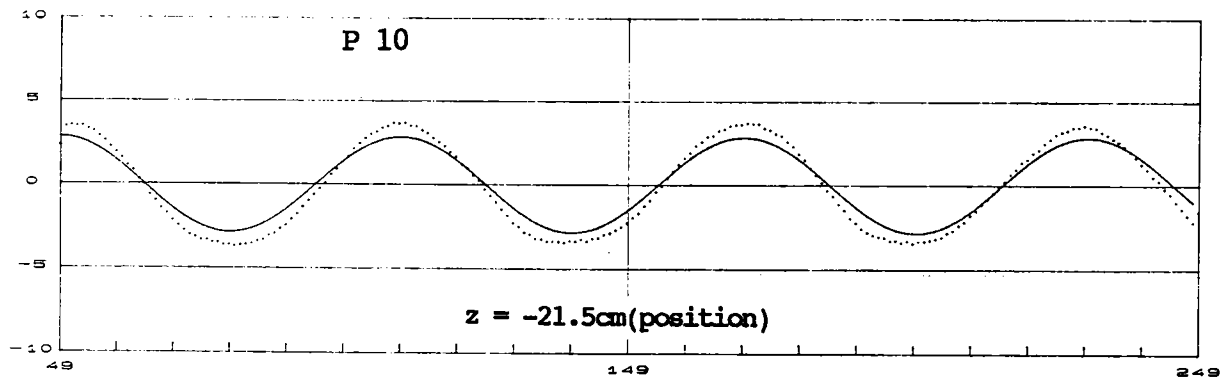
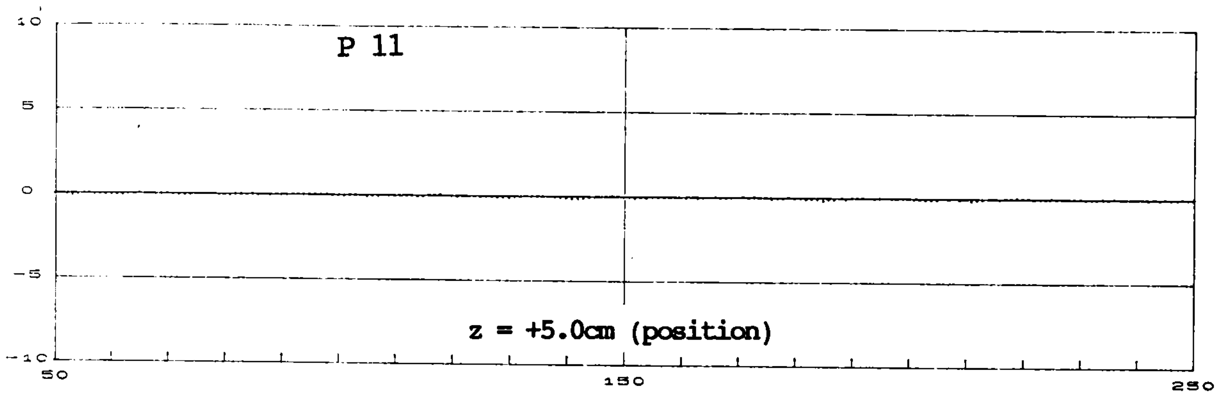


圖11-19


P 9
z = -16.5cm(position)
10
5
0
-5
-10
49
149
249


P 10
z = -21.5cm(position)
10
5
0
-5
-10
49
149
249


P 11
z = +5.0cm (position)
10
5
0
-5
-10
50
150
250

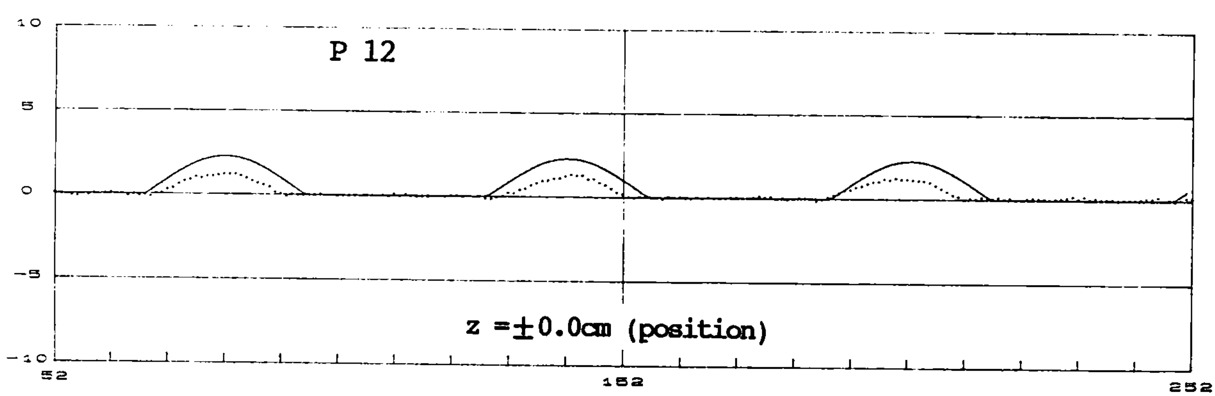

P 12
z =±0.0cm (position)
10
5
0
-5
-10
52
152
252

圖11-19

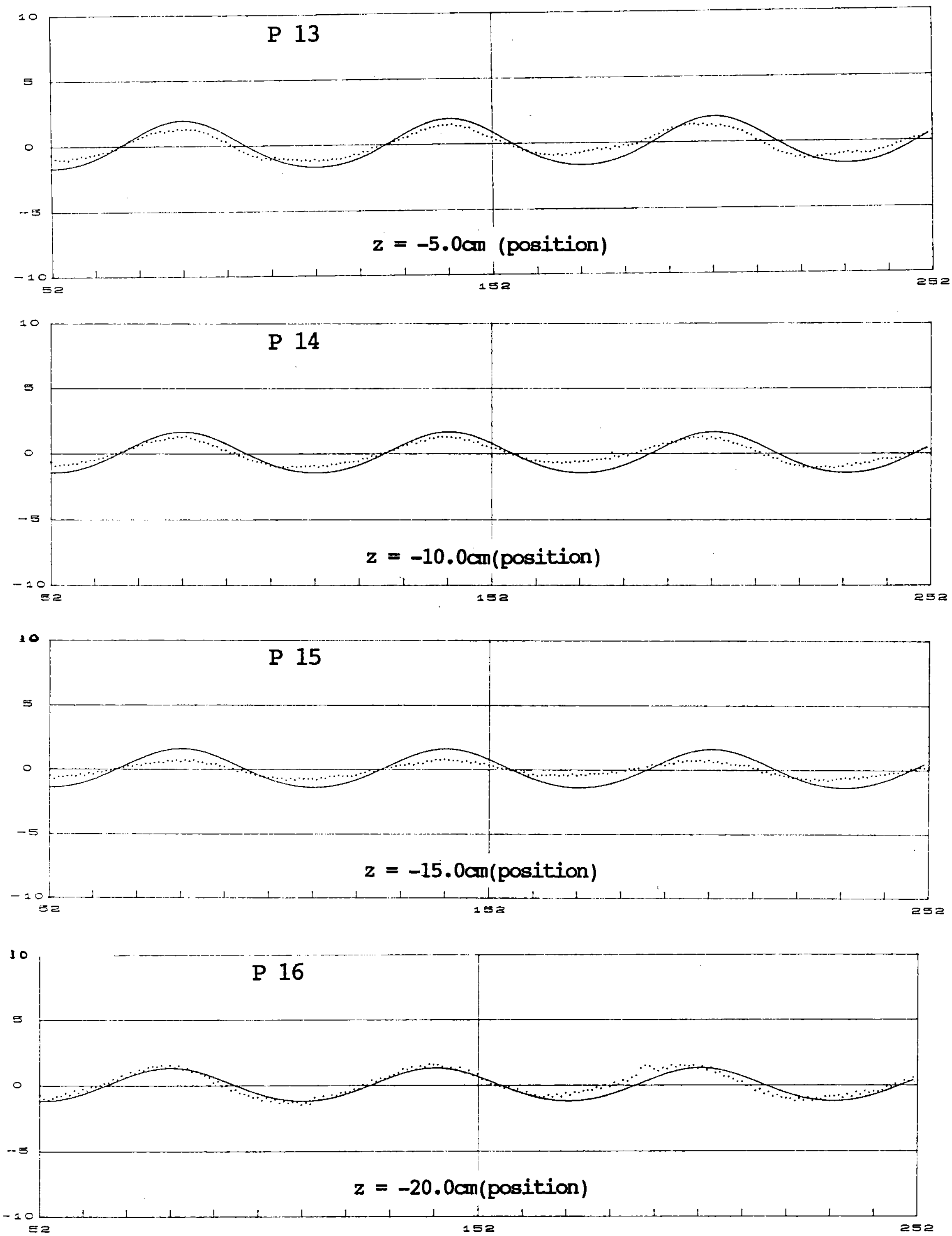

P 13
z = -5.0cm (position)
10
5
0
-5
-10
52
152
252
P 14
z = -10.0cm(position)
10
5
0
-5
-10
52
152
252
P 15
z = -15.0cm(position)
10
5
0
-5
-10
52
152
252
P 16
z = -20.0cm(position)
10
5
0
-5
-10
52
152
252

圖11-19

θ =45° (intersection angle), d =40cm(water depth), T_1=T_2=1.20sec(incident wave period 1 & 2) H_1=10.27cm(incident wave height 1) H_2=3.21cm(incident wave height 2)

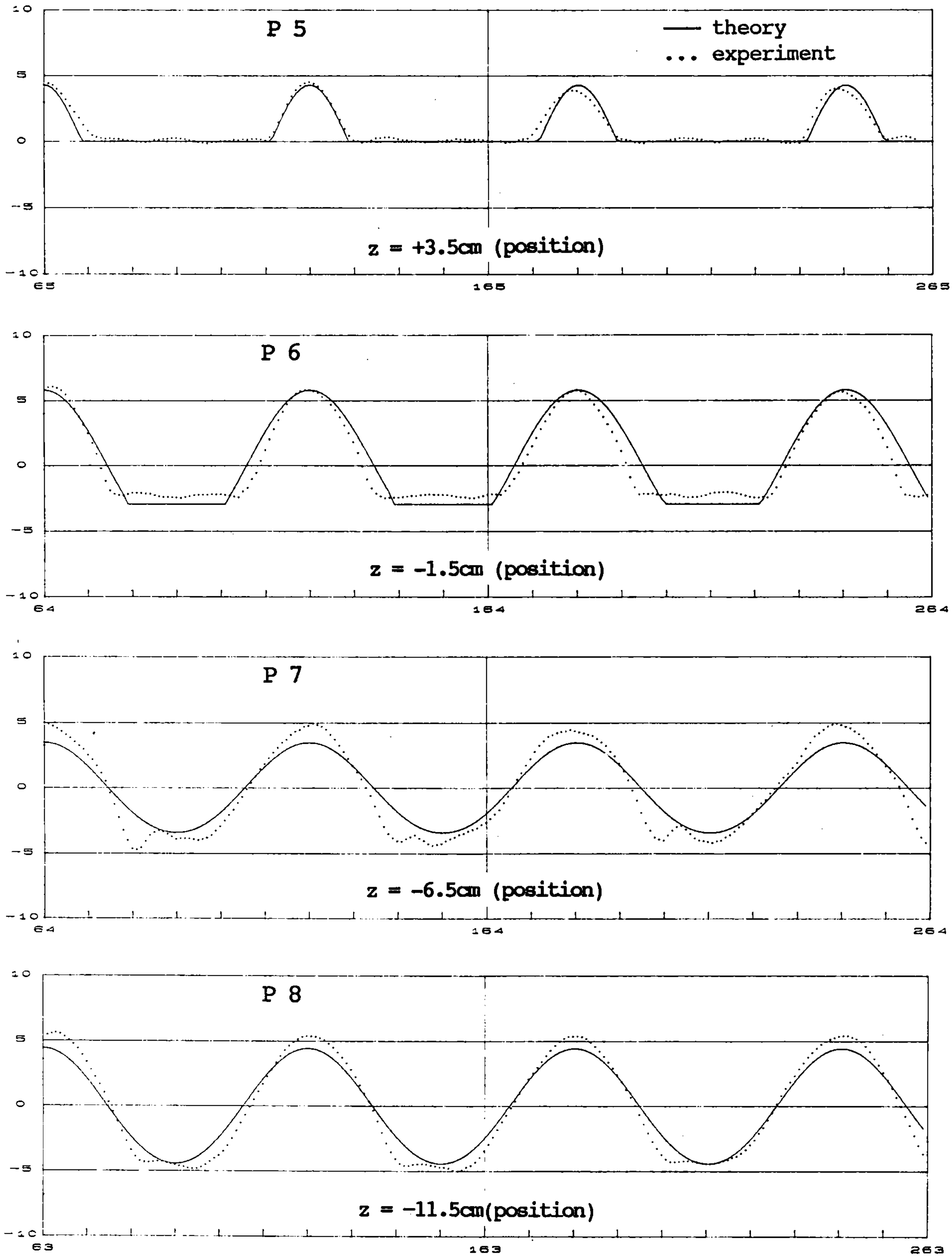


圖11-20

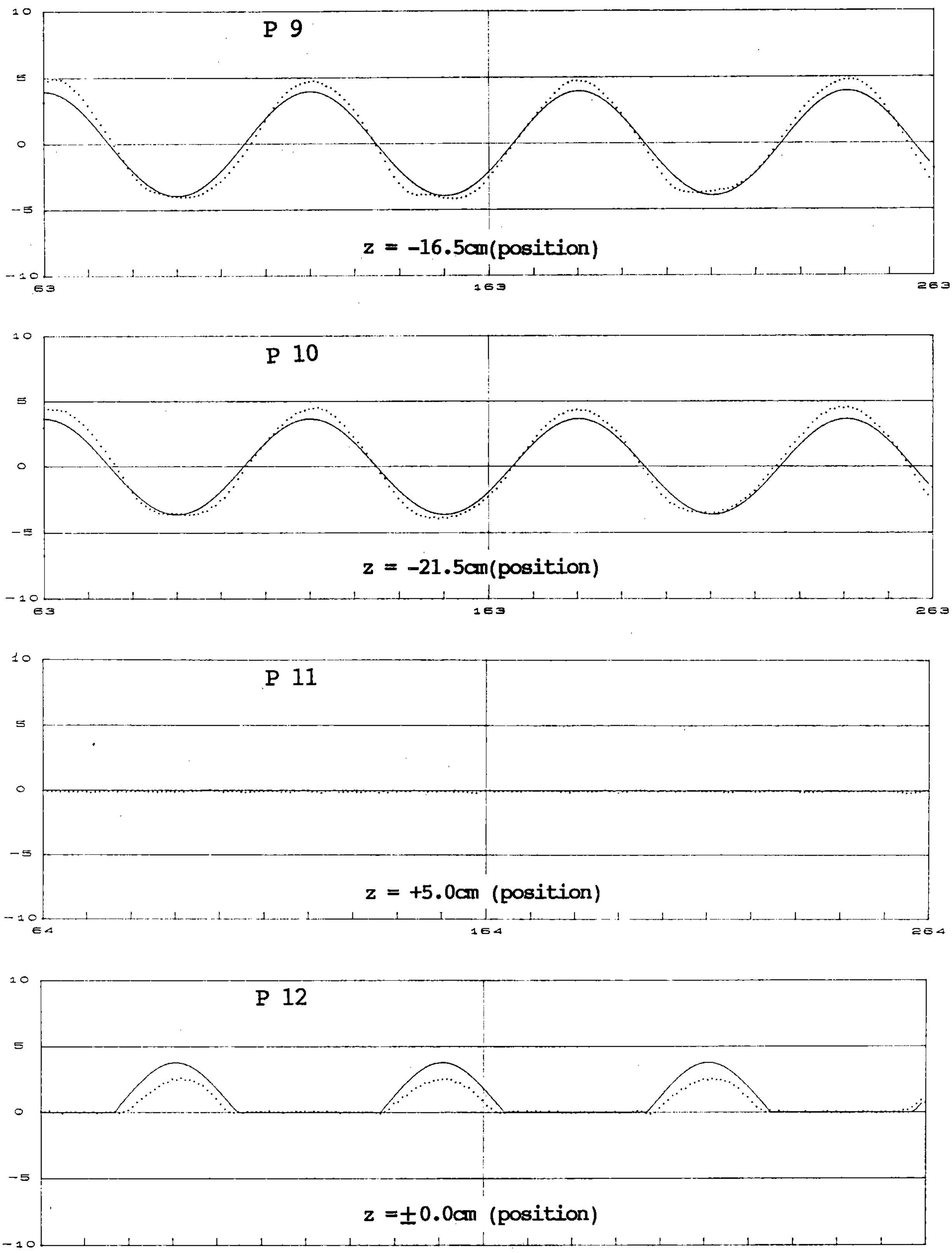

P 9
z = -16.5cm(position)
10
5
0
-5
-10
63
163
263
P 10
z = -21.5cm(position)
10
5
0
-5
-10
63
163
263
P 11
z = +5.0cm (position)
10
5
0
-5
-10
64
164
264
P 12
z =±0.0cm (position)
10
5
0
-5
-10
64
164
264

圖11-20

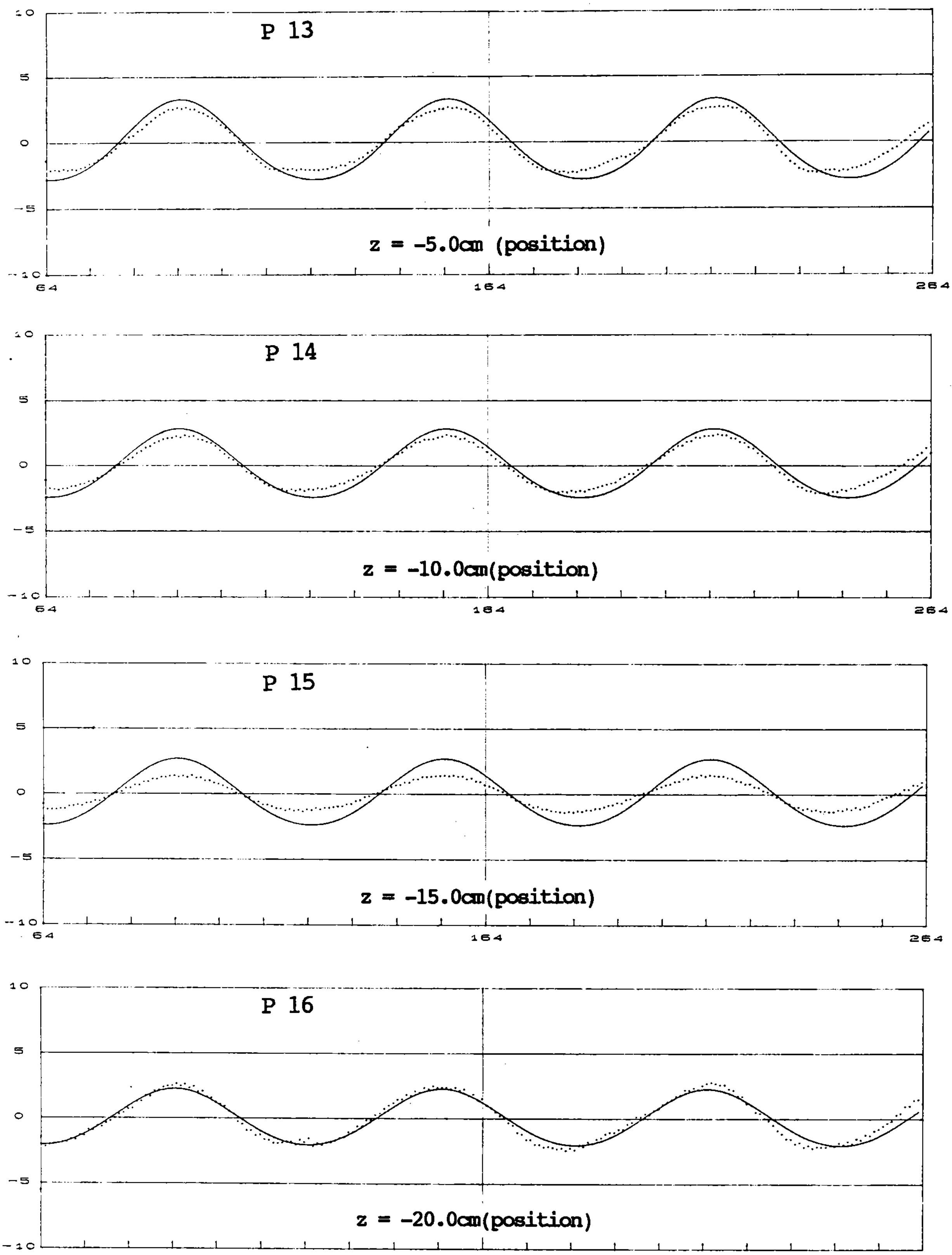

P 13
z = -5.0cm (position)
P 14
z = -10.0cm(position)
P 15
z = -15.0cm(position)
P 16
z = -20.0cm(position)
10
5
0
-5
-10
64
164
264

圖11-20

θ =45° (intersection angle), T_1=T_2=1.60sec(incident wave period 1 & 2)
d =40cm(water depth), H_1=6.87cm(incident wave height 1)
H_2=3.96cm(incident wave height 2)

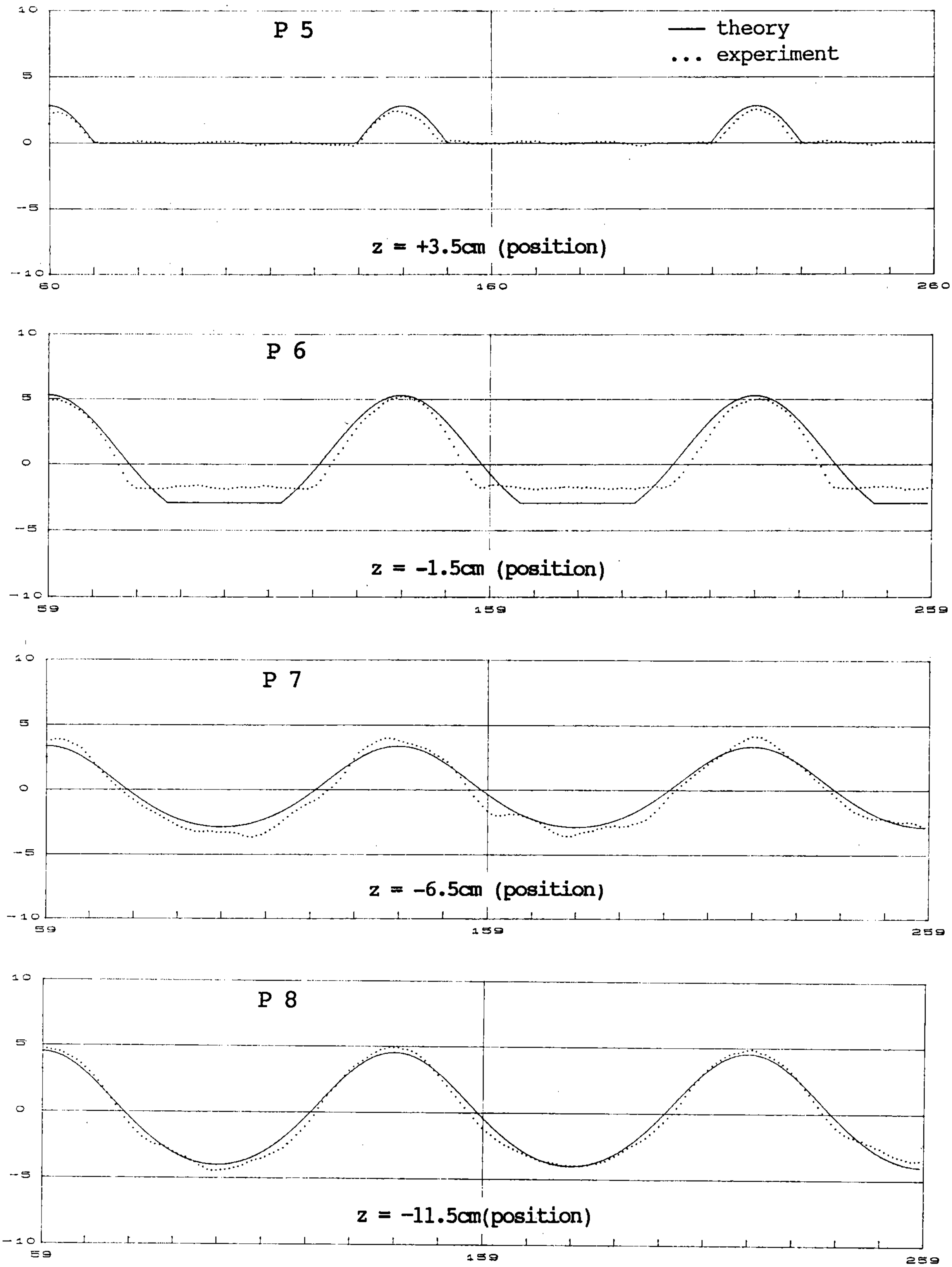


圖11-21

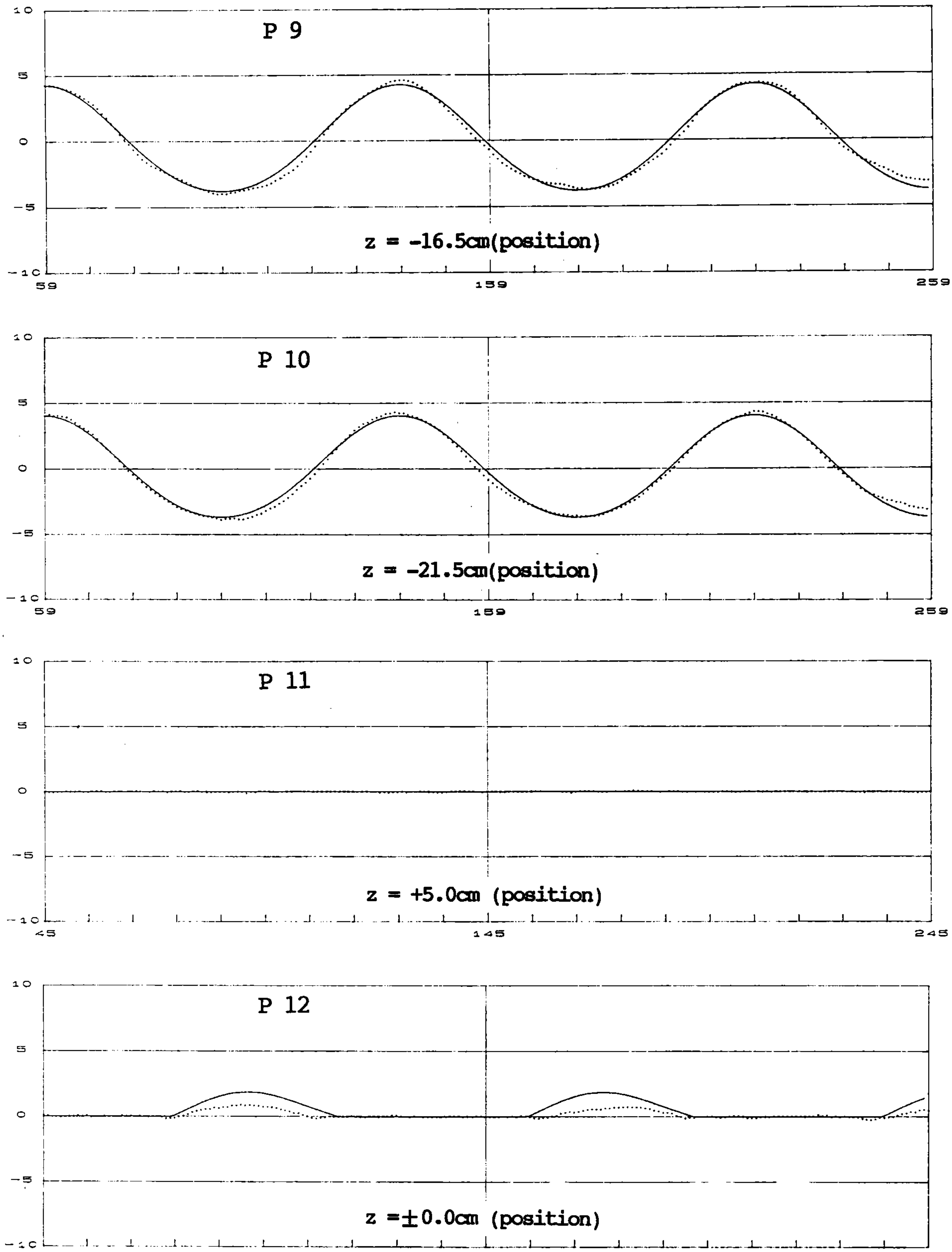
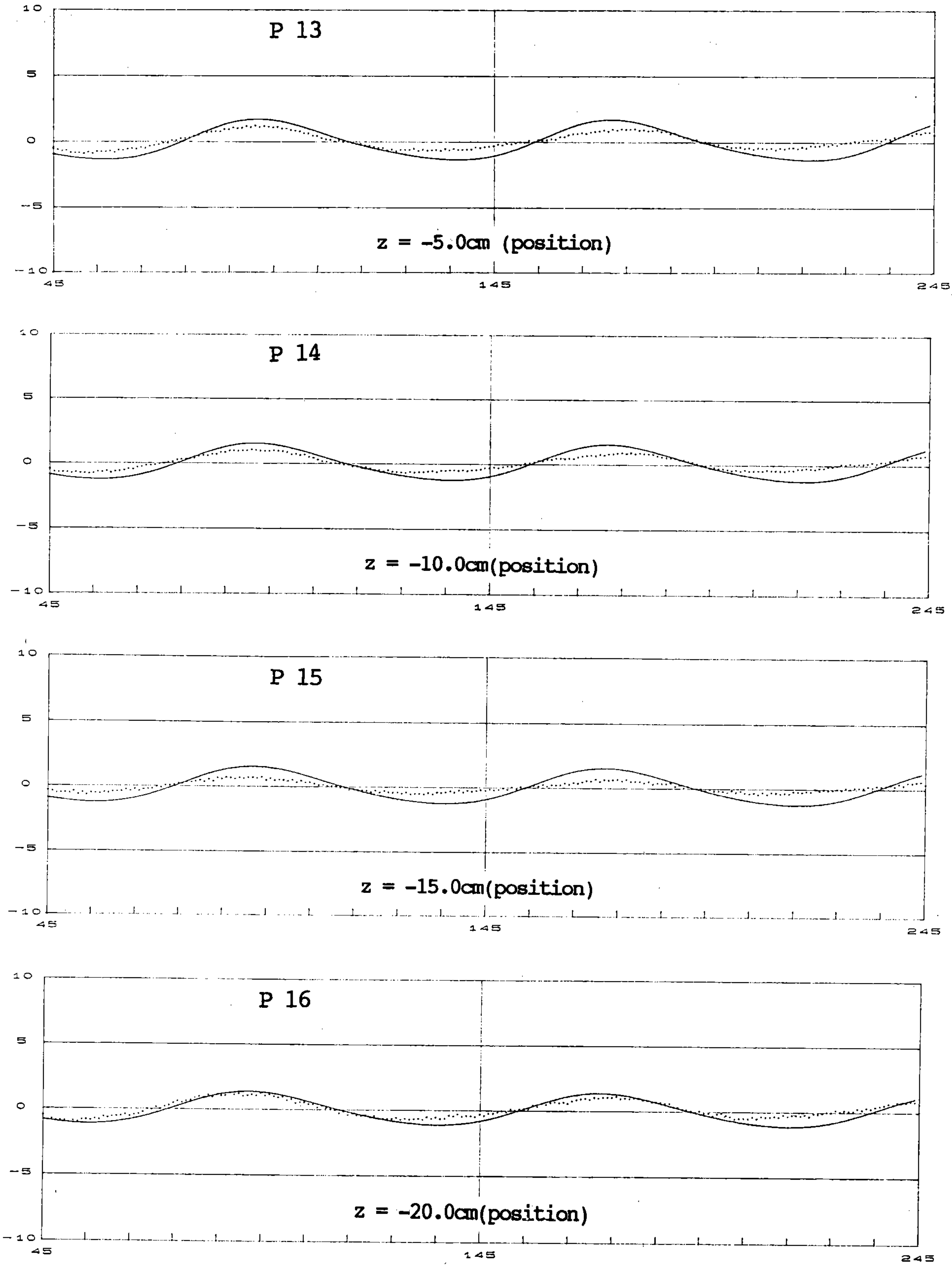

P 9
z = -16.5cm(position)
P 10
z = -21.5cm(position)
P 11
z = +5.0cm (position)
P 12
z =±0.0cm (position)

圖11-21



P 13
z = -5.0cm (position)
P 14
z = -10.0cm(position)
P 15
z = -15.0cm(position)
P 16
z = -20.0cm(position)
10
5
0
-5
-10
45
145
245

圖11-21

θ =45° (intersection angle), T_1=T_2=1.60sec(incident wave period 1 & 2)
d =40cm(water depth), H_1=11.35cm(incident wave height 1)
H_2=3.40cm(incident wave height 2)

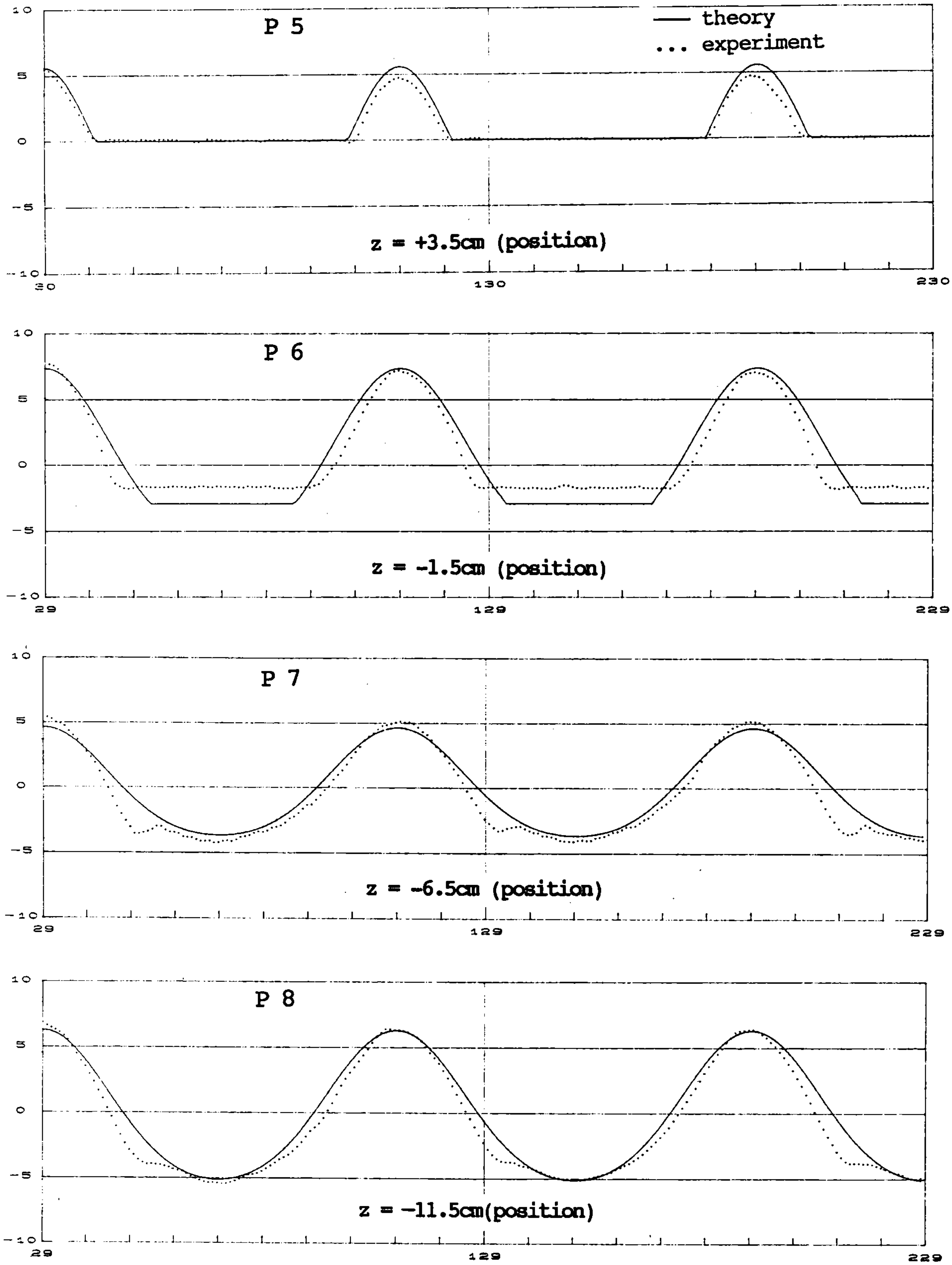


圖11-22

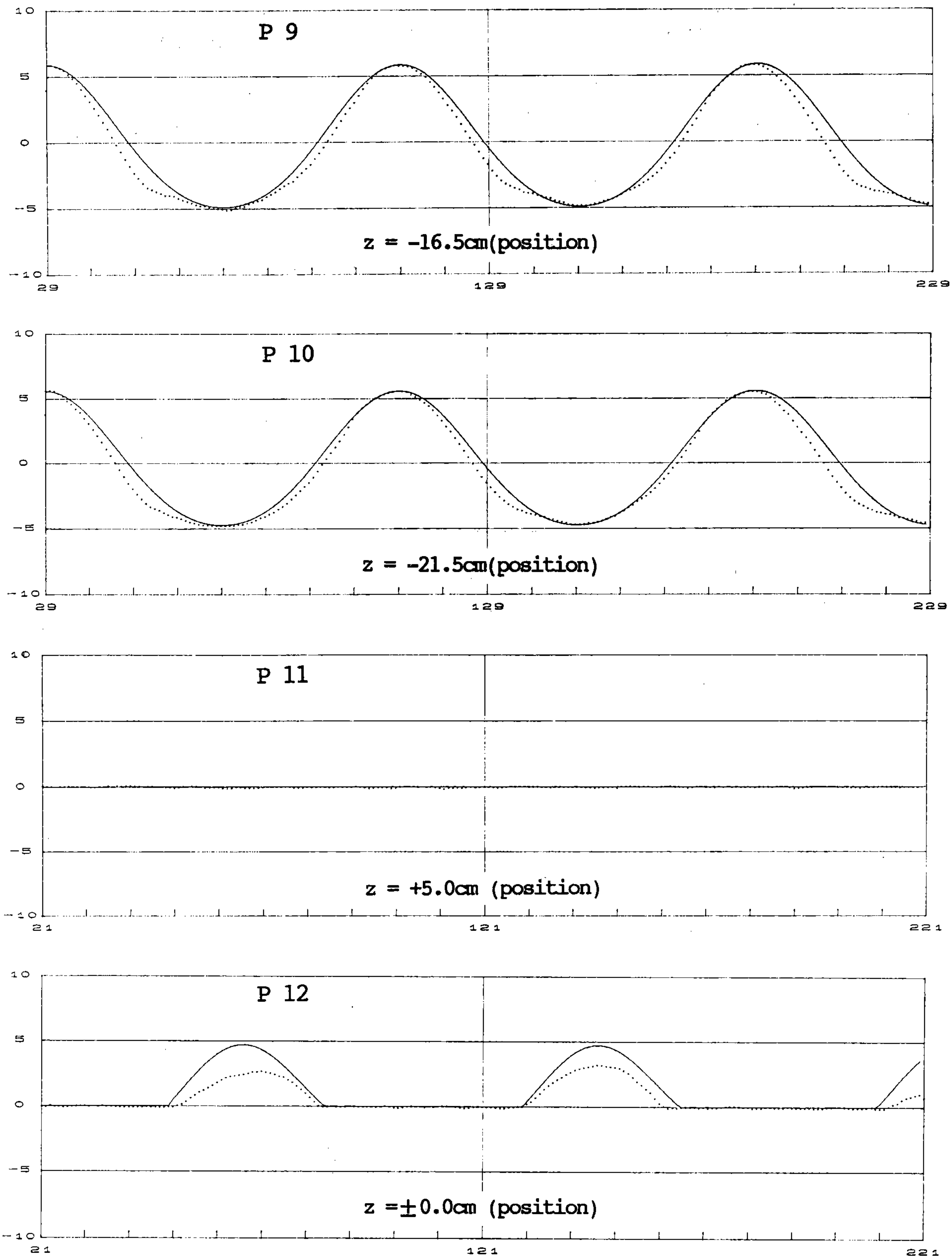
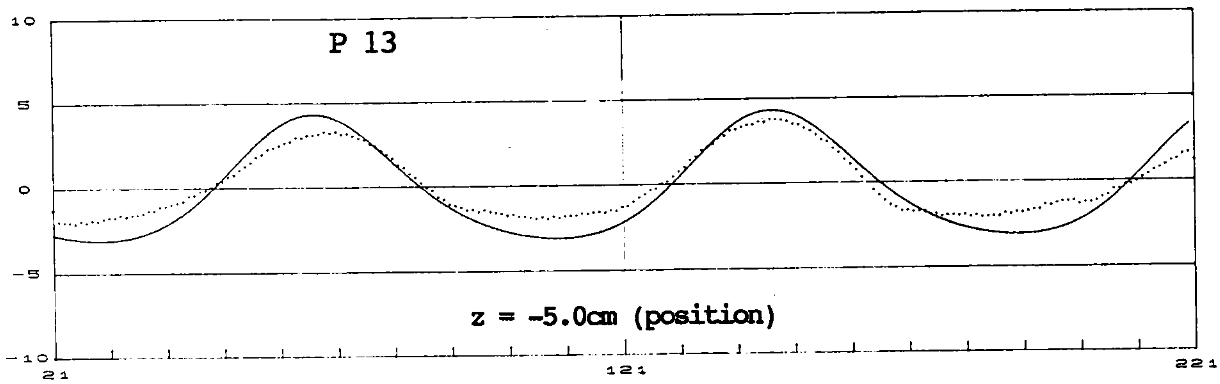
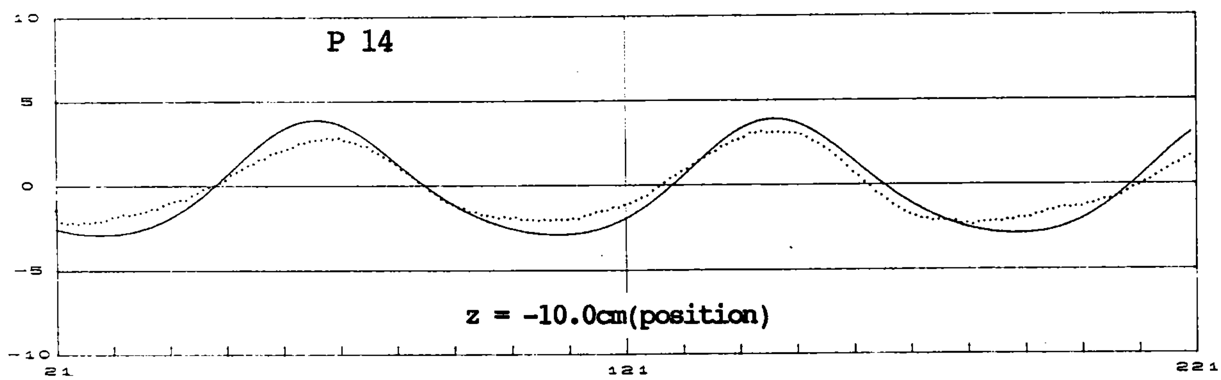
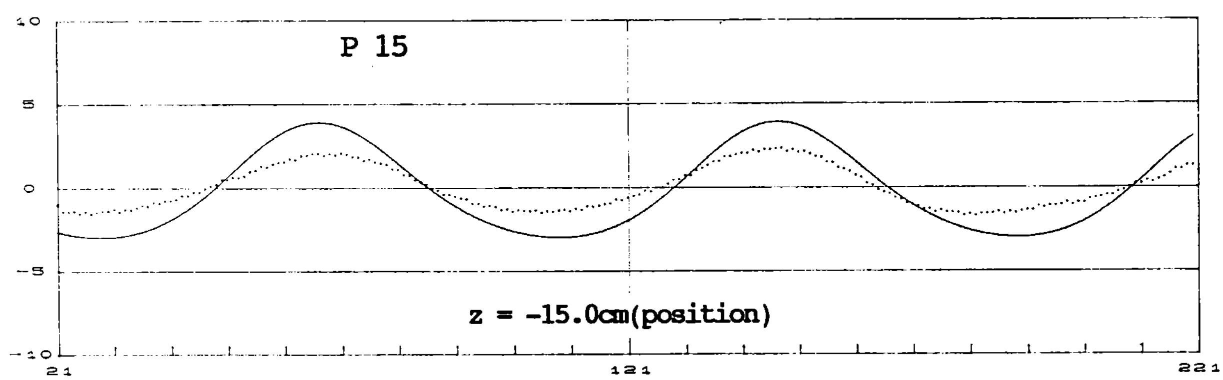

P 9
z = -16.5cm(position)
P 10
z = -21.5cm(position)
P 11
z = +5.0cm (position)
P 12
z =±0.0cm (position)
10
5
0
-5
-10
29
129
229
21
121
221

圖11-22


P 13
z = -5.0cm (position)
10
5
0
-5
-10
21
121
221


P 14
z = -10.0cm(position)
10
5
0
-5
-10
21
121
221


P 15
z = -15.0cm(position)
10
5
0
-5
-10
21
121
221

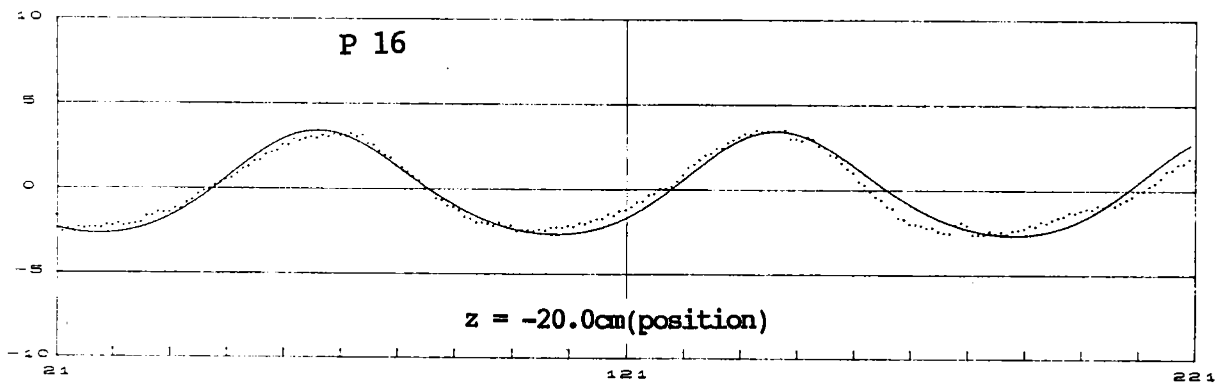

P 16
z = -20.0cm(position)
10
5
0
-5
-10
21
121
221

圖11-22

θ=45° (intersection angle), T_1=T_2=2.00sec(incident wave period 1 & 2)
d =40cm(water depth), H_1=6.48cm(incident wave height 1)
H_2=3.59cm(incident wave height 2)

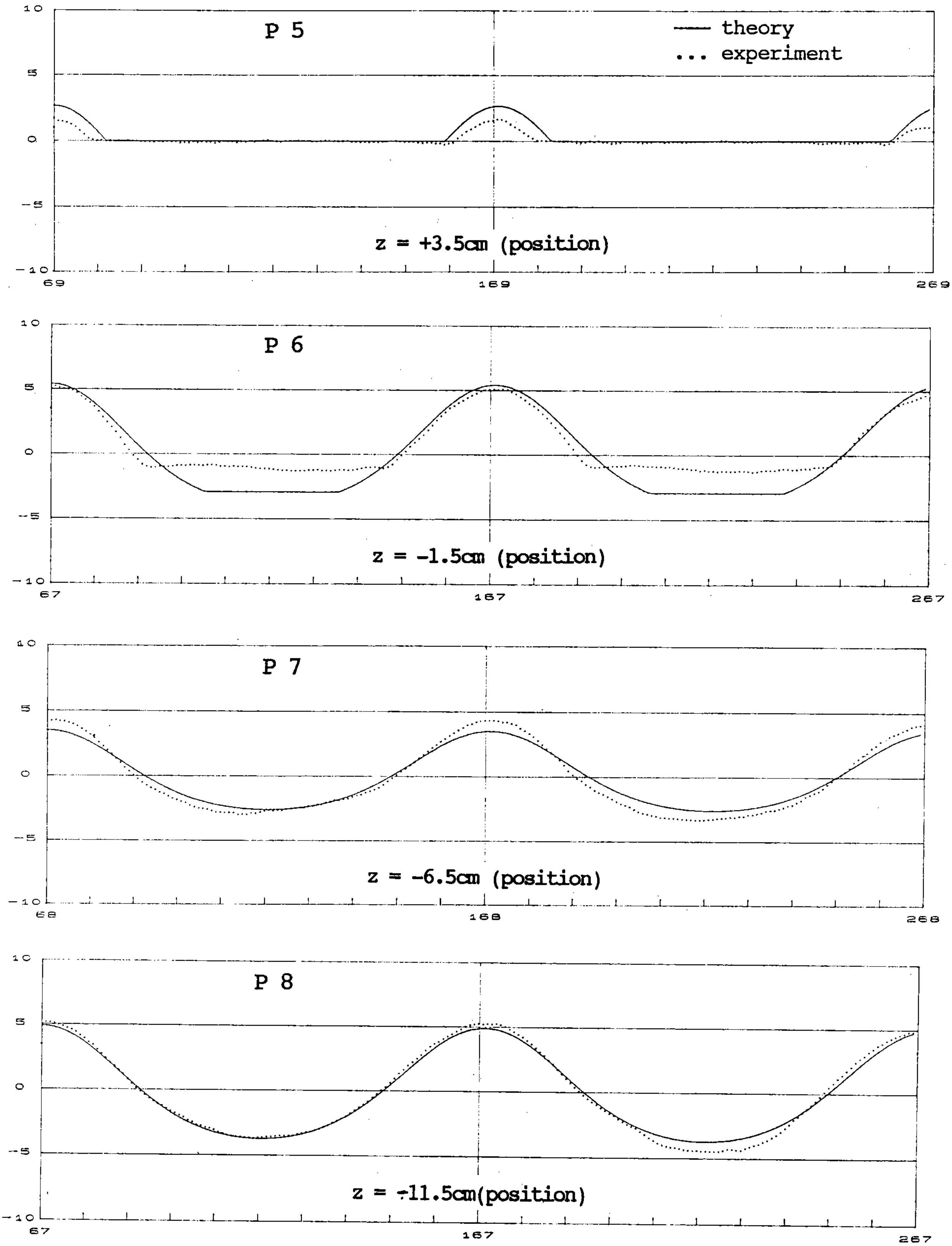
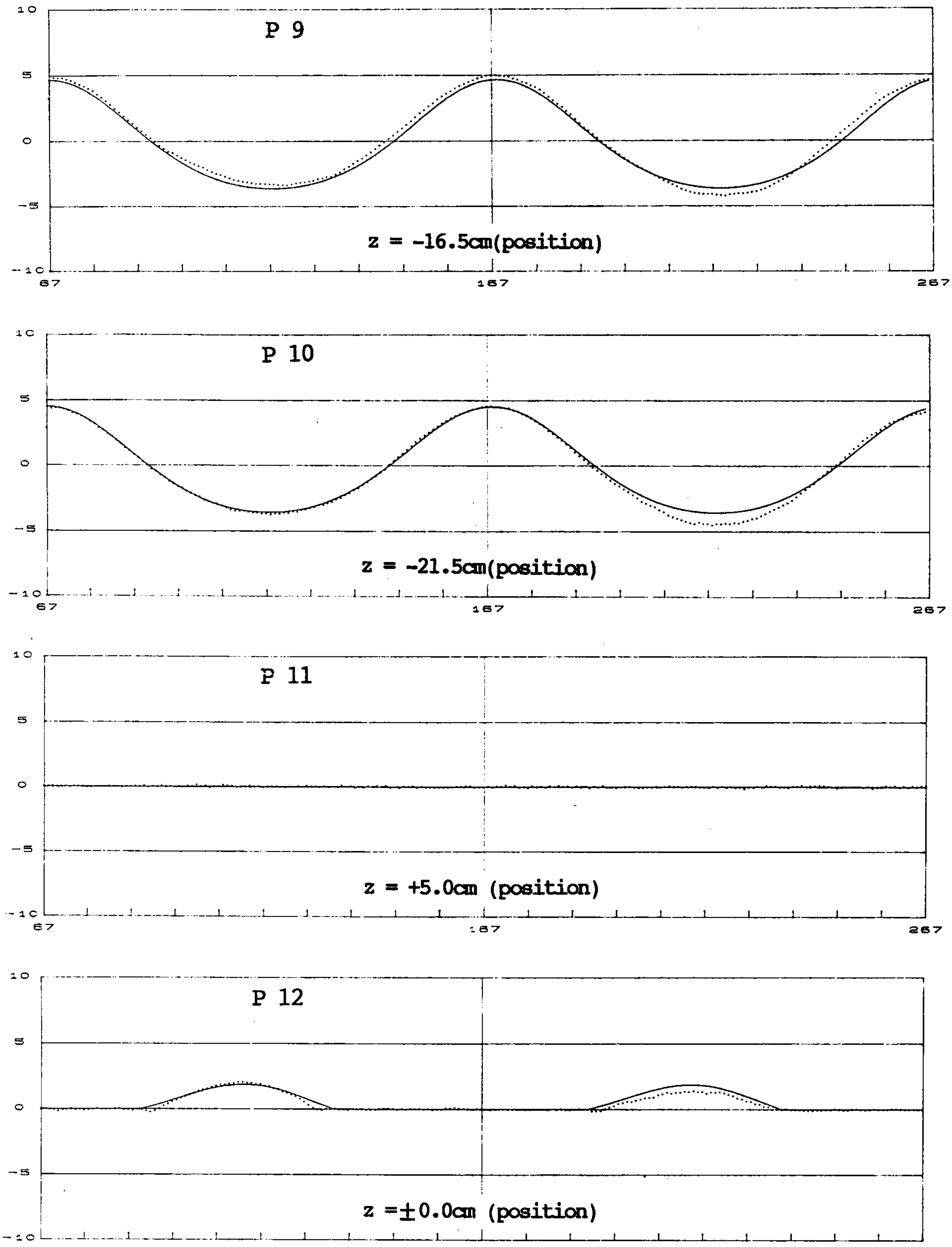
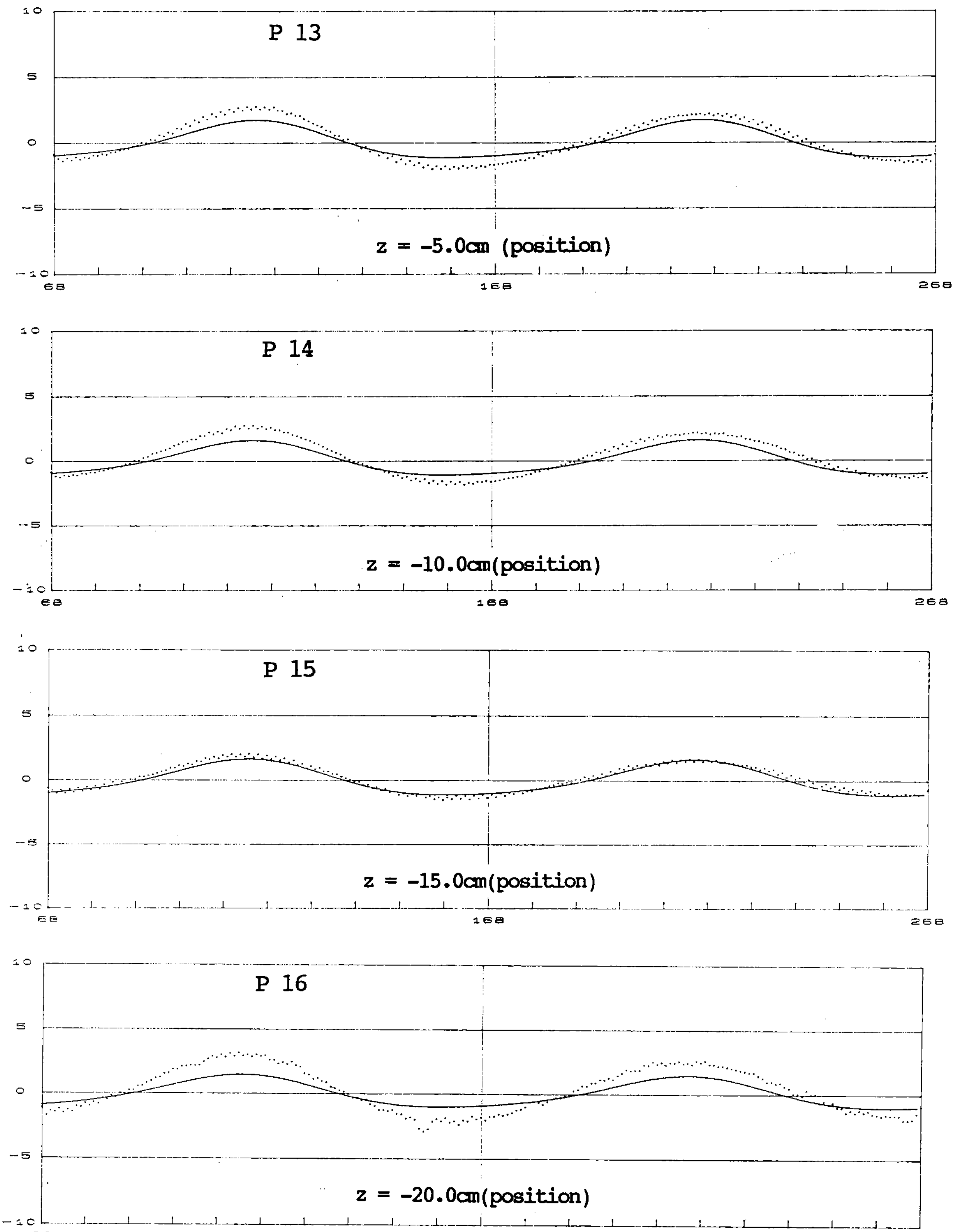


圖11-23



P 9
z = -16.5cm(position)
10
5
0
-5
-10
67
167
267
P 10
z = -21.5cm(position)
10
5
0
-5
-10
67
167
267
P 11
z = +5.0cm (position)
10
5
0
-5
-10
67
167
267
P 12
z =±0.0cm (position)
10
5
0
-5
-10
68
168
268

圖11-23



10
5
0
-5
-10
P 13
z = -5.0cm (position)
68
168
268
P 14
z = -10.0cm(position)
P 15
z = -15.0cm(position)
P 16
z = -20.0cm(position)

圖11-23

θ =45° (intersection angle), T_1=T_2=2.00sec(incident wave period 1 & 2)
d =40cm(water depth), H_1=11.56cm(incident wave height 1)
H_2=3.93cm(incident wave height 2)

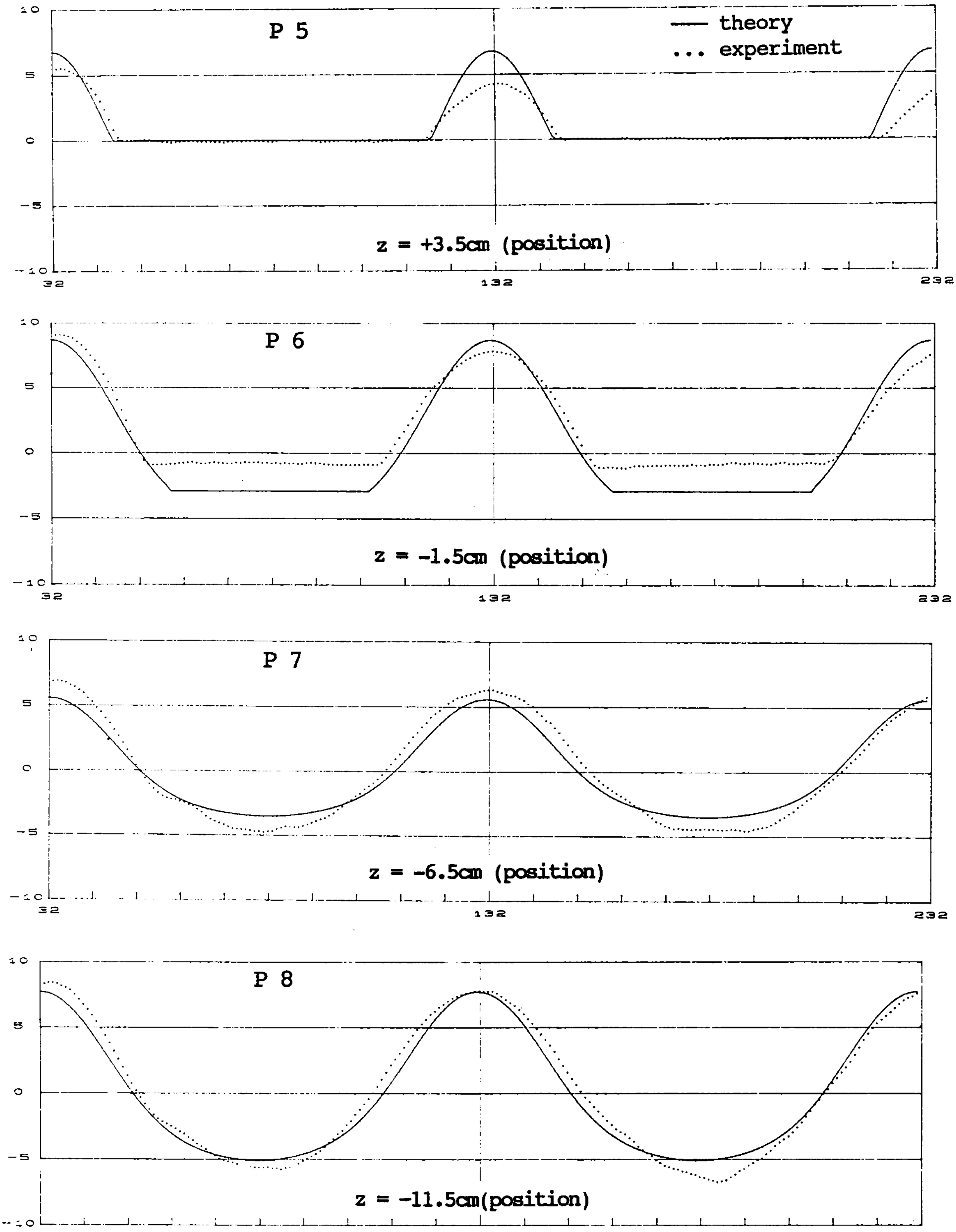


圖11-24

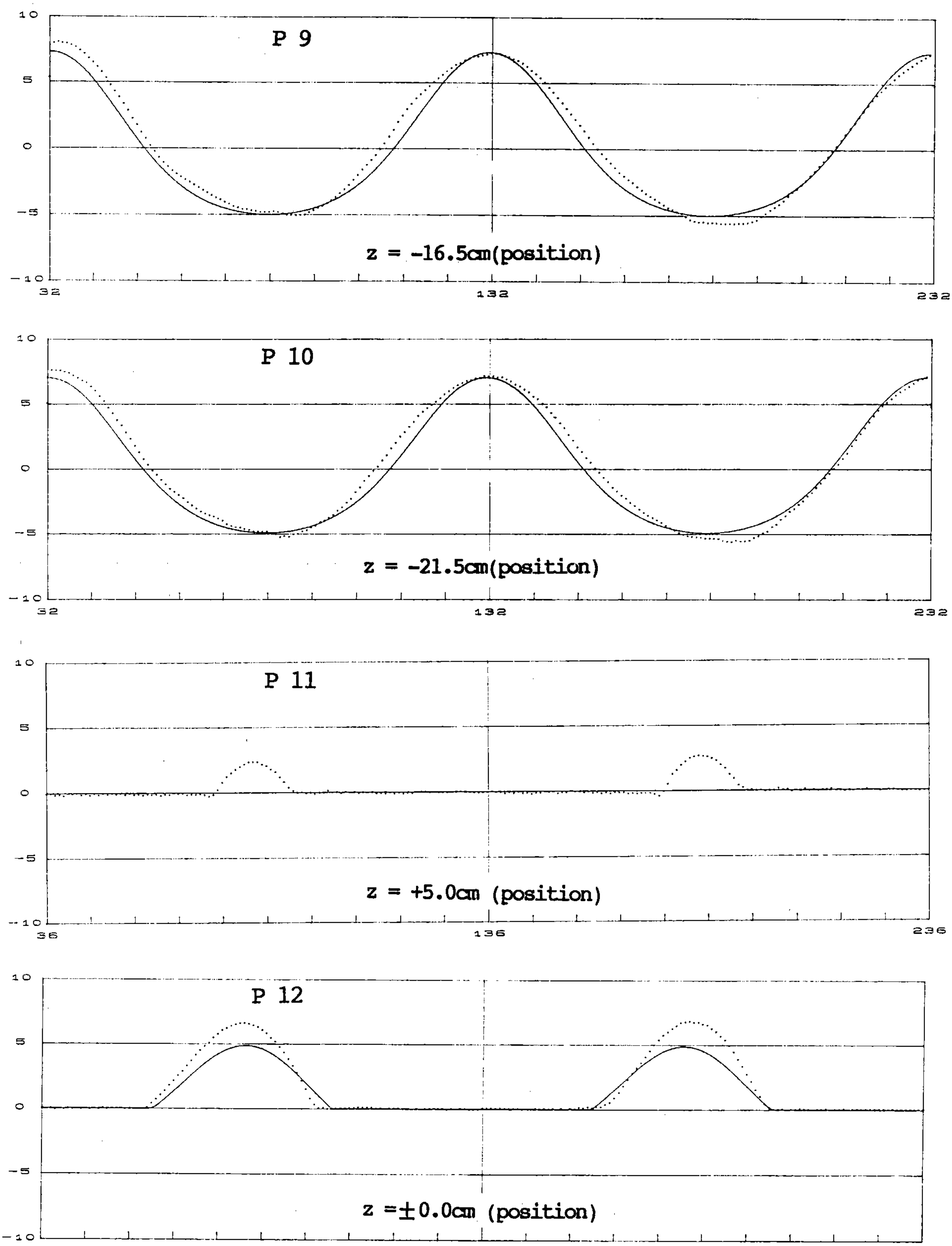

P 9
z = -16.5cm(position)
10
5
0
-5
-10
32
132
232
P 10
z = -21.5cm(position)
10
5
0
-5
-10
32
132
232
P 11
z = +5.0cm (position)
10
5
0
-5
-10
36
136
236
P 12
z =±0.0cm (position)
10
5
0
-5
-10
37
137
237

圖11-24

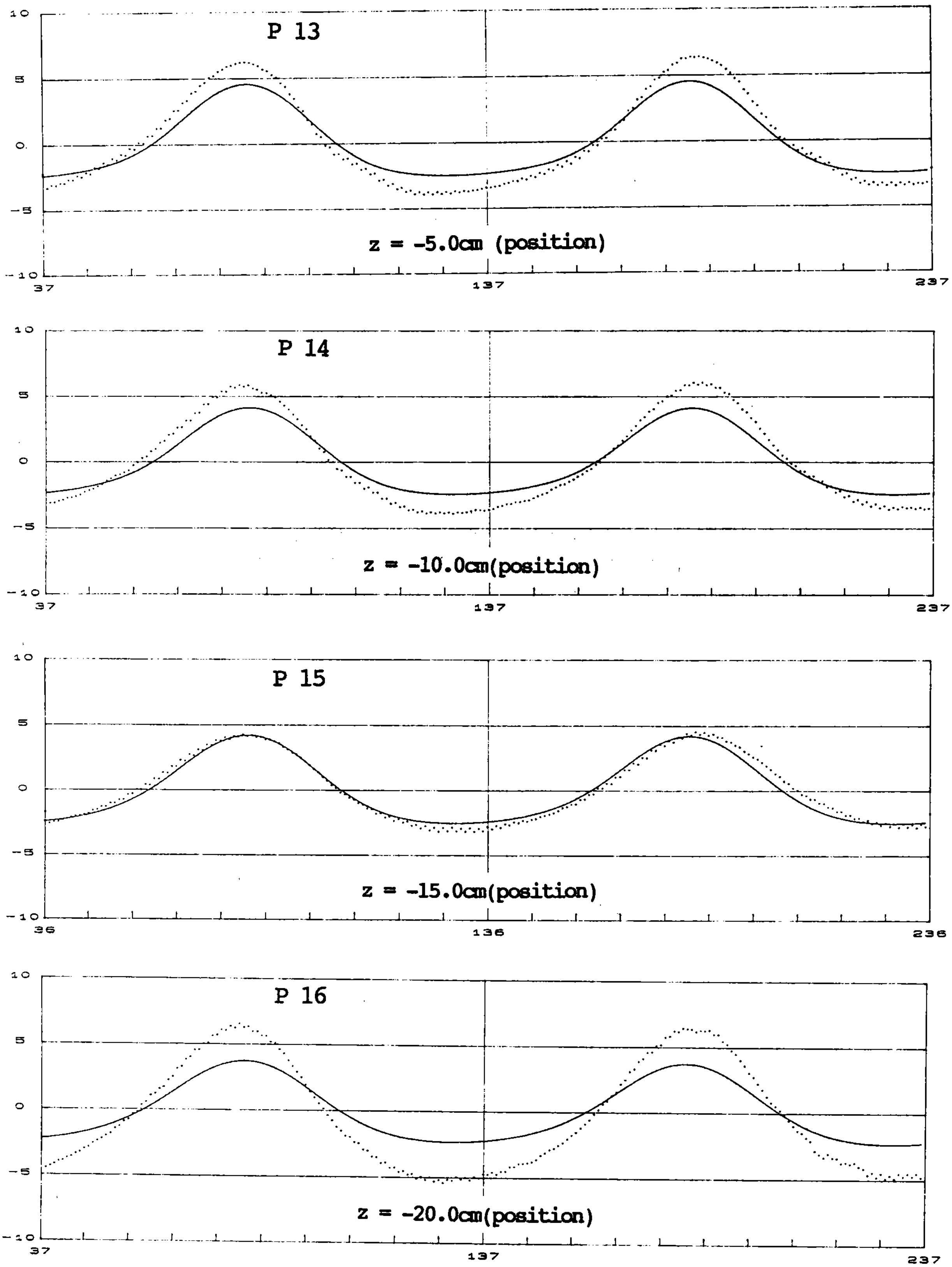

P 13
z = -5.0cm (position)
10
5
0.
-5
-10
37
137
237
P 14
z = -10.0cm(position)
10
5
0
-5
-10
37
137
237
P 15
z = -15.0cm(position)
10
5
0
-5
-10
36
136
236
P 16
z = -20.0cm(position)
10
5
0
-5
-10
37
137
237

圖11-24

θ=45° (intersection angle), T_1=T_2=2.40sec(incident wave period 1 & 2)
d =40cm(water depth), H_1=6.82cm(incident wave height 1)
H_2=3.14cm(incident wave height 2)

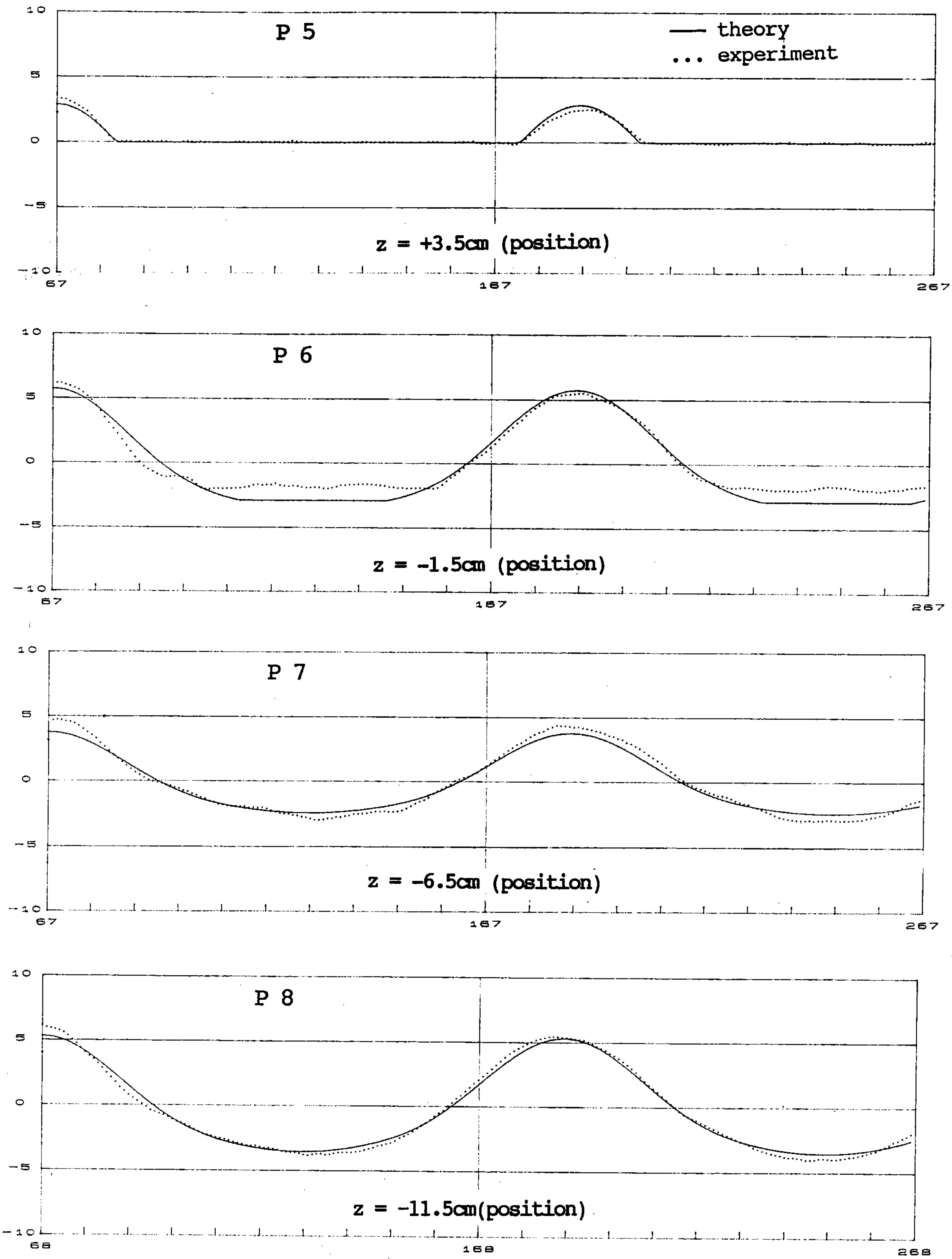


圖11-25

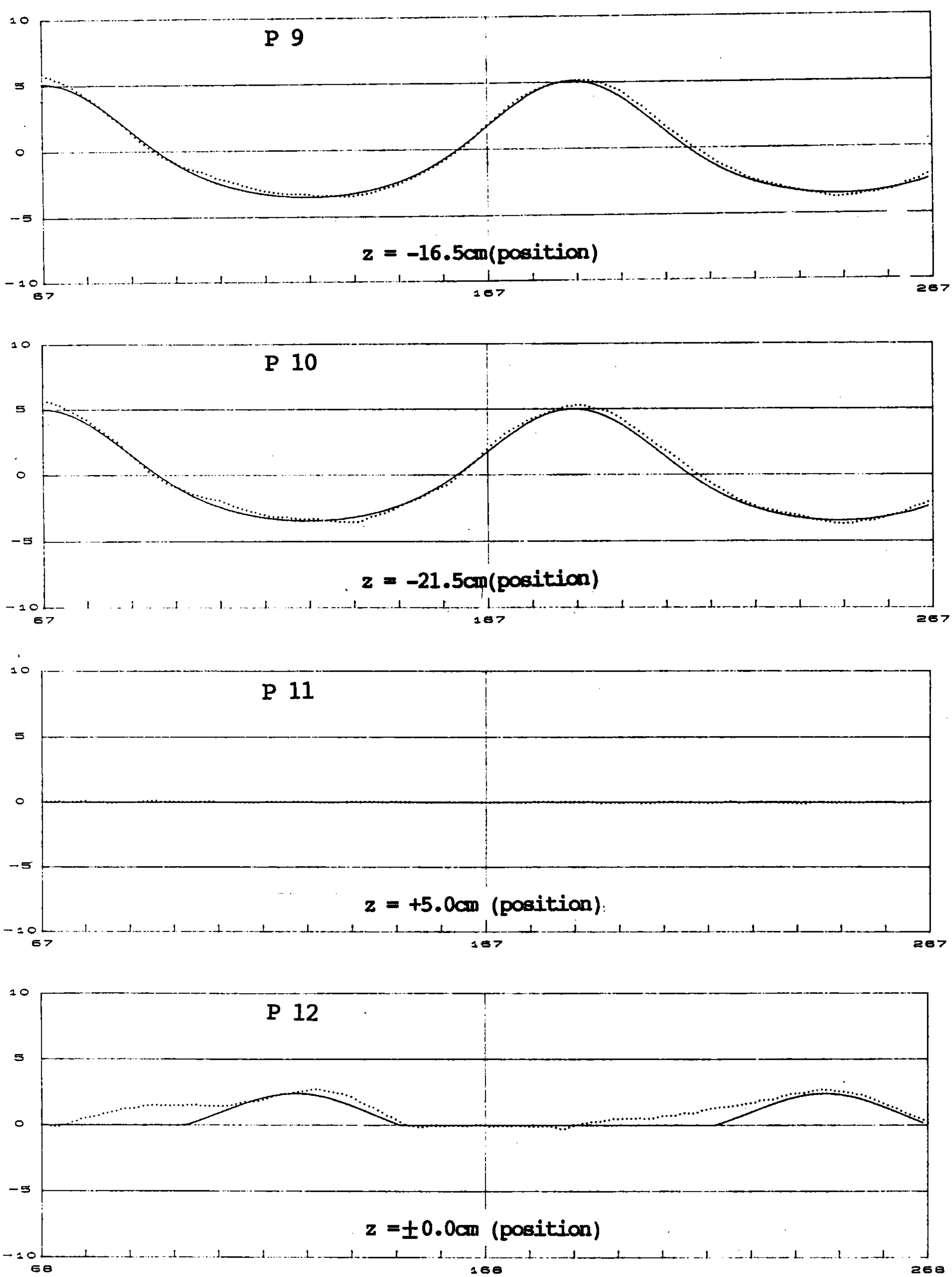

P 9
z = -16.5cm(position)
P 10
z = -21.5cm(position)
P 11
z = +5.0cm (position)
P 12
z =±0.0cm (position)

圖11-25

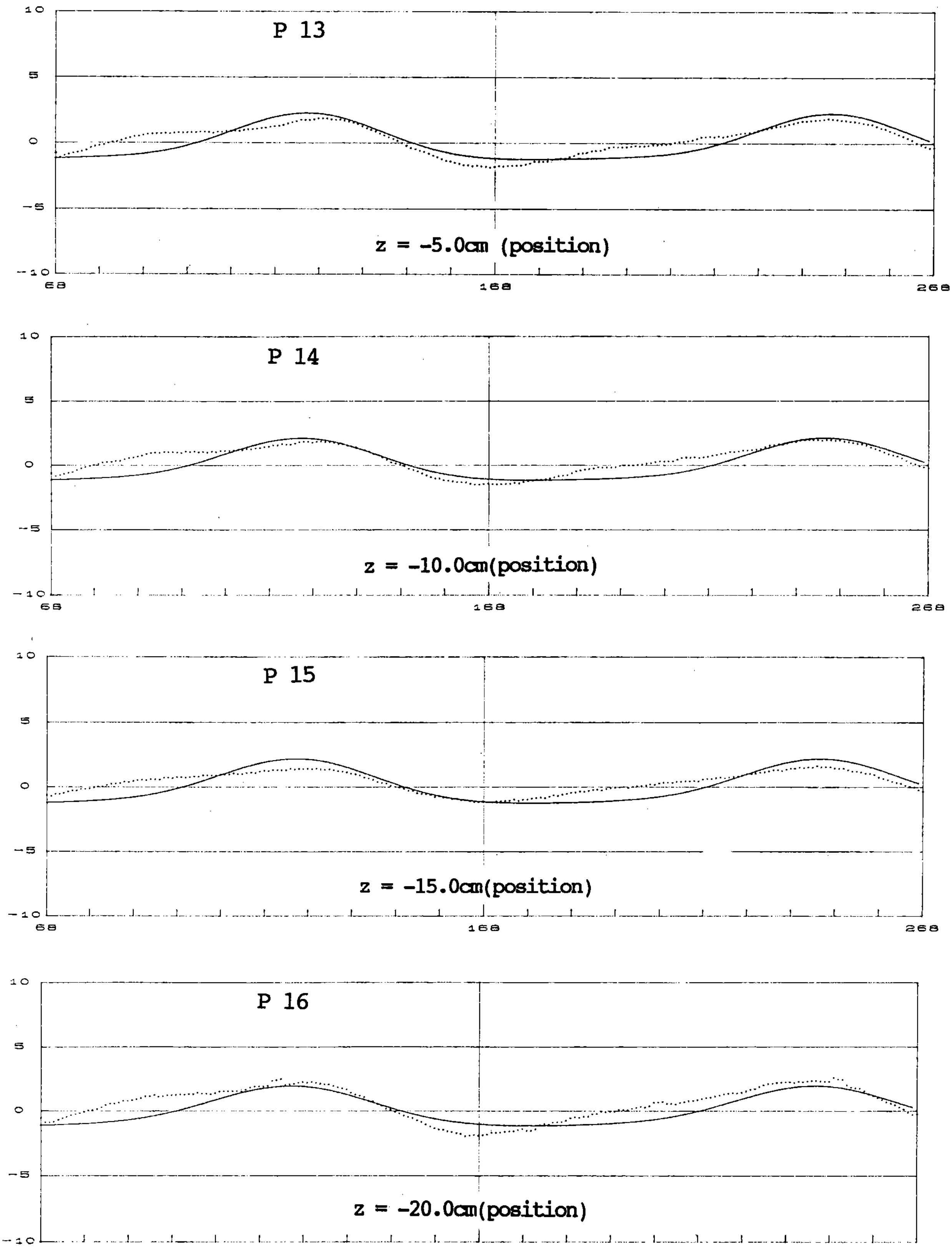

P 13
z = -5.0cm (position)
P 14
z = -10.0cm(position)
P 15
z = -15.0cm(position)
P 16
z = -20.0cm(position)
10
5
0
-5
-10
68
168
268

圖11-25

θ =45° (intersection angle), T_1=T_2=2.40sec(incident wave period 1 & 2)
d =40cm(water depth), H_1=9.80cm(incident wave height 1)
H_2=4.29cm(incident wave height 2)

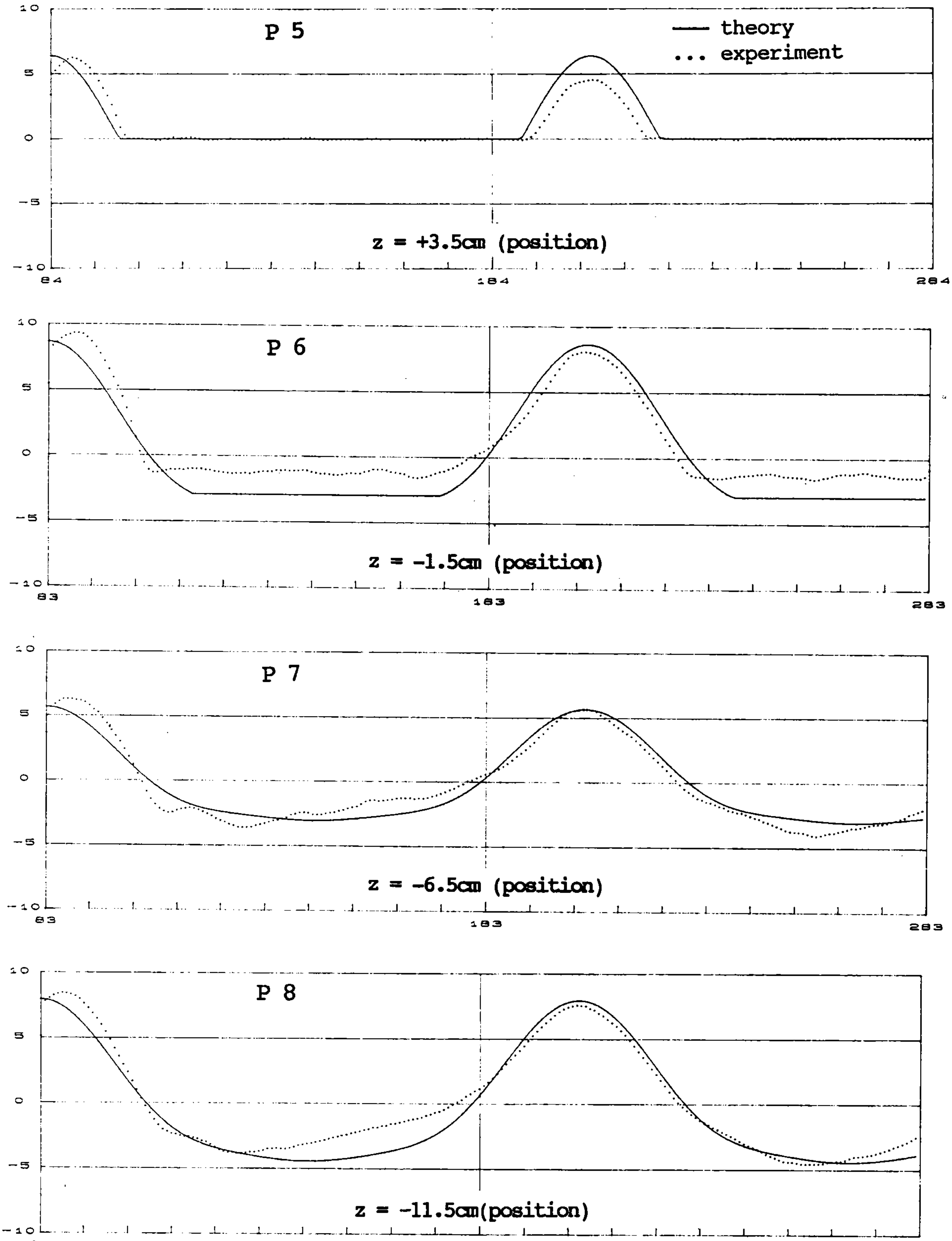


圖11-26

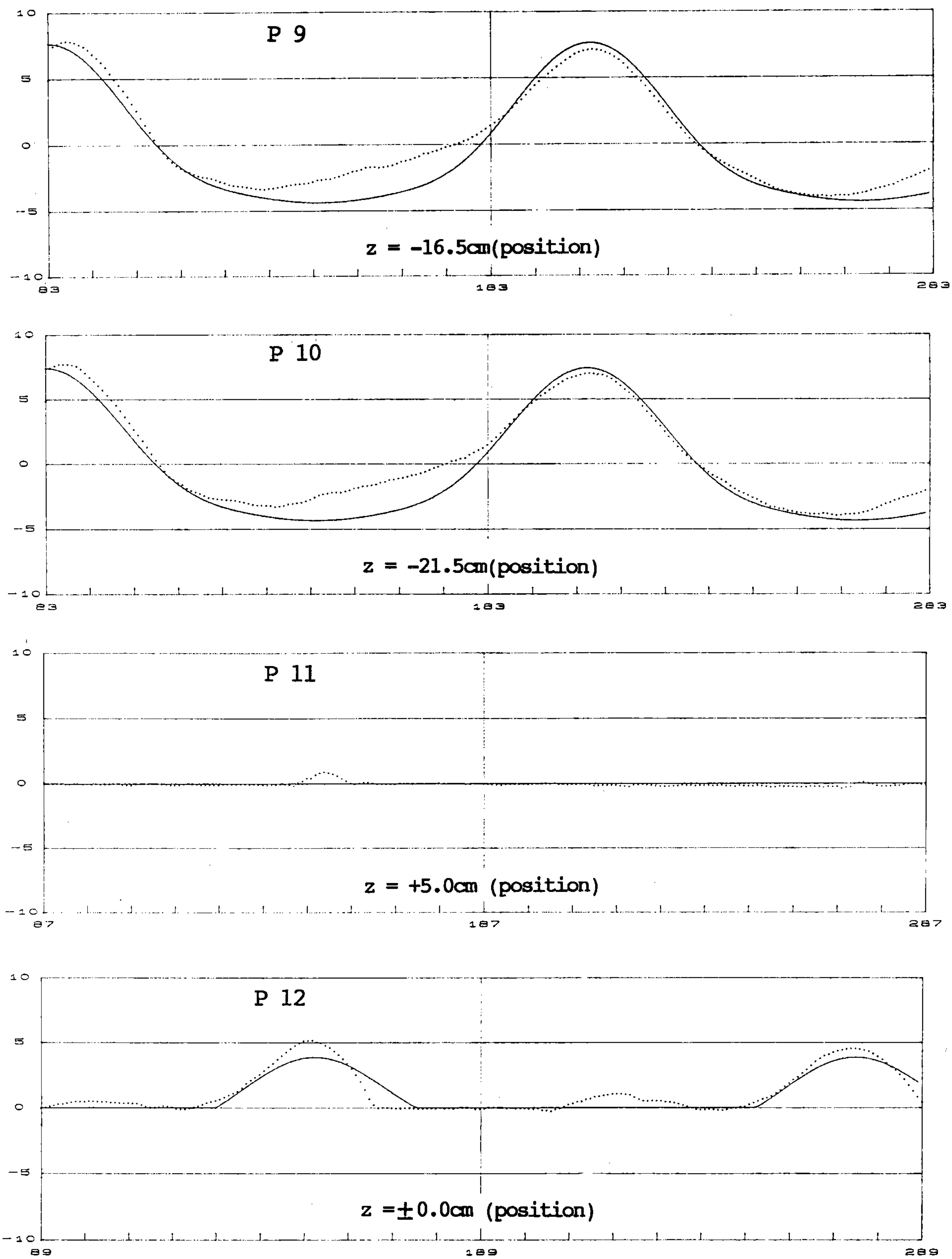
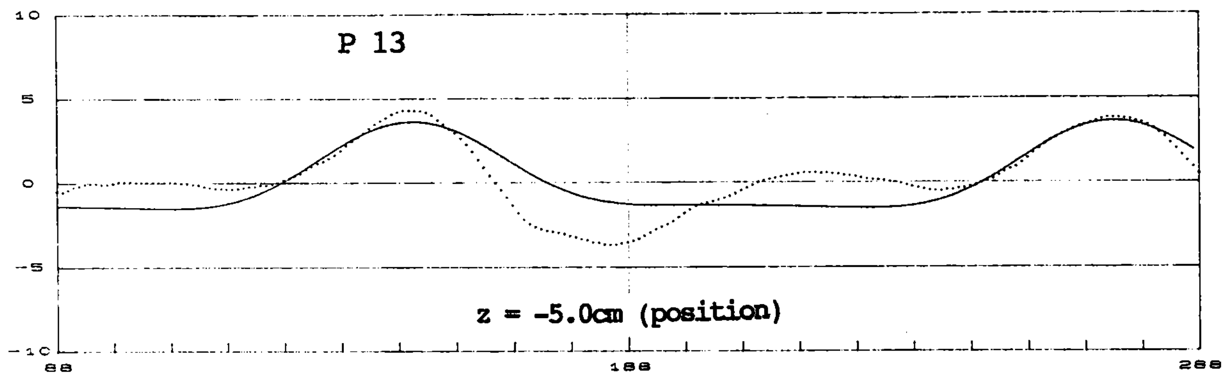
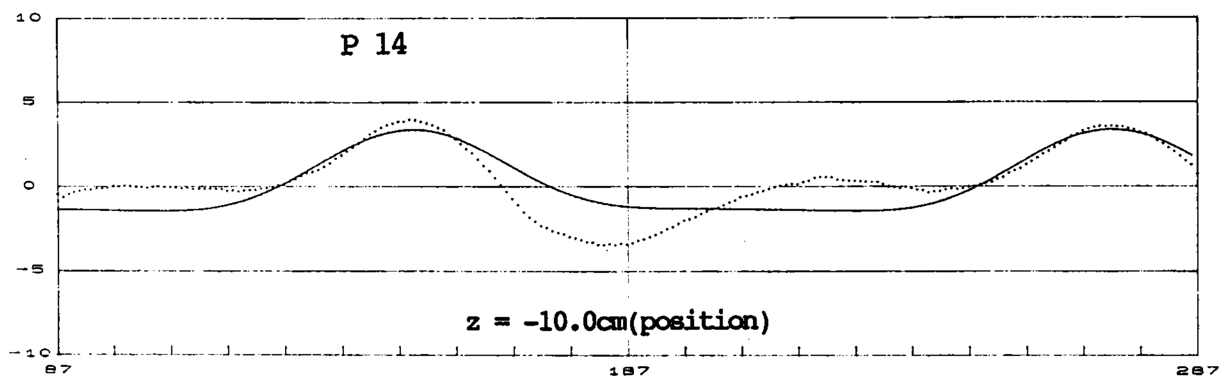
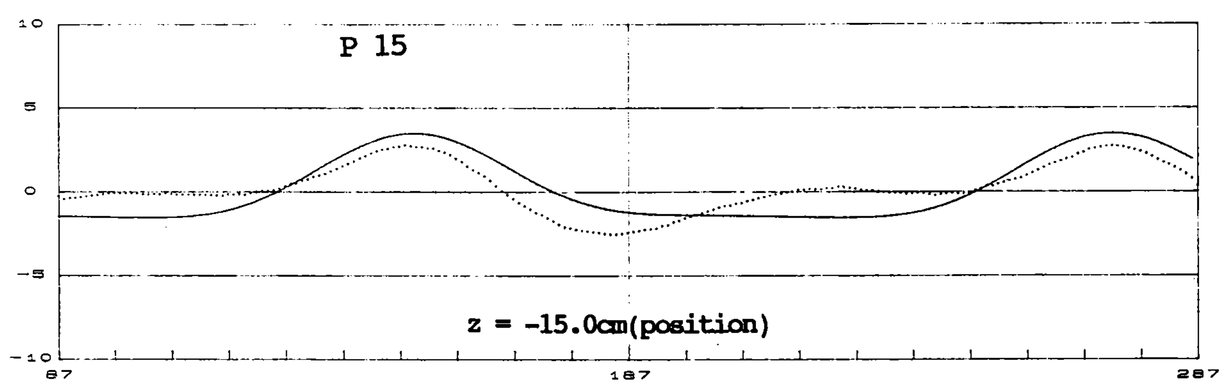

P 9
z = -16.5cm(position)
10
5
0
-5
-10
83
183
283
P 10
z = -21.5cm(position)
10
5
0
-5
-10
83
183
283
P 11
z = +5.0cm (position)
10
5
0
-5
-10
87
187
287
P 12
z =±0.0cm (position)
10
5
0
-5
-10
89
189
289

圖11-26


10
P 13
5
0
-5
z = -5.0cm (position)
-10
88
188
288


10
P 14
5
0
-5
z = -10.0cm(position)
-10
87
187
287


10
P 15
5
0
-5
z = -15.0cm(position)
-10
87
187
287

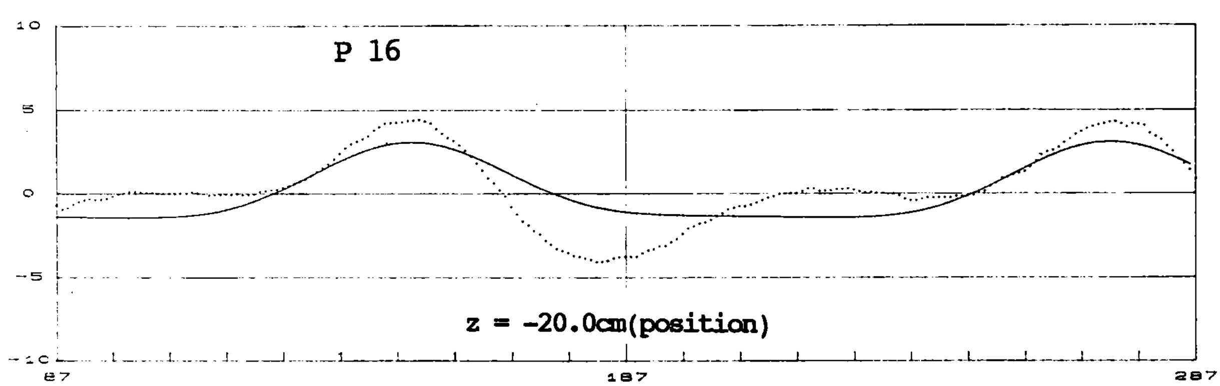

10
P 16
5
0
-5
z = -20.0cm(position)
-10
87
187
287

圖11-26

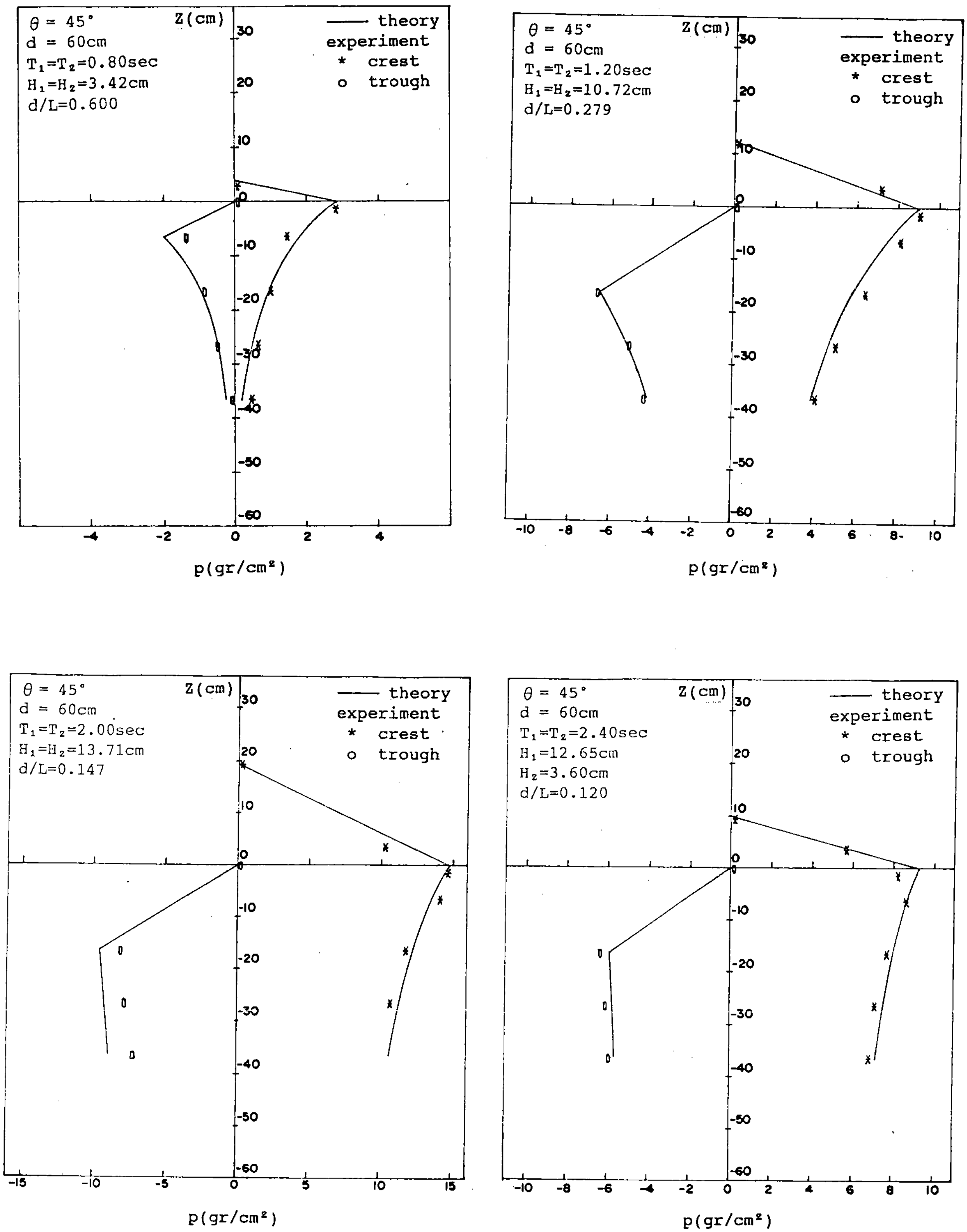

θ = 45°
d = 60cm
T₁=T₂=0.80sec
H₁=H₂=3.42cm
d/L=0.600
Z(cm)
theory
experiment
* crest
o trough
p(gr/cm²)
θ = 45°
d = 60cm
T₁=T₂=1.20sec
H₁=H₂=10.72cm
d/L=0.279
Z(cm)
theory
experiment
* crest
o trough
p(gr/cm²)
θ = 45°
d = 60cm
T₁=T₂=2.00sec
H₁=H₂=13.71cm
d/L=0.147
Z(cm)
theory
experiment
* crest
o trough
p(gr/cm²)
θ = 45°
d = 60cm
T₁=T₂=2.40sec
H₁=12.65cm
H₂=3.60cm
d/L=0.120
Z(cm)
theory
experiment
* crest
o trough
p(gr/cm²)

圖11-27

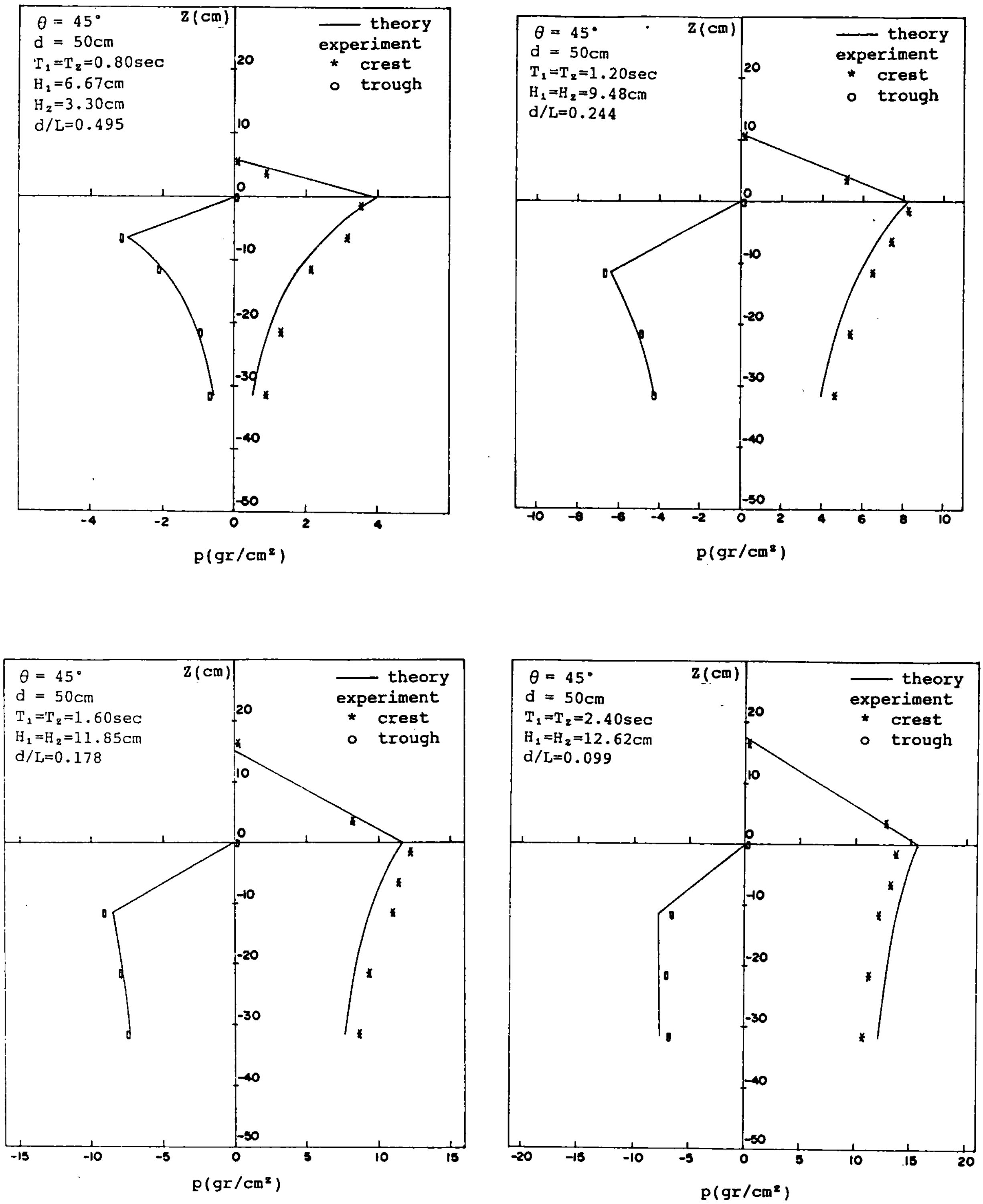

θ = 45°
d = 50cm
T₁=T₂=0.80sec
H₁=6.67cm
H₂=3.30cm
d/L=0.495
Z(cm)
theory
experiment
* crest
o trough
p(gr/cm²)
θ = 45°
d = 50cm
T₁=T₂=1.20sec
H₁=H₂=9.48cm
d/L=0.244
Z(cm)
theory
experiment
* crest
o trough
p(gr/cm²)
θ = 45°
d = 50cm
T₁=T₂=1.60sec
H₁=H₂=11.85cm
d/L=0.178
Z(cm)
theory
experiment
* crest
o trough
p(gr/cm²)
θ = 45°
d = 50cm
T₁=T₂=2.40sec
H₁=H₂=12.62cm
d/L=0.099
Z(cm)
theory
experiment
* crest
o trough
p(gr/cm²)

圖11-28

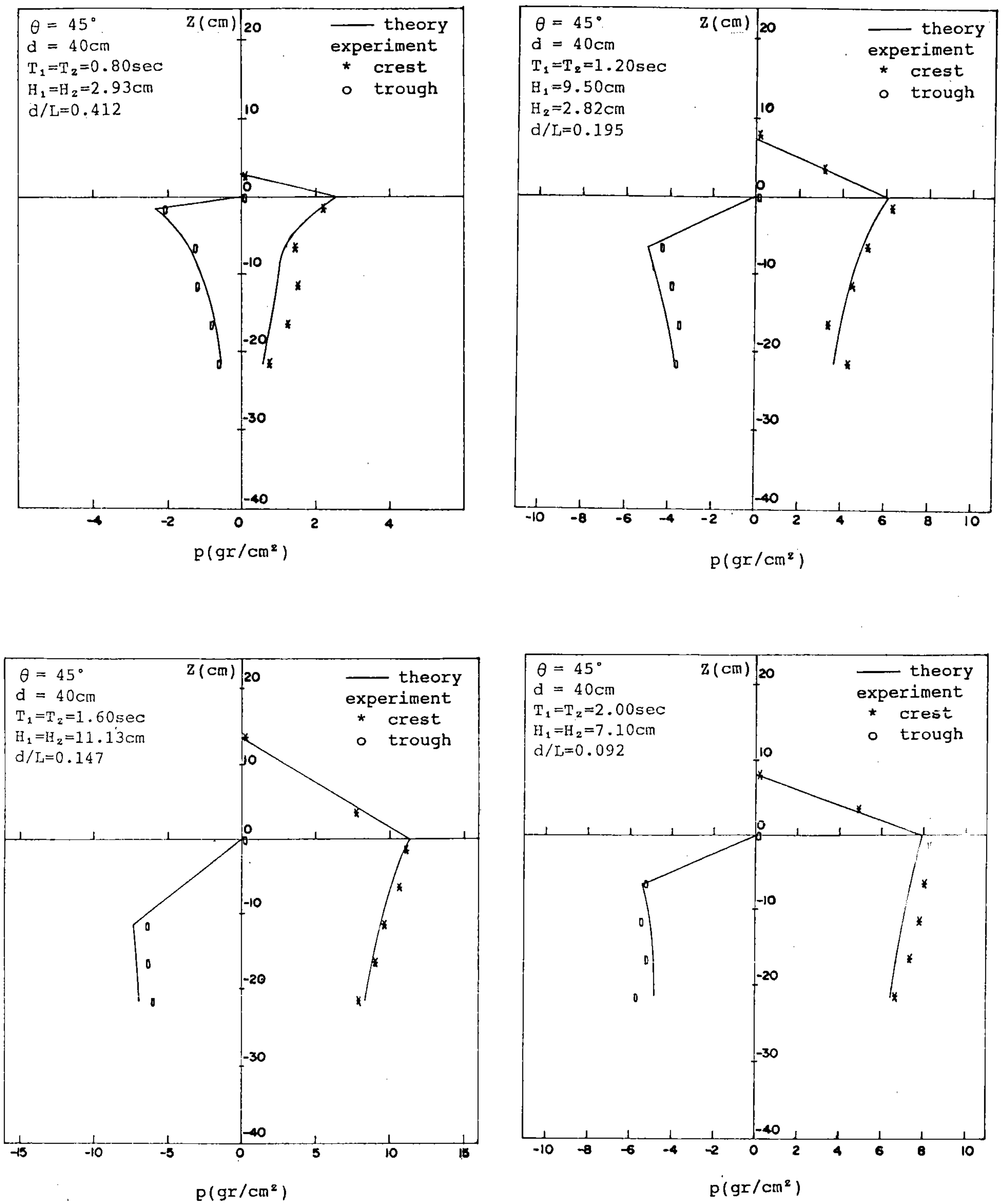

θ = 45°
d = 40cm
T₁=T₂=0.80sec
H₁=H₂=2.93cm
d/L=0.412
Z(cm)
theory
experiment
* crest
o trough
p(gr/cm²)
θ = 45°
d = 40cm
T₁=T₂=1.20sec
H₁=9.50cm
H₂=2.82cm
d/L=0.195
Z(cm)
theory
experiment
* crest
o trough
p(gr/cm²)
θ = 45°
d = 40cm
T₁=T₂=1.60sec
H₁=H₂=11.13cm
d/L=0.147
Z(cm)
theory
experiment
* crest
o trough
p(gr/cm²)
θ = 45°
d = 40cm
T₁=T₂=2.00sec
H₁=H₂=7.10cm
d/L=0.092
Z(cm)
theory
experiment
* crest
o trough
p(gr/cm²)

圖11-29

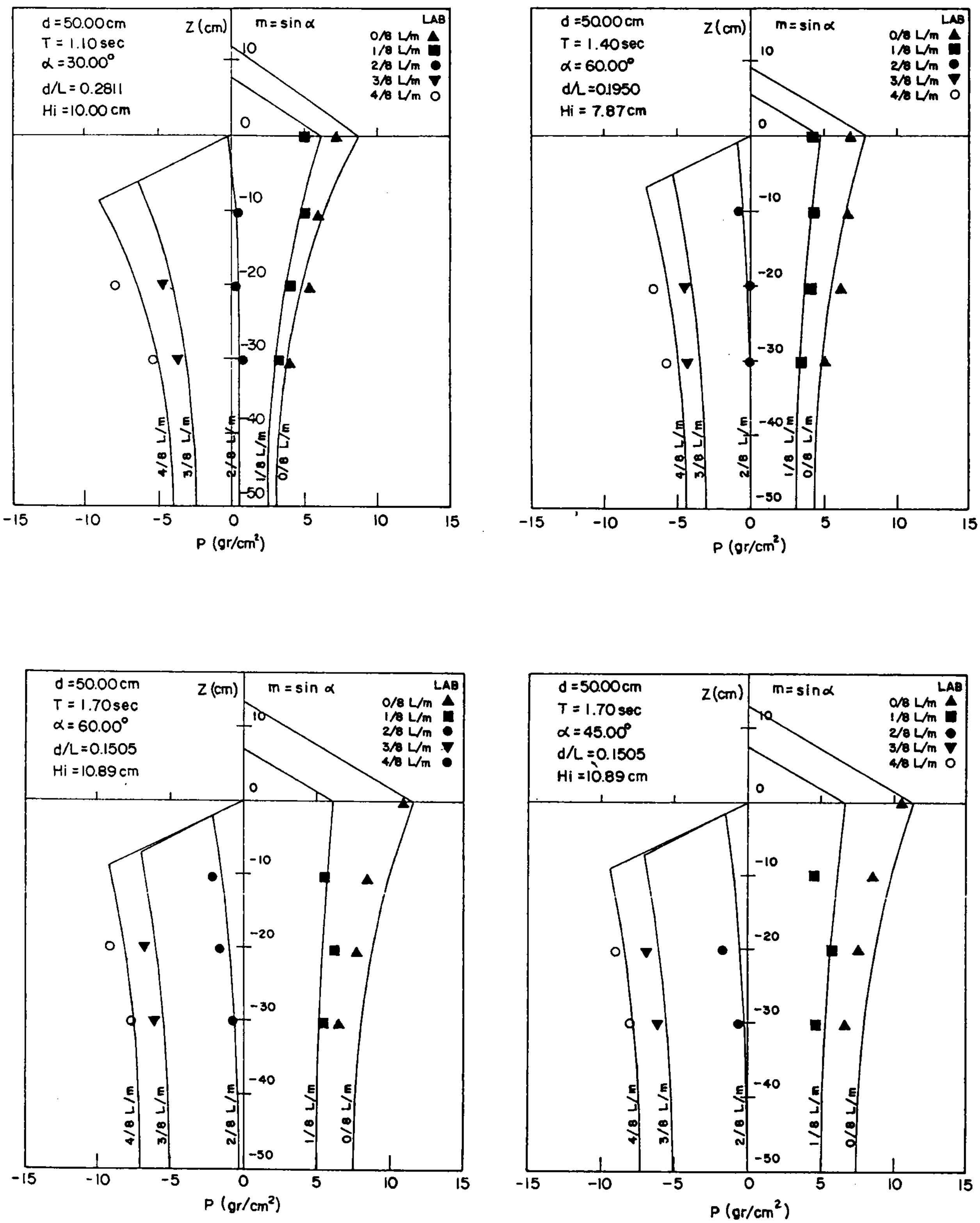
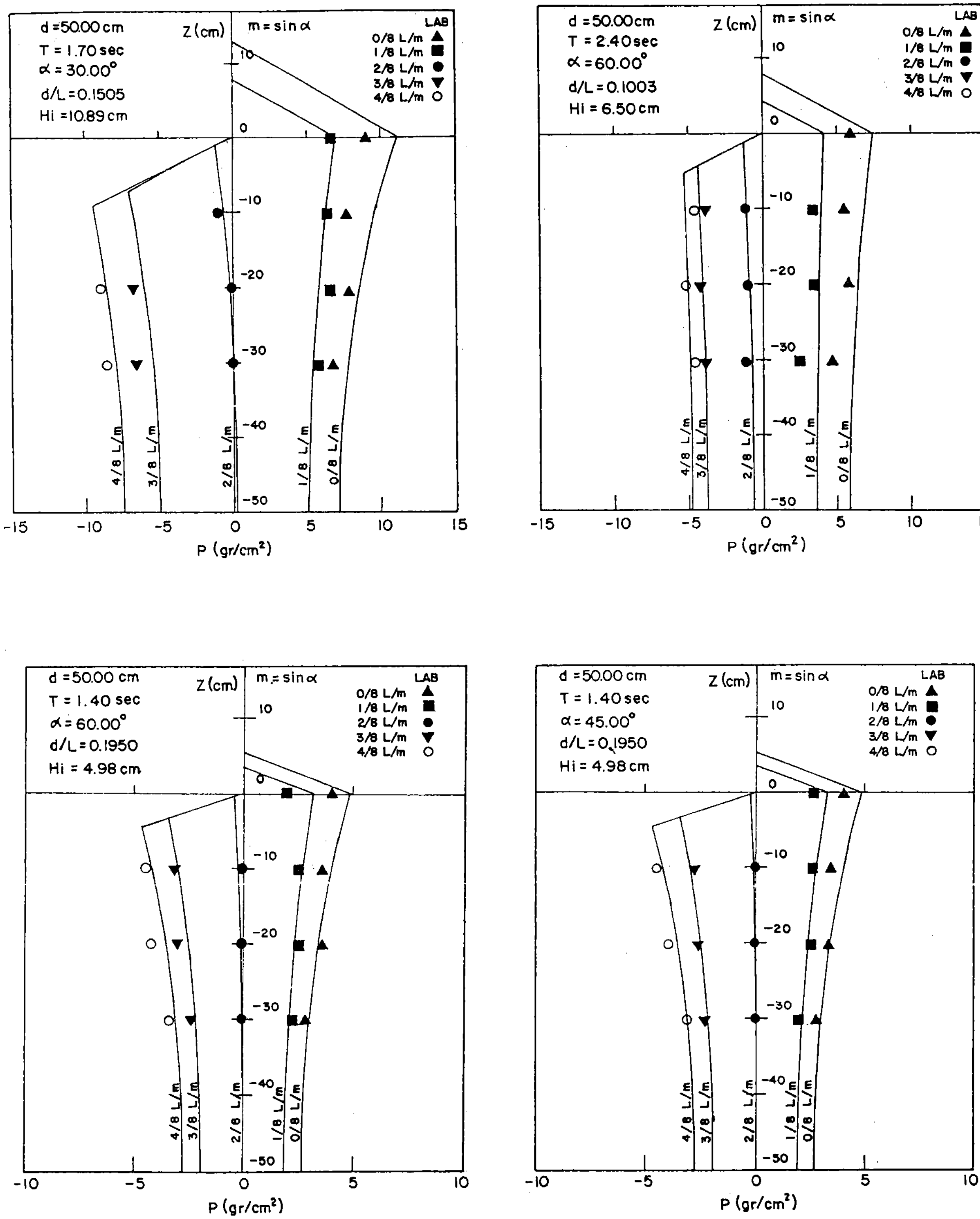

d = 50.00 cm
T = 1.10 sec
α = 30.00°
d/L = 0.2811
Hi = 10.00 cm
Z (cm)
m = sin α
LAB
0/8 L/m ▲
1/8 L/m ■
2/8 L/m ●
3/8 L/m ▼
4/8 L/m ○
P (gr/cm²)
d = 50.00 cm
T = 1.40 sec
α = 60.00°
d/L = 0.1950
Hi = 7.87 cm
Z (cm)
m = sin α
LAB
0/8 L/m ▲
1/8 L/m ■
2/8 L/m ●
3/8 L/m ▼
4/8 L/m ○
P (gr/cm²)
d = 50.00 cm
T = 1.70 sec
α = 60.00°
d/L = 0.1505
Hi = 10.89 cm
Z (cm)
m = sin α
LAB
0/8 L/m ▲
1/8 L/m ■
2/8 L/m ●
3/8 L/m ▼
4/8 L/m ●
P (gr/cm²)
d = 50.00 cm
T = 1.70 sec
α = 45.00°
d/L = 0.1505
Hi = 10.89 cm
Z (cm)
m = sin α
LAB
0/8 L/m ▲
1/8 L/m ■
2/8 L/m ●
3/8 L/m ▼
4/8 L/m ○
P (gr/cm²)

圖11-30



d = 50.00 cm
T = 1.70 sec
α = 30.00°
d/L = 0.1505
Hi = 10.89 cm
d = 50.00 cm
T = 2.40 sec
α = 60.00°
d/L = 0.1003
Hi = 6.50 cm
d = 50.00 cm
T = 1.40 sec
α = 60.00°
d/L = 0.1950
Hi = 4.98 cm
d = 50.00 cm
T = 1.40 sec
α = 45.00°
d/L = 0.1950
Hi = 4.98 cm
Z (cm)
m = sin α
LAB
0/8 L/m
1/8 L/m
2/8 L/m
3/8 L/m
4/8 L/m
P (gr/cm²)

圖11-31

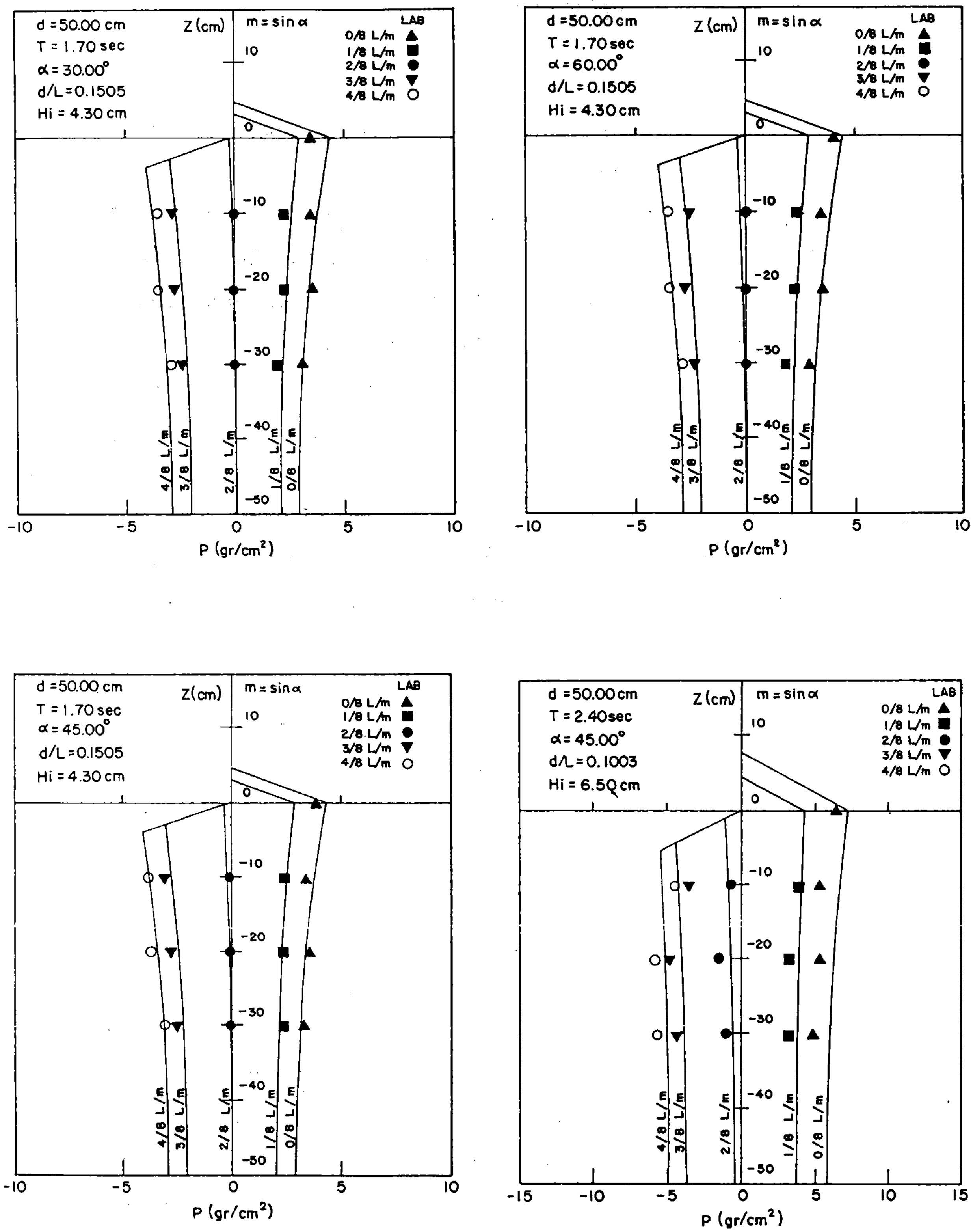

d = 50.00 cm
T = 1.70 sec
α = 30.00°
d/L = 0.1505
Hi = 4.30 cm
Z (cm)
m = sin α
LAB
0/8 L/m ▲
1/8 L/m ■
2/8 L/m ●
3/8 L/m ▼
4/8 L/m ○
P (gr/cm²)
d = 50.00 cm
T = 1.70 sec
α = 60.00°
d/L = 0.1505
Hi = 4.30 cm
d = 50.00 cm
T = 1.70 sec
α = 45.00°
d/L = 0.1505
Hi = 4.30 cm
d = 50.00 cm
T = 2.40 sec
α = 45.00°
d/L = 0.1003
Hi = 6.50 cm

圖11-32

Reference

Benney, D. T. 1962 Non-linear gravity wave interactions, J. Fluid Mech. 14, 577-584.

Bretherton, F.P. 1964 Resonant interaction between waves. The case of discrete oscillations, J. Fluid Mech. 20, 457-479.

Bryant, P.J. 1982 Two-dimensional periodic permanent waves in shallow water, J. Fluid Mech. 115, 525-532.

Chappelear, J. E. 1961 On the description of short-crested waves. Beach Erosion Board, U.S. Army, Corps. Engineerings, Tech. Memo. No.125.

Chen, Y.Y. 1983 Theoretical researches on the universal model and maximum height of water waves (in Chinese), Thesis for Doctor of Eng., National Cheng Kung University, Taiwan, R. O.C.

Chen, Y. Y. 1985 Investigation on the highest gravity waves of permanent type in deep water, Research Paper of Basic Theory, The Institute of Harbour & Marine Technology, Wuchi, Taichung, Taiwan, R.O.C.

Chen, Y.Y. 1988 Investigation on the finite-amplitude standing gravity waves in water of any uniform depth, Research Paper of Basic Theory Ⅲ, The Institute of Harbor & Marine Technology , Wuchi, Taichung, Taiwan, R.O.C.

Chen, Y.Y. and Hsu, J.R.C. 1988 A note on the intersection of two progressive gravity wave trains (in Chinese), Proc. 10th Conf. on Ocean Eng., Taiwan, R.O.C., 79-113.

Chen, Y. Y. and Juang, W. J. 1988 The properties of interaction between two gravity wave trains in deep water (in Chinese), Proc. 12th National Conf. on Theoretical and Applied Mechanics, Taiwan, R.O.C. 19-31.

Chen, Y. Y. 1989 Study on finite amplitude standing waves, Report 78-9 (in Chinese), The Institute of Harbor & Marine Technology, Wuchi, Taichung, Taiwan, R.O.C.

Chen, Y. Y. 1989 Systematic approach to the interactions between two gravity wave trains and its mechanism (in Chinese), J. Harbour Technology, Taiwan, R.O.C., 4,27-60.

Chen, Y. Y. Chiu, Y. F. and Juang, W. J. 1989 Relationship between the progressive and standing gravity waves (in Chinese), Proc. 11th Conf. On Ocean Eng., Taiwan, R.O.C., 273-298.

Chen, Y. Y. 1990 Nonlinear interaction between two gravity wave trains in water of uniform depth (in Chinese), J. Harbor Technology, Taiwan, R.O.C., 5, 1-45.

Chen, Y. Y. and Chiu, Y. F. 1990 Short-crested waves — one case of the interaction between two gravity wave trains (in Chinese), J. Harbour Technology, Taiwan, R.O.C., 125-162.

Chen, Y.Y., Chang, S.G., Ho, L.S.,Hwung, H.H. and Chiu, Y.F. 1990 Experimental studies on wave elevation and pressure of two wave interaction (in Chinese), Proc. 12th Conf. on Ocean Eng., Taiwan, R.O.C. 309-335.

Cokelet, E.D. 1977 Steep gravity waves in water of arbitrary uniform depth, Phil. Trans. R. Soc. Lond A. 286, 183-230.

Fenton, J. D. 1985 A fifth-order Stokes theory for steady waves, J. Waterway, Port, Coastal and Ocean Engineering, ASCE. Vol. 111, No.2, 216-234.

Fenton, J. D. 1985 Wave forces on vertical walls, J. Waterway, Port, Coastal and Ocean Engineering, ASCE, Vol. 111, No.4, 693-718.

Fuchs, R.A. 1952 On the theory of short-crested oscillatory waves, Gravity Waves, U.S. Nat. Bur. Stand. Circular 521, pp.187-200.

Goda, Y. and kakizaki, S. 1966 Study on finite amplitude standing waves and their pressure upon a vertical wall (in Japanese), Rep. Port harbour Res. Inst., Japan, Vol.5, No. 10.

Hasselmann, K. 1962 On the nonlinear energy transfer in a gravity-wave spectrum, Part 1, General theory, J. Fluid Mech. 12, 481-500.

Hasselmann, K. 1963 On the nonlinear energy transfer in a gravity-wave spectrum, Part 2, Conservation theorem; wave particle analogy; irreversibility, J. Fluid mech. 15. 273-281.

Hasselmann, K. 1963 On the nonlinear energy transfer in a gravity-wave spectrum, Part 3, Evaluation of the energy flux and swell sea interaction for a Neumann spectrum, J. Fluid Mech. 15, 385-398.

Hasselmann, K. 1967 Nonlinear interactions treated by the method of theoretical physics (with application to the generation of wave by wind), Proc. R. Soc. Lond. A. 299, 77-100.

Ho, L.S. and Chen, Y.Y. 1990 The wave pressure between two wave interaction (In Chinese), Proc. 12th Conf. on Ocean Eng., Taiwan, R.O.C., 291-308.

Hogen, S. J., Idith Gruman and Stiassnie, M. 1988 On the changes in phase speed of one train of water waves in the presence of another, J. fluid Mech. 192, 97-114.

Hsu, J.R.C. Tsuchiya, Y. and Silvester, R. 1979 Third-order approximation to short-crested waves, J. Fluid Mech. 90, 179-196.

Hsu, J.R.C., Silvester, R. and Tsuchiya, Y. 1980 Boundary layer velocities and mass transport in short-crested waves, J. Fluid Mech. 99, 321-342.

Jeffrays, H. 1924 On water waves near the coast, Phil. Mag. Ser. Vol. 6, No.48.

Lin, S.C. Chen, Y.Y. and Chang, S.G. 1987 Numerical verification on a 90° interior crest angle in limiting gravity waves, Journal of the Chinese Institute of Engineers, Vol. 10, No.5, 539-542.

Lin S.C. 1988 Numerical analysis of nonlinear gravity waves (in Chinese), Thesis for Doctor of Eng., National Cheng Kung University, Taiwan, R.O.C.

Longuet-Higgins, M.S. and Stewart, R.W. 1960 Changes in the form of short gravity waves on long waves and tidal current, J. Fluid Mech. 8, 565-583.

Longuet-Higgins, M.S. 1962 Resonant interactions between two trains of gravity waves, J. Fluid Mech. 12, 321-332.

Longuet-Higgins, M.S. and Phillips, O.M. 1962 Phase velocity effects in tertiary interactions, J. Fluid Mech. 12, 333-336.

Longuet-Higgins, M.S. and Smith, N.D. 1966 An experiment on third-order resonant wave interactions, J. Fluid Mech. 25, 417-456.

Phillips, O. M. 1960 On the dynamics of unsteady gravity waves of finite amplitude, Part 1, The elementary interactions, J. Fluid Mech. 9, 193-217.

Phillips, O.M. 1967 Theoretical and experimental studies of gravity wave interactions, Proc. R. Soc. Lond. A. 299, 104-119.

Phillips, O. M. 1981 Wave interaction—the evolution of an idea, J. Fluid Mech. 106, 215-227.

Rienecker, M. M. and Fenton, J. D. 1981 A Fourier approximation method for steep water waves, J. Fluid Mech.104, 119-137.

Roberts, A. J. and Schwartz, L. W. 1983 The calculation of nonlinear short-crested gravity waves, Phys. Fluids, Vol. 26(9), 2388-2392.

Roberts, A. J. 1983 Highly nonlinear short-crested water waves, J. Fluid Mech. 135, 301-321.

Simmons, W. F. 1969 A Variational method for weak resonant wave interactions, Proc. R. Soc. Lond. A. 309, 551-575.

Stokes, G.G. 1847 On the theory of oscillatory waves, Trans. Camb. Phil. Soc. Vol.8, 441-473.

Tadjbakhsh, I. and Keller, J.B. 1960 Standing surface waves of finite amplitude, J. Fluid Mech. 8, 442-451.

Whitham, G. B. 1974 Linear and nonlinear waves. John Wiley & Sons. Inc.

Whilliams, J. M. 1981 Limiting gravity waves in water of finite depth, Phil. Trans. R. Soc. Lond. A. 302, 139-188.

林銘崇，劉沉榮，莊明暉987 短峰波基本特性之數值計算，中國造船工程學刊第六卷第一期，269-281。

鄭東昇，蔡清標 1988 傅立葉近似法於短峰波之解析，中華民國第十屆海洋工程研討會論文集，115-125。

張憲國 1991 有限水深下三重力波列交會波場之非線性交互作用，國立成功大學水利及海洋工程研究所，博士論文初稿。