國 立 交 通 大 學
運 輸 與 物 流 管 理 學 系

博 士 論 文

No.001

路線別模糊資料包絡分析模式評估運輸服務效率

Route-based Fuzzy Data Envelopment Analysis Models for Evaluating Transport Service Efficiency

研 究 生 ： 閻 姿 慧
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藍武王 博士

中 華 民 國 一百零二 年 八 月

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國 立 交 通 大 學

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博 士 論 文

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中文摘要

本論文研提新型資料包絡分析(DEA)模式評估運輸服務效率，主要針對路線別績效分析及納入模糊變數時進行 DEA 模式建構。首先，針對路線別績效分析提出路線別資料包絡分析(RDEA)模式。RDEA 模式包含兩類模式，分別為 RCCR 與 RBCC 模式。此兩類路線別模式均可同時分析公司與路線別之績效值。其中，RDEA 模式是利用三階段求解法求算公司績效值、路線績效值以及最佳共同成本配置比例。本研究驗證三階段求解之績效排序一致性，而後利用台灣城際公路客運公司之績效評估驗證 RDEA 模式之適用性。

其次，針對模糊變數提出整合式模糊資料包絡分析(IFDEA)模式。IFDEA 模式包含兩類型之模式，分別為 IFCCR 與 IFBCC 模式，分別用於衡量固定規模報酬及變動規模報酬下之效率值。IFDEA 模式利用績效值上下限整合概念，可針對模糊變數之上下限進行不同的差額變數分析。本論文亦利用相同之公路客運案例驗證 IFDEA 模式之適用性。

最後，經整合 RDEA 模式與 IFDEA 模式進而提出兩類型之整合式路線別模糊資料包絡分析(IRFDEA)模式，分別為 IRFCCR 與 IRFBCC 模式。IRFDEA 模式可同時考量路線別與模糊變數，此模式利用 RDEA 模式之概念提出，因此 IRFDEA 模式亦為三階段之整合模式，如同 RDEA 模式可同時求解公司績效值、路線績效值及最佳共同成本配置比例。同時，IRFDEA 模式亦保有公司別與路線別績效一致性之特性，最後亦利用相同之公路客運案例驗證 IRFDEA 之適用性。

關鍵詞：整合式資料包絡分析、共同成本配置、整合式模糊資料包絡分析、整合式路線別模糊資料包絡分析

Route-based Fuzzy Data Envelopment Analysis Models for Evaluating Transport Service Efficiency

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Abstract

This study proposes three different types of data envelopment analysis (DEA) modeling to remedy two research gaps: lacking of route performance evaluation and including vagueness of some variables measurement. First, route-based data envelopment analysis (RDEA) modeling is proposed. This study develops two novel RDEA models, termed RCCR and RBCC, that jointly measure the route-level and company-level efficiencies amongst transport carriers. The core logics comprise a three-stage procedure that determines company efficiency, route efficiency and optimal allocation ratios for the common inputs. We prove that the ranking order of company performance determined by the route-based DEA model is identical to that determined by the company-based DEA model. An empirical study of intercity bus transport companies in Taiwan demonstrates the superiority of the proposed models in identifying the less efficient routs/companies as well as in reducing the input slacks without subjective conjectures.

Second, integrated fuzzy data envelopment analysis (IFDEA) modeling is proposed. This study develops two IFDEA models, termed IFCCR and IFBCC, by combining both lower- and upper-bound efficiency frontiers into a single one under a specific α-cut. The proposed IFDEA models can simultaneously determine the slack values for both lower- and upper-bound input/output variables. A numerical example shows that the proposed IFDEA models are more generalized and have greater simplicity than an existent FDEA model. An empirical study of the same case further demonstrates the superiority of the proposed IFDEA models, which have successfully dealt with both quantitative (crisp) and qualitative (fuzzy) variables.

Third, integrated route-based fuzzy data envelopment analysis (IRFDEA) modeling is proposed. This study develops two IRFDEA models, termed IRFCCR and IRFBCC, which jointly measure the route-level and company-level efficiencies with both crisp and fuzzy variables. The proposed models also comprise three stages. The first stage uses an integrated company-based IFDEA model to acquire a set of optimal multipliers. The second stage uses the solved multipliers to determine its optimal allocation ratios for the common inputs among the routes within a company to maximize the efficiency of all routes. The third stage further determines the relative efficiency for all routes across the companies. An empirical study of the same case demonstrates the superiority of the proposed models in pining down the less efficient routes/companies and in suggesting how much the inputs of less efficient routes/companies should be improved.

Keywords: Route-based data envelopment analysis, common inputs allocation, integrated fuzzy data envelopment analysis, integrated route-based fuzzy data envelopment analysis.

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Barbara T.H. Yen
10 August 2013

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CHAPTER 1. INTRODUCTION

1.1 Background

Data Envelopment Analysis (DEA) is a well-established technique that can provide a comprehensive insight into how comparatively well an organization has performed in comparison to its peers within or across the industries. It can be used to rank quality level and analyze the performance with multiple inputs and outputs simultaneously. DEA imposes neither a specific functional relationship between outputs and inputs, nor any assumptions on the specific statistical distribution of the error terms. DEA can be defined as a nonparametric method of measuring the efficiency of a Decision Making Unit (DMU).

Over the past three decades, various DEA models have been developed and used to evaluate the efficiency of DMUs in transport sectors. Most of previous studies evaluated the efficiency at the company level by viewing each transport company as a DMU. However, the scheduled carriers such as airline, maritime, transit, and railway are usually operated under a fixed-route basis. Each route faces different competition and operating environments, which should be evaluated individually. The carriers' overall performances should be reflected by the efficiency values delivered by all routes. In practice, an efficient carrier may operate some inefficient routes; likewise, an inefficient carrier may run some efficient routes. Moreover, the improvement strategies based on a company-based DEA model would provide little information for devising useful strategies to improve the inefficient routes. In other words, using a company-based DEA model can only identify the inefficient companies among the industry; it cannot reveal the problematic routes within a company because the company-based DEA approach views companies, rather routes, as the analysis units. For instance, if overstaffing is a problem to an inefficient carrier, the company-based DEA slack analysis would provide a clue of how much the labor force in the company should be reduced; but the managers may have difficulties to determine which routes' staff and by how much should be curtailed. An inappropriate reduction in the inputs of any individual routes may result in even worse overall performance. Furthermore, most of the government regulations or periodical evaluations on the franchised transport are also route-based, for example, new routes franchised, old routes renewed, deficit routes subsidized, and periodical service quality evaluation. In this circumstance, developing an appropriate route-based DEA model is absolutely necessary for the transport industry. In addition, to assess the system performance for any scheduled transport carriers, it would become more informative if one could jointly measure the company-based and route-based performances.

While assessing the transport performance, the qualitative measures can be as important as the quantitative ones. For example, one may wish to evaluate the transport performance by incorporating some qualitative variables (e.g., operator's attitude, vehicle's quality, and passenger's satisfaction) into the DEA modeling, in addition to the quantitative variables (e.g., labor, vehicle, fuel consumption, service frequency, vehicle-kilometers, ton-kilometers, and passenger-kilometers). However, conventional DEA models are often formulated with quantitative variables measured in a "crisp" manner. The qualitative variables are in nature characterized with "vagueness" due mainly to the subjective judgment from customers, thus developing appropriate fuzzy DEA models that take into account both quantitative (crisp) and qualitative (vague) variables deserve further exploration.

1.2 Motivations

A considerable number of studies have employed the DEA models to evaluate the relative performance of transport carriers in different contexts, including airline (*e.g.* Schefczyk, 1993;

Charnes *et al*., 1996; Sengupta, 1999; Alder and Golany, 2001; Chiou and Chen, 2006), airport (*e.g.* Peck *et al*., 1998; Salazar de La Cruz, 1999; Tzeng and Chiang, 2000; Sarkis, 2000; Martin and Roman, 2001; Adler and Berechman, 2001; Barros and Dieke, 2007), maritime (e.g. Tongzon, 2001; Cullinane, *et al.*, 2006), transit (*e.g.* Fielding *et al*., 1984; Fielding, 1987; Nolan, 1996; Kerstens, 1996; Viton, 1998; Odeck and Alkadi, 2001; Nolan *et al.,* 2002; Karlaftis, 2003, 2004; Sheth *et al.*, 2007; Margari *et al*., 2007; Chiou *et al*., 2010, 2013), and railway (*e.g.* Oum and Yu, 1994; Coelli and Perelman, 1999; Lan and Lin, 2003, 2005). Most of the above literature measured the DMUs' efficiency and/or effectiveness on a company basis in terms of crisp input/output variables. A few have looked into the relative performance among the carriers, along with the detailed performance of each carrier's subordinated routes simultaneously. Even fewer have taken qualitative (vague) variables into account. As explained above, simultaneously measuring the route-based and company-based performance can be crucial to a transportation carrier. Therefore, this study, firstly, aims to develop a route-based DEA (termed as RDEA) modeling approach that can clearly indentify the inefficient routes and propose more rational countermeasures accordingly.

In addition, assessment for transport service requires considering not only quantitative measures but also qualitative measures. As explained, the qualitative measures have been ignored in the most crisp DEA (CDEA) models (*e.g.* Schefczyk, 1993; Charnes *et al*., 1996; Sengupta, 1999; Adler and Golany, 2001; Chiou and Chen, 2006). In the past decade, several fuzzy DEA (FDEA) models have been proposed and most of which adopted two CDEA modeling approaches by separately determining the evaluation results of lower- and upper-bound under a specific α-cut level (*e.g.*, Despotis and Smirlis, 2002; Kao and Liu, 2000; Guh *et al*., 2001; Smirlis *et al*., 2006; Karsak, 2008; Azadeh *et al.*, 2008; Azadeh and Alem, 2010). Repeating the modeling with different α-cuts, the final evaluation results (efficiency scores) can be determined by a reformulated fuzzy number. The major drawbacks of such FDEA modeling, however, are inconsistent efficiency rankings and unreasonable efficiency scores due to the distorted fuzzy number. The numerical data provided by León *et al.* (2003), which is used to generate the lower- and upper-bound frontiers by using two separated CDEA models proposed by Kao and Liu (2000), can depict such problems. Figure 1.1 displays the results, which clearly show that the lower-bound efficiency scores are greater than the upper-bound efficiency scores for DMUs D, E, F and G under α =0, which is obviously unreasonable.

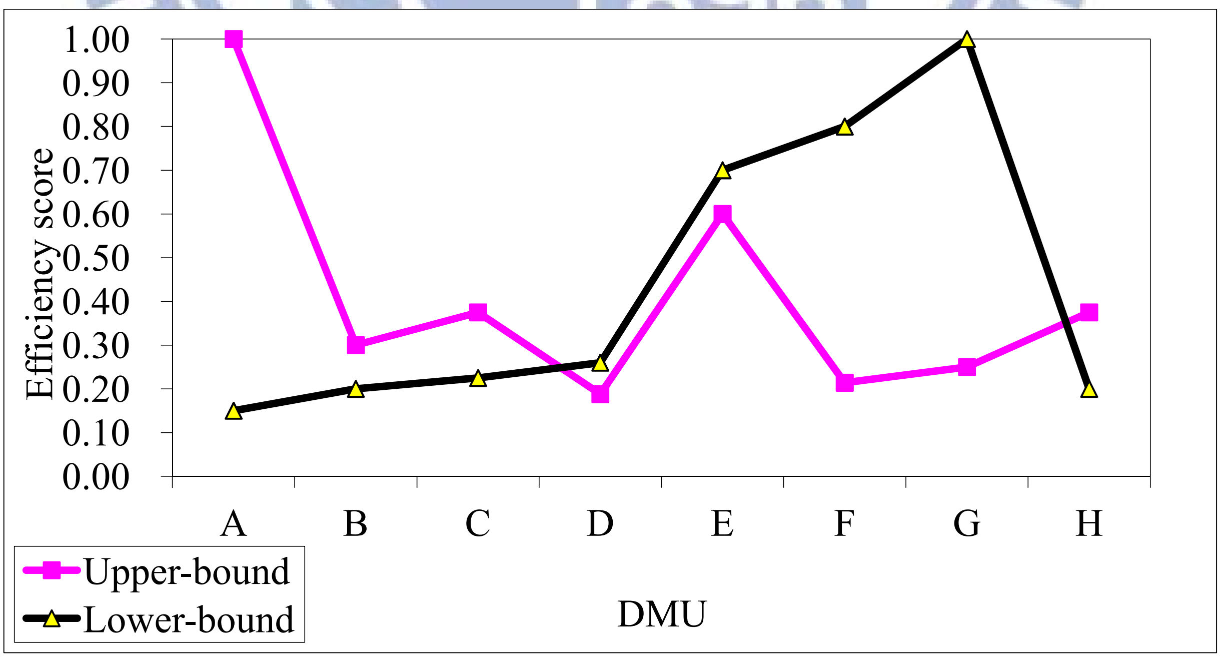

Figure 1. 1 Efficiency frontiers determined by separate CDEA models under α=0.

Furthermore, the computation process of fuzzy efficiency score, which is repeatedly determined from the interval values (lower- and upper-bound) under various α-cuts, is too cumbersome—making the scale and slack analyses too difficult to compute. As such, this study, secondly, aims to develop an integrated fuzzy DEA (IFDEA) modeling approach that can simultaneously optimize both lower-bound and upper-bound under a specific α-cut and further to derive a crisp efficient frontier without the needs of additional fuzzy ranking.

To rectify the aforementioned problems altogether, this study, thirdly, aims to develop an integrated route-based fuzzy DEA (IRFDEA) approach which decomposes company efficiency into route efficiency by simultaneously optimizing the allocation of common inputs with both crisp and fuzzy variables. The proposed IRFDEA approach contains three stages. The first stage uses an integrated company-based fuzzy DEA (ICFDEA) model to acquire a set of optimal (objective) input/output multipliers. The second stage uses the corresponding objective multipliers to determine its optimal allocation ratios of common inputs among routes so as to maximize the average efficiency of all routes. Once the optimal allocation ratios are determined, an integrated route-based fuzzy DEA (IRFDEA) model treating each route as a DMU is developed in the third stage to determine the efficiency scores of all routes across all companies. In other words, the proposed three-stage IRFDEA approach can jointly determine the efficiency values on a company level as well as a route level, along with the optimal allocation ratios of common inputs among different routes.

1.3 Research Purposes

Based on the abovementioned background and motivations, the main purposes of this study are listed as follows:

1. To review and summarize the related studies in evaluating the performance of transportation industry by applying DEA model and FDEA model.
2. To respectively propose two DEA models to remedy two gaps of decomposition and vagueness.
3. To propose an integrated DEA model by integrating two models.
4. To develop solution algorithms for these models.
5. To implement the proposed models on several empirical cases.

According to abovementioned research purposes, the research flow chat of this study can be depicted in Figure 1.2.

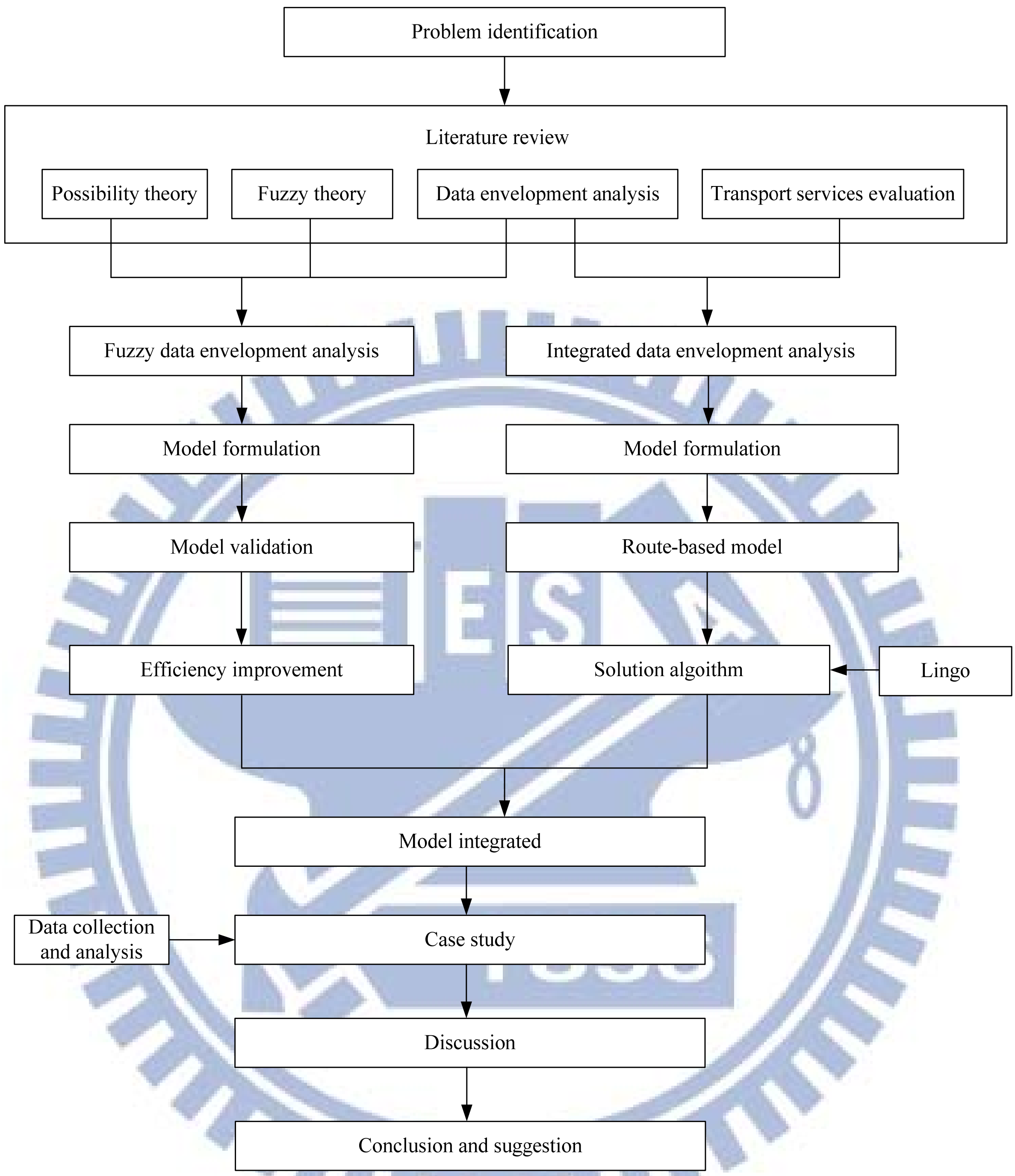

Figure 1. 2 Research flowchart

1.4 Organization of Report

This study is organized as follows. Chapter 2 briefly reviews of the applications in DEA models. Chapter 3 outlines the different DEA formulations including conventional CDEA models with the introduction of two basic CCR and BCC models in various forms and FDEA models. Chapter 4 presents the RDEA modeling and its applications. Chapter 5 demonstrates the IFDEA modeling and its applications. Chapter 6 reports the IRFDEA modeling and its applications. Finally, the conclusions and recommendations for future research are presented in chapter 7.

CHAPTER 2. LITERATURE REVIEW

This study reviews several related works, including DEA application in transport, DEA modeling and fuzzy DEA modeling. The details of these works are elaborated as follows.

2.1 Application of DEA models in transportation

Over the past decades, various DEA models have been widely used to evaluate the efficiency of DMUs in different transport fields, including airline, airport, maritime, transit, and railway. The corresponding concept of relative studies is introducing in following section.

2.1.1 Air transportation

2.1.1.1 Airline

Schefczyk (1993) evaluated the efficiencies of airlines with DEA model and the reference data set contains 1990 information on 15 international airlines. This study used constant returns to scale technical to evaluate the performance of airline because they thaught airlines had the opportunity to influence their own scale over a time frame of a few years. Sengupta (1999) utilized the time-serious data set for international airlines used by Schefczyk (1993). Sengupta (1999) further considered the unit price for inputs and outputs in the DEA model.

Charnes *et al*. (1996) chose the Latin American airline industry for application. Given the differences in competition, regulation, growth, route length and passenger characteristics between domestic and international operations, two separate models were obtained, one for domestic and one for international operations.

Chiou and Chen (2006) employed DEA approach to evaluate the performance of domestic air routes from the perspectives of cost efficiency, cost effectiveness and service effectiveness. The cost efficiency indicated the relative efficiency in the production; while the service effectiveness stood for the relative efficiency in the sale. The cost effectiveness therefore represented a combined effect of the relative efficiency in both production and sale. This framework (Figure 2.1) was used to evaluate air route performance.

There were three input variables: fuel cost (FC), personnel cost (PC), including the salaries of cabin and ground-handling crews, and aircraft cost (AC), including maintenance costs, depreciation costs and interest payments. The production variables include number of flights (FL) and seat-mile (SM). The service variables include passenger-mile (PM) and embarkation passengers (EP). This study also used Tobit regression to identify variables significant or not.

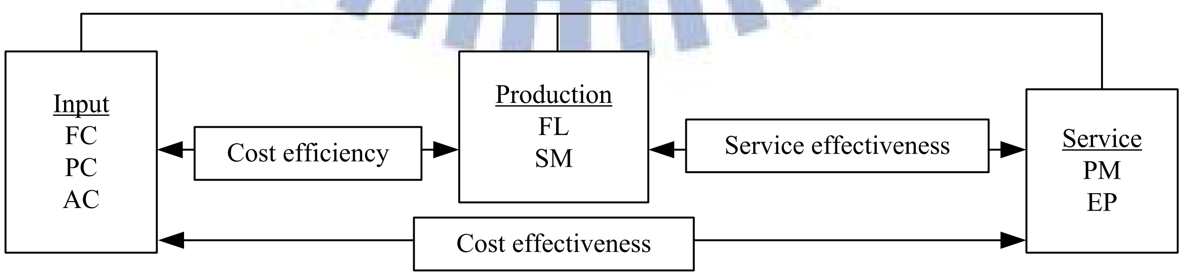

Figure 2. 1 The analysis framework

2.1.1.1 Airport

Peck *et al.* (1998) focused on discretionary maintenance strategies and their relationship to aircraft reliability, as measured by the percentage of scheduled flights delayed because of mechanical problems. They introduced DEA model to identify the various strategies employed by the major airlines over the time period 1990-1994. The output variable was defined to be the percentage of all scheduled flights arrivals delayed for mechanical reasons not including weather or scheduling problems. The input variables represented all of the reported non-overlapping categories of maintenance expenses.

Tzeng and Chiang (2000) proposed an efficiency measure in DEA: the efficiency achievement measure. Comparing with the traditional radial measure and distance measure proposed by Chang and Guh (1995) using different sets of multipliers to compute the efficiency ratio, the efficiency achievement measure did so by using the common multipliers that obtained easily by solving fuzzy multiple objectives programming.

Sarkis (2000) evaluated the operational efficiencies of 44 major U.S. airports by using DEA. Various airport characteristics were evaluated to determine their relationship to an airport's efficiency. Efficiency measures were based on four resource input measures including airport operational costs, number of airport employees, gates and runways, and five output measures including operational revenue, passenger flow, commercial and general aviation movement, and total cargo transportation.

Adler and Berechman (2001) used DEA to determine the relative efficiency or quality ranking of various West-European and other airports. The main source of data for this study was a questionnaire whose objective was to evaluate the quality level of 26 airports.

Barros and Dieke (2007) addressed empirically financial and operational performance of Italian airports by using DEA. With panel data for 2001–2003, this study tested variable relationships—the relative roles of dimension, managerial status and workload unit—to measure the proximity of the airports to the frontiers of best practices.

2.1.2 Maritime transportation

Tongzon (2001) applied DEA to provide an efficiency measurement for four Australian and twelve other international container ports. This study used two output and six input measures of port performance. The output measures were cargo throughput and ship working rate. Based on the production framework, port inputs can be generalized as land, labor and capital. The major capital inputs in port operations were the number of berths, cranes and tugs. This study had shown the suitability of DEA for port efficiency evaluation.

Cullinane *et al*. (2006) applied two leading approaches to efficiency measurement, DEA and SFA, to the same data set for the container port industry. This study ranked the technical efficiency and the ranges was from 0.63 to 1.00, indicating that these approaches yield similar efficiency rankings.

2.1.3 Transit

Fielding *et al*. (1984) used three categories of statistics-service inputs, service outputs and service consumption-provided the framework to organize the much larger set of data. Cost-efficiency indicators measured service inputs (labor, capital, fuel) to the amount of service produced (service outputs: vehicle hours, vehicle miles, capacity miles, service reliability). Cost-effectiveness indicators measured the level of service consumption (passengers, passenger miles, operating revenue) against service inputs. Finally, service-effectiveness indicators measured the extent to which service outputs were consumed.

Viton (1998) examined the claim that US bus transit productivity had declined in recent years. These systems operated either conventional motor-bus (MB) or demand-responsive (DR) services (or both), but no other form of public transit. This study used a piecewise-linear best-practice DEA production frontier, computed for multi-modal bus transit between 1988 and 1992. The output variables were vehicle-miles, vehicle hours and passenger trips. The input variables come from three sources. First was a set of variables describing the situation in which the system finds itself. These included the average fleet age and the number of directional miles provided by the MB. Second, they used a number of conventional inputs: the fleet sizes, and the number of gallons of fuel. It distinguished four kinds of labor inputs: the number of person-hours of transportation, maintenance, administrative, capital and labor used by each mode in providing service. The final inputs were those for which there was no obvious summary physical measure. For these they used a cost measure. In this category they had the cost of tires and other materials and supplies, of services, of utilities, and of insurance. The results did not support the pessimistic view of changes in the industry because both the efficiency and productivity approaches suggested an improvement.

Cowie and Asenova (1999) claimed that the ideal output measure was passenger kilometers, unfortunately due to commercial sensitivity such figures were unavailable. Nevertheless, clearly related to passenger kilometers was operating revenue. The inputs for each company reflect capital and labor elements. Labor was simply the total staff employed, both management and operational. This study showed strong evidence of increasing returns for smaller companies. This study used technical, managerial and organizational efficiency. The technical efficiency of each company was assessed by a comparison of all companies in the data set. The level of managerial efficiency, however, can be further isolated from overall technical efficiency by separating DMUs into the different sets of interest. The difference between technical and managerial efficiency represented the level of inefficiency attributed to the organizational structure.

Odeck and Alkadi (2001) focused on the performance of Norwegian bus companies subsidized by the government. The performance was evaluated from a productive efficiency with DEA approach. In this study, the output variables were seat kilometers, vehicle kilometers, passenger kilometers, and passengers and the input variables were the total number of seats offered by the company, fuel consumption in liters and equipment such as oil and tires. The average bus company was found to be exhibiting increasing return to scale. This means that the average company was smaller than the optimal size.

Karlaftis (2003) uncovered production characteristics of transit firms by relating efficiency with production in a less constraining environment. In this study used DEA to rank efficient subsets of transit systems and then based on the results of the DEA analysis, built globally efficient frontier production functions. The results indicated that when jointly considered, there was an improvement on both the theoretical and empirical aspects of examining efficiency and production in transit systems.

Karlaftis (2004) used DEA approach and globally efficient frontier production functions to investigate two important issues in transit operations: first, the relationship between the two basic dimensions of performance, namely efficiency and effectiveness; second, the relationship between performance and scale economies. This study found that systems performing well in one dimension (e.g. efficiency) generally perform well in the other dimensions (e.g. effectiveness). This was important since the performance scores can be useful in describing transit system performance both for internal and external purposes. This study used two outputs: vehicle-miles (often referred to as ‘‘produced output type’’) and passenger-miles (often referred to as ‘‘consumed output type’’). Transit systems most frequently used three input quantities, namely labor, fuel, and capital to produce output.

Chiou *et al*. (2010) proposed two novel integrated data envelopment analysis (IDEA) approaches to jointly analyze the overall performance of intercity bus company. The proposed models were simultaneously determining the virtual multipliers associated with inputs, outputs, and consumption by additive specifications for technical efficiency and service effectiveness terms with equal weights.

2.1.4 Railway

Coelli and Perelman (1999) discussed and compared a number of the different methods that had been used to estimate multi-output distance functions. This study focused upon the three most commonly used estimation methods:

(1) A parametric frontier using linear programming methods;

(2) A non-parametric piece-wise linear frontier using the linear programming method known as DEA; and

(3) A parametric frontier using corrected ordinary least squares (COLS).

These three different estimation methods provided similar information on the relative productive performance. The correlations between the various sets of technical efficiency predictions were all positive and significant. Furthermore, the parameter estimates obtained using two parametric estimates were also quite similar in many respects. Given these observations, it appeared that a researcher can safely select one of these methods without too much concern for their choice having a large influence upon results.

Conventional DEA approaches neither considered the environmental differences across the DMUs nor accounted for the statistical error (data noise) and slack effects. Thus, the comparison can be seriously biased because all DMUs were not brought into a common platform. Lan and Lin (2005) proposed a four-stage DEA approach with further adjustment of slack effects. The empirical results showed that proposed four-stage DEA approach had slightly more reasonable efficiency and effectiveness scores than those measured by Fried's three-stage DEA approach. This study measured the technical efficiency by selecting number of passenger cars per kilometer of lines, number of freight cars per kilometer of lines, and number of employees per kilometer of lines as input factors and passenger train-kilometer per kilometer of lines and freight-train-kilometer per kilometer of lines as output variables. In measuring the service effectiveness, they chose passenger-kilometers and ton-kilometers as two consumptions and passenger train kilometers and freight train-kilometers as two outputs.

Lan and Lin (2003) investigated the technical efficiency and service effectiveness for 76 railways by employing two-stage DEA approach. At the technical efficiency analysis stage, they used input orientation DEA by selecting length of lines, number of locomotives and cars, and number of employees as inputs and train-kilometer as output. At the service effectiveness analysis stage, this study used output orientation DEA by selecting train-kilometer as input and passenger-kilometer and ton-kilometer as outputs. In addition, they performed a technical effectiveness analysis with one-stage DEA by choosing the same input factors and outputs.

2.2 Different DEA modeling approaches

El-Mahgary and Lahdlma (1995) examined various two-dimensional charts for illustrating the DEA efficiency results. The identification of reference units provided a general framework that can be used to define guideline for the inefficient units. Visualizing such results should help decision-maker to better understand the result of a DEA assessment.

Cooper *et al.* (2001) examined two approaches that were presently available in the DEA literature for using in identifying and analyzing congestion. These two approaches were due to Färe *et al*. (1985) and Cooper *et al*. (1996). This study showed that FGL (Färe-Grosskopf-Lovell) model might fail to give correct result.

Cherchye *et al.* (2001) responded the problem that FGL procedure failed to identify congestion in Cooper *et al*. examples. Because FGL model was originally proposed for measuring structural efficiency rather than detecting congestion.

Yun *et al.* (2004) suggested a model called generalized DEA (GDEA) model, which can treat the basic DEA models (CCR model, BCC model and FDH model) in a unified way. GDEA model can make a quantitative analysis for inefficiency on the basis of surplus of inputs and slack of outputs.

Appa and Williams (2006) provided an alternative framework for solving DEA models which in comparison with the standard linear programming (LP) based approach that solved one LP for each DMU. The method of projection was Fourier–Motzkin (F–M) elimination. It is shown that the output from the F–M method improves on existing methods of (i) establishing the returns to scale status of each DMU, (ii) calculating cross-efficiencies and (iii) dealing with weight flexibility.

In the past, several FDEA models have been proposed by different researchers. Most of the FDEA models, however, generally adopted two DEA modeling approaches, which separately determine the evaluation results of lower- and upper-bound under a specific α-cut level. Repeat the modeling approach under different α-cut levels, the final evaluation result (efficiency score) was determined by a reformulated (distorted) fuzzy number. The details of these studies were shown below.

Cooper *et al*. (1999) developed the imprecise DEA model which permits mixtures of imprecisely- and exactly-known data and the imprecise DEA models can transform into ordinary linear programming forms.

Kao and Liu (2000) proposed a procedure to measure the efficiencies of DMUs with fuzzy observations. The basic idea was to transform a fuzzy DEA model to a family of conventional crisp DEA models by applying the α-cut approach. A pair of parametric programs was formulated to describe that family of crisp DEA models, via which the membership functions of the efficiency measures were derived.

Guh *et al*. (2001) proposed an approach which was based on α-cuts to obtain the different intervals and combinatorial interval analysis to avoid some of the difficulties such as the multiple occurrence of variables.

Guo and Tanaka (2001) proposed a fuzzy DEA model to deal with the efficiency evaluation problem with the given fuzzy input and output data. Furthermore, an extension of the fuzzy DEA model to a more general form was also proposed with considering the relationship between DEA and Regression Analysis. Using the proposed fuzzy DEA models, the crisp efficiency in CCR model was extended to be a fuzzy number to reflect the inherent uncertainty in real evaluation problems. They also introduced the possibility approach to define the relationship between two fuzzy numbers.

Despotis and Smirlis (2002) developed an approach for dealing with imprecise data in DEA. This approach was to transform a non-linear DEA model to a linear programming equivalent by applying transformations only on the variables. Upper and lower bounds for the efficiency scores of the units are then defined as natural outcomes of our formulations. Compare to the model proposed by Cooper *et al*. (1999), the efficiency determined by their model would have its lower bound and upper bound.

Lertworasirikul *et al*. (2003) developed DEA models using imprecise data represented by fuzzy sets. It was shown that fuzzy DEA models took the form of fuzzy linear programming which typically was solved with the aid of some methods to rank fuzzy sets. As an alternative, a possibility approach was introduced in which constraints were treated as fuzzy events.

León *et al*. (2003) developed some fuzzy versions of the classical DEA models by using some ranking methods based on the comparison of α-cuts. Their models were proposed base on the concept of possibility approach as well.

Jahanshahloo *et al*. (2004) developed a fuzzy DEA model in which a fuzzy comparison of fuzzy numbers was defined and a slack-based measure in DEA was extended to be a fuzzy DEA model. This model used possibilitic mean value and possibilitic variance to construct the membership function for fuzzy number.

Smirlis *et al*. (2006) introduced an approach based on interval DEA that allowed the evaluation of the units with missing values along with the other units with available crisp data. The missing values were replaced by intervals in which the unknown values were likely to belong. For the units with missing values, the proposed models were able to identify an upper- and a lower-bound of their efficiency scores. The efficiency analysis was further extended by estimating new values for the initial interval bounds that may turn the unit to an efficient one.

Jahanshahloo *et al*. (2009) suggested a model with interval data called interval generalized DEA model, which can treat the stated basic DEA models with interval data in a unified way. The input and output values of any DMU were set to be located in a certain interval and utilized its lower and upper bound into the model to determine the efficiency.

Mostafaee and Saljooghi (2010) developed a method for the estimation of upper- and lower-bounds for the cost efficiency measure in situations of uncertain input and output data and developed the theory of efficiency measurement so as to accommodate incomplete price information by deriving upper and lower bounds for the cost efficiency measure.

Azadeh and Alem (2010) utilized three types of vendor selection models in supply chains and presented a decision making scheme for choosing appropriate method for supplier selection under certainty, uncertainty and probabilistic conditions. These models were, DEA, FDEA, and Chance Constraint DEA. The basic idea of FDEA model was to transform the fuzzy CCR model into a crisp linear programming problem by applying an alternative α-cut approach. Thereby, the problem was converted to an interval programming.

2.3 Summary

Table 2.1 and Table 2.2 summarize the literature review from which one can notice several points. First, most studies only used company as DMUs to evaluate the performance of transport industries. That means these studies did not consider route performance of transport industries. However, route service is the frontline of a transport company, it could determine service quality and operating efficiency of the whole company. Second, most FDEA models used two separate crisp DEA models to respectively determine the lower- and upper-bound of fuzzy efficiency scores under various α-levels and it may lead to inconsistent and unreasonable results. In order to remedy these research gaps, this study proposes a route-based DEA modeling and integrated fuzzy modeling. The formulations of the proposed modeling are elaborated in the following chapter.

Furthermore, quite a number of variables have been used to evaluate performance for transportation industry. In transit related research, the following variables are chosen the most.

- Input variable: operating cost, number of vehicles/number of buses, fuel usage/fuel cost, total employee/personal cost/labor, length of operating network/length of line/lines, and/or capital cost.
- Output variable: vehicle-miles/vehicle-kilometers, passenger-kilometers, bus-kilometers, operating revenue, and/or total revenue/revenue.

According to the literature review, this study chooses the variables within this scope. Limit to the data availability, we do not select all these variables in the empirical study.

Table 2. 1 Summary of literature review for DEA model in transportation

No	Author	Industry	Approach	Input variables	Output variables	Service variables	Model	DMU
1	Cullinane *et al.* (2006)	Port	DEA SFA	terminal length terminal area quayside gantry yard gantry straddle carrier	container throughput	-	CCR BCC	Country

2	Pels *et al*. (2001)	Airport	DEA SFA	Terminal size	Air transport movement	-	BCC	City
				aircraft parking positions at the terminal	Passenger movement			
				remote aircraft parking positions	-			
				number of check-in desks				
				number of baggage claim				
3	Karlaftis (2003)	Transit	DEA	Operating cost	Vehicle-miles travelled	-	CCR	US City
				Number of vehicles	Passengers			
				Gallons of fuel	-			
				Total employees				
4	Adler and Berechman (2001)	Airport	DEA	Questionnaire	Service satisfaction	-	BCC	City
				Haul charge	-			
				Connection times				
				Average delay time				
				Number of terminals				
				Number of runways				
				Distance to the nearest major city-center				
5	Tongzon(2001)	Port	DEA	number of berths, cranes and tugs	cargo throughput	-	CCR	City
				number of port authority employees	ship working rate			
				terminal area of the ports	-			
6	Chiou and Chen(2006)	Airline	DEA	fuel cost	number of flights	Passenger mile	CCR BCC	Airline
				personnel cost	seat-mile	embarkation passengers		
				aircraft cost	-	-		
				-				
7	Fielding *et al*. (1984)	Transit	DEA	labor	vehicle hours	passengers	CCR	US City
				capital	vehicle miles	Passenger miles		
				fuel	capacity miles	operating revenue		
				-	service reliability	-		
					-			
8	Yun *et al*. (2004)	Bank	DEA	Non-interest expense	Deposits	-	CCR BCC FDH	Bank
			GDEA	Interest income plus	-			
				non-interest income				
9	Peck *et al*. (1998)	Airport	DEA	labor expenses on airframes	flights arrivals delayed	-	BCC	Airlines
				labor expenses on aircraft engines	for mechanical reasons			
				expenditures on airframe repairs	-			
				expenditures on engine repairs				

				material expenditures on airframes				
				material expenditures on engines				
10	Odeck and Alkadi (2001)	Transit	DEA	total number of seats	seat kilometers	-	BCC	Bus company
				fuel consumption	Vehicle kilometers			
				consumption equipment	Passenger kilometers			
				-	passengers			
11	Philip A. Viton (1998)	Transit	DEA	fleet sizes	vehicle-miles	-	BCC	Transit industry
				number of gallons of fuel	Passenger trips			
				number of person-hours of transportation	vehicle hours			
				number of person-hours	-			
				of maintenance				
				number of person-hours of administrative				
				capital				
				the cost of tires and other materials				
				the cost of services				
				the cost of utilities				
				the cost of insurance				
12	Cowie and Asenova (1999)	Transit	DEA	total staff employed	Operating revenue	-	BCC	Bus company
				fleet size	-			
13	Lan and Lin (2003)	Railway	DEA	length of lines	Train kilometer	Passenger kilometer	CCR BCC EXO CAT	Railway
				number of locomotives and cars	-	Ton kilometer		
				number of employees		-		
14	Lan and Lin (2005)	Railway	DEA	Lines	passenger train-kilometer	Passenger kilometers	BCC (Four Stage)	Railway
				Passenger cars	freight-train-kilometer	Ton kilometers		
				Freight cars	-	-		
				Employees				
15	Tzeng and Chiang (2000)	Airport	DEA	total capital	net operation revenue	-	CCR BCC	Airline company
				number of employees	Passenger kilometers			
				total number of seats				
16	Karlaftis (2004)	Transit	DEA	Number of vehicles	Vehicle miles	Passenger miles	BCC	City
				gallons of fuel	-	-		
				Total employees				
17	Coelli and Perelman (1999)	Railway	DEA	annual mean of monthly data on staff levels	passenger services	-	BCC	Company
			SFA	available freight wagons	freight services			
			COLS	coach transport capacities in tones	-			

				coach transport capacities in seats				
				total length of lines				
18	Schefczyk (1993)	Airline	DEA	available ton kilometers	revenue passenger kilometers	-	CCR	Airlines
				facilities	cargo revenue			
				current assets	other revenue			
				other assets				
				labor				
				fuel				
				commissions to agents				
19	Charnes *et al*. (1996)	Airline	DEA	seat-kilometer available	Passenger kilometer		BCC	Airlines
			REPF	cargo-ton kilometer available				
				fuel				
				labor				
20	Sengupta (1999)	Airline	DEA	available ton kilometers	revenue passenger kilometers	-	CCR	Airlines
				facilities	cargo revenue			
				current assets	other revenue			
				other assets				
				labor				
				fuel				
21	Sarkis (2000)	Airport	DEA	airport operational costs	operational revenue		CCR BCC	Airport
				number of airport employees	passenger flow			
				gates	commercial			
				runways	general aviation movement			
					total cargo transportation			
22	chiou *et al*. (2010	transit	IDEA	number of buses	number of bus runs	operating revenue	CCR BCC	Bus company
				operating network	Bus kilometer	number of passengers		
						Passenger kilometer		
						average number of on-board passengers per run		
23	Barros and Dieke (2007)	Airport	DEA	labor costs	number of planes		CCR BCC	Airport
				capital invested	number of passenger			
				operational costs	cargo			
					aeronautical receipts			
					handling receipts			

					commercial receipts			

Note:
EXO DEA: exogenously fixed inputs model
CAT DEA: To compare the performance measurements in a homogeneous environment can be formulated according to appropriate categorical variables.
COLS: A parametric frontier using corrected ordinary least squares
REPF: Robustly Efficient Parametric Frontier
IDEA: integrated data envelopment analysis
EXO DEA: exogenously fixed inputs model
CAT DEA: To compare the performance measurements in a homogeneous environment can be formulated according to appropriate categorical variables.
COLS: A parametric frontier using corrected ordinary least squares

Table 2. 2 Summary of literature for FDEA model

No	Author	Data form	Approach	Model formulation tool	Efficiency value form
1	Cooper *et al*. (1999)	Crisp data Interval data Ordinal data Ratio data	CCR	assurance region approach	crisp data
2	Kao and Liu (2000)	Crisp data Fuzzy data	CCR BCC	α-cut	interval values (lower- and upper-bound) under various α-cut levels
3	Guh *et al*. (2001)	Interval data	CCR	α-cut	
4	Guo and Tanaka	Fuzzy data	CCR	h-level	
5	Despotis and Smirlis (2002)	Crisp data Interval data	CCR	α-cut	interval values (lower- and upper-bound)
6	Lertworasirikul *et al*. (2003)	Fuzzy data	CCR	α-cut	interval values (lower- and upper-bound) under various α-cut levels
7	Leon *et al*. (2003)	Fuzzy data	CCR	h-level	
8	Jahanshahloo(2004)	Fuzzy data	CCR	α-cut	
9	Smirlis *et al*. (2006)	Crisp data Interval data	CCR	α-cut	interval values (lower- and upper-bound)
10	Mostafaee and Saljooghi (2009)	Interval data	CCR	α-cut	interval values interval values (lower- and upper-bound)
11	Jahanshahloo *et al*. (2009)	Interval data	CCR	α-cut	
12	Azadeh and Alem (2010)	Fuzzy data	CCR	α-cut	crisp data

CHAPTER 3. DEA FORMULATIONS

This chapter introduces different DEA modeling forms including conventional DEA models and fuzzy DEA models.

3.1 Conventional DEA models

DEA was initially developed as a method for assessing the comparative efficiencies of organizational units. The key feature which makes the units comparable is that they perform the same function in terms of the kinds of inputs they use and the types of outputs they produce.

The CCR model (Charnes *et al*., 1978) generalized the single output/single input ratio efficiency measure for DMU to multiple outputs/multiple inputs situations by forming the ratio of a weighted sum of outputs to a weighted sum of inputs. Based on the CCR model, the BCC model (Banker *et al*., 1984) relaxed the constant returns to scale assumption of the CCR model and made it possible to investigate whether the performance of each DMU was conducted in region of increasing, constant or decreasing returns to scale in multiple outputs and multiple inputs situations.

The main characteristics of DEA are that (i) it can be applied to analyze multiple outputs and multiple inputs without pre-assigned weights, (ii) it can be used for measuring a relative efficiency based on the observed data without knowing information on the production function.

Two basic and conventional DEA models are CCR model and BCC model. These two basic model forms are illustrated as following.

3.1.1 CCR model

DMU k is assumed to be evaluated. And there are i DMUs, each utilizes j kinds of inputs, $(x_{1i}, x_{2i}, \ldots, x_{ji})$, and purchases r kinds of outputs, $(y_{1i}, y_{2i}, \ldots, y_{ri})$. The efficiency of DMU k can be estimated by following programming.

$$[\text{CCR}]\ \underset{u,v}{Max} \quad h_k = \frac{\sum_{r=1}^{s} u_r y_{kr}}{\sum_{j=1}^{m} v_j x_{kj}} \tag{3.1}$$

$$\text{s.t.} \quad \frac{\sum_{r=1}^{s} u_r y_{ir}}{\sum_{j=1}^{m} v_j x_{ij}} \le 1, \quad i = 1,2,\cdots,n \tag{3.2}$$

$$v_j \ge 0, \quad j = 1,2,\cdots,m \tag{3.3}$$

$$u_r \ge 0, \quad r = 1,2,\cdots,s \tag{3.4}$$

The model [CCR] is an input oriented programming problem, which can be formulated as output oriented problem [CCR-O] by following programming.

$$[\text{CCR-O}] \quad \underset{\omega,\mu}{Min} \quad g_k = \frac{\sum_{j=1}^{m} \omega_j x_{kj}}{\sum_{r=1}^{s} \mu_r y_{kr}} \tag{3.5}$$

$$\text{s.t.} \quad \frac{\sum_{j=1}^{m} \omega_j x_{ij}}{\sum_{r=1}^{s} \mu_r y_{ir}} \geq 1, \quad i = 1,2,\cdots,n \tag{3.6}$$

$$\omega_j \geq 0, \quad j = 1,2,\cdots,m \tag{3.7}$$

$$\mu_r \geq 0, \quad r = 1,2,\cdots,s \tag{3.8}$$

Then, one can transform above [CCR] model into an ordinary linear problem [CCR-L], show as following.

$$[\text{CCR-L}] \quad \underset{u,v}{Max} \quad h_k = \sum_{r=1}^{s} u_r y_{kr} \tag{3.9}$$

$$\text{s.t.} \quad \sum_{r=1}^{s} u_r y_{ir} - \sum_{j=1}^{m} v_j x_{ij} \leq 0, \quad i = 1,2,\cdots,n \tag{3.10}$$

$$\sum_{j=1}^{m} v_j x_{kj} = 1, \tag{3.11}$$

$$v_j \geq 0, \quad j = 1,2,\cdots,m \tag{3.12}$$

$$u_r \geq 0, \quad r = 1,2,\cdots,s \tag{3.13}$$

Because [CCR-L] model is a linear problem, one can transform it into dual problem [CCR-D] as follows.

$$[\text{CCR-D}] \quad \underset{z,\lambda_i}{Min} \quad z \tag{3.14}$$

$$\text{s.t.} \quad z x_{kj} - \sum_{i=1}^{n} x_{ij} \lambda_i \geq 0, \quad j = 1,2,\cdots,m \tag{3.15}$$

$$- y_{kr} + \sum_{i=1}^{n} y_{ir} \lambda_i \geq 0, \quad r = 1,2,\cdots,s \tag{3.16}$$

$$\lambda_i \geq 0, \quad i = 1,2,\cdots,n \tag{3.17}$$

z is a scalar, which is the efficiency of k^{th} firm, and it ranges from zero to unity. If z equals to one, the firm is efficient. And if z is less than one, the firm is inefficient.

One also can transform [CCR-O] model into linear problem [CCR-O-D] and then one can find its dual problem [CCR-O-D], show as follows.

$$\text{[CCR-O-D]} \quad \underset{z,\lambda_i}{Max} \quad \phi \tag{3.18}$$

$$\text{s.t.} \quad -\phi \cdot y_{kr} + \sum_{i=1}^{n} y_{ir}\lambda_i \geq 0, \quad r = 1,2,\cdots,s \tag{3.19}$$

$$x_{kj} - \sum_{i=1}^{n} x_{ij}\lambda_i \geq 0, \quad j = 1,2,\cdots,m \tag{3.20}$$

$$\lambda_i \geq 0, \quad i = 1,2,\cdots,n \tag{3.21}$$

3.1.2 BCC model

Model [CCR-D] and [CCR-O-D] are input and output oriented DEA models under the assumption of constant returns to scale (CRS) production technology. Banker, Charnes and Cooper (1984) relaxed this CRS constrain to variable returns to scale (VRS) technology by adding convexity constraint, as following models. Then one can get BCC input ([BCC]) and output oriented model ([BCC-O]) as following.

$$\text{[BCC]} \quad \underset{z,\lambda_i}{Min} \quad z \tag{3.22}$$

$$\text{s.t.} \quad zx_{kj} - \sum_{i=1}^{n} x_{ij}\lambda_i \geq 0, \quad j = 1,2,\cdots,m \tag{3.23}$$

$$-y_{kr} + \sum_{i=1}^{n} y_{ir}\lambda_i \geq 0, \quad r = 1,2,\cdots,s \tag{3.24}$$

$$\lambda_i \geq 0, \quad i = 1,2,\cdots,n \tag{3.25}$$

$$\sum_{i=1}^{n} \lambda_i = 1 \tag{3.26}$$

$$\text{[BCC-O]} \quad \underset{z,\lambda_i}{Max} \quad \phi \tag{3.27}$$

$$\text{s.t.} \quad -\phi \cdot y_{kr} + \sum_{i=1}^{n} y_{ir}\lambda_i \geq 0, \quad r = 1,2,\cdots,s \tag{3.28}$$

$$x_{kj} - \sum_{i=1}^{n} x_{ij}\lambda_i \geq 0\,, \quad j = 1,2,\cdots,m \tag{3.29}$$

$$\lambda_i \geq 0\,, \quad i = 1,2,\cdots,n \tag{3.30}$$

$$\sum_{i=1}^{n} \lambda_i = 1 \tag{3.31}$$

Once one knows the basic models for DEA, one can use these models to evaluate relative efficiency for each DMU.

3.2 Fuzzy DEA models

In recent years, fuzzy set theory has been proposed as a way to quantify imprecise and vague data in DEA models. Fuzzy DEA models take the form of fuzzy linear programming models. The CCR model with fuzzy coefficients is given in following model ([FCCR]).

$$\text{[FCCR]} \quad \underset{u_r, v_i}{Max} \quad \tilde{h}_k = \sum_{r=1}^{s} u_r \tilde{y}_{rk} \tag{3.32}$$

$$s.t. \quad \sum_{i=1}^{m} v_i \tilde{x}_{ik} = \tilde{1} \tag{3.33}$$

$$\sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \leq \tilde{0},\ j = 1,\ldots,n \tag{3.34}$$

$$u_r, v_i \geq \varepsilon > 0, r = 1,\ldots,s, i = 1,\ldots,m \tag{3.35}$$

where $\tilde{h}_k$ is the fuzzy efficiency score of DMU *k*. $\tilde{x}_{ik}$ is the fuzzy input *i* of DMU *k*. $\tilde{y}_{rk}$ is the fuzzy output *r* of DMU *k*. u_r and v_i are the multipliers corresponding to output *r* and input *i*, respectively.

Similar to the CCR model, the constraints $\sum_{i=1}^{m} v_i \tilde{x}_{ik} = \tilde{1}$ and $\sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \leq 0$ in the model [FCCR] are used for normalization of the value $\sum_{r=1}^{s} u_r \tilde{y}_{rk}$. However, the objective value $\sum_{r=1}^{s} u_r \tilde{y}_{rk}$ can now exceed one since the second and third constraints of model [FCCR] are satisfied "possibilistically". That is, since their parameters are fuzzy sets, $\sum_{i=1}^{m} v_i \tilde{x}_{ik}$ is "approximately equal to one", which implies that $\sum_{r=1}^{s} u_r \tilde{y}_{rj} \Big/ \sum_{i=1}^{m} v_i \tilde{x}_{ij}$ is "approximately less than or equal to one".

The fuzzy CCR models cannot be solved by a standard LP solver like a crisp CCR model because coefficients in the fuzzy CCR model are fuzzy sets. With the fuzzy inputs and fuzzy outputs, the optimality conditions for the crisp DEA model need to be clarified and generalized. The corresponding fuzzy linear programming problem is usually solved using some ranking methods for fuzzy sets.

Many studies (Guo and Tanaka, 2001; Lertworasirikul *et al*., 2003; Leon *et al*., 2003; Jahanshahloo *et al*., 2004) apply probability approach. Possibility theory was formulated in terms of fuzzy set theory by Zadeh (1978) and has been developed by many researchers. Zadeh (1978) suggested that fuzzy sets can be used as a basis for the theory of possibility similar to the way that measure theory provides the basis for the theory of probability. He introduced the "fuzzy variable", which is associated with a possibility distribution in the same manner that a random variable is associated with a probability distribution. In the fuzzy linear programming model, each fuzzy coefficient can be viewed as a fuzzy variable and each constraint can be considered to be a fuzzy event. Using possibility theory, possibilities of fuzzy events (i.e., fuzzy constraints) can be determined. Using the Fuzzy DEA based on CCR model proposed by Guo and Tanaka (2001) as an example.

First, they give the Definition 1 to define the two symmetric triangular fuzzy variables which are $Z_1 = (z_1, w_1)$ and $Z_2 = (z_2, w_2)$, the relation $Z_1 \le Z_2$ is defined by the following inequalities:

$$Z_1 - (1-h)w_1 \le Z_2 - (1-h)w_2,$$

$$Z_1 + (1-h)w_1 \le Z_2 + (1-h)w_2,$$

Where, $0 \le h \le 1$ is a predefined possibility level by decision-makers. Maximizing a symmetrical triangular fuzzy variable $Z = (z, w)$ can be explained as simultaneously maximizing $Z-(1-h)w$ and $Z+(1-h)w$. Here, a weighted function $\lambda_1(Z-(1-h)w)+\lambda_2(Z+(1-h)w)$ is introduced to obtain some compromise solution where $\lambda_1 \ge 0$ and $\lambda_2 \ge 0$ are the weights of left and right endpoints of the h-level set of *Z*, respectively, with $\lambda_1 + \lambda_2 = 1$. Taking $\lambda_1 = 1$ is regarded as a pessimistic opinion of maximizing *Z* because the worst situation is considered, whereas taking $\lambda_2 = 1$ is regarded as an optimistic opinion because the best situation is concerned with. In the study of Guo and Tanaka (2001), λ_1 is taken as 1, that is, $\max\ z-(1-h)w$.

Next, consider the relation $\sum_{i=1}^{m} v_i \tilde{x}_{ik} = \tilde{1}$ in model [FCCR] which plays the same role as $\sum_{i=1}^{m} v_i x_{ik} = 1$ in model [CCR-L]. As a result, finding out v_i to make $\sum_{i=1}^{m} v_i \tilde{x}_{ik} = \tilde{1}$ is approximated as finding out v_i to make $\sum_{i=1}^{m} v_i \tilde{x}_{ik}$ approach $\tilde{1}$ as much as possible, simply denoted as $\sum_{i=1}^{m} v_i \tilde{x}_{ik} = \tilde{1}$. Considering the Definition 1, $\sum_{i=1}^{m} v_i \tilde{x}_{ik}$ that satisfies $\sum_{i=1}^{m} v_i \tilde{x}_{ik} = \tilde{1}$ can be regarded as an upper bound subject to $\sum_{i=1}^{m} v_i \tilde{x}_{ik} < \tilde{1}$. It means that the left endpoints of the h-level sets of $\sum_{i=1}^{m} v_i \tilde{x}_{ik}$ and $\tilde{1} = (1, e)$ overlap while the right endpoint of $\sum_{i=1}^{m} v_i \tilde{x}_{ik}$ expands rightwards as much as possible but is not larger than that of the h-level set of $\tilde{1} = (1, e)$. Thus, the problem for finding out v_i such that $\sum_{i=1}^{m} v_i \tilde{x}_{ik} = \tilde{1}$ would be the problem can be converted into the following optimization problem ([FCCR-*x*]).

$$\text{[FCCR-}x\text{]}\quad \underset{u_r, v_i}{Max} \ \sum_{i=1}^{m} v_i c_{ik} = \tilde{1} \tag{3.36}$$

$$s.t.\quad \sum_{i=1}^{m} v_i \tilde{x}_{ik} - (1-h)\sum_{i=1}^{m} v_i c_{ik} = 1-(1-h)e \tag{3.37}$$

$$\sum_{i=1}^{m} v_i \tilde{x}_{ik} + (1-h)\sum_{i=1}^{m} v_i c_{ik} = 1+(1-h)e \tag{3.38}$$

$$v_i \geq \varepsilon > 0 \tag{3.39}$$

It can be seen that model [FCCR-*x*] is used to find out $Z = \sum_{i=1}^{m} v_i \tilde{x}_{ik}$ constrained by $\sum_{i=1}^{m} v_i \tilde{x}_{ik} \leq \tilde{1}$ with the largest spread $\sum_{i=1}^{m} v_i c_{ik}$ and the same left endpoint as the one of fuzzy number $\tilde{1}$ in h-level sets. The obtained v_i from model [FCCR-*x*] is denoted as v_i^*. Using model [FCCR-*x*], the fuzzy optimization problem model [FCCR] can be transformed into the following LP problem ([FCCR-L]) with a primary objective function and a secondary objective function:

$$\text{[FCCR-L]}\quad \underset{u_r, v_i}{Max}\ \tilde{h}_k = \sum_{r=1}^{s} u_r \tilde{y}_{rk} - (1-h)\sum_{r=1}^{s} u_r d_{rk} \tag{3.40}$$

$$s.t.\quad \underset{u_r, v_i}{Max}\ \sum_{i=1}^{m} v_i c_{ik} = \tilde{1} \tag{3.41}$$

$$s.t.\quad \sum_{i=1}^{m} v_i \tilde{x}_{ik} - (1-h)\sum_{i=1}^{m} v_i c_{ik} = 1-(1-h)e \tag{3.42}$$

$$\sum_{i=1}^{m} v_i \tilde{x}_{ik} + (1-h)\sum_{i=1}^{m} v_i c_{ik} = 1+(1-h)e \tag{3.43}$$

$$v_i \geq \varepsilon > 0 \tag{3.44}$$

$$\sum_{r=1}^{s} u_r \tilde{y}_{rj} + (1-h)\sum_{r=1}^{s} u_r d_{rj} \leq \sum_{i=1}^{m} v_i \tilde{x}_{ij} + (1-h)\sum_{i=1}^{m} v_i c_{ij},\ j=1,\ldots,n \tag{3.45}$$

$$\sum_{r=1}^{s} u_r \tilde{y}_{rj} - (1-h)\sum_{r=1}^{s} u_r d_{rj} \leq \sum_{i=1}^{m} v_i \tilde{x}_{ij} - (1-h)\sum_{i=1}^{m} v_i c_{ij},\ j=1,\ldots,n \tag{3.46}$$

$$u_r, v_i \geq \varepsilon > 0,\ r=1,\ldots,s,\ i=1,\ldots,m \tag{3.47}$$

Considering *n* DMUs, *e* is taken as $e = \max_{j=1,\ldots,n}(\max_{k=1,\ldots,m} c_{ik}/x_{ik})$ in the optimization problem in [FCCR-L] model. Assuming that the optimal value of the objective function of model [FCCR-*x*] is g_0, the optimization problem in [FCCR-L] can be rewritten as the following LP problem (FCCR-L-g_0):

$$\text{[FCCR-L-}g_0\text{]}\quad \underset{u_r, v_i}{Max}\ \tilde{h}_k = \sum_{r=1}^{s} u_r \tilde{y}_{rk} - (1-h)\sum_{r=1}^{s} u_r d_{rk} \tag{3.48}$$

$$s.t.\quad \sum_{i=1}^{m} v_i c_{ik} \geq g_0 \tag{3.49}$$

$$\sum_{i=1}^{m} v_i \tilde{x}_{ik} - (1-h)\sum_{i=1}^{m} v_i c_{ik} = 1-(1-h)e \tag{3.50}$$

$$\sum_{i=1}^{m} v_i \tilde{x}_{ik} + (1-h)\sum_{i=1}^{m} v_i c_{ik} = 1+(1-h)e \tag{3.51}$$

$$\sum_{r=1}^{s} u_r \tilde{y}_{rj} + (1-h)\sum_{r=1}^{s} u_r d_{rj} \le \sum_{i=1}^{m} v_i \tilde{x}_{ij} + (1-h)\sum_{i=1}^{m} v_i c_{ij}, j=1,\ldots,n \tag{3.52}$$

$$\sum_{r=1}^{s} u_r \tilde{y}_{rj} - (1-h)\sum_{r=1}^{s} u_r d_{rj} \le \sum_{i=1}^{m} v_i \tilde{x}_{ij} - (1-h)\sum_{i=1}^{m} v_i c_{ij}, j=1,\ldots,n \tag{3.53}$$

$$u_r, v_i \ge \varepsilon > 0, r=1,\ldots,s, i=1,\ldots,m \tag{3.54}$$

The fuzzy efficiency of an evaluated DMU with the symmetrical triangular fuzzy input vector $X_{ik} = (x_{ik}, c_{ik})$ and output vector $Y_{rk} = (y_{rk}, d_{rk})$ is defined as a non-symmetrical triangular fuzzy number $E = (w_l, \eta, w_r)$ as follows:

$$\eta = \frac{u_r^* y_{rk}}{v_i^* x_{ik}}, \quad w_l = \eta = \frac{u_r^*(y_{rk} - d_{rk}(1-h))}{v_i^*(x_{ik} + c_{ik}(1-h))}, w_r = \eta = \frac{u_r^*(y_{rk} + d_{rk}(1-h))}{v_i^*(x_{ik} - c_{ik}(1-h))},$$

The DUM with $\eta + w_r \ge 1$ for the h possibility level is called an h-possibilistic D efficient DMU (PD DMU). On the contrary, the DMU with $\eta + w_r < 1$ for the h possibility level is called an h-possibilistic D inefficient DMU (PDI DMU). The set of all PD DMUs is called the h-possibilistic nondominated set.

CHAPTER 4. ROUTE-BASED DEA MODELS

This chapter proposes two similar RDEA models under CRS and VRS contexts, termed as route-based CCR (RCCR) model and route-based BCC (RBCC) model, respectively. Both of the proposed RCCR and RBCC models comprise a three-stage procedure that decomposes company efficiency into individual route efficiencies with simultaneously optimizing the allocation of common inputs. The formulations of the proposed RCCR and RBCC models are elaborated as follows.

4.1 Proposed Route-based DEA models

The model structure of three-stage route-based DEA modeling is shown in Figure 4.1.

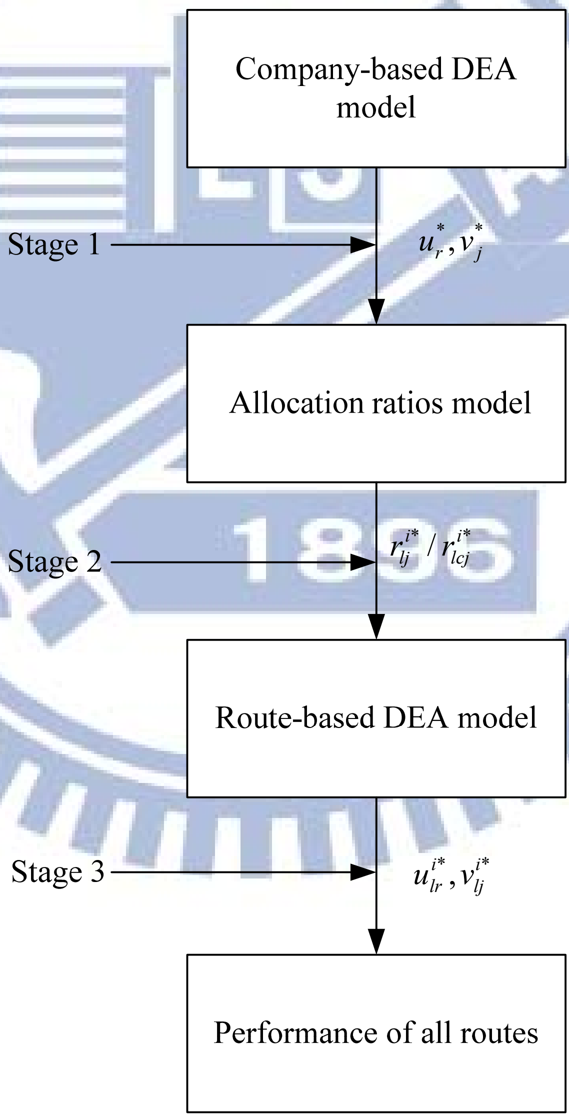

Figure 4. 1 The structure of three-stage route-based DEA modeling

4.1.1 RCCR model

The first stage uses the following company-based CCR model to determine a set of optimal input/output multipliers:

$$[\text{CCR}] \quad \underset{u,v}{Max} \quad h_q = \sum_{r=1}^{R} u_r y_{qr} \tag{4.1}$$

$$s.t. \quad \sum_{r=1}^{R} u_r y_{ir} - \sum_{j=1}^{J} v_j x_{ij} \le 0, \quad i = 1,2,\cdots,I \tag{4.2}$$

$$\sum_{j=1}^{J} v_j x_{sq} = 1 \tag{4.3}$$

$$v_j \ge 0, \quad j = 1,2,\cdots,J \tag{4.4}$$

$$u_r \ge 0, \quad r = 1,2,\cdots,R \tag{4.5}$$

where h_s is the efficiency score of company *q*. Supposed that there are totally *I* companies to be evaluated, each of which utilizes *J* types of inputs and produces *R* kinds of outputs. u_r and v_j are the multipliers corresponding to output *r* and input *j* of company *q*, respectively. From the above [CCR] model, the optimal input/output multipliers can be determined.

The second stage then uses the solved multipliers to determine its optimal allocation ratios for the common inputs among the routes within a company to maximize the overall efficiency of all routes. In practice, however, some portions of the inputs can be clearly attributed to only a specific route; some other portions should be regarded as common inputs for all routes of a company. For instance, the drivers are responsible for and should be reasonably attributed to a specific route; the administrative staff and the managers, however, are the common inputs—not readily attributed to any specific route. In determining the optimal allocation ratios, the following develops two models: [AR1] and [AR2]. [AR1] model is for the case when all the route attributed inputs cannot be identified, whereas [AR2] model is for the case when a portion of the route attributed inputs can be identified.

The [AR1] model is expressed as follows:

$$[\text{AR1}] \quad \underset{s}{Max} \quad h_i = \frac{1}{L_i}\left(\sum_{l=1}^{L_i} \frac{\sum_{r=1}^{R} u_r y_{lr}^i}{\sum_{j=1}^{J} v_j s_{lj}^i x_j^i} \right) \tag{4.6}$$

$$s.t. \quad \frac{\sum_{r=1}^{R} u_r y_{lr}^i}{\sum_{j=1}^{J} v_j s_{lj}^i x_j^i} \le 1, \quad l = 1,2,\cdots,L_i \tag{4.7}$$

$$\sum_{l=1}^{L_i} s_{lj}^i = 1, \quad j = 1,2,\cdots,J \tag{4.8}$$

$$s_{lj}^i \ge 0, \quad l = 1,2,\cdots,L_i \;; \quad j = 1,2,\cdots,J \tag{4.9}$$

where h_i is the average of efficiency scores for all routes of company *i* which operates totally L_i routes and each route utilizes *J* types of inputs and produces *R* kinds of outputs. u_r and v_j are the multipliers determined by the [CCR] model. Besides, each output is assumed route attributable, *i.e.,* can be clearly identified as a route output (y_{lr}^i). Since all the routes attributed inputs cannot be identified, the inputs (x_j^i) to be allocated is based on the optimally solved ratio (s_{lj}^i)—an allocation ratio of route l for input j of company i. Eq. (4.8) ensures that each common input is completely allocated to all routes.

On the other hand, the [AR2] model is expressed as follows:

$$\text{[AR2]} \quad \underset{s}{Max} \quad h_i = \frac{1}{L_i}\left(\sum_{l=1}^{L_i}\frac{\sum_{r=1}^{R}u_r y_{lr}^i}{\sum_{j=1}^{J}v_j(x_{lj}^i + s_{lj}^i x_{cj}^i)}\right) \tag{4.10}$$

$$s.t. \quad \frac{\sum_{r=1}^{R}u_r y_{lr}^i}{\sum_{j=1}^{J}v_j(x_{lj}^i + s_{lj}^i x_{cj}^i)} \le 1, \quad l = 1,2,\cdots,L_i \tag{4.11}$$

$$\sum_{l=1}^{L_i} s_{lj}^i = 1, \quad j = 1,2,\cdots,J \tag{4.12}$$

$$s_{lj}^i > 0, \quad l = 1,2,\cdots,L_i; \quad j = 1,2,\cdots,J \tag{4.13}$$

where the input j of company i is divided into two parts: the attributable part (x_{lj}^i) and the common part (x_{cj}^i); $x_j^i = \sum_{l=1}^{L_i} x_{lj}^i + x_{cj}^i$. Only the common part ($x_{cj}^i$) requires an optimally solved allocation ratio (s_{lj}^i) to assign to route l. To determine the optimal allocation ratio of common input, however, only the routes operated by the same company are considered; namely, the route allocation ratios for one company are irrelevant to the routes operated by other companies. With the optimal allocation ratios (s_{lj}^i), the inputs of route l under evaluation can be computed by $x_l^i = x_{lj}^i + s_{lj}^i x_{cj}^i$.

Finally, based on the computed inputs, the third stage is to optimally determine the route efficiency by treating each route (could be operated by different companies) as a DMU, expressed as follows.

$$\text{[RCCR]} \quad \underset{u,v}{Max} \quad h_k^i = \sum_{r=1}^{R} u_{kr}^i y_{kr}^i \tag{4.14}$$

$$s.t. \quad \sum_{r=1}^{R} u_{lr}^i y_{lr}^i - \sum_{j=1}^{J} v_{lj}^i x_{lj}^i \le 0, \quad i = 1,2,\cdots,I; \quad l = 1,2,\cdots,L \tag{4.15}$$

$$\sum_{j=1}^{J} v_{kj}^i x_{kj}^i = 1 \tag{4.16}$$

$$v_{lj}^i \ge 0, \quad j = 1,2,\cdots,J; \quad l = 1,2,\cdots,L \tag{4.17}$$

$$u_{lr}^i \ge 0, \quad r = 1,2,\cdots,R; \quad l = 1,2,\cdots,L \tag{4.18}$$

where h_k^i is the efficiency score of route k operated by company i. u_{lr}^i and v_{lj}^i are the multipliers corresponding to output r and input j for route l operated by company i, respectively. There are a total of L routes under evaluation, $L=L_1+L_2+\ldots+L_I$. Unlike [AR1] or [AR2] model wherein the routes sequence are ordered only within the same company, the routes sequence of [RCCR] here are ordered among all routes across all companies.

4.1.2 RBCC model

Following the same vein of the above RCCR modeling procedures, the RBCC model simply adds a convexity constraint. In the first stage, the following company-based BCC model is used to determine the optimal multipliers.

$$\text{[BCC]} \quad \underset{u,v}{Max} \quad h_q = \sum_{r=1}^{R} u_r y_{qr} - u \tag{4.19}$$

$$s.t. \quad \sum_{r=1}^{R} u_r y_{ir} - u - \sum_{j=1}^{J} v_j x_{ij} \le 0, \quad i = 1,2,\cdots,I \tag{4.20}$$

$$\sum_{j=1}^{J} v_j x_{qj} = 1 \tag{4.21}$$

$$v_j \ge 0, \quad j = 1,2,\cdots,J \tag{4.22}$$

$$u_r \ge 0, \quad r = 1,2,\cdots,R \tag{4.23}$$

where u is efficiency scale of company q. In the second stage, the corresponding allocation ratio models can be expressed as follows.

$$\text{[AR1']} \quad \underset{s}{Max} \quad h_i = \frac{1}{L_i}\left(\sum_{l=1}^{L_i} \frac{\sum_{r=1}^{R} u_r y_{lr}^i - u}{\sum_{j=1}^{J} v_j s_{lj}^i x_j^i} \right) \tag{4.24}$$

$$s.t. \quad \frac{\sum_{r=1}^{R} u_r y_{lr}^i - u}{\sum_{j=1}^{J} v_j s_{lj}^i x_j^i} \le 1, \quad l = 1,2,\cdots,L_i \tag{4.25}$$

$$\sum_{l=1}^{L_i} s_{lj}^i = 1, \quad j = 1,2,\cdots,J \tag{4.26}$$

$$s_{lj}^i \ge 0, \quad l = 1,2,\cdots,L_i \;; \quad j = 1,2,\cdots,J \tag{4.27}$$

and

$$\text{[AR2']} \quad \underset{s}{Max} \quad h_i = \frac{1}{L_i}\left(\sum_{l=1}^{L_i} \frac{\sum_{r=1}^{R} u_r y_{lr}^i - u}{\sum_{j=1}^{J} v_j (x_{lj}^i + s_{lj}^i x_{cj}^i)} \right) \tag{4.28}$$

$$s.t. \quad \frac{\sum_{r=1}^{R} u_r y_{lr}^i - u}{\sum_{j=1}^{J} v_j (x_{lj}^i + s_{lj}^i x_{cj}^i)} \le 1, \quad l = 1,2,\cdots,L_i \tag{4.29}$$

$$\sum_{l=1}^{L_i} s_{lj}^i = 1, \quad j = 1,2,\cdots,J \tag{4.30}$$

$$s_{lj}^i > 0, \quad l = 1,2,\cdots,L_i ; \quad j = 1,2,\cdots,J \tag{4.31}$$

In the third stage, the corresponding [RBCC] model can be written as follows.

$$\text{[RBCC]} \quad \underset{u,v}{Max} \quad h_k^i = \sum_{r=1}^{R} u_{kr}^i y_{kr}^i - u_k^i \tag{4.32}$$

$$s.t. \quad \sum_{r=1}^{R} u_{lr}^i y_{lr}^i - u_k^i - \sum_{j=1}^{J} v_{lj}^i x_{lj}^i \le 0, \quad i = 1,2,\cdots,I ; \quad l = 1,2,\cdots,L \tag{4.33}$$

$$\sum_{j=1}^{J} v_{kj}^{i} x_{kj}^{i} = 1 \tag{4.34}$$

$$v_{lj}^{i} \geq 0, \quad j = 1,2,\cdots,J; \quad l = 1,2,\cdots,L \tag{4.35}$$

$$u_{lr}^{i} \geq 0, \quad r = 1,2,\cdots,R; \quad l = 1,2,\cdots,L \tag{4.36}$$

where u_{k}^{i} is the scale of route *k* of company *i*.

4.2 Properties

4.2.1. Slack analysis

Definition: the slack value of the route is the difference between the shared input of the route and that of its benchmark routes.

The following two slack analyses should be used depending on whether or not the attributed inputs are known:

Case (1) When attributed inputs are unknown, the shared input value $s_{lj}^{i} x_{j}^{i}$ determined by the [AR1] or [AR1'] model are used as the inputs of the [RCCR] or [RBCC] model to evaluate the route efficiency and to determine the corresponding benchmark routes.

Case (2) When attributed inputs are known, with the allocation ratios determined by the [AR2] or [AR2'] model, the shared input value $x_{lj}^{i} + s_{lcj}^{i} x_{cj}^{i}$ is used as the inputs of the [RCCR] or [RBCC] model to evaluate the route efficiency and to determine the corresponding benchmark routes. For instance, if route *r* is benchmarked by route *i*, the slack value for the attribute part of input *j* is $x_{rj}^{i} - x_{lj}^{i}$ and for the common part is $s_{rj}^{i} x_{cj}^{i} - s_{lj}^{i} x_{cj}^{i}$.

4.2.2. Consistency of ranking order

Property: the ranking order of company's performance represented by the efficiency value determined by the company-based DEA model is identical to the average of route efficiency values determined by the route-based DEA model.

[proof] Without loss of generality, consider two companies—company 1 and company 2, each operates two routes. According to Charnes *et al.* (1978), the company efficiency can be defined as $E_i = \dfrac{y_c^i}{y_R}$, $y_R \geq y_c^i$, where y_R is the maximum outputs produced from given inputs and y_c^i is the actual outputs rated from the same inputs for company *i*. We use this concept to derive the company efficiency with the company-based DEA model as follows:

Let u_c^{i*}, v_c^{i*} represent the optimal set of corresponding values. Then $x_R = x_c^i$ implies $v_c^{i*} x_c^i = v_c^{i*} x_R$. By definition, the efficiency score of the benchmark company is equal to 1, implying $v_c^{i*} x_R = u_c^{i*} y_R$. Thus, the following relationship holds:

$$E_i = \frac{u_c^{i1*} y_c^i}{v_c^{i*} x_c^i} = \frac{u_c^{i*} y_c^i}{v_c^{i*} x_R} = \frac{u_c^{i*} y_c^i}{u_c^{i*} y_R} = \frac{y_c^i}{y_R} \tag{4.37}$$

Without loss of generality, assuming company 1 performs better than company 2, then we obtain the result: $\frac{y_c^1}{y_R} = E_1 > E_2 = \frac{y_c^2}{y_R}$, implying $y_c^1 - y_c^2 > 0$.

Similarly, the route efficiency can be defined as $E_l^i = \frac{y_l^i}{y_r}$, $y_r \geq y_l^i$, where y_r is the maximum outputs of the benchmark route produced by the given inputs and y_l^i is the actual outputs rated from the same inputs for route *l* in company *i*. We use this concept to derive the route efficiency with the route-based DEA model as follows:

Let u_l^{i*} and v_l^{i*} represent the optimal set of corresponding values. $x_r = x_l^i$ implies $v_l^{i*} x_l^i = v_l^{i*} x_r$. By definition, the efficiency score of benchmark route is equal to 1, implying $v_l^{i*} x_r = u_r^{i*} y_r$. Thus, the following relationship holds:

$$E_{route}^i = \frac{u_1^{i*} y_1^i}{v_1^{i*} r_1^{i*} x_1^i} + \frac{u_2^{i*} y_2^i}{v_2^{i*} r_1^{i*} x_2^i} = \frac{u_1^{i*} y_1^i}{v_1^{i*} x_r} + \frac{u_2^{i*} y_2^i}{v_2^{i*} x_r} = \frac{u_1^{i*} y_1^i}{u_1^{i*} y_r} + \frac{u_2^{i*} y_2^1}{u_2^{i*} y_r} = \frac{y_1^i + y_2^i}{y_r} = \frac{y_c^i}{y_r} \tag{4.38}$$

From the company-based DEA model, $y_c^1 - y_c^2 > 0$, therefore we can further derive $\frac{y_c^1}{y_r} = E_{route}^1 > E_{route}^2 = \frac{y_c^2}{y_r}$. Namely, the ranking order of company performance represented by the efficiency value determined by the company-based DEA model is identical to the average of route efficiency values determined by the route-based DEA model.

4.3 An Empirical Study

To implement the proposed RDEA models, an empirical study on 1,035 routes currently operated by 37 intercity bus companies in Taiwan is conducted. Referring to relevant literature (e.g., Gillen and Lall, 1997a,b; Lan and Lin, 2005; Chiou and Chen, 2006; Bhadra, 2009; Greer, 2009; Lin and Lan, 2009), we utilize fuel cost, number of employees (hereinafter, labor), and number of buses (hereinafter, bus) as the input variables; operating revenue and passenger-km as the output variables. Note that the number of employees in input variable includes both operating related and other business related labor (e.g., driver and salesperson). The operating revenue and passenger-km are considered as two output variables since the carriers' operating revenue in Taiwan includes not only the fare box revenue, which may have direct correlation with passenger-km, but also other business revenue (e.g., real estate rent, advertisements, etc.), which may not be directly related to passenger-km. Besides, each company may charge the fare box differently due to various fare discounting strategies (e.g., frequent passengers). In order to present the difference between these two variables, operating revenue per passenger-km for each company has been calculated and shown in Table 4.1. The operating revenue per passenger-km is largely varied. Furthermore, the correlation coefficients among operating revenue and passenger-km is 0.86 (in Table 4.2). Base on these reasons, we think it is necessary to take these two variables into account.

In current practice, some buses are exclusively used in a specific route, but some others may be used flexibly in different routes. It suggests that the input of bus fleet contains two parts: attribute part and common part. The other input variables are regarded as unknown attributed inputs.

Table 4. 1 Operating revenue per passenger-km of 37 bus companies

Company	operating revenue/ passenger km	Company	operating revenue/ passenger km
1	1.90	20	1.90
2	1.36	21	37.17
3	9.92	22	9.98
4	3.88	23	1.86
5	25.25	24	1.32
6	90.55	25	6.76
7	33.07	26	10.51
8	5.83	27	9.52
9	1.91	28	5.59
10	1.07	29	61.35
11	1.43	30	12.98
12	1.16	31	12.90
13	1.47	32	1.93
14	1.15	33	4.18
15	153.96	34	4.99
16	24.71	35	1.57
17	1.14	36	110.13
18	7.63	37	39.18
19	1.36		

4.3.1 Data

Our dataset came from the annual report published by the Institute of Transportation, Ministry of Transportation and Communications in 2005. It contained the above-mentioned detailed inputs and outputs information for the 1,035 routes operated by 37 companies. To save space, the detailed information for each of the 1,035 routes is not presented here. Table 4.2 displays the correlation coefficients among input and output variables at the company level. Note that all correlation coefficients between input and output variables are significantly positive, suggesting that the dataset satisfies the isotonicity property.

To ensure the selected input/output variables important and relevant, regression analyses are further conducted and Table 4.3 presents the results. Note that all the explanatory variables show positive and significant effects on at least one of the associated dependent variables, suggesting the appropriateness of the above selected variables.

Table 4. 2 Correlation coefficients among input and output variables

Variable	Output		Input		
	Operating revenue	Passenger-km	Fuel cost	Labor	Bus
Operating revenue	1.00				
Passenger-km	0.86	1.00			
Fuel cost	0.98	0.89	1.00		
Labor	0.97	0.77	0.96	1.00	
Bus	0.94	0.69	0.90	0.95	1.00

Table 4. 3 Regression results for input and output variables

Dependent variables	Independent variables		
	Fuel cost	Labor	Bus
Operating revenue	2.311 (8.734)	340118.244 (2.184)	518999.382 (2.827) R^2 =0.987
Passenger-km	5.457 (8.815)	790009.511 (2.169)	776740.479 (1.809) R^2 =0.889

Note: t values in parentheses.

In order to evaluate the severity of multicollinearity for above regression, the variance inflation factor (VIF) has been calculated. When operating revenue/passenger-km is viewed as dependent variable, the VIF factor is 76.92/9.00. Even the VIF of operating revenue is larger than 10, we still select it as an output variable because of the importance of the variable and the limitation of variable availability.

4.3.2. Results

In the first stage, a CDEA model is used to evaluate the company-level efficiency. The efficiency scores of 37 companies are summarized in Table 4.4. For confidential reasons, , the names of company are anonymous in this study. Note that only 4 companies are evaluated as efficient. Most of the inefficient companies, characterized as IRS, need to enlarge their scales.

Table 4. 4 Efficiency scores and scale efficiencies of 37 bus companies

Company	CRS	VRS	Scale	Company	CRS	VRS	Scale
1	0.564	0.565	0.999 IRS	20	0.586	0.594	0.987 IRS
2	0.938	1.000	0.938 DRS	21	0.965	1.000	0.965 IRS
3	0.879	0.919	0.957 DRS	22	0.875	0.943	0.928 IRS
4	1.000	1.000	1.000 CRS	23	0.686	0.901	0.762 DRS
5	0.787	0.912	0.863 DRS	24	0.465	0.491	0.949 DRS
6	0.812	0.921	0.881 DRS	25	0.559	0.585	0.956 IRS
7	0.741	0.833	0.889 DRS	26	0.464	0.479	0.970 DRS
8	0.439	0.449	0.978 IRS	27	0.417	0.445	0.937 IRS
9	0.387	0.397	0.973 IRS	28	0.581	0.604	0.962 IRS
10	0.678	0.828	0.820 IRS	29	0.645	0.774	0.833 IRS
11	0.902	1.000	0.902 IRS	30	0.525	1.000	0.525 IRS
12	0.877	1.000	0.877 IRS	31	0.320	0.342	0.933 IRS
13	0.995	0.996	0.999 IRS	32	0.457	0.467	0.980 IRS
14	1.000	1.000	1.000 CRS	33	1.000	1.000	1.000 CRS
15	0.837	1.000	0.837 DRS	34	0.464	0.506	0.917 DRS
16	0.828	0.954	0.867 DRS	35	0.468	0.531	0.881 DRS
17	0.554	0.763	0.726 IRS	36	1.000	1.000	1.000 CRS
18	0.958	0.981	0.977 IRS	37	0.487	0.500	0.974 DRS
19	0.769	0.911	0.844 DRS				

Note: CRS, IRS and DRS represent constant, increasing and decreasing returns to scale, respectively.

In the second stage, the optimal allocation ratios among different routes within each company are determined. For brevity, Table 4.5 only illustrates the detailed allocation ratios for the 17 routes operated by company 1.

Figure 4.2 displays the allocation ratios of inputs and shares of outputs for the 17 routes of company 1. The detailed allocation ratios for the routes operated by the remaining 16 companies are not presented here.

It should be noted that the number of buses is the only input variable that has both attributed and common parts. The route with low allocation ratio of buses in the common part is not necessarily associated with low allocation ratio of fuel cost or labor force because the route still has an attributed part—the buses exclusively used in that route. Our results indicate that the total number of buses of a route (including both attributed and allocated common buses) is in effect proportional to the fuel cost and labor force allocated. As shown in Table 4.5 and Figure 4.2, the allocation ratios of three inputs exhibit similar patterns to the shares of two outputs, suggesting that our proposed model tends to allocate larger amount of inputs to those routes with larger amount of outputs, such as routes 1, 4 and 12. This rationale is logical because the route with larger production generally requires more inputs. In other words, the proposed model will not allocate more inputs to lower productive routes. The correlation coefficients of allocated fuel cost and labor associated with allocated bus (combined with common and attributed parts) are 0.90 and 0.95, respectively, further suggesting reasonability of the determined allocation ratios—more buses used in a route requires more fuel cost and labor.

Table 4. 5 Optimal allocation ratios for the 17 routes operated by company 1

Route	Fuel cost	Labor	Bus		
			Common part	Attribute part	Total
1	14.28%	16.59%	11.03%	14.00%	13.68%
2	11.87%	9.48%	7.40%	8.69%	8.55%
3	0.07%	0.28%	2.01%	0.71%	0.85%
4	13.86%	11.98%	8.12%	10.67%	10.39%
5	0.07%	0.52%	1.81%	0.70%	0.82%
6	0.07%	0.58%	2.34%	0.71%	0.89%
7	0.07%	0.32%	1.78%	0.70%	0.82%
8	0.08%	3.16%	4.63%	0.78%	1.20%
9	9.13%	4.02%	5.51%	6.03%	5.97%
10	0.06%	0.24%	0.48%	0.67%	0.65%
11	0.07%	3.51%	4.57%	0.78%	1.19%
12	21.60%	25.79%	27.68%	37.30%	36.26%
13	9.39%	3.81%	3.15%	2.70%	2.75%
14	0.07%	3.98%	5.04%	0.80%	1.26%
15	10.38%	6.82%	4.01%	6.63%	6.35%
16	0.10%	4.98%	6.01%	3.43%	3.71%
17	8.83%	3.94%	4.43%	4.69%	4.66%
Total	100.00%	100.00%	100.00% (23 buses)	100.00% (189 buses)	100.00% (212 buses)

Note: The allocation ratio of attribute part of bus is computed according to the data (i.e. the number of buses exclusively used in the route) not determined by the model.

The optimal allocation ratios in Table 4.5 are representing the unique characteristics for each route. Take route 12 as an example, it is the most profitable route in company 1 and it contributed 31.5% to company operating revenue. Furthermore, this route has the highest allocation ratio of bus

(37.3%, which is the number of buses exclusively used in this route). These special characteristics can be reflected on the route properties which can be shown as follows.

1. The highest frequency in peak hour (headway is 5-10 minutes).
2. The lowest frequency in off-peak hour (headway is 40-50 minutes).
3. Short route length (The average route length for the 17 routes operated by company 1 is 28km and for route 12 is 20.1km).
4. Freeway route.

Because of these characteristics and properties, the proposed model allocated the largest bus number to route 12 automatically. Similar situation exists in route 16.

As for route 13, it only contributed 0.2% to company operating revenue. Besides, the route properties are special as well. It is a city bus with fixed schedule. It only has 16 runs per day with the longest route length (60.5 km). Base on these route characteristics and properties, this route has higher percentages of fuel but a tiny part of total buses.

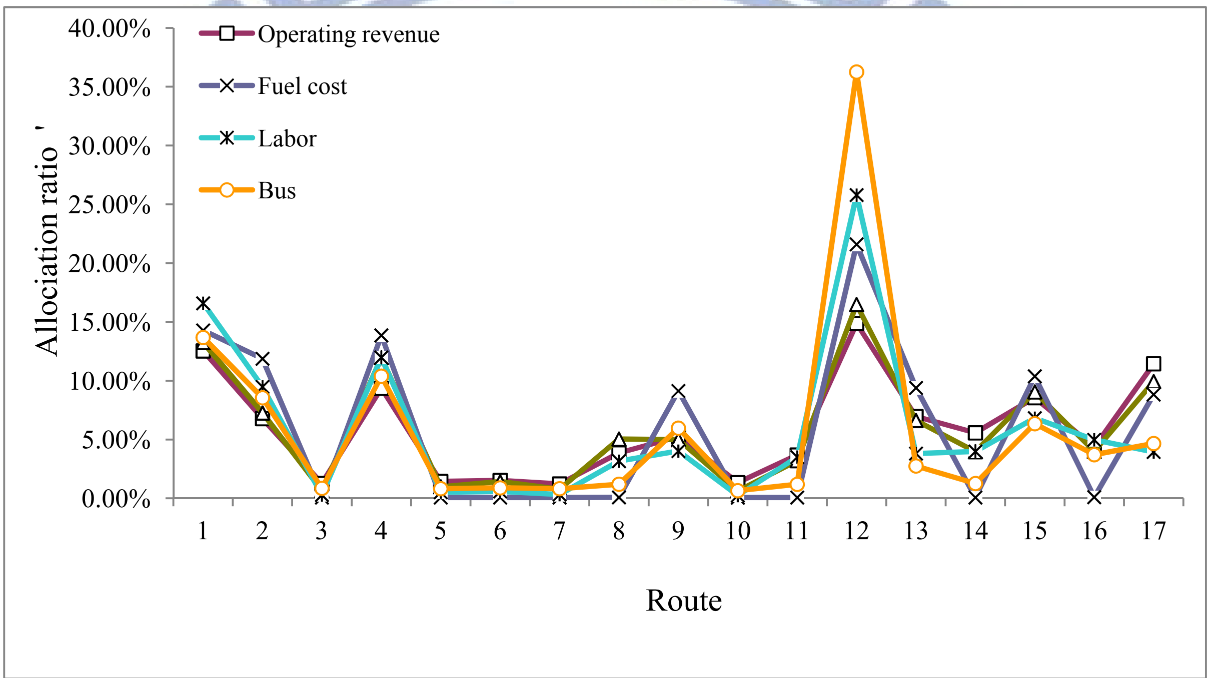

Figure 4. 2 Allocation ratios of inputs and shares of outputs for all routes of company 1

In the third stage, the proposed RCCR and RBCC models are used to determine the route-level efficiency for all routes, under CRS and VRS contexts, within an individual company and across all companies. For brevity, Table 4.6 only illustrates the results for the 17 routes operated by company 1. Details of the route-level efficiency scores for the remaining 16 companies are not presented here.

It is interesting to note from Table 4.4 that the results based on company-based DEA model have revealed that company 1 is in effect inefficient due to its overall scale of IRS. However, it does not mean that all of its subordinated routes require scaling up. By further looking into the details of the route efficiencies obtained from the RCCR and RBCC models (Table 4.6), we can scrutinize the insights: of the 17 routes, only twelve with IRS need to be scaled up; one with CRS should remain unchanged, and four with DRS even require downsizing. This evidence manifestly indicates the importance of jointly evaluating the company-level and route-level performance for the carriers at the same time. It would facilitate the managers to exercise more accurate tactics to improve the performance for the inefficient individual routes and for the whole company.

Table 4. 6 Efficiency scores for the 17 routes operated by company 1

Route	CRS	VRS	Scale	
1	0.596	0.600	0.993	DRS
2	0.557	0.561	0.993	IRS
3	0.233	0.656	0.355	IRS
4	0.605	0.613	0.987	IRS
5	0.302	0.486	0.621	IRS
6	0.356	0.625	0.570	IRS
7	0.235	0.676	0.348	IRS
8	0.800	0.801	0.999	IRS
9	0.426	0.437	0.975	IRS
10	0.613	1.000	0.613	IRS
11	0.897	0.911	0.985	DRS
12	0.385	0.455	0.846	DRS
13	0.662	0.701	0.944	IRS
14	0.960	0.998	0.962	DRS
15	0.503	0.516	0.975	IRS
16	1.000	1.000	1.000	CRS
17	0.408	0.454	0.899	IRS
Average	0.561	0.676	-	-

To propose the improvement tactics for the inefficient companies or inefficient routes, slack values for each of the input variables are computed. Taking company 1 as an example, the results are reported in Table 4.7. For those inputs (such as bus) that can be distinguished into attributed and common parts, two slack values will be generated; in contrast, for those inputs (such as fuel cost, labor) that cannot be separated from attributed to common part, the proposed model will determine an overall improvement for those inputs. For instance, route 9 has used too much input resource; one should reduce the fuel cost by 11.18%, labor force by 5.66%, and bus fleet by 17.47% (the attributed part takes 7.27%, while the common part takes 10.20%) so as to achieve the efficiency frontier.

Table 4. 7 Slack values for inputs of the 17 routes operated by company 1

Route	Fuel cost	Labor	Bus	
			Attributed part	Common part
1	12.42%	16.60%	12.05%	7.26%
2	11.33%	10.41%	8.19%	7.96%
3	0.05%	0.24%	0.49%	6.25%
4	11.66%	11.60%	8.88%	7.02%
5	0.08%	0.67%	0.74%	9.34%
6	0.06%	0.54%	1.08%	6.81%
7	0.05%	0.26%	0.46%	5.88%
8	0.03%	1.57%	2.00%	3.61%
9	11.18%	5.66%	7.27%	10.20%
10	0.00%	0.00%	0.00%	0.00%
11	0.01%	0.78%	0.89%	1.62%
12	25.60%	35.16%	43.78%	9.89%
13	6.11%	2.85%	1.72%	5.43%
14	0.00%	0.02%	0.02%	0.04%

15	10.93%	8.26%	6.94%	8.79%
16	0.00%	0.00%	0.00%	0.00%
17	10.48%	5.38%	5.48%	9.91%
Total	100.00%	100.00%	100.00%	100.00%

4.4 Discussion

To further identify the external factors affecting the route efficiency, a Tobit regression is conducted. We choose the following four factors as the explanatory variables: load factor (*LF*), subsidy from government (*SG*), freeway route (*FW*), and connection to major cities (*CM*). Where, *LF* is defined as seat-km/passenger-km. The route with higher *LF* is anticipated to have higher route efficiency (a positive sign is expected). *SG* is a binary variable representing whether the route is being subsidized by the government. If yes, *SG*=1; otherwise, *SG*=0 (a positive sign is expected). *FW* is a binary variable indicating that the route is operated on the freeways (*FW*=1) or on the ordinary surface roadways (*FW*=0). The freeway buses are more fuel efficient than those on the surface roadways (a positive sign is expected). *CM* is also a binary variable representing whether the route connects the major cities (If yes, *CM*=1; otherwise, *CM*=0). The five major cities in Taiwan include Taipei City, New Taipei City, Taichung City, Tainan City and Kaohsiung City, which cover approximately 27% of the total island area but inhabit about 60% of the total population. Generally, the bus routes connecting the populated areas can attract more patronage (a positive sign is expected).

Tobit model allows us to incorporate only one bound of the dependent variable while DEA efficiency score is constrained to fall between zero and one. Therefore, by taking the logarithm of the DEA efficiency scores, one could convert the dependent variable so that it has only one bound (Oum and Yu, 1994). For ease of interpretation, however, the signs of the regression coefficients are reported in accordance with the original form. By regressing the logarithm of route efficiency scores on the above four explanatory variables, the estimation result is shown below:

$$\underset{}{ln\ (Efficiency\ score)} = \underset{(9.791)}{0.0365} + \underset{(5.962)}{0.0867\ ln\ (LF)} + \underset{(2.572)}{0.0585\ SG} + \underset{(3.304)}{0.2899\ FW} + \underset{(7.173)}{0.2704\ CM} \tag{4.39}$$

R^2=0.7256

where *ln* (*Efficiency score)* denotes the logarithm of the route efficiency score. The *t*-values are given in parentheses. From Eq. (4.39), all estimated parameters are statistically significant with positive values as anticipated, suggesting that these variables have positive contributions to route efficiency. On average, one percent increased in load factor (*LF*) will lead to an increase of the route efficiency score by 0.0867%. The remaining three explanatory variables are binary. According to their associated estimated parameters, *FW* has the largest contribution to route efficiency, followed by *CM*, then by *SG*.

Basically, *FW* can be viewed as a proxy variable for better service quality in terms of speedy, smooth and reliable services. Therefore, to enhance the operation efficiency for the ordinary surface roadway routes, providing bus exclusive lanes with preemption signals in congested urbanized areas can be an effective strategy to improve the route efficiency. *CM* is a proxy variable for higher transportation demand. Thus, a concept of transit-oriented development (TOD) land use or traffic management would invite more public transport patronage. Meanwhile, the government should grant the carriers more concession to run the freeway routes connecting the major cities. Finally, *SG* represents the government financial subsidy. The result shows that government subsidy can raise the route efficiency but its effect is relatively small in comparison with both *FW* and *CM*.

4.5 Summary

This section has proposed two route-based DEA models, RCCR and RBCC, under CRS and VRS contexts. The proposed two novel models have contributed to the literature with several merits. First, the proposed RDEA models can jointly measure the route- and company-level efficiency at the same time, which is superior to the previous DEA modeling approaches. Next, we prove that the ranking order of company performance determined by the route-based DEA model is identical to that determined by the company-based DEA model, and this adds a significant contribution to the DEA theories. Third, the empirical study results supported the argument that an efficient carrier may operate some inefficient routes and that an inefficient carrier may run some efficient routes. Based on the empirical results, one can easily pinpoint the less efficient routes and/or less efficient companies and exercise more accurate improvement tactics. Fourth, the route-based allocation ratios of all common inputs are optimally determined without subjective conjectures. It also greatly contributes to the practices. Last, the Tobit regression results provide useful information to the regulation agencies for better decision making to help improve the carriers' efficiencies.

In sum, this chapter has remedied the first research gap by proposing a route-based DEA modeling. The next chapter would target on the second research gapby proposing an integrated fuzzy DEA modeling.

CHAPTER 5. INTEGRATED FUZZY DEA MODELS

Two basic IFDEA formulations, hereinafter termed as Integrated Fuzzy CCR (IFCCR) model and Integrated Fuzzy BCC (IFBCC) model, are developed in this chapter.

5.1 IFCCR model

Consider n DMUs to be evaluated, each DMU utilizes m inputs to produce s outputs, and some of the inputs and the outputs are characterized with fuzziness. To develop the IFCCR model, we first look into the fuzzy CCR ([FCCR]) model, which can be formulated as follows:

$$\text{[FCCR]} \qquad \underset{u,v}{Max} \quad \tilde{h}_q = \sum_{r=1}^{R} u_r \tilde{y}_{qr} \tag{5.1}$$

$$s.t. \quad \sum_{j=1}^{J} v_j \tilde{x}_{qj} = \tilde{1} \tag{5.2}$$

$$\sum_{r=1}^{R} u_r \tilde{y}_{ir} - \sum_{j=1}^{J} v_j \tilde{x}_{ij} \le \tilde{0}, \ i = 1,2,\cdots,I \tag{5.3}$$

$$u_r, v_j \ge \varepsilon > 0, \ r = 1,\cdots,R; \ j = 1,2,\cdots,J \tag{5.4}$$

where $\tilde{h}_q$ is the fuzzy efficiency score of DMU k. $\tilde{x}_{qj}$ is the fuzzy input j of DMU q. $\tilde{y}_{qr}$ is the fuzzy output r of DMU q. u_r and v_j are the multipliers corresponding to output r and input j, respectively. To solve [FCCR] problem, α-cut technique (Dubois and Prade, 1980) is adopted to convert associated fuzzy numbers into its crisp formulation. The α-cut of $\tilde{x}_{ij}$ and $\tilde{y}_{ir}$ are defined as follow.

$$\tilde{x}_{ij\alpha} = \left\{ x_{ij} \in S(\tilde{x}_{ij}) \middle| u_{\tilde{x}_{ij}}(x_{ij}) \ge \alpha \right\}, \forall i, j \tag{5.5}$$

$$\tilde{y}_{ir\alpha} = \left\{ y_{ir} \in S(\tilde{y}_{ir}) \middle| u_{\tilde{y}_{ir}}(y_{ir}) \ge \alpha \right\}, \forall i, r \tag{5.6}$$

where $u_{\tilde{x}_{ij}}$ and $u_{\tilde{y}_{ir}}$ are the membership functions of $\tilde{x}_{ij}$ and $\tilde{y}_{ir}$. $S(\tilde{x}_{ij})$ and $S(\tilde{y}_{ir})$ denote the support of $\tilde{x}_{ij}$ and $\tilde{y}_{ir}$. The α-cut of a fuzzy number is an interval number defined by lower-bound and upper-bound. That is, $\tilde{x}_{ij\alpha} = \left[x^L_{ij\alpha}, x^U_{ij\alpha}\right]$ and $\tilde{y}_{ir\alpha} = \left[y^L_{ir\alpha}, y^U_{ir\alpha}\right]$ under α-cut level, where $x^L_{ij\alpha}$, $x^U_{ij\alpha}$ and $y^L_{ir\alpha}, y^U_{ir\alpha}$ respectively denote the lower-bound and upper-bound of $\tilde{x}_{ij\alpha}$ and $\tilde{y}_{ir\alpha}$.

Without loss of generality, the values of all inputs and outputs can be regarded as fuzzy numbers because any crisp value can be represented by a degenerated membership function, which has only one value in its domain. Hence, previous relevant works formulated the FCCR model in two separated crisp CCR models can be respectively associated with lower-bound and upper-bound. However, as demonstrated by the above illustration, this may lead to inconsistent evaluation results. Our proposed IFDEA model, therefore, combines both lower-bound and upper-bound into a single model as depicted in Figure 5.1 and Figure 5.2 where five DMUs (A, B, C, D and E) have been considered. For simplicity, each DMU here is assumed using two inputs to produce one output. Figure 5.1 demonstrates the projection of membership function for DMU B. Assuming the efficiency frontier is formed by DMUs A, C, D and E. Under a specific α-cut level, the lower-, center-, and upper-bound efficiency frontiers are respectively denoted as F^L_α, F^C_α, and F^U_α as shown in Figure 5.2. The range between F^L_α and F^U_α represents the bandwidth of the efficiency frontiers. In order to integrate the lower- and upper-bound efficiency frontiers, a preference weight is further introduced to generate a weighted efficiency frontier; the crisp efficiency can therefore be determined by the IFCCR model, explained as follows.

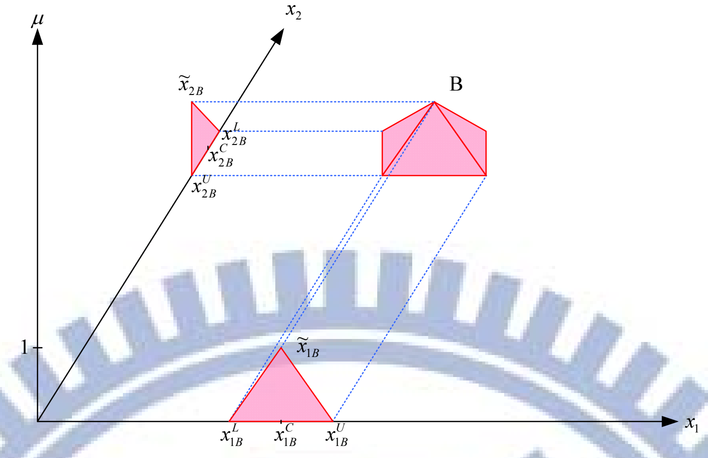

Figure 5. 1 Projection of membership function of DMU *B*

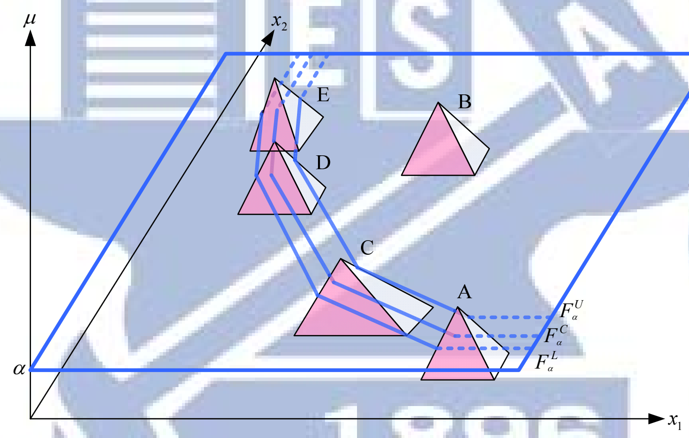

Figure 5. 2 Efficiency frontier formed by DMUs *A, C, D, E*

To maximize Eq. (5.1) is equivalent to simultaneously maximize the summed lower-bound ($\sum_{r=1}^{R} u_r y_{ir\alpha}^L$) and the summed upper-bound ($\sum_{r=1}^{R} u_r y_{ir\alpha}^U$), as depicted in Figure 5.3 and expressed below:

$$\underset{u,v}{Max}\ (\tilde{h}_q)_\alpha = \underset{u,v}{Max} \sum_{r=1}^{R} u_r \left[y_{qr\alpha}^L, y_{qr\alpha}^U \right] = \begin{cases} \underset{u,v}{Max} \sum_{r=1}^{R} u_r y_{qr\alpha}^L \\ \underset{u,v}{Max} \sum_{r=1}^{R} u_r y_{qr\alpha}^U \end{cases} \tag{5.7}$$

In order to combine the objection function, the preference weight β is introduced. The preference weights β is the weight of lower-bound under certain α-cut of $\tilde{y}_{qr}$. Taking β =1 is regarded as a pessimistic opinion of maximizing $\tilde{y}_{qr}$ because the worst situation is considered, whereas taking β =0 is regarded as an optimistic opinion because the best situation is concerned with. Furthermore, the sum of the preference weights must be 1 in order to ensure the optimal combination value of lower- and upper-bound for $\tilde{y}_{qr\alpha}$ would be located between $y_{qr\alpha}^L$ and $y_{qr\alpha}^U$. By introducing a preference weight, β ($0 \le \beta \le 1$), Eq. (5.7) can be converted into a single objective function:

$$\underset{u,v}{Max}\ \left\{\sum_{r=1}^{R} u_r \beta y_{qr\alpha}^{L} + \sum_{r=1}^{R} u_r (1-\beta) y_{qr\alpha}^{U}\right\} = \underset{u,v}{Max}\ \left\{\sum_{r=1}^{R} u_{r1} y_{qr\alpha}^{L} + \sum_{r=1}^{R} u_{r2} y_{qr\alpha}^{U}\right\} \quad (5.8)$$

where $u_{r1}=u_r\beta$ and $u_{r2}=u_r(1-\beta)$. Since $0 \le \beta \le 1$ and $u_r \ge 0$, both u_{r1} and u_{r2} are non-negative.

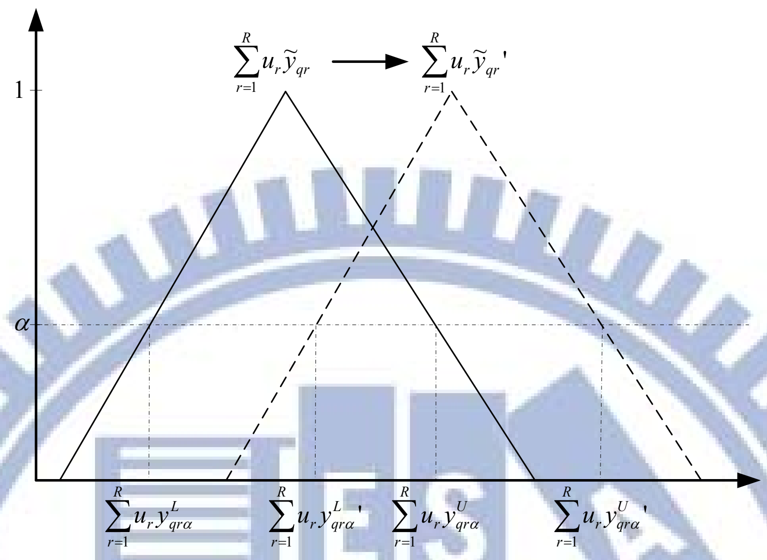

Figure 5. 3 The summed fuzzy output of DMU k towards maximization

Similarly, by substituting the α-cut interval number into Eq. (5.2), we obtain an equivalent but crisp constraint as:

$$\sum_{j=1}^{J} v_j \tilde{x}_{qj} = \sum_{j=1}^{J} v_j \left[x_{qj\alpha}^{L}, x_{qj\alpha}^{U}\right] = \tilde{1} \quad (5.9)$$

where, $\tilde{1}$ represents a fuzzy number distributed within proximity of 1. The constraint of $\sum_{j=1}^{J} v_j \tilde{x}_{qj}$ equal to $\tilde{1}$ indicates that the range between the summed lower-bound ($\sum_{j=1}^{J} v_j x_{qj\alpha}^{L}$) and the summed upper-bound ($\sum_{j=1}^{J} v_j x_{qj\alpha}^{U}$) should contain the value of 1. Hence, Eq. (5.9) can be expressed by two inequalities:

$$\sum_{j=1}^{J} v_j x_{qj\alpha}^{L} \le 1 \quad (5.10)$$

$$\sum_{j=1}^{J} v_j x_{qj\alpha}^{U} \ge 1 \quad (5.11)$$

Following the same vein in converting the objective function, a preference weight variable γ ($0 \le \gamma \le 1$) is introduced to integrate both Eq. (5.10) and Eq. (5.11) into one equation as below:

$$\sum_{j=1}^{J} v_j \gamma x_{qj\alpha}^{L} + \sum_{j=1}^{J} v_j (1-\gamma) x_{qj\alpha}^{U} = 1 \quad (5.12)$$

Let $v_{j1} = v_j \gamma$ and $v_{j2} = v_j(1-\gamma)$, Eq. (5.12) can be rewritten as:

$$\sum_{j=1}^{J} v_{j1} x_{qj\alpha}^{L} + \sum_{j=1}^{J} v_{j2} x_{qj\alpha}^{U} = 1 \quad (5.13)$$

Both v_{j1} and v_{j2} are non-negative because $v_j \geq 0$ and $0 \leq \gamma \leq 1$.

By substituting α-cut interval numbers of inputs and outputs into Eq. (5.3), the constraint Eq. (5.3) can be expressed as:

$$\sum_{r=1}^{R} u_r \left[y_{ir\alpha}^{L}, y_{ir\alpha}^{U}\right] - \sum_{j=1}^{J} v_j \left[x_{ij\alpha}^{L}, x_{ij\alpha}^{U}\right] \leq \tilde{0} \tag{5.14}$$

Using the addition operation of interval numbers, Eq. (5.14) can further be expressed as:

$$\left[\sum_{r=1}^{R} u_r y_{ir\alpha}^{L}, \sum_{r=1}^{R} u_r y_{ir\alpha}^{U}\right] - \left[\sum_{j=1}^{J} v_j x_{ij\alpha}^{L}, \sum_{j=1}^{J} v_j x_{ij\alpha}^{U}\right] \leq \tilde{0} \tag{5.15}$$

Note that the left-hand side of Eq. (5.15) is a minus of two interval numbers. To satisfy an interval number always smaller than the other, we let any arbitrary value in the former interval number be smaller than that in the latter interval number. That is:

$$\left(\beta \sum_{r=1}^{R} u_r y_{ir\alpha}^{L} + (1-\beta) \sum_{r=1}^{R} u_r y_{ir\alpha}^{U}\right) \leq \left(\gamma \sum_{j=1}^{J} v_j x_{ij\alpha}^{L} + (1-\gamma) \sum_{j=1}^{J} v_j x_{ij\alpha}^{U}\right) \tag{5.16}$$

Eq. (5.16) can therefore be expressed as:

$$\left(\sum_{r=1}^{R} u_{r1} y_{ir\alpha}^{L} + \sum_{r=1}^{R} u_{r2} y_{ir\alpha}^{U}\right) \leq \left(\sum_{j=1}^{J} v_{j1} x_{ij\alpha}^{L} + \sum_{j=1}^{J} v_{j2} x_{ij\alpha}^{U}\right) \tag{5.17}$$

With Eq. (5.8), Eq. (5.13), and Eq. (5.17), the above [FCCR] model can be easily transformed into our proposed [IFCCR] model as follows:

$$\text{[IFCCR]} \quad \underset{u,v}{Max} \quad (\tilde{h}_q)_\alpha = \left\{\sum_{r=1}^{R} u_{r1} y_{qr\alpha}^{L} + \sum_{r=1}^{R} u_{r2} y_{qr\alpha}^{U}\right\} \tag{5.18}$$

$$s.t. \quad \sum_{j=1}^{J} v_{j1} x_{qj\alpha}^{L} + \sum_{j=1}^{J} v_{j2} x_{qj\alpha}^{U} = 1 \tag{5.19}$$

$$\left(\sum_{r=1}^{R} u_{r1} y_{ir\alpha}^{L} + \sum_{r=1}^{R} u_{r2} y_{ir\alpha}^{U}\right) \leq \left(\sum_{j=1}^{J} v_{j1} x_{ij\alpha}^{L} + \sum_{j=1}^{J} v_{j2} x_{ij\alpha}^{U}\right) \tag{5.20}$$

$$u_{r1}, u_{r2}, v_{j1}, v_{j2} \geq 0,\ j = 1, \cdots J; r = 1, \cdots, R; i = 1, \cdots, I \tag{5.21}$$

where $h_{k\alpha}$ represents the crisp efficiency score of DMU *k*. If h_k equals 1, the DMU is regarded as relatively efficient; otherwise, it is relatively inefficient. The variables $u_{r1}, u_{r2}, v_{i1}, v_{i2}$ are the corresponding virtual multipliers of the r^{th} output and the i^{th} input. *n, m* and *s* denote the number of DMUs, inputs and outputs, respectively.

The dual form of our proposed [IFCCR] model can be expressed as follows:

$$\text{[IFCCR-D]} \quad \underset{\theta,\lambda}{Min} \quad \theta - \varepsilon\left(\sum_{j=1}^{J} s_{j1}^{-} + \sum_{j=1}^{J} s_{j2}^{-} + \sum_{r=1}^{R} s_{r1}^{+} + \sum_{r=1}^{R} s_{r2}^{+}\right) \tag{5.22}$$

$$\text{s.t.} \quad \theta x_{qj\alpha}^{L} - \sum_{i=1}^{I} \lambda_i x_{ij\alpha}^{L} - s_{j1}^{-} = 0 \tag{5.23}$$

$$\theta x_{qj\alpha}^{U} - \sum_{i=1}^{I} \lambda_i x_{ij\alpha}^{U} - s_{j2}^{-} = 0 \tag{5.24}$$

$$\sum_{i=1}^{I} \lambda_i y_{ir\alpha}^{L} - y_{qr\alpha}^{L} - s_{r1}^{+} = 0 \tag{5.25}$$

$$\sum_{i=1}^{I} \lambda_i y_{ir\alpha}^{U} - y_{qr\alpha}^{U} - s_{r1}^{+} = 0 \tag{5.26}$$

$$\lambda_i, s_{j1}^-, s_{j2}^-, s_{r1}^+, s_{r2}^+ \geq 0, j = 1, \cdots, J; r = 1, \cdots, R; i = 1, \cdots, I \tag{5.27}$$

$$\theta \text{ unrestricted in sign} \tag{5.28}$$

where θ represents the efficiency score of DMU q. If θ equals 1, the DMU is regarded as relatively efficient; otherwise, it is relatively inefficient. λ_i is the influence from DMU j. $s_{i_1}^-, s_{i_2}^-$ are slack variables of the j^{th} input and $s_{r_1}^+, s_{r_2}^+$ are slack variables of the r^{th} output for lower-bound and upper-bound corresponding to a specific α-cut, respectively.

5.2 IFBCC Model

Following the above [IFCCR-D] procedures, the [IFBCC] model for VRS technology can be easily derived by simply adding a convexity constraint. The dual form of the proposed [IFBCC] model can be expressed as follows:

$$\text{[IFBCC-D]} \quad \underset{\theta,\lambda}{Min} \quad \theta - \varepsilon\left(\sum_{j=1}^{J} s_{j1}^- + \sum_{j=1}^{J} s_{j2}^- + \sum_{r=1}^{R} s_{r1}^+ + \sum_{r=1}^{R} s_{r2}^+\right) \tag{5.29}$$

$$\text{s.t.} \quad \theta x_{qj\alpha}^L - \sum_{i=1}^{I} \lambda_i x_{ij\alpha}^L - s_{j1}^- = 0 \tag{5.30}$$

$$\theta x_{qj\alpha}^U - \sum_{i=1}^{I} \lambda_i x_{ij\alpha}^U - s_{j2}^- = 0 \tag{5.31}$$

$$\sum_{i=1}^{I} \lambda_i y_{ir\alpha}^L - y_{qr\alpha}^L - s_{r1}^+ = 0 \tag{5.32}$$

$$\sum_{i=1}^{I} \lambda_i y_{ir\alpha}^U - y_{qr\alpha}^U - s_{r1}^+ = 0 \tag{5.33}$$

$$\sum_{i=1}^{I} \lambda_i = 1 \tag{5.34}$$

$$\lambda_i, s_{j1}^-, s_{j2}^-, s_{r1}^+, s_{r2}^+ \geq 0, j = 1, \cdots, J; r = 1, \cdots, R; i = 1, \cdots, I \tag{5.35}$$

$$\theta \quad \text{unrestricted in sign} \tag{5.36}$$

5.3 Efficiency and Slack Analyses

5.3.1. Technical Efficiency

The crisp efficiency score for each DMU can be determined by the proposed [IFCCR-D] and [IFBCC-D] models. Three types of efficiency scores are addressed below.

(1) If $\theta_q^* < 1$, DMU q is defined as relatively inefficient. Eq. (5.23) and Eq. (5.24) show that $\sum_{i=1}^{I} \lambda_i x_{ij\alpha}^L + s_{j1}^- = \theta x_{qj\alpha}^L < x_{qj\alpha}^L$ and $\sum_{i=1}^{I} \lambda_i x_{ij\alpha}^U + s_{j2}^- = \theta x_{qj\alpha}^U < x_{qj\alpha}^U$, suggesting that DMU q needs to reduce some amount of input so as to achieve the efficiency frontier (e.g., DMU B in Figure 5.1).

(2) If $\theta_q^* = 1$ and $s_{j1}^-, s_{j2}^-, s_{r1}^+, s_{r2}^+$ are not all equal to zero, DMU q is defined having radical efficiency. If $\theta_q^* = 1$ and $s_{j1}^- \neq 0$, Eq. (5.23) and Eq. (5.24) show that $\sum_{i=1}^{I} \lambda_i x_{ij\alpha}^L + s_{j1}^- = x_{qj\alpha}^L$ and

$\sum_{i=1}^{I}\lambda_i x_{ij\alpha}^U = x_{qj\alpha}^U$, suggesting that the lower-bound of input j of DMU q is larger than the weighted lower-bound of input j of DMUs on the efficiency frontier. If $\theta_q^* = 1$ and $s_{j2}^- \neq 0$, Eq. (5.23) and Eq. (5.24) show that $\sum_{i=1}^{I}\lambda_i x_{ij\alpha}^L = x_{qj\alpha}^L$ and $\sum_{i=1}^{I}\lambda_i x_{ij\alpha}^U + s_{j2}^- = x_{qj\alpha}^U$, suggesting that the upper-bound of input j of DMU q is larger than the weighted upper-bound of input j of DMUs on the efficiency frontier. If $\theta_q^* = 1$ and $s_{r1}^+ \neq 0$, Eq. (5.25) shows that $\sum_{i=1}^{I}\lambda_i y_{ir\alpha}^L > y_{qr\alpha}^L$, suggesting that the lower-bound of output r of DMU q is less than the weighted lower-bound of output r of DMUs on the efficiency frontier. If $\theta_q^* = 1$ and $s_{r2}^+ \neq 0$, Eq. (5.26) show that $\sum_{i=1}^{I}\lambda_i y_{ir\alpha}^U > y_{qr\alpha}^U$, suggesting that the upper-bound of output r of DMU q is less than the weighted upper-bound of output r of DMUs on the efficiency frontier. These DMUs are defined as relatively inefficient (e.g., DMUs A and E in Figure 5.1).

(3) If $\theta_q^* = 1$ and $s_{j1}^-, s_{j2}^-, s_{r1}^+, s_{r2}^+$ are all equal to zero, DMU q is defined as relatively efficient. Eq. (5.23) through Eq. (5.26) show that $\sum_{i=1}^{I}\lambda_i x_{ij\alpha}^L = x_{qj\alpha}^L$, $\sum_{i=1}^{I}\lambda_i x_{ij\alpha}^U = x_{qj\alpha}^U$, $\sum_{i=1}^{I}\lambda_i y_{ir\alpha}^L = y_{qr\alpha}^L$, and $\sum_{i=1}^{I}\lambda_i y_{ir\alpha}^U = y_{qr\alpha}^U$, suggesting that the lower-bound and upper-bound of fuzzy inputs and outputs of DMU k are equal to the weighted lower-bound and upper-bound of inputs and outputs of DMUs on the efficiency frontier. There is no need to do any improvement. Such DMUs are defined as relatively efficient (e.g., DMUs C and D in Figure 5.1).

5.3.2 Scale Efficiency

To tackle with both crisp and fuzzy data, the above [IFBCC-D] model can be further transformed into the following [IFBCC-D*] model, where Eq. (5.38) through Eq. (5.41) are for fuzzy data, and Eq. (5.43) and Eq. (5.44) are for crisp data.

[IFBCC-D*] $$\underset{\theta,\lambda}{Min}\ \theta - \varepsilon\left(\sum_{j=1}^{J}s_{j1}^- + \sum_{j=1}^{J}s_{j2}^- + \sum_{r=1}^{R}s_{r1}^+ + \sum_{r=1}^{R}s_{r2}^+\right) \quad (5.37)$$

$$s.t.\quad \theta x_{qj\alpha}^L - \sum_{i=1}^{I}\lambda_i x_{ij\alpha}^L - s_{j1}^- = 0 \quad (5.38)$$

$$\theta x_{qj\alpha}^U - \sum_{i=1}^{I}\lambda_i x_{ij\alpha}^U - s_{j2}^- = 0 \quad (5.39)$$

$$\sum_{i=1}^{I}\lambda_i y_{ir\alpha}^L - y_{qr\alpha}^L - s_{r1}^+ = 0 \quad (5.40)$$

$$\sum_{i=1}^{I}\lambda_i y_{ir\alpha}^U - y_{qr\alpha}^U - s_{r1}^+ = 0 \quad (5.41)$$

$$\sum_{i=1}^{I}\lambda_i = 1 \quad (5.42)$$

$$\theta x_{qj} - \sum_{i=1}^{I} \lambda_i x_{ij} - s_j^- = 0 \quad (5.43)$$

$$\sum_{i=1}^{I} \lambda_i y_{ir} - y_{qr} - s_r^+ = 0 \quad (5.44)$$

$$\lambda_i, s_{j1}^-, s_{j2}^-, s_j^-, s_{r1}^+, s_{r2}^+, s_r^+ \geq 0, j = 1,\cdots,J; r = 1,\cdots,R; i = 1,\cdots,I \quad (5.45)$$

$$\theta \quad \text{unrestricted in sign} \quad (5.46)$$

The DMU with $\sum_{i=1}^{I} \lambda_i < 1$ determined by the [IFCCR-D*] model is referred as IRS; if $\sum_{i=1}^{I} \lambda_i > 1$, the DMU is DRS; if $\sum_{i=1}^{I} \lambda_i = 1$, it is CRS.

5.3.3 Slack Analysis

Improvement strategies for any of the inefficient DMUs can be proposed based on slack analysis. For inefficient DMU k, its fuzzy input and output under α-cut can be expressed as $\left(\left[x_{ij\alpha}^L, x_{ij\alpha}^U\right], \left[y_{ir\alpha}^L, y_{ir\alpha}^U\right]\right)$. If the efficiency score and optimal multipliers of DMU q are $\theta^*, \lambda_i^*, s_{j1}^{-*}, s_{j2}^{-*}, s_{r1}^{+*}, s_{r2}^{+*}$, the shadows of DMU q on the efficiency frontier are:

$$x_{ij\alpha}^{L*} = \theta^* x_{qj\alpha}^L - s_{i1}^{-*} \quad (5.47)$$

$$x_{ij\alpha}^{U*} = \theta^* x_{qj\alpha}^U - s_{i2}^{-*} \quad (5.48)$$

$$y_{ir\alpha}^{L*} = y_{qr\alpha}^L + s_{r1}^{+*} \quad (5.49)$$

$$y_{ir\alpha}^{U*} = y_{qr\alpha}^U + s_{r2}^{+*} \quad (5.50)$$

The DMUs with $\lambda_i^* \neq 0$ determined by the [IFBCC-D*] model form a reference set—the efficiency frontier of DMU q. The coordinates of these benchmarking DMUs are denoted as:

$$\left(\left[\sum_{i=1}^{I} \lambda_i^* x_{ij\alpha}^L, \sum_{i=1}^{I} \lambda_i^* x_{ij\alpha}^U\right], \left[\sum_{i=1}^{I} \lambda_i^* y_{ir\alpha}^L, \sum_{i=1}^{I} \lambda_i^* y_{ir\alpha}^U\right]\right) \quad (5.51)$$

From Eq. (5.47) through Eq. (5.50), the slack values of DMU k can be expressed as follows.

$$\Delta x_{qj\alpha}^L = x_{qj\alpha}^L - x_{ij\alpha}^{L*} \quad (5.52)$$

$$\Delta x_{qj\alpha}^U = x_{qj\alpha}^U - x_{ij\alpha}^{U*} \quad (5.53)$$

$$\Delta y_{qr\alpha}^L = y_{ir\alpha}^{L*} - y_{qr\alpha}^L \quad (5.54)$$

$$\Delta y_{qr\alpha}^U = y_{ir\alpha}^{U*} - y_{qr\alpha}^U \quad (5.55)$$

where $\Delta x_{qj\alpha}^L$ and $\Delta x_{qj\alpha}^U$ are the slack values of the lower-bound and upper-bound of input j of DMU q, respectively. $\Delta y_{qr\alpha}^L$ and $\Delta y_{qr\alpha}^U$ are the slack values of the lower-bound and upper-bound of output j of DMU q, respectively.

5.4 Numerical Example

5.4.1 Comparison with existent FDEA model

To demonstrate the applicability and superiority of our proposed IFDEA models, a comparison with the FDEA model using the same numerical data proposed by León's *et al*. (2003) is conducted. Table 5.1 presents the same numerical data.

Table 5. 1 Input and output data from León *et al*. (2003)

DMU	x	y
A	$\tilde{3} = (3, 2)$	$\tilde{3} = (3, 1)$
B	$\tilde{4} = (4, 0.5)$	$\tilde{2.5} = (2.5, 1)$
C	$\tilde{4.5} = (4.5, 1.5)$	$\tilde{6} = (6, 1)$
D	$\tilde{6.5} = (6.5, 0.5)$	$\tilde{4} = (4, 1.25)$
E	$\tilde{7} = (7, 2)$	$\tilde{5} = (5, 0.5)$
F	$\tilde{8} = (8, 0.5)$	$\tilde{3.5} = (3.5, 0.5)$
G	$\tilde{10} = (10, 1.0)$	$\tilde{6} = (6, 0.5)$
H	$\tilde{6} = (6, 0.5)$	$\tilde{2} = (2, 1.5)$

Table 5.2 presents the efficiency scores under CRS determined by the proposed [IFCCR] model under various α-cuts. As noted from Table 5.2, only DMU C is benchmarked as efficient by the proposed [IFCCR] model under all α-cuts and DMU A is evaluated as efficient for $\alpha \leq 0.5$.

Table 5. 2 Efficiency scores under various α-cuts determined by the IFCCR model

α-cut	DMU							
	A	B	C	D	E	F	G	H
0.0	1.0000	0.6667	1.0000	0.6429	0.6000	0.4235	0.6000	0.4615
0.1	1.0000	0.6478	1.0000	0.6252	0.5931	0.4140	0.5841	0.4403
0.2	1.0000	0.6287	1.0000	0.6074	0.5863	0.4045	0.5684	0.4191
0.3	1.0000	0.6094	1.0000	0.5895	0.5797	0.3950	0.5529	0.3979
0.4	1.0000	0.5899	1.0000	0.5715	0.5732	0.3855	0.5377	0.3766
0.5	1.0000	0.5701	1.0000	0.5534	0.5668	0.3760	0.5227	0.3554
0.6	0.9418	0.5502	1.0000	0.5352	0.5604	0.3665	0.5079	0.3342
0.7	0.8839	0.5301	1.0000	0.5169	0.5542	0.3570	0.4932	0.3130
0.8	0.8337	0.5098	1.0000	0.4985	0.5480	0.3474	0.4787	0.2919
0.9	0.7895	0.4894	1.0000	0.4801	0.5418	0.3378	0.4643	0.2709
1.0	0.7500	0.4688	1.0000	0.4615	0.5357	0.3281	0.4500	0.2500

By using the proposed [IFBCC] model and León's *et al.* model, the efficiency scores under VRS are presented in Tables 5.3 and 5.4. As to León's *et al.* model (Table 5.3), two DMUs (A and C) are evaluated as efficient under all α-cuts. Furthermore, G and B become efficient as $\alpha \leq 0.9$ and $\alpha \leq 0.3$, respectively. From Tables 5.3 and 5.4, one can discover that the evaluation scores of the [IFBCC] model (Table 5.4) are almost the same as those of the León's *et al.* model (Table 5.3). Besides, the comparison between DMU efficiency, which gets from the [IFBCC] model, under specific α-cuts could be determined easily and without any ranking method. Take DMUs D and E as an example. E has better performance than D for $\alpha \geq 0.8$ but worse performance then D for $\alpha < 0.8$.

Table 5. 3 Efficiency scores under various *α*-cuts determined by the León's *et al.* model

α-cut	DMU							
	A	B	C	D	E	F	G	H
0.0	1.0000	1.0000	1.0000	0.7500	0.6429	0.6050	1.0000	0.6923
0.1	1.0000	1.0000	1.0000	0.7399	0.6398	0.5952	1.0000	0.6899
0.2	1.0000	1.0000	1.0000	0.7292	0.6369	0.5857	1.0000	0.6875
0.3	1.0000	1.0000	1.0000	0.7084	0.6310	0.5660	1.0000	0.6850
0.4	1.0000	0.9767	1.0000	0.6853	0.6244	0.5446	1.0000	0.6667
0.5	1.0000	0.9412	1.0000	0.6623	0.6172	0.5227	1.0000	0.6400
0.6	1.0000	0.9048	1.0000	0.6383	0.6094	0.5004	1.0000	0.6129
0.7	1.0000	0.8675	1.0000	0.6144	0.6010	0.4776	1.0000	0.5854
0.8	1.0000	0.8293	1.0000	0.5894	0.5919	0.4543	1.0000	0.5574
0.9	1.0000	0.7901	1.0000	0.5645	0.5821	0.4305	1.0000	0.5289
1.0	1.0000	0.7500	1.0000	0.5385	0.5714	0.4062	0.4500	0.5000

Table 5. 4 Efficiency scores under various α-cuts determined by the proposed [IFBCC] model

α-cut	DMU							
	A	B	C	D	E	F	G	H
0.0	1.0000	1.0000	1.0000	0.7500	0.6429	0.6050	1.0000	0.6923
0.1	1.0000	1.0000	1.0000	0.7396	0.6398	0.5953	1.0000	0.6899
0.2	1.0000	1.0000	1.0000	0.7292	0.6369	0.5857	1.0000	0.6875
0.3	1.0000	1.0000	1.0000	0.7081	0.6311	0.5660	1.0000	0.6850
0.4	1.0000	0.9767	1.0000	0.6853	0.6244	0.5446	1.0000	0.6667
0.5	1.0000	0.9412	1.0000	0.6620	0.6172	0.5227	1.0000	0.6400
0.6	1.0000	0.9048	1.0000	0.6383	0.6094	0.5004	1.0000	0.6129
0.7	1.0000	0.8675	1.0000	0.6141	0.6010	0.4776	1.0000	0.5854
0.8	1.0000	0.8293	1.0000	0.5894	0.5919	0.4543	1.0000	0.5574
0.9	1.0000	0.7901	1.0000	0.5642	0.5821	0.4305	1.0000	0.5289
1.0	1.0000	0.7500	1.0000	0.5385	0.5714	0.4063	0.4500	0.5000

Although, our proposed model and León's model have similar results, the proposed model could do further analyses, such as scale and slack analyses. By using the proposed [IFBCC] model, the corresponding scale efficiency scores are presented in Table 5.5. In term of scale efficiency scores in Table 5.5, except that DMU *G* is characterized with DRS for $\alpha \leq 0.9$ and DMUs *A* and *C* are characterized with CRS for $\alpha \leq 0.5$ and for all *α*-cuts, respectively, other DMUs are all characterized with IRS for all *α*-cuts, including *B*, *D*, *E*, *F*, and *H*, suggesting most of the DMUs need to expand their scales.

Table 5. 5 Scale efficiency scores under various α-cuts determined by the [IFDEA] model

α-cut	DMU															
	A		B		C		D		E		F		G		H	
0.0	1.00	CRS	0.50	IRS	1.00	CRS	0.75	IRS	0.90	IRS	0.60	IRS	1.10	DRS	0.50	IRS
0.1	1.00	CRS	0.49	IRS	1.00	CRS	0.74	IRS	0.89	IRS	0.60	IRS	1.09	DRS	0.49	IRS
0.2	1.00	CRS	0.49	IRS	1.00	CRS	0.74	IRS	0.88	IRS	0.60	IRS	1.08	DRS	0.47	IRS
0.3	1.00	CRS	0.48	IRS	1.00	CRS	0.73	IRS	0.88	IRS	0.59	IRS	1.07	DRS	0.46	IRS
0.4	1.00	CRS	0.47	IRS	1.00	CRS	0.72	IRS	0.87	IRS	0.59	IRS	1.06	DRS	0.44	IRS
0.5	1.00	CRS	0.46	IRS	1.00	CRS	0.71	IRS	0.86	IRS	0.59	IRS	1.05	DRS	0.42	IRS
0.6	0.53	IRS	0.45	IRS	1.00	CRS	0.70	IRS	0.86	IRS	0.59	IRS	1.04	DRS	0.41	IRS
0.7	0.52	IRS	0.44	IRS	1.00	CRS	0.69	IRS	0.85	IRS	0.59	IRS	1.03	DRS	0.31	IRS
0.8	0.52	IRS	0.44	IRS	1.00	CRS	0.69	IRS	0.84	IRS	0.59	IRS	1.02	DRS	0.37	IRS
0.9	0.51	IRS	0.43	IRS	1.00	CRS	0.68	IRS	0.84	IRS	0.58	IRS	1.01	DRS	0.35	IRS
1.0	0.50	IRS	0.42	IRS	1.00	CRS	0.67	IRS	0.83	IRS	0.58	IRS	1.00	CRS	0.33	IRS

5.4.2 Slack analysis

Slack values of each input variable provide useful information for proposing improvement strategies for the inefficient DMUs. Since the input variable of the numerical example is fuzzy, two slack values are respectively determined for its lower- and upper-bound under various α-cuts. The results are shown in Tables 5.6 and 5.7. It should be noted that for α=1.0 (representing a crisp input data), the slack values for lower- and upper-bound will be the same. From Tables 5.6 and 5.7, except for the efficient DMUs (*A* and *C* for all α-cuts; B for $\alpha \le 0.3$; *G* for $\alpha \le 0.9$), all inefficient DMUs are required to reduce input amounts to achieve efficiency frontier. Taking DMU *D* as an example, one requires decreasing input amounts by 1.50 to 3.00 for the lower-bound and by 1.75 to 3.00 for the upper-bound. By considering all required reductions in lower-bound and upper-bound under various α-cuts, the fuzzy input for DMU *D* has to decrease to the value of $\tilde{3} = (3, 0.375)$ to achieve efficiency, suggesting that both the cortex and the spread of fuzzy input should be simultaneously decreased.

Table 5. 6 Slack values for the lower-bound of input variable under various α-cuts

α-cut	DMU							
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
0.0	0.0000	0.0000	0.0000	1.5000	1.7857	3.0557	0.0000	1.4109
0.1	0.0000	0.0000	0.0000	1.5756	1.8732	3.0557	0.0000	1.4109
0.2	0.0000	0.0000	0.0000	1.6519	1.9609	3.1486	0.0000	1.4375
0.3	0.0000	0.0000	0.0000	1.7952	2.0661	3.3203	0.0000	1.4646
0.4	0.0000	0.0860	0.0000	1.9512	2.1785	3.5067	0.0000	1.5667
0.5	0.0000	0.2206	0.0000	2.1123	2.2969	3.6989	0.0000	1.7100
0.6	0.0000	0.3619	0.0000	2.2787	2.4217	3.8968	0.0000	1.8581
0.7	0.0000	0.5102	0.0000	2.4505	2.5537	4.1008	0.0000	2.0110
0.8	0.0000	0.6659	0.0000	2.6279	2.6935	4.3109	0.0000	2.1689
0.9	0.0000	0.8290	0.0000	2.8110	2.8420	4.5272	0.0000	2.3318
1.0	0.0000	1.0000	0.0000	3.0000	3.0000	4.7500	5.5000	2.5000

Table 5. 7 Slack values for the upper-bound of input variable under various α-cuts

α-cut	DMU							
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
0.0	0.0000	0.0000	0.0000	1.7500	3.2143	3.3571	0.0000	1.6923
0.1	0.0000	0.0000	0.0000	1.8100	3.1700	3.4200	0.0000	1.6899
0.2	0.0000	0.0000	0.0000	1.8686	3.1229	3.4800	0.0000	1.6875
0.3	0.0000	0.0000	0.0000	1.9996	3.0992	3.6242	0.0000	1.6850
0.4	0.0000	0.1000	0.0000	2.1400	3.0800	3.7800	0.0000	1.7667
0.5	0.0000	0.2500	0.0000	2.2813	3.0625	3.9375	0.0000	1.8900
0.6	0.0000	0.4000	0.0000	2.4233	3.0467	4.0967	0.0000	2.0129
0.7	0.0000	0.5500	0.0000	2.5663	3.0325	4.2575	0.0000	2.1354
0.8	0.0000	0.7000	0.0000	2.7100	3.0200	4.4200	0.0000	2.2574
0.9	0.0000	0.8500	0.0000	2.8546	3.0092	4.5842	0.0000	2.3789
1.0	0.0000	1.0000	0.0000	3.0000	3.0000	4.7500	5.5000	2.5000

5.5 An Empirical Study

To demonstrate the applicability of the proposed IFDEA models, an empirical study on 35 intercity bus companies in Taiwan is conducted. The data and evaluation results are delineated below.

5.5.1 Data

The crisp data base is the same as the case study in RDEA models. However, referring to previous relevant literature (e.g., Gillen and Lall, 1997a,b; Lan and Lin, 2005; Chiou and Chen, 2006; Bhadra, 2009; Greer, 2009; Lin *et al*., 2010), this section selects different variables to verify the IFDEA models. The selected variables are number of buses, number of employees, length of operating network, capital cost and fuel cost as the input variables and total passenger-km, total bus-km, total revenue, and passenger satisfaction as the output variables. Note that passenger satisfaction is the only fuzzy qualitative variable in this section, which is obtained from a questionnaire survey conducted by Ministry of Transportation and Communications (Taiwan) in 2005 while evaluating the performance of intercity bus carriers. The remaining quantitative variables are all crisp and they are available from the annual report published by the Institute of Transportation, Ministry of Transportation and Communications (Taiwan) in 2005. Because the passenger satisfaction survey only conduct for 35 intercity bus companies, the data we use in this section will only contain 35 intercity bus companies rather than 37 in chapter 4.

In order to quantify the importance and relevance of the selected variables, the correlation coefficients analyses and regression analyses have been conducted. Table 5.8 gives the correlation coefficients among crisp variables. All correlation coefficients between input and output variables are significantly positive, confirming that the dataset satisfies the isotonicity property. To ensure the selected input/output variables important and relevant, regression analyses are further conducted and Table 5.9 presents the results. Note that all the explanatory variables show positive and significant effects on at least one of the associated dependent variables, suggesting the appropriateness of the above selected variables.

The fuzzy variable, passenger satisfaction, is represented by three linguistic degrees: poor service, fair service and good service, with half-overlapped triangular membership functions. The original data of fuzzy variable is shown in Table 5.10.

Table 5. 8 Correlation coefficients among crisp input and output variables

Variable	Input					Output		
	Bus	Labor	Operating network	Capital cost	Fuel cost	Bus-km	Passenger -km	Revenue
Bus	1.00							
Labor	0.95	1.00						
Operating network	0.52	0.61	1.00					
Capital cost	0.53	0.51	0.25	1.00				
Fuel cost	0.90	0.96	0.54	0.52	1.00			
Bus-km	0.84	0.90	0.39	0.58	0.96	1.00		
Passenger-km	0.72	0.81	0.43	0.54	0.91	0.96	1.00	
Revenue	0.94	0.98	0.55	0.52	0.98	0.95	0.87	1.00

Table 5. 9 Regression results for input and output variables

Dependent variables	Independent variables				
	Bus	Labor	Operating network	Capital cost	Fuel cost
Bus-km	26991.389 (8.351)	7304.43 (2.409)	3872.509 (3.500)	0.015 (2.801)	0.217 (7.700) R^2 =0.979
Passenger-km	790437.011 (2.385)	693665.200 (1.395)	27678.15 (3.885)	0.258 (2.730)	4.842 (6.014) R^2 =0.921
Revenue	551132.550 (3.245)	127018.628 (2.421)	25300.793 (4.042)	0.015 (3.132)	2.524 (4.041) R^2 =0.970

Note: t values in parentheses.

Table 5. 10 Passenger satisfaction for 35 intercity bus companies

DMU	Passenger satisfaction	DMU	Passenger satisfaction	DMU	Passenger satisfaction
1	Fair service	13	Fair service	25	Fair service
2	Fair service	14	Fair service	26	Fair service
3	Fair service	15	Good service	27	Poor service
4	Poor service	16	Fair service	28	Good service
5	Poor service	17	Poor service	29	Fair service
6	Poor service	18	Poor service	30	Poor service
7	Fair service	19	Poor service	31	Fair service
8	Fair service	20	Poor service	32	Fair service
9	Fair service	21	Poor service	33	Fair service
10	Fair service	22	Fair service	34	Fair service
11	Poor service	23	Fair service	35	Poor service
12	Poor service	24	Poor service		

5.5.2 Efficiency scores

Table 5.11 presents the efficiency scores of the bus companies under CRS and VRS technologies, respectively. From Table 5.11, a total of 16 and 21 companies are benchmarked as efficient with [IFCCR] and [IFBCC] models, respectively. It is interesting to note that the efficiency scores do not vary much with different α-cuts. Similar to the evaluation results of the numerical example presented in Section 5.4, the efficiency scores of inefficient bus companies increase as the α-cut goes higher. Table 5.12 further gives the scale efficiency scores of these bus companies. We note that most of the bus companies are characterized with DRS, implying the necessity to downsize their scale. Only three bus companies are characterized with IRS, suggesting that they have the advantages to scale up.

Table 5. 11 Efficiency scores of 35 intercity bus companies under various α-cuts

DMU	CRS				VRS			
	α=0.0	α=0.4	α=0.8	α=1.0	α=0.0	α=0.4	α=0.8	α=1.0
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	0.4640	0.4640	0.4640	0.4640	0.4645	0.4645	0.4645	0.4645
5	0.5436	0.5436	0.5436	0.5436	0.6753	0.6753	0.6753	0.6753
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	0.8904	0.8904	0.8903	0.8902	0.9452	0.9452	0.9452	0.9452
8	0.6915	0.6915	0.6915	0.6915	0.8702	0.8702	0.8702	0.8702
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	0.5613	0.5613	0.5613	0.5613	0.8262	0.8262	0.8262	0.8262
12	0.9468	0.9468	0.9468	0.9468	0.9842	0.9842	0.9842	0.9842
13	0.6669	0.6668	0.6667	0.6666	0.7942	0.7942	0.7942	0.7942
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	0.5843	0.5842	0.5842	0.5842	0.7826	0.7826	0.7826	0.7826
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
22	0.6075	0.6075	0.6075	0.6075	0.7943	0.7943	0.7943	0.7943
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
24	0.9127	0.9127	0.9127	0.9127	1.0000	1.0000	1.0000	1.0000
25	0.7768	0.7768	0.7768	0.7768	1.0000	1.0000	1.0000	1.0000
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
27	0.5784	0.5783	0.5783	0.5782	0.5813	0.5813	0.5813	0.5813
28	0.4789	0.4789	0.4789	0.4789	1.0000	1.0000	1.0000	1.0000
29	0.5457	0.5457	0.5457	0.5457	0.5571	0.5571	0.5571	0.5571
30	0.8027	0.8027	0.8027	0.8027	0.9213	0.9213	0.9213	0.9213
31	0.8051	0.8051	0.8051	0.8051	1.0000	1.0000	1.0000	1.0000
32	0.9978	0.9978	0.9977	0.9977	1.0000	1.0000	1.0000	1.0000
33	0.8003	0.8003	0.8003	0.8003	1.0000	1.0000	1.0000	1.0000
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
35	0.4704	0.4703	0.4702	0.4701	0.4759	0.4759	0.4759	0.4759

Table 5. 12 Scale efficiency scores of 35 intercity bus companies under various α-cuts

DMU	α-cut							
	0.0		0.4		0.8		1.0	
1	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
2	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
3	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
4	0.9167	IRS	0.9148	IRS	0.9128	IRS	0.9118	IRS
5	1.5088	DRS	1.5088	DRS	1.5088	DRS	1.5088	DRS
6	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
7	1.0190	DRS	1.0185	DRS	1.0181	DRS	1.0179	DRS
8	2.0600	DRS	2.0600	DRS	2.0600	DRS	2.0600	DRS
9	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
10	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
11	1.1935	DRS	2.2882	DRS	2.2882	DRS	2.2882	DRS
12	1.7514	DRS	1.7514	DRS	1.7514	DRS	1.7514	DRS
13	1.1128	DRS	1.1102	DRS	1.1077	DRS	1.1065	DRS
14	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
15	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
16	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
17	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
18	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
19	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
20	1.1211	DRS	1.1181	DRS	1.1151	DRS	1.1137	DRS
21	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
22	1.0693	DRS	1.0676	DRS	1.0660	DRS	1.0652	DRS
23	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
24	4.4237	DRS	4.4237	DRS	4.4237	DRS	4.4237	DRS
25	3.5736	DRS	3.5736	DRS	3.5736	DRS	3.5736	DRS
26	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
27	0.9416	IRS	0.9403	IRS	0.9389	IRS	0.9382	IRS
28	1.3082	DRS	1.3082	DRS	1.3082	DRS	1.3082	DRS
29	1.3515	DRS	1.3515	DRS	1.3515	DRS	1.3515	DRS
30	1.6929	DRS	1.6929	DRS	1.6929	DRS	1.6929	DRS
31	1.2837	DRS	1.2837	DRS	1.2837	DRS	1.2837	DRS
32	1.0099	DRS	1.0097	DRS	1.0095	DRS	1.0094	DRS
33	2.7128	DRS	2.7128	DRS	2.7128	DRS	2.7128	DRS
34	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
35	0.9564	IRS	0.9554	IRS	0.9544	IRS	0.9539	IRS

5.5.3 Slack analysis

To propose improvement directions for the inefficient bus companies, slack values for the input variables are computed. Table 5.13 gives the slack values for the input variables under α=0.8. From Table 5.13, it is found that for the inefficient bus companies the percentages of input amounts reduction can range from 4.62% to 94.88%. Taking Company 11 as an example, Table 5.13 suggests that reducing the fleet size by 40.06%, the labor workforce by 43.03%, the operating network by 55.90%, the capital by 91.45%, and the fuel by 47.18% will move the company towards efficiency.

Table 5. 13 Slack values of input variables for 35 intercity bus companies (α=0.8)

DMU	Bus	Labor	Operating network	Capital cost	Fuel cost
1	0.00%	0.00%	0.00%	0.00%	0.00%
2	0.00%	0.00%	0.00%	0.00%	0.00%
3	0.00%	0.00%	0.00%	0.00%	0.00%
4	60.75%	63.10%	63.89%	71.11%	66.15%
5	46.12%	44.16%	36.83%	58.23%	45.18%
6	0.00%	0.00%	0.00%	0.00%	0.00%
7	37.00%	34.70%	4.62%	14.42%	63.19%
8	16.61%	15.40%	43.03%	67.43%	22.97%
9	0.00%	0.00%	0.00%	0.00%	0.00%
10	0.00%	0.00%	0.00%	0.00%	0.00%
11	40.06%	43.03%	55.90%	91.45%	47.18%
12	29.68%	23.37%	39.25%	81.48%	12.67%
13	27.83%	4.73%	5.48%	40.32%	16.15%
14	0.00%	0.00%	0.00%	0.00%	0.00%
15	0.00%	0.00%	0.00%	0.00%	0.00%
16	0.00%	0.00%	0.00%	0.00%	0.00%
17	0.00%	0.00%	0.00%	0.00%	0.00%
18	0.00%	0.00%	0.00%	0.00%	0.00%
19	0.00%	0.00%	0.00%	0.00%	0.00%
20	41.04%	39.79%	27.09%	9.09%	40.15%
21	0.00%	0.00%	0.00%	0.00%	0.00%
22	47.57%	39.01%	28.18%	52.64%	41.57%
23	0.00%	0.00%	0.00%	0.00%	0.00%
24	0.00%	0.00%	0.00%	0.00%	0.00%
25	0.00%	0.00%	0.00%	0.00%	0.00%
26	0.00%	0.00%	0.00%	0.00%	0.00%
27	50.43%	68.37%	68.60%	79.64%	73.01%
28	0.00%	0.00%	0.00%	0.00%	0.00%
29	68.51%	64.02%	89.35%	94.88%	51.98%
30	42.72%	42.26%	54.25%	48.53%	30.97%
31	0.00%	0.00%	0.00%	0.00%	0.00%
32	0.00%	0.00%	0.00%	0.00%	0.00%
33	0.00%	0.00%	0.00%	0.00%	0.00%
34	0.00%	0.00%	0.00%	0.00%	0.00%
35	66.77%	58.66%	82.03%	71.32%	70.72%

5.5.4 Discussion

The major merit of the proposed IFDEA models is to integrate the lower- and upper-bound efficiency frontiers to generate a crisp efficiency value. With the determined crisp efficiency frontier, the scale efficiency scores and the slack values for DMUs can be easily computed. As such, and the improvement directions for the inefficient DMUs can be clearly identified.

To further highlight the advantages of the proposed [IFCCR] model, a comparison with the FDEA models proposed by Kao and Liu (2000) is conducted. Table 5.14 presents the slack values of input variables for DMU 13 determined by the [IFCCR] model. Figure 5.4 further compares the efficiency scores for DMU 13 by the FDEA model (Kao and Liu, 2000) and by the proposed [IFCCR] model. We note that the efficiency value for DMU 13 decreases as α gets larger, showing that the proposed [IFCCR] model computes lower efficiency value with higher α value (i.e. more pessimistic than FDEA). The proposed [IFCCR] model becomes a crisp model and shows DMU 13 being inefficient as α=1. From Figure 5.4, it is apparent that the results of [IFCCR] model lie between lower- and upper-efficiency frontiers, which are in effect derived from two CDEA models (Kao and Liu, 2000). In contrast, the proposed [IFCCR] model has reasonably integrated the lower- and upper-efficiency frontiers.

Table 5. 14 Slack values of input variables for DMU 13 by the [IFCCR] model

α value	Bus	Labor	Operating network	Capital cost	Fuel cost
0.0	31.57%	10.09%	5.66%	44.71%	22.95%
0.4	32.18%	10.97%	5.69%	45.43%	24.06%
0.8	32.79%	11.85%	5.72%	46.14%	25.17%
1.0	33.41%	12.73%	5.74%	46.86%	26.29%

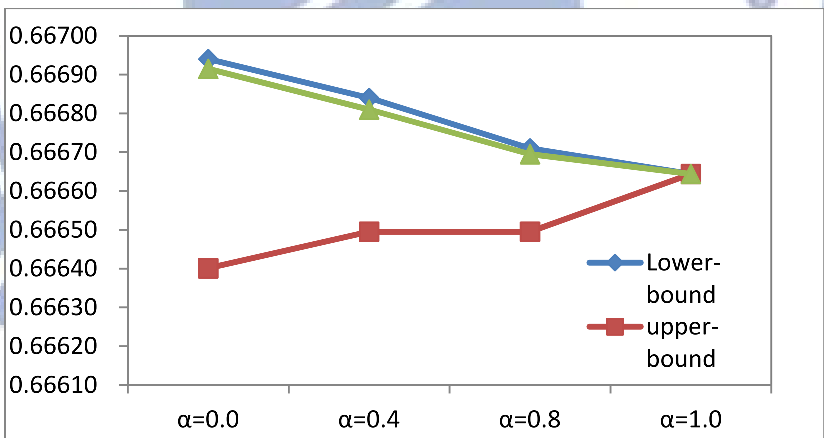

Figure 5. 4 Efficiency scores for DMU 13 by the FDEA model (Kao and Liu, 2000) and by the [IFCCR] model

Figure 5.5 further displays the slack values of input variables for the inefficient DMU 13 under different α values by the [IFCCR] model. When α value becomes larger, DMU 13 requires curtailing more amounts of its inputs. Hence, a pessimistic decision maker may choose a larger α value by which the inefficient DMUs will be improved more remarkably, and vice versa. With this procedure, the decision maker can easily determine how to improve the inefficient DMUs' performance in a context containing crisp and fuzzy input/output measures. With flexible settings of α values, the proposed IFDEA models can facilitate the managers to make more flexible and correct decisions, based on informative and useful evaluation results.

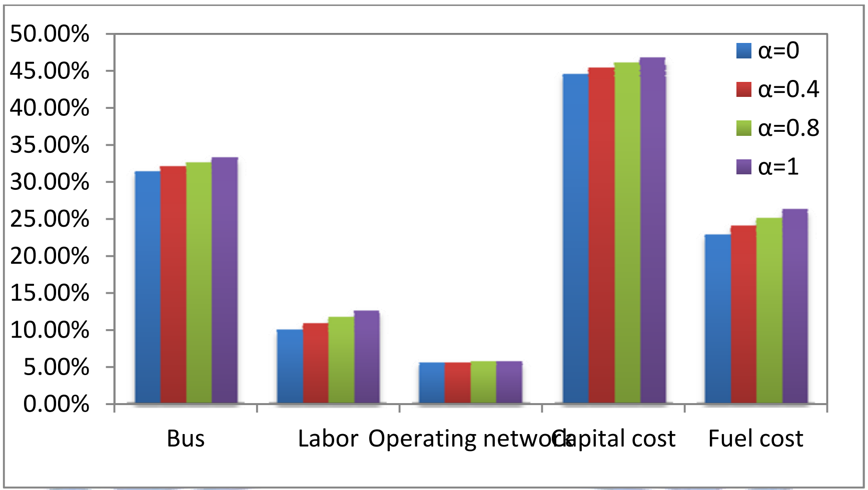

Figure 5. 5 Slack values of input variables for DMU 13 under different α values by the [IFCCR] model

5.6 Summary

Previous FDEA models have separately determined the lower- and upper-bound efficiency scores under various α-cut levels by using subjective ranking methods to find the crisp evaluation results. This can often lead to unreasonable frontiers—with lower-bound efficiency scores greater than upper-bound efficiency scores. This study contributes two IFDEA models (IFCCR and IFBCC) that have successfully overcome this problem. The proposed IFDEA models can determine crisp evaluation scores under various α-cuts with CRS and VRS technologies. In addition, the proposed IFDEA models can easily determine the slack values for both lower- and upper-bound input/output variables simultaneously. With the computed slack values under various α-cut levels, the associated fuzzy values for input variables can be determined to achieve efficiency. The numerical example has illustrated that the proposed IFDEA models are more generalized and with greater simplicity than an existent FDEA model. The case study has also demonstrated the proposed IFDEA modeling approach can satisfactorily evaluate the relative efficiency for DMUs with a portion of qualitative variables measured with vagueness.

In sum, this chapter has remedied the second research gap by constructing an integrated fuzzy DEA modeling. The next chapter would propose an integrated route-based fuzzy DEA modeling to cope with both research gaps 1 and 2.

CHAPTER 6. INTEGRATED ROUTE-BASED FUZZY DEA MODELS

This chapter proposes two similar integrated route-based fuzzy DEA (IRFDEA) models under CRS and VRS contexts, termed as integrated route-based fuzzy CCR (IRFCCR) model and integrated route-based fuzzy BCC (IRFBCC) model, respectively. The proposed IRFDEA models can both consider the importance of route performance and qualitative variables when evaluate the performance in transport field.

6.1 IRFCCR model

The first stage uses the following integrated company-based fuzzy CCR model to determine a set of optimal input/output multipliers:

$$\text{[CFCCR]} \quad \underset{u,v}{Max}\ \tilde{h}_q = \sum_{r=1}^{R} u_r \tilde{y}_{qr} \tag{6.1}$$

$$s.t.\ \sum_{j=1}^{J} v_j \tilde{x}_{qj} = \tilde{1} \tag{6.2}$$

$$\sum_{r=1}^{R} u_r \tilde{y}_{ir} - \sum_{j=1}^{J} v_j \tilde{x}_{ij} \le \tilde{0},\ \ i = 1,2,\cdots,I \tag{6.3}$$

$$v_j \ge 0,\ \ j = 1,2,\cdots,J \tag{6.4}$$

$$u_r \ge 0,\ \ r = 1,2,\cdots,R \tag{6.5}$$

where $\tilde{h}_q$ is the fuzzy efficiency score of company q. If $\tilde{h}_q = \tilde{1}$, the DMU is defined relatively efficient; otherwise the DMU is relatively inefficient. X_{qj} represents the j^{th} input of DMU q. y_{qr} denotes the r^{th} output of DMU q. The variables v_j, u_r are corresponding virtual multipliers of the j^{th} input, and the r^{th} output. J and R are the number of companies' inputs and outputs, respectively.

To solve [CFCCR] problem, following the same vein in converting the IFDEA model in chapter 5. The [CFCCR] model can be easily transformed into our proposed [ICFCCR] model as follows:

$$\text{[ICFCCR]} \quad \underset{u_{r1},u_{r2},v_{j1},v_{j2}}{Max}\ \left(\tilde{h}_q\right)_\alpha = \sum_{r=1}^{R} u_{r1} y_{qr\alpha}^{L} + \sum_{r=1}^{R} u_{r2} y_{qr\alpha}^{U} \tag{6.6}$$

$$s.t.\ \sum_{j=1}^{J} v_{j1} x_{iq\alpha}^{L} + \sum_{j=1}^{J} v_{j2} x_{iq\alpha}^{U} = 1 \tag{6.7}$$

$$\left(\sum_{r=1}^{R} u_{r1} y_{ir\alpha}^{L} + \sum_{r=1}^{R} u_{r2} y_{ir\alpha}^{U}\right) \le \left(\sum_{j=1}^{J} v_{j1} x_{ij\alpha}^{L} + \sum_{j=1}^{J} v_{j2} x_{ij\alpha}^{U}\right) \tag{6.8}$$

$$v_{j1}, v_{j2} \ge 0,\ \ j = 1,2,\cdots,J \tag{6.9}$$

$$u_{r1}, u_{r2} \ge 0,\ \ r = 1,2,\cdots,R \tag{6.10}$$

The second stage then uses the solved multipliers in the first stage to determine its optimal allocation ratios for the common inputs among the routes within a company to maximize the efficiency of all routes. According to attributed inputs can be identified or not, two models are developed in determining the allocation ratio: [AR1] and [AR2]. [AR1] model is for the case when all the route attributed inputs cannot be identified, whereas [AR2] model is for the case when a portion of the route attributed inputs can be identified.

The [AR1] model is expressed as follows:

$$[\text{AR1}]\quad \underset{s_{lj}^i}{Max}\quad h_i=\frac{1}{L_i}\left[\sum_{l=1}^{L_i}\left(\frac{\sum_{r=1}^{R}u_{r1}(y_{lr}^i)_\alpha^L+\sum_{r=1}^{R}u_{r2}(y_{lr}^i)_\alpha^U}{\sum_{j=1}^{J}v_{j1}s_{lj}^i(x_{lj}^i)_\alpha^L+\sum_{j=1}^{J}v_{j2}s_{lj}^i(x_{lj}^i)_\alpha^U}\right)\right] \tag{6.11}$$

$$s.t.\quad \left(\sum_{r=1}^{R}u_{r1}(y_{lr}^i)_\alpha^L+\sum_{r=1}^{R}u_{r2}(y_{lr}^i)_\alpha^U\right)\le\left(\sum_{j=1}^{J}v_{j1}s_{lj}^i(x_{lj}^i)_\alpha^L+\sum_{j=1}^{J}v_{j2}s_{lj}^i(x_{lj}^i)_\alpha^U\right),\ l=1,2,\cdots,L_i \tag{6.12}$$

$$\sum_{l=1}^{L_i}s_{lj}^i=1,\ j=1,2,\cdots,J \tag{6.13}$$

$$s_{lj}^i\ge 0,\ l=1,2,\cdots,L_i;\ j=1,2,\cdots,J \tag{6.14}$$

where h_i is the average of efficiency scores for all routes of company i which operates totally L_i routes and each route utilizes J kinds of inputs, produces R kinds of outputs. u_r, and v_j are the multipliers determined by the [ICFCCR] model. Besides, under α-cut level, where $\left[(x_{lj}^i)_\alpha^L,(x_{lj}^i)_\alpha^U\right]$ and $\left[(y_{lr}^i)_\alpha^L,(y_{lr}^i)_\alpha^U\right]$ respectively denote the lower- and upper-bound of routes inputs ($\tilde{x}_{lj}^i$) and outputs ($\tilde{y}_{lr}^i$). Furthermore, each output is assumed route attributable. Since all the route attributed inputs cannot be identified, the inputs ($(x_{lj}^i)_\alpha^L$ and $(x_{lj}^i)_\alpha^U$) to be allocated is based on the optimally solved ratio (s_{lj}^i)—an allocation ratio of route l for input j of company i. Eq. (6.14) ensures that each common input is completely allocated to all routes.

On the other hand, the [AR2] model is expressed as follows:

$$[\text{AR2}]\quad \underset{s_{lj}^i}{Max}\quad h_i=\frac{1}{L_i}\left[\sum_{l=1}^{L_i}\left(\frac{\sum_{r=1}^{R}u_{r1}(y_{lr}^i)_\alpha^L+\sum_{r=1}^{R}u_{r2}(y_{lr}^i)_\alpha^U}{\sum_{j=1}^{J}v_{j1}((x_{lj}^i)_\alpha^L+s_{lj}^i(x_{cj}^i)_\alpha^L)+\sum_{j=1}^{J}v_{j2}((x_{lj}^i)_\alpha^U+s_{lj}^i(x_{cj}^i)_\alpha^U)}\right)\right] \tag{6.15}$$

$$s.t.\quad \sum_{r=1}^{R}u_{r1}(y_{lr}^i)_\alpha^L+\sum_{r=1}^{R}u_{r2}(y_{lr}^i)_\alpha^L\le\sum_{j=1}^{J}v_{j1}((x_{lj}^i)_\alpha^L+s_{lj}^i(x_{cj}^i)_\alpha^L)+\sum_{j=1}^{J}v_{j2}((x_{lj}^i)_\alpha^U+s_{lj}^i(x_{cj}^i)_\alpha^U),\ l=1,2,\cdots,L_i \tag{6.16}$$

$$\sum_{l=1}^{L_i}s_{lj}^i=1,\quad j=1,2,\cdots,J \tag{6.17}$$

$$s_{lj}^i>0,\quad l=1,2,\cdots,L_i;\quad j=1,2,\cdots,J \tag{6.18}$$

where the input j of company i is divided into two parts: the attributable part ($(x_{lj}^i)_\alpha^L$ & $(x_{lj}^i)_\alpha^U$) and the common part $((x_{cj}^i)_\alpha^L \& (x_{cj}^i)_\alpha^U)$; $(x_j^i)_\alpha^L = \sum_{l=1}^{L_i}(x_{lj}^i)_\alpha^L + (x_{cj}^i)_\alpha^L$ and $(x_j^i)_\alpha^U = \sum_{l=1}^{L_i}(x_{lj}^i)_\alpha^U + (x_{cj}^i)_\alpha^U$. Only the common part $((x_{cj}^i)_\alpha^L \& (x_{cj}^i)_\alpha^U)$ requires an optimally solved allocation ratio (s_{lj}^i) to assign to route l. To determine the optimal allocation ratio of common input, however, only the routes operated by the same company are considered. With the optimal allocation ratios (s_{lj}^i), the inputs of route l under evaluation can be computed by $(x_{lj}^i)_\alpha^L = (x_{lj}^i)_\alpha^L + s_{lj}^i (x_{cj}^i)_\alpha^L$ and $(x_{lj}^i)_\alpha^U = (x_{lj}^i)_\alpha^U + s_{lj}^i (x_{cj}^i)_\alpha^U$.

Finally, based on the computed inputs, the third stage is to optimally determine the route efficiency by treating each route (could be operated by different companies) as a DMU, expressed as follows.

$$\text{[IRFCCR]} \quad \underset{u_{lr1}^i, u_{lr2}^i}{Max} \; h_k^i = \sum_{r=1}^{R} u_{kr1}^i (y_{kr}^i)_\alpha^L + \sum_{r=1}^{R} u_{kr2}^i (y_{kr}^i)_\alpha^U \tag{6.19}$$

$$s.t. \quad \sum_{j=1}^{J} v_{kj1}^i (x_{kj}^i)_\alpha^L + \sum_{j=1}^{J} v_{kj2}^i (x_{kj}^i)_\alpha^U = 1 \tag{6.20}$$

$$\sum_{r=1}^{R} u_{lr1}^i (y_{lr}^i)_\alpha^L + \sum_{r=1}^{R} u_{lr2}^i (y_{lr}^i)_\alpha^U \le \sum_{j=1}^{J} v_{lj1}^i (x_{lj}^i)_\alpha^L + \sum_{j=1}^{J} v_{lj2}^i (x_{lj}^i)_\alpha^U, i = 1,2,\cdots,I \tag{6.21}$$

$$v_{kj1}^i, v_{kj2}^i \ge 0, j = 1,2,\cdots,J; l = 1,2,\cdots,L \tag{6.22}$$

$$u_{lr1}^i, u_{lr2}^i \ge 0, r = 1,2,\cdots,R; l = 1,2,\cdots,L \tag{6.23}$$

where h_k^i is the efficiency score of route k operated by company i. $u_{lr1}^i \& u_{lr2}^i$ and $v_{kj1}^i \& v_{kj2}^i$ are the multipliers corresponding to the lower- and upper-bound of output r and input j for route l operated by company i, respectively. There are a total of L routes under evaluation, $L=L_1+L_2+\ldots+L_I$. Unlike [AR1] and [AR2] models where the route sequence is ordered only within the same company, the route sequence of [IRFCCR] here is ordered among all routes across all companies.

6.2 IRFBCC model

Following the same vein of the above modeling procedures, the [IRFBCC] model simply adds the convexity constraint. In the first stage, the following integrated company-based FBCC model is used to determine the optimal multipliers.

$$\text{[ICFBCC]} \quad \underset{u,v,w}{Max} \; \left(\tilde{h}_q\right)_\alpha = \sum_{r=1}^{R} u_{r1} y_{qr\alpha}^L + \sum_{r=1}^{R} u_{r2} y_{qr\alpha}^U - u \tag{6.24}$$

$$s.t. \quad \sum_{j=1}^{J} v_{j1} x_{iq\alpha}^L + \sum_{j=1}^{J} v_{j2} x_{iq\alpha}^U = 1 \tag{6.25}$$

$$\left(\sum_{r=1}^{R} u_{r1} y_{ir\alpha}^L + \sum_{r=1}^{R} u_{r2} y_{ir\alpha}^U - u\right) \le \left(\sum_{j=1}^{J} v_{j1} x_{ij\alpha}^L + \sum_{j=1}^{J} v_{j2} x_{ij\alpha}^U\right), \quad i = 1,2,\cdots,I \tag{6.26}$$

$$v_{j1}, v_{j2} \ge 0, \quad j = 1,2,\cdots,J \tag{6.27}$$

$$u_{r1}, u_{r2} \ge 0, \quad r = 1,2,\cdots,R \tag{6.28}$$

where u is efficiency scale for efficiency of company q. In the second stage, the corresponding allocation ratio models can be expressed as follows.

$$\text{[AR1']} \quad \underset{s}{Max} \quad h_i = \frac{1}{L_i}\left[\sum_{l=1}^{L_i}\left(\frac{\sum_{r=1}^{R}u_{r1}(y_{lr}^i)_\alpha^L + \sum_{r=1}^{R}u_{r2}(y_{lr}^i)_\alpha^U - u}{\sum_{j=1}^{J}v_{j1}s_{lj}^i(x_{lj}^i)_\alpha^L + \sum_{j=1}^{J}v_{j2}s_{lj}^i(x_{lj}^i)_\alpha^U}\right)\right] \tag{6.29}$$

$$s.t. \quad \left(\sum_{r=1}^{R}u_{r1}y_{qr\alpha}^{Li} + \sum_{r=1}^{R}u_{r2}y_{qr\alpha}^{Ui} - u\right) \le \left(\sum_{j=1}^{J}v_{j1}s_{lj}^i x_{qj\alpha}^L + \sum_{j=1}^{J}v_{j2}s_{lj}^i x_{qj\alpha}^U\right), \quad l = 1,2,\cdots,L_i \tag{6.30}$$

$$\sum_{l=1}^{L_i}s_{lj}^i = 1, \quad j = 1,2,\cdots,J \tag{6.31}$$

$$s_{lj}^i \ge 0, \quad l = 1,2,\cdots,L_i \;\; ; \;\; j = 1,2,\cdots,J \tag{6.32}$$

and

$$\text{[AR2']} \quad \underset{s}{Max} \quad h_i = \frac{1}{L_i}\left[\sum_{l=1}^{L_i}\left(\frac{\sum_{r=1}^{R}u_{r1}(y_{lr}^i)_\alpha^L + \sum_{r=1}^{R}u_{r2}(y_{lr}^i)_\alpha^U - u}{\sum_{j=1}^{J}v_{j1}((x_{lj}^i)_\alpha^L + s_{lj}^i(x_{cj}^i)_\alpha^L) + \sum_{j=1}^{J}v_{j2}((x_{lj}^i)_\alpha^U + s_{lj}^i(x_{cj}^i)_\alpha^U)}\right)\right] \tag{6.33}$$

$$s.t. \quad \sum_{r=1}^{R}u_{r1}(y_{lr}^i)_\alpha^L + \sum_{r=1}^{R}u_{r2}(y_{lr}^i)_\alpha^U - u \le \sum_{j=1}^{J}v_{j1}((x_{lj}^i)_\alpha^L + s_{lj}^i(x_{cj}^i)_\alpha^L) + \sum_{j=1}^{J}v_{j2}((x_{lj}^i)_\alpha^U + s_{lj}^i(x_{cj}^i)_\alpha^U),$$
$$l = 1,2,\cdots,L_i \tag{6.34}$$

$$\sum_{l=1}^{L_i}s_{lj}^i = 1, \quad j = 1,2,\cdots,J \tag{6.35}$$

$$s_{lj}^i > 0, \quad l = 1,2,\cdots,L_i; \quad j = 1,2,\cdots,J \tag{6.36}$$

In the third stage, the corresponding [IRFBCC] model can be written as follows.

$$\text{[IRFBCC]} \quad \underset{u,v}{Max} \quad h_k^i = \sum_{r=1}^{R}u_{kr1}^i(y_{kr}^i)_\alpha^L + \sum_{r=1}^{R}u_{kr2}^i(y_{kr}^i)_\alpha^U - u_k^i \tag{6.37}$$

$$s.t. \quad \sum_{j=1}^{J}v_{kj1}^i(x_{kj}^i)_\alpha^L + \sum_{j=1}^{J}v_{kj2}^i(x_{kj}^i)_\alpha^U = 1 \tag{6.38}$$

$$\sum_{r=1}^{R}u_{lr1}^i(y_{lr}^i)_\alpha^L + \sum_{r=1}^{R}u_{lr2}^i(y_{lr}^i)_\alpha^U - u_k^i \le \sum_{j=1}^{J}v_{lj1}^i(x_{lj}^i)_\alpha^L + \sum_{j=1}^{J}v_{lj2}^i(x_{lj}^i)_\alpha^U, \quad i = 1,2,\cdots,I;$$
$$l = 1,2,\cdots,L \tag{6.39}$$

$$v_{kj1}^i, v_{kj2}^i \ge 0, \quad j = 1,2,\cdots,J; \quad l = 1,2,\cdots,L \tag{6.40}$$

$$u_{lr1}^i, u_{lr2}^i \ge 0, \quad r = 1,2,\cdots,R; \quad l = 1,2,\cdots,L \tag{6.41}$$

where u_k^i is the scale variables of route *k* of company *i*.

6.3 Properties

6.3.1 Slack analysis

Definition: the slack value of the route is the difference between the shared input of the route and that of its benchmark routes.

The following two slack analyses should be used depending on whether or not the attributed inputs are known:

Case (1) When attributed inputs are unknown, the shared input value $s_{lj}^{i}(x_{lj}^{i})_{\alpha}^{L}$ and $s_{lj}^{i}(x_{lj}^{i})_{\alpha}^{U}$ determined by the [AR1] or [AR1'] model are used as the inputs of the [IRFCCR] or [IRFBCC] model to evaluate the route efficiency and to determine the corresponding benchmark routes.

Case (2) When attributed inputs are known, with the allocation ratios determined by the [AR2] or [AR2'] model, the shared input value $(x_{lj}^{i})_{\alpha}^{L} = (x_{lj}^{i})_{\alpha}^{L} + s_{lj}^{i}(x_{cj}^{i})_{\alpha}^{L}$ and $(x_{lj}^{i})_{\alpha}^{U} = (x_{lj}^{i})_{\alpha}^{U} + s_{lj}^{i}(x_{cj}^{i})_{\alpha}^{U}$ is used as the inputs of the [IRFCCR] or [IRFBCC] model to evaluate the route efficiency and to determine the corresponding benchmark routes.

For instance, if route *r* is benchmarked by route *i*, the lower-bound (upper-bound) slack value for the attribute part of input *j* is $(x_{rj}^{i})_{\alpha}^{L} - (x_{lj}^{i})_{\alpha}^{L}$ ($(x_{rj}^{i})_{\alpha}^{U} - (x_{lj}^{i})_{\alpha}^{U}$) and for the common part is $s_{rj}^{i}(x_{cj}^{i})_{\alpha}^{L} - s_{lj}^{i}(x_{cj}^{i})_{\alpha}^{L}$ ($s_{rj}^{i}(x_{cj}^{i})_{\alpha}^{U} - s_{lj}^{i}(x_{cj}^{i})_{\alpha}^{U}$).

6.3.2 Consistency of ranking order

Property: the ranking order of company's performance represented by the efficiency value determined by the integrated company-based fuzzy DEA model is identical to the average of route efficiency values determined by the integrated route-based fuzzy DEA model.

[proof] Without loss of generality, consider two companies—company 1 and company 2, each operates two routes. According to Charnes *et al.* (1978), the company efficiency can be defined as follows:

$$E_i = \frac{y_c^i}{y_R} \tag{6.42}$$

where y_R is the maximum outputs produced from given inputs and y_c^i is the actual outputs rated from the same inputs for company *i*. Because $y_R \geq y_c^i$, the company efficiency $E_i = \frac{y_c^i}{y_R} \leq 1$. The company efficiency can be transformed into fuzzy form by introducing the preference weight and can be defined as follows:

$$E_i = \frac{y_c^i}{y_R} = \frac{\beta_c^i (y_c^i)_{\alpha}^{L} + (1-\beta_c^i)(y_c^i)_{\alpha}^{U}}{\beta_c^R (y_R)_{\alpha}^{L} + (1-\beta_c^R)(y_R)_{\alpha}^{U}} \tag{6.43}$$

We use this concept to derive the company efficiency with the integrated company-based FDEA model as follows:

Let u_{c1}^{i*}, u_{c2}^{i*}, v_{c1}^{i*} and v_{c2}^{i*} represent the optimal set of corresponding values.

By introducing the preference weight γ, the fuzzy input can be expressed as:

$$\gamma_c^R (x_R)_{\alpha}^{L} + (1-\gamma_c^R)(x_R)_{\alpha}^{U} = \gamma_c^i (x_c^i)_{\alpha}^{L} + (1-\gamma_c^i)(x_c^i)_{\alpha}^{U} \tag{6.44}$$

Eq. (6.44) implies $v_c^{i*}\gamma_c^R (x_R)_{\alpha}^{L} + v_c^{i*}(1-\gamma_c^R)(x_R)_{\alpha}^{U} = v_c^{i*}\gamma_c^i (x_c^i)_{\alpha}^{L} + v_c^{i*}(1-\gamma_c^i)(x_c^i)_{\alpha}^{U}$. Let $v_c^{i*}\gamma_c^R = v_{c1}^{i*}$ and $v_c^{i*}(1-\gamma_c^R) = v_{c2}^{i*}$, Eq. (6.44) can be rewrote as:

$$v_{c1}^{i*}(x_R)_{\alpha}^{L} + v_{c2}^{i*}(x_R)_{\alpha}^{U} = v_{c1}^{i*}(x_c^i)_{\alpha}^{L} + v_{c2}^{i*}(x_c^i)_{\alpha}^{U} \tag{6.45}$$

By definition, the efficiency score of the benchmark company is equal to 1, implying $v_{c1}^{i*}(x_R)_\alpha^L + v_{c2}^{i*}(x_R)_\alpha^U = u_{c1}^{i*}(y_R)_\alpha^L + u_{c2}^{i*}(y_R)_\alpha^U$. Thus, the following relationship holds:

$$E_i = \frac{u_{c1}^{i*}(y_c^i)_\alpha^L + u_{c2}^{i*}(y_c^i)_\alpha^U}{v_{c1}^{i*}(x_c^i)_\alpha^L + v_{c2}^{i*}(x_c^i)_\alpha^U} = \frac{u_{c1}^{i*}(y_c^i)_\alpha^L + u_{c2}^{i*}(y_c^i)_\alpha^U}{v_{c1}^{i*}(x_R)_\alpha^L + v_{c2}^{i*}(x_R)_\alpha^U} = \frac{u_{c1}^{i*}(y_c^i)_\alpha^L + u_{c2}^{i*}(y_c^i)_\alpha^U}{u_{c1}^{i*}(y_R)_\alpha^L + u_{c2}^{i*}(y_R)_\alpha^U} \tag{6.46}$$

Introduce the preference weight back to Eq. (6.6), and expressed below:

$$E_i = \frac{u_c^{i*}\beta_c^i(y_c^i)_\alpha^L + u_c^{i*}(1-\beta_c^i)(y_c^i)_\alpha^U}{u_c^{i*}\beta_c^R(y_R)_\alpha^L + u_c^{i*}(1-\beta_c^R)(y_R)_\alpha^U} = \frac{\beta_c^i(y_c^i)_\alpha^L + (1-\beta_c^i)(y_c^i)_\alpha^U}{\beta_c^R(y_R)_\alpha^L + (1-\beta_c^R)(y_R)_\alpha^U} = \frac{y_c^i}{y_R} \tag{6.47}$$

Without loss of generality, assuming company 1 performs better than company 2, then we obtain the result: $\frac{y_c^1}{y_R} = E_1 > E_2 = \frac{y_c^2}{y_R}$, implying $y_c^1 - y_c^2 > 0$.

Similarly, the route efficiency can be defined as $E_l^i = \frac{y_l^i}{y_r}$, $y_r \ge y_l^i$, where y_r is the maximum outputs of the benchmark route produced by the given inputs and y_l^i is the actual outputs rated from the same inputs for route *l* in company *i*. The route efficiency can be transformed into fuzzy form by introducing the preference weight , β, and can be defined as follows:

$$E_l^i = \frac{y_l^i}{y_r} = \frac{\beta_l^i(y_l^i)_\alpha^L + (1-\beta_l^i)(y_l^i)_\alpha^U}{\beta_l^r(y_r)_\alpha^L + (1-\beta_l^r)(y_r)_\alpha^U} \tag{6.48}$$

We use this concept to derive the route efficiency with the integrated route-based FDEA model as follows:

Let u_{l1}^{i*}, u_{l2}^{i*}, v_{l1}^{i*} and v_{l2}^{i*} represent the optimal set of corresponding values.

By introducing the preference weight, γ, the fuzzy input can be expressed as:

$$\gamma_l^r(x_r)_\alpha^L + (1-\gamma_l^r)(x_r)_\alpha^U = \gamma_l^i(x_l^i)_\alpha^L + (1-\gamma_l^i)(x_l^i)_\alpha^U \tag{6.49}$$

Eq. (6.49) implies $v_l^{i*}\gamma_l^r(x_r)_\alpha^L + v_l^{i*}(1-\gamma_l^r)(x_r)_\alpha^U = v_l^{i*}\gamma_l^i(x_l^i)_\alpha^L + v_l^{i*}(1-\gamma_l^i)(x_l^i)_\alpha^U$. Let $v_l^{i*}\gamma_l^r = v_{l1}^{i*}$ and $v_l^{i*}(1-\gamma_l^r) = v_{l2}^{i*}$, Eq. (6.49) can be rewrote as:

$$v_{l1}^{i*}(x_r)_\alpha^L + v_{l2}^{i*}(x_r)_\alpha^U = v_{l1}^{i*}(x_l^i)_\alpha^L + v_{l2}^{i*}(x_l^i)_\alpha^U \tag{6.50}$$

By definition, the efficiency score of benchmark route is equal to 1, implying $v_{l1}^{i*}(x_r)_\alpha^L + v_{l2}^{i*}(x_r)_\alpha^U = u_{l1}^{i*}(y_r)_\alpha^L + u_{l2}^{i*}(y_r)_\alpha^U$. Thus, the following relationship holds:

$$\begin{aligned} E_{route}^i &= \frac{u_{11}^{i*}(y_1^i)_\alpha^L + u_{12}^{i*}(y_1^i)_\alpha^U}{v_{11}^{i*}r_1^{i*}(x_1^i)_\alpha^L + v_{12}^{i*}r_1^{i*}(x_1^i)_\alpha^U} + \frac{u_{21}^{i*}(y_2^i)_\alpha^L + u_{22}^{i*}(y_2^i)_\alpha^U}{v_{21}^{i*}r_2^{i*}(x_2^i)_\alpha^L + v_{22}^{i*}r_2^{i*}(x_2^i)_\alpha^U} \\ &= \frac{u_{11}^{i*}(y_1^i)_\alpha^L + u_{12}^{i*}(y_1^i)_\alpha^U}{v_{11}^{i*}(x_r)_\alpha^L + v_{12}^{i*}(x_r)_\alpha^U} + \frac{u_{21}^{i*}(y_2^i)_\alpha^L + u_{22}^{i*}(y_2^i)_\alpha^U}{v_{21}^{i*}(x_r)_\alpha^L + v_{22}^{i*}(x_r)_\alpha^U} \\ &= \frac{u_{11}^{i*}(y_1^i)_\alpha^L + u_{12}^{i*}(y_1^i)_\alpha^U}{u_{11}^{i*}(y_r)_\alpha^L + u_{12}^{i*}(y_r)_\alpha^U} + \frac{u_{21}^{i*}(y_2^i)_\alpha^L + u_{22}^{i*}(y_2^i)_\alpha^U}{u_{21}^{i*}(y_r)_\alpha^L + u_{22}^{i*}(y_r)_\alpha^U} \end{aligned} \tag{6.51}$$

Introduce the preference weight back to Eq. (6.19), and expressed below:

$$\begin{aligned} E_{route}^i &= \frac{u_1^{i*}\beta_1^i(y_1^i)_\alpha^L + u_1^{i*}(1-\beta_1^i)(y_1^i)_\alpha^U}{u_1^{i*}\beta_l^r(y_r)_\alpha^L + u_1^{i*}(1-\beta_l^r)(y_r)_\alpha^U} + \frac{u_2^{i*}\beta_2^i(y_2^i)_\alpha^L + u_2^{i*}(1-\beta_2^i)(y_2^i)_\alpha^U}{u_2^{i*}\beta_2^r(y_r)_\alpha^L + u_2^{i*}(1-\beta_2^r)(y_r)_\alpha^U} \\ &= \frac{\beta_1^i(y_1^i)_\alpha^L + (1-\beta_1^i)(y_1^i)_\alpha^U}{\beta_1^r(y_r)_\alpha^L + (1-\beta_1^r)(y_r)_\alpha^U} + \frac{\beta_2^i(y_1^i)_\alpha^L + (1-\beta_2^i)(y_1^i)_\alpha^U}{\beta_2^r(y_r)_\alpha^L + (1-\beta_2^r)(y_r)_\alpha^U} = \frac{y_1^i}{y_r} + \frac{y_2^i}{y_r} = \frac{y_c^i}{y_r} \end{aligned} \tag{6.52}$$

From the integrated company-based FDEA model, $y_c^1 - y_c^2 > 0$, therefore we can further derive $\frac{y_c^1}{y_r} = E_{route}^1 > E_{route}^2 = \frac{y_c^2}{y_r}$. Namely, the ranking order of company performance represented by the efficiency value determined by the integrated company-based FDEA model is identical to the average of route efficiency values determined by the integrated route-based FDEA model.

6.4 An Empirical Study

To demonstrate the applicability of the proposed IRFDEA models, an empirical study on 35 intercity bus companies in Taiwan is conducted. The data are the same as IFDEA models and delineated as Tables 5.8, 5.9 and 5.10. The evaluation results are delineated below.

In the first stage, a [ICFDEA] model is used to evaluate the efficiency of companies. The efficiency scores of 35 companies under variable returns to scale are presented in Table 6.1. Note that 16 and 22 companies have been benchmarked as efficient with [ICFCCR] and [ICFBCC] models, respectively. Interestingly, the efficiency scores do not vary much with different α-cuts just like the results from IFDEA model. The efficiency scores of inefficient companies decrease as the α-cut goes higher.

Table 6. 1 Efficiency scores of Taiwanese intercity bus companies under various α-cuts

DMU	CRS				VRS			
	α=0.0	α=0.4	α=0.8	α=1.0	α=0.0	α=0.4	α=0.8	α=1.0
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	0.4640	0.4640	0.4640	0.4640	0.4645	0.4645	0.4645	0.4645
5	0.5436	0.5436	0.5436	0.5436	0.6753	0.6753	0.6753	0.6753
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	0.8904	0.8904	0.8903	0.8902	0.9452	0.9452	0.9452	0.9452
8	0.6915	0.6915	0.6915	0.6915	0.8702	0.8702	0.8702	0.8702
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	0.5613	0.5613	0.5613	0.5613	0.8262	0.8262	0.8262	0.8262
12	0.9468	0.9468	0.9468	0.9468	0.9842	0.9842	0.9842	0.9842
13	0.6669	0.6668	0.6667	0.6666	0.7942	0.7942	0.7942	0.7942
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	0.5843	0.5842	0.5842	0.5842	0.7826	0.7826	0.7826	0.7826
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
22	0.6075	0.6075	0.6075	0.6075	0.7943	0.7943	0.7943	0.7943
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
24	0.9127	0.9127	0.9127	0.9127	1.0000	1.0000	1.0000	1.0000
25	0.7768	0.7768	0.7768	0.7768	1.0000	1.0000	1.0000	1.0000
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
27	0.5784	0.5783	0.5783	0.5782	0.5813	0.5813	0.5813	0.5813
28	0.4789	0.4789	0.4789	0.4789	1.0000	1.0000	1.0000	1.0000
29	0.5457	0.5457	0.5457	0.5457	0.5571	0.5571	0.5571	0.5571

30	0.8027	0.8027	0.8027	0.8027	0.9213	0.9213	0.9213	0.9213
31	0.8051	0.8051	0.8051	0.8051	1.0000	1.0000	1.0000	1.0000
32	0.9978	0.9978	0.9977	0.9977	1.0000	1.0000	1.0000	1.0000
33	0.8003	0.8003	0.8003	0.8003	1.0000	1.0000	1.0000	1.0000
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
35	0.4704	0.4703	0.4702	0.4701	0.4759	0.4759	0.4759	0.4759

Table 6.2 further gives the scale efficiency scores of these bus companies. We note that most of the bus companies are characterized with DRS, implying the necessity to downsize their scale. Only three bus companies (4, 27 and 35) are characterized with IRS, suggesting that they have the advantages to scale up.

Table 6. 2 Scale efficiency scores of Taiwanese intercity bus companies under various α-cuts

DMU	α-cut							
	0.0		0.4		0.8		1.0	
1	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
2	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
3	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
4	0.9167	IRS	0.9148	IRS	0.9128	IRS	0.9118	IRS
5	1.5088	DRS	1.5088	DRS	1.5088	DRS	1.5088	DRS
6	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
7	1.0190	DRS	1.0185	DRS	1.0181	DRS	1.0179	DRS
8	2.0600	DRS	2.0600	DRS	2.0600	DRS	2.0600	DRS
9	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
10	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
11	1.1935	DRS	2.2882	DRS	2.2882	DRS	2.2882	DRS
12	1.7514	DRS	1.7514	DRS	1.7514	DRS	1.7514	DRS
13	1.1128	DRS	1.1102	DRS	1.1077	DRS	1.1065	DRS
14	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
15	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
16	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
17	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
18	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
19	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
20	1.1211	DRS	1.1181	DRS	1.1151	DRS	1.1137	DRS
21	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
22	1.0693	DRS	1.0676	DRS	1.0660	DRS	1.0652	DRS
23	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
24	4.4237	DRS	4.4237	DRS	4.4237	DRS	4.4237	DRS
25	3.5736	DRS	3.5736	DRS	3.5736	DRS	3.5736	DRS
26	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
27	0.9416	IRS	0.9403	IRS	0.9389	IRS	0.9382	IRS
28	1.3082	DRS	1.3082	DRS	1.3082	DRS	1.3082	DRS
29	1.3515	DRS	1.3515	DRS	1.3515	DRS	1.3515	DRS
30	1.6929	DRS	1.6929	DRS	1.6929	DRS	1.6929	DRS
31	1.2837	DRS	1.2837	DRS	1.2837	DRS	1.2837	DRS
32	1.0099	DRS	1.0097	DRS	1.0095	DRS	1.0094	DRS
33	2.7128	DRS	2.7128	DRS	2.7128	DRS	2.7128	DRS
34	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
35	0.9564	IRS	0.9554	IRS	0.9544	IRS	0.9539	IRS

The slack values for the input variables of inefficient companies are computed by the [ICFBCC] model. Table 6.3 gives the slack values for the input variables under α=0.8, from which one can notice that the percentages of input amounts reduction for the inefficient companies can range from

4.62% to 94.88%. Taking Company 11 as an example, reducing the fleet size by 40.06%, the labor force by 43.03%, the operating network by 55.90%, the capital by 91.45%, and the fuel by 47.18% will move the company towards efficiency.

Table 6. 3 Slack values of input variables for 35 intercity bus companies (α=0.8)

DMU	Bus	Labor	Operating network	Capital cost	Fuel cost
1	0.00%	0.00%	0.00%	0.00%	0.00%
2	0.00%	0.00%	0.00%	0.00%	0.00%
3	0.00%	0.00%	0.00%	0.00%	0.00%
4	60.75%	63.10%	63.89%	71.11%	66.15%
5	46.12%	44.16%	36.83%	58.23%	45.18%
6	0.00%	0.00%	0.00%	0.00%	0.00%
7	37.00%	34.70%	4.62%	14.42%	63.19%
8	16.61%	15.40%	43.03%	67.43%	22.97%
9	0.00%	0.00%	0.00%	0.00%	0.00%
10	0.00%	0.00%	0.00%	0.00%	0.00%
11	40.06%	43.03%	55.90%	91.45%	47.18%
12	29.68%	23.37%	39.25%	81.48%	12.67%
13	27.83%	4.73%	5.48%	40.32%	16.15%
14	0.00%	0.00%	0.00%	0.00%	0.00%
15	0.00%	0.00%	0.00%	0.00%	0.00%
16	0.00%	0.00%	0.00%	0.00%	0.00%
17	0.00%	0.00%	0.00%	0.00%	0.00%
18	0.00%	0.00%	0.00%	0.00%	0.00%
19	0.00%	0.00%	0.00%	0.00%	0.00%
20	41.04%	39.79%	27.09%	9.09%	40.15%
21	0.00%	0.00%	0.00%	0.00%	0.00%
22	47.57%	39.01%	28.18%	52.64%	41.57%
23	0.00%	0.00%	0.00%	0.00%	0.00%
24	0.00%	0.00%	0.00%	0.00%	0.00%
25	0.00%	0.00%	0.00%	0.00%	0.00%
26	0.00%	0.00%	0.00%	0.00%	0.00%
27	50.43%	68.37%	68.60%	79.64%	73.01%
28	0.00%	0.00%	0.00%	0.00%	0.00%
29	68.51%	64.02%	89.35%	94.88%	51.98%
30	42.72%	42.26%	54.25%	48.53%	30.97%
31	0.00%	0.00%	0.00%	0.00%	0.00%
32	0.00%	0.00%	0.00%	0.00%	0.00%
33	0.00%	0.00%	0.00%	0.00%	0.00%
34	0.00%	0.00%	0.00%	0.00%	0.00%
35	66.77%	58.66%	82.03%	71.32%	70.72%

In the second stage, the optimal allocation ratios among different routes are determined. For brevity, Table 6.4 only presents the detailed allocation ratios for the 17 routes operated by Company 1. Figure 6.1 displays the allocation ratios of inputs and shares of outputs for all routes of Company 1. The detailed allocation ratios for the routes operated by the remaining companies are not presented.

Table 6. 4 Optimal allocation ratios for the 17 routes operated by Company 1

Route	Fuel cost	Labor	Operating network	Capital cost	Bus		
					Common part	Attribute part	Total
1	15.76%	14.78%	15.27%	13.59%	12.93%	11.11%	11.31%
2	10.99%	9.37%	10.18%	9.03%	7.85%	7.69%	7.71%
3	0.46%	0.35%	0.41%	0.52%	0.69%	0.71%	0.71%
4	13.52%	13.63%	13.58%	12.05%	8.74%	9.87%	9.75%
5	0.49%	0.48%	0.49%	0.58%	0.85%	0.70%	0.72%
6	0.70%	0.66%	0.68%	0.70%	0.92%	0.71%	0.73%
7	0.41%	0.36%	0.39%	0.49%	0.35%	0.71%	0.67%
8	2.53%	2.37%	2.45%	1.87%	3.19%	0.78%	1.04%
9	5.06%	4.72%	4.89%	5.46%	6.04%	6.43%	6.39%
10	0.19%	0.18%	0.19%	0.42%	0.85%	0.77%	0.78%
11	2.84%	2.65%	2.75%	2.44%	3.99%	1.78%	2.02%
12	23.88%	22.42%	23.15%	28.99%	27.64%	39.40%	38.12%
13	4.97%	4.66%	4.82%	4.06%	5.58%	2.70%	3.01%
14	3.33%	2.97%	3.15%	2.33%	3.72%	0.89%	1.20%
15	7.40%	10.14%	8.77%	7.90%	8.21%	5.63%	5.91%
16	3.50%	5.23%	4.37%	4.56%	4.29%	4.43%	4.41%
17	3.97%	5.04%	4.50%	5.02%	4.16%	5.69%	5.52%
Total	100.00%	100.00%	100.00%	100.00%	100.00% (23 buses)	100.00% (189 buses)	100.00% (212 buses)

Note from Table 6.5 and Figure 6.1 that the allocation ratios of three inputs exhibit similar patterns to the shares of three outputs, suggesting that our proposed model tends to allocate larger amount of inputs to those routes with larger amount of outputs, such as Routes 1, 4 and 12. This rationale is logical because the route with larger production generally consumes more inputs. From this inspection, the proposed IRFDEA model could generate the logical results as proposed RDEA models.

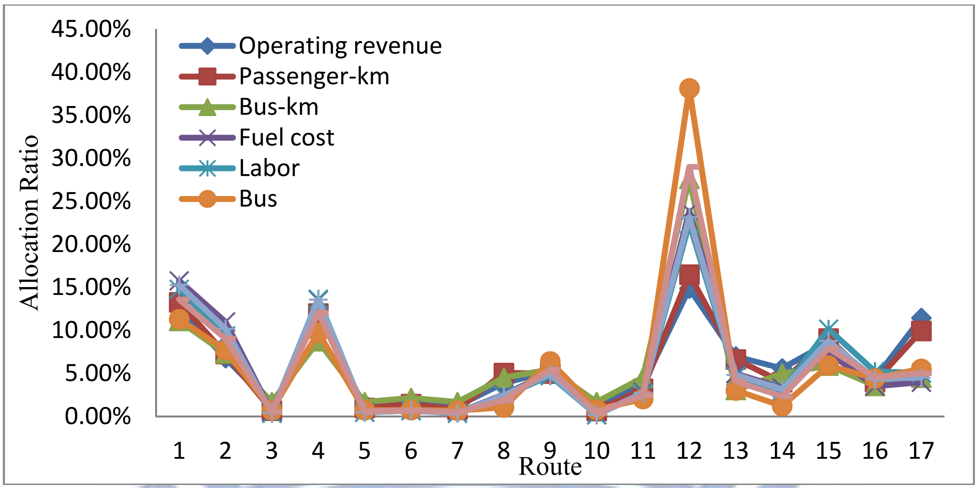

Figure 6. 1 Allocation ratios of inputs and shares of outputs for all routes of Company 1

In the third stage, the proposed IRFDEA models are used to determine the route efficiency for all routes operated by all companies. Table 6.5 only present the results for the 17 routes operated by Company 1. For brevity, the detailed results of the routes operated by the remaining companies are not presented.

Table 6. 5 Efficiency scores for the 17 routes operated by Company 1

Route	α=0.0	α=0.4	α=0.8	α=1.0	Scale	
1	0.5864	0.5864	0.5864	0.5864	2.1203	DRS
2	0.5473	0.5473	0.5473	0.5473	-0.0352	IRS
3	0.2271	0.2271	0.2271	0.2271	-0.0754	IRS
4	0.5949	0.5949	0.5949	0.5949	-0.1395	IRS
5	0.3031	0.3031	0.3031	0.3031	-0.0637	IRS
6	0.3508	0.3507	0.3506	0.3505	-0.3899	IRS
7	0.2306	0.2306	0.2305	0.2305	-0.1106	IRS
8	0.7838	0.7838	0.7838	0.7838	-0.0014	IRS
9	0.4173	0.4173	0.4173	0.4173	-1.0000	IRS
10	0.6026	0.6026	0.6026	0.6026	-0.2898	IRS
11	0.8801	0.8801	0.8801	0.8801	2.3522	DRS
12	0.3774	0.3774	0.3773	0.3772	-0.1703	IRS
13	0.6497	0.6497	0.6497	0.6497	0.0000	CRS
14	0.9432	0.9432	0.9432	0.9432	0.0000	CRS
15	0.5321	0.5321	0.5321	0.5321	-1.0000	IRS
16	1.0000	1.0000	1.0000	1.0000	0.0000	CRS
17	0.3987	0.3986	0.3985	0.3984	2.8756	DRS

Note from Table 6.2 that the results based on [ICFDEA] model have revealed that Company 1 is an efficient company. However, it does not mean that all of its subordinated routes are efficient. By further looking into the details of the route efficiencies based on integrated route-based fuzzy DEA model (Table 6.5), we can scrutinize the insights: of the 17 routes, only eleven with (IRS) need to be scaled up; three with (CRS) should remain unchanged, and three with (DRS) even require downsizing. This evidence manifestly indicates the importance of evaluating the company-based and route-based performance for the transport carriers simultaneously.

To propose improvement tactics for the inefficient companies and routes, slack values for each of the input variables are computed. Take Company 1 as an example, the results are reported in Table 6.6. For instance, route 9 has used too much of the overall input resources; therefore, it should reduce the fuel cost by 9.74%, labor force by 6.84%, operating network by 7.41%, capital cost by 7.85%, and bus fleet by 19.18% (of which, the attributed part takes only 10.12% while the common part takes 9.06%) so as to achieve the efficiency frontier.

Table 6. 6 Slack values for inputs of the 17 routes operated by Company 1

Route	Fuel cost	Labor	Operating network	Capital cost	Bus	
					Common part	Attribute part
1	14.01%	16.07%	14.19%	15.35%	12.01%	13.69%
2	10.42%	10.13%	9.91%	10.23%	9.81%	10.19%
3	0.27%	0.36%	0.75%	0.33%	2.21%	1.72%
4	11.24%	12.19%	11.83%	11.86%	10.08%	11.18%
5	0.27%	0.45%	0.80%	0.39%	2.21%	1.76%
6	0.63%	0.98%	1.18%	0.86%	2.93%	2.38%
7	0.27%	0.38%	0.75%	0.34%	2.21%	1.73%
8	1.13%	1.96%	1.90%	1.67%	2.08%	1.99%
9	9.74%	6.84%	7.41%	7.85%	10.12%	9.06%
10	0.00%	0.00%	0.12%	0.00%	0.00%	0.00%
11	0.27%	0.49%	1.29%	0.41%	0.49%	0.48%
12	30.12%	35.65%	28.19%	33.73%	23.05%	25.16%
13	3.82%	2.27%	3.86%	2.81%	4.35%	3.97%
14	0.00%	0.00%	1.10%	0.00%	0.00%	0.00%
15	9.14%	6.99%	8.36%	7.74%	9.01%	8.51%
16	0.00%	0.00%	1.78%	0.00%	0.00%	0.00%
17	8.70%	5.26%	6.59%	6.46%	9.45%	8.19%
Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%

6.5 Summary

To shed insightful light on improving the less efficient transport carriers, it is beneficial to evaluate the efficiency at the route and company level with both crisp and fuzzy variables, in lieu of conventional evaluation—only measuring the efficiencies at the company level and only paying attention to the influence of crisp variables. This section has proposed two integrated route-based fuzzy data envelopment analysis (IRFDEA) models, [IRFCCE] and [IRFBBC], under CRS and VRScontexts. The proposed models have contributed with the same merits as the proposed RDEA models. Moreover, the proposed IRFDEA models further consider the importance of both crisp and fuzzy variables. Our empirical case study demonstrates the superiority of the proposed models in identifying the less efficient routes/companies, in suggesting improvement strategies for inefficient routes/companies, and in utilizing different type of variables.

In sum, this chapter has remedied two research gaps together. Following this chapter would introduce the conclusions and directions for future studies.

CHAPTER 7. CONCLUSIONS AND FUTURE STUDIES

Following would introduce the conclusion of this study and the direction for future works.

7.1 Conclusions

To remedy two gaps, importance of route performance evaluation and vagueness in variable measurement, a three stage route-based DEA model, integrated fuzzy DEA model and integrated route-based fuzzy DEA model are proposed.

The proposed route-based DEA model can jointly measure both company-level and route-level efficiencies and this model exhibit following properties. First, it can provide an integrated evaluation result for both company-level and route-level simultaneously. Second, the detailed improvement strategies can be proposed accordingly. Third, the optimal allocation ratios of all inputs for each route can be determined. Moreover, we prove that the ranking order of company's performance represented by the efficiency value determined by the company-based model and the sum of route efficiency values determined by the route-based model is consistency. Finally, to examine the applicability of the proposed RDEA model, a case study on 37 Taiwanese intercity bus companies operating 1,035 routes is carried out. Based on the results, the managers can propose more insightful improvement strategies to ameliorate the route efficiency as well as the company performance as a whole.

Furthermore, this study proposes the IFDEA models based on α-cut technique and linear combination concept. The proposed IFDEA models can determine crisp evaluation scores under various α-cuts and production technologies (CRS and VRS). In addition, the proposed IFDEA models can simultaneously determine the slack values for both lower- and upper-bound of input/output variables. With the computed slack values under various α-cuts, the fuzzy value of the corresponding input variable for achieving efficiency can then be determined. To validate the proposed models, a numerical example is tested against the León's *et al*. (2003) model under VRS technology. The results show that our proposed IFBCC model generates very similar efficiency scores to those determined by the León's *et al*. model. Furthermore, the propose IFCCR and IFBCC model could give the improvement directions for the inefficient bus companies through the scale and slack analysis. A case study on 35 Taiwanese intercity bus companies is also performed using the proposed IFDEA models. The results have provided useful directions for improving the inefficient intercity bus companies.

Finally, the three stage integrated route-based fuzzy DEA models have been constructed. Our proposed IRFDEA models have contributed the same advantages as proposed RDEA model. Moreover, the proposed IRFDEA models consider the importance of both crisp and fuzzy variables. As well as, our empirical case study demonstrates the superiority of the proposed models in pining down the less efficient routes/companies, in suggesting how much the inputs for the less efficient routes/companies should be improved and in considering the importance of both qualitative and quantitative variables.

7.2 Directions for Future Studies

Some directions for future studies can be identified. First, this study proposes a three-stage approach to separately determine the optimal multipliers (at stage 3) and the optimal allocation ratios (at stage 2). One may argue that it would be more logical to start the performance evaluation at the route level and to end at the company level by simply averaging the efficiency values of routes operated by the corresponding company. With this rationale, an integrated modeling approach that simultaneously determines the optimal multipliers of input/output variables and optimal allocation ratios has to be developed. However, the integrated model may involve with a greater number of constraints, increasing the complexity in modeling. Moreover, the integrated model is in essence nonlinear due to the multiplication terms of allocation ratios and the multipliers in the denominator of Eqs. (4.6), (4.7), (4.10), and (4.11), making the integrated model rather difficult to solve. Nonetheless, the formulation and solving algorithm for such an integrated model deserves further exploration. As well as, it is interesting to compare the optimal allocation ratios determined by the proposed RDEA model with those determined by other common-cost allocation principles. Besides, application of the proposed RDEA models to other transport practices or service industries are also calling for further studies.

Second, the route-based models are proposed with conventional DEA modeling form. One can try different DEA modeling form to proposed similar model, such as slack-based measurement or directional distant function. Moreover, before analyzing the empirical study, the DMUs can be clustered first in order to reduce the difference between DMUs.

Third, the proposed IFDEA models are to determine the efficiency score under a pre-specified α-cut. In practice, however, it might be difficult for a decision maker to preset the α-cut. Therefore, it is worthy to further develop more comprehensive IFDEA models which can determine the efficiency score by simultaneously considering all possible α-cuts. Moreover, more comparisons with other existent FDEA models deserve further exploration to test the superiority of the proposed IFDEA models. Turning to the empirical applications in bus transport, in addition to the passenger satisfaction, other qualitative (fuzzy) data such as driver attitudes, vehicle amenity, and passenger complaints can also affect the effectiveness of bus services. Therefore, in the future study, including these qualitative (fuzzy) input and output data in the IFDEA models will make the performance evaluation more comprehensive and impartial.

Furthermore, the data of empirical study are all using single-year data. One can try multiple-year data with the proposed route-based DEA modeling and integrate fuzzy DEA modeling to capture the dynamic effects of DMUs. This is important to identify the "sustained" best performers for strategic benchmarking. Besides, the results of the proposed models could be compared with other research or government evaluation results to gain more policy implications.

Last but not least, measuring efficiency in general contains two streams: non-parametric approach and parametric approach—the former is best represented by the DEA method; while the latter is best represented by the stochastic frontier analysis (SFA) method. However, to estimate the inefficiency term, one has to impose an appropriate distribution form (e.g., truncated-normal, half-normal, exponential, gamma) if a SFA model were to build. Aside from the current study with different novel DEA models, it is interesting to elaborate the corresponding novel SFA models to jointly estimate the bus company-based and route-based efficiencies in the future study.

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Yun, Y.B., Nakayama, H., Tanino, T., 2004. A generalized model for data envelopment analysis. European Journal of Operational Research 157, 87-105.
Zadeh, L. A., 1978. Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets and Systems 1, 3-28.

CURRICULUM VITAE

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Qualifications

Sep 2007 – Jul 2013 National Chiao Tung University Taiwan
Ph.D. in Transportation and Logistics Management

- Distinguished awards in Academic Performance
- Scholarship -- China Engineering Consultants, INC. (2008 and 2009)
- Fellowship -- Graduate Students Study Abroad Program, National Science Council, ROC
- Published 3 academic journal papers, and 3 international conference papers
- Participated in 6 research projects

Sep 2005 – Jul 2007 National Chiao Tung University Taiwan
Master in Traffic and Transportation

- Distinguished awards in Academic Performance
- Scholarship -- China Engineering Consultants, INC. (2006)
  -- National Chiao Tung University Presidential Award (2006 and 2007)
- Master Thesis Award for 1st place
- Graduated in 1st place with the score of 92.65
- Published 2 academic journal papers, and 1 international conference paper
- Participated in 3 research projects

Sep 2001 – Jul 2005 National Central University Taiwan
Bachelor of Business Administration in Economics

- Distinguished awards in Academic Performance
- Scholarship -- National Central University Presidential Award (2004 and 2005)
- Participated in 1 research project

Research Interests

- Transportation Economics
- Econometric Modeling and Forecasting
- Performance Evaluation
- Operations Research

Research Experience

Sep 2007 – Jul 2013
Ph.D. in Traffic and Transportation

In the field of performance evaluation and operation research
Thesis: "*Route-based Performance Measurement for Transport Carriers with Integrated Fuzzy Data Envelopment Analysis Approaches.*"

Sep 2005 – Jul 2007
Master in Traffic and Transportation
In the field of performance evaluation
Thesis: "*A joint measurement of efficiency and effectiveness for non-storable commodities: Integrated data envelopment analysis approaches.*"

Sep 2001 – Jul 2005
Bachelor of Business Administration in Economics
Project: *Barbara T.H. Yen, 2004. The competition and/or complement between shuttle and conventional buses. National Science Council Undergraduate Research Project, ROC.*

Publications

Refereed Papers:

- Lawrence W. Lan, Yu-Chiun Chiou and Barbara T. H. Yen, 2013 (forthcoming). Integrated fuzzy data envelopment analysis models: Case of bus transport performance assessment. **Transportmetrica Part A: Transport Science**, DOI:10.1080/23249935.2013.775611.
- Yu-Chiun Chiou, Lawrence W. Lan and Barbara T. H. Yen, 2012. Route-based data envelopment analysis models. **Transportation Research Part E**, Vol.48, 415-425.
- Yu-Chiun Chiou, Lawrence W. Lan and Barbara T. H. Yen, 2010. Joint Measurement of Efficiency and Effectiveness for Non-storable Commodities: Integrated Data Envelopment Analysis. **European Journal of Operational Research**, Vol. 201, Issue 2, 477-489.
- Yu-Chiun Chiou, Chieh-Hua Wen, I-Chang Chen, Barbara T. H. Yen, Shyh-Shyang Yuh, 2009. A Management Strategy Decision Support System for Energy Consumption and Emissions of Cars and Motorcycles. **Transportation Planning Journal**, Vol. 38, 323-354.
- Yu-Chiun Chiou, Lawrence W. Lan and Barbara T. H. Yen, 2007. Integrated data envelopment analysis models for measuring transport efficiency and effectiveness. **Journal of the Eastern Asia Society for Transportation Studies**, Vol. 7, 427-440.

Conference Papers:

- Yu-Chiun Chiou, Barbara T.H. Yen, Chih-Wei Hsieh National, 2012. Modeling airline competition in an airfare regulated domestic market. Air Transport Research Society world conference, Taiwan, June 27-30.
- Yu-Chiun Chiou, Lawrence W. Lan and Barbara T. H. Yen, 2010. Route-Based Data Envelopment Analysis Model. The 15TH HKSTS International Conference, Hong Kong, December 11-14.
- Yu-Chiun Chiou, Tzyy-Tyng Shyu, Barbara T. H. Yen, 2009. An integrated Fuzzy Data Envelopment Analysis Model. The 2009 International Conference and Annual Meeting for Chinese Institute of Transportation, Taoyuan, Taiwan, December.
- Yu-Chiun Chiou, Lawrence W. Lan and Barbara T. H. Yen, 2007. An Integrated Data Envelopment Analysis Model to Evaluate the Performance of Non-storable Commodities. Conference of Productivity Growth and Efficiency Measurement, Academia Sinica, Taipei, Taiwan, March 8-9.

Research Projects

- Lawrence W. Lan, Yu-Chiun Chiou, 2012-2013. Multi-period Performance Measurement for Non-storable Product Firms: Dynamic Integrated Data Envelopment Analysis Models. National Science Council Research Project, ROC.
- Barbara T.H. Yen, 2012. Route-Based Efficiency Analysis for Transport Performance Measure with New Fuzzy Data Envelopment Analysis Approach. Graduate Students Study Abroad Program, National Science Council, ROC. (Performance letter offered by Prof. Wu is attached behind)
- Lawrence W. Lan, Yu-Chiun Chiou, 2009-2012. Route-based efficiency analysis for transport performance measure with new fuzzy data envelopment analysis approach (I , Ⅱ and III). National Science Council Research Project, ROC.
- Yu-Chiun Chiou, 2010-2011. The benefit evaluation of public transportation development policy - A preliminary study on mode choice models and questionnaire survey (I and Ⅱ). The research project of Institute of Transportation, Ministry of Transportation and Communications, ROC.
- Cheng-Min Feng, Yu-Chiun Chiou, 2009-2011. Marketing strategies planning and analysis of distance-based toll collection with differential pricing for freeway electronic toll collection system. The Research Project of Far Eastern Electronic Toll Collection Co..
- Yu-Chiun Chiou, 2007-2011. Transportation Research Statistics. The research project of Institute of Transportation, Ministry of Transportation and Communications, ROC.
- Yu-Chiun Chiou, 2007-2009. Integrated Modelling for Energy Consumption and Pollutant Emissions in Correlation with Vehicle Usage (I , Ⅱ and III). The research project of Institute of Transportation, Ministry of Transportation and Communications, ROC.
- Yuan-Ching Hsu, Wan-Hui Chen, 2006-2008. Comprehensive planning for the coming of aging society in Taiwan: a vision for the first quarter of the 21st century. National Science Council Research Project, ROC.
- Barbara T.H. Yen, 2004. The competition and/or complement between shuttle and conventional buses. National Science Council Undergraduate Research Project, ROC.

Work Experience

Aug 2013 – current
Research Fellow at Urban Research Program, Griffith University.

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