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靜態自行車調度與維護問題

STATIC BIKE REPOSITIONING AND MAINTENANCE PROBLEM

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摘 要

本研究提出一個全新、考慮破損自行車維護的靜態自行車調度問題：系統當中的破損自行車不但可以經由調度車收集並運送回倉庫進行整修，而且能以本研究首先提出的現場維修方法來進行維護。接著，本文構建了一道混合整數線性規劃問題，同時決定調度車輛數量、所有自行車站點上正常自行車的裝卸數量、以及所有自行車站點上已收集和實地維修的破損自行車的數量，以最小化總服務時間和所有站點與其目標庫存水平之差的懲罰成本的加權總和。透過臺灣 Youbike 的實際案例，數據實驗顯示現場維修比單純收集更能減少調度成本和與目標庫存水平的偏差。敏感性分析顯示較短的維修時間、較大的車輛以及較高的偏離目標庫存水平的權重，能達至與目標庫存水平偏離較小和行駛時間較短的調度策略。同時，本研究提出增加了限制服務時間的延伸問題，並透過數值研究顯示更長的服務時間能減少調配車輛的數量和總服務時間。

關鍵詞： 自行車調度、破損自行車維護、共享自行車系統、實地維修

ABSTRACT

This paper proposes a new static bike repositioning problem that maintenance operation is simultaneously considered. In this problem, a broken bike at a station can be either collected on the relocation vehicle to be sent back to the depot or repaired on-site to restore its function during repositioning. This

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on-site repair becomes a new strategy that is examined in this paper and new to the literature. The problem is formulated as a mixed-integer linear programming problem that determines the number of deployed vehicles, the loading and unloading quantities for normal bikes at all stations, and the number of collected and repaired broken bikes at all stations such that the weighted sum of the total service time and the penalties associated with the deviations from the targeted inventory level is minimized. Using the real-world case of Youbike in Taiwan, this paper demonstrates the on-site repair strategy can reduce the repositioning cost and deviations from the target inventory levels more than mere collection strategy. Sensitivity analyses show that the shorter repairing time, larger vehicle, and higher penalty for deviations from the target inventory level can obtain strategies with smaller deviation and shorter travel time. Moreover, a revised problem which includes service time constraint is proposed and the numerical studies show that the increase in allowable service time can reduce the number of deployed vehicles and the total service time.

Key Words： *Bike repositioning; Broken bike maintenance; Bike sharing system; On-site repair*

I. Introduction

A bike-sharing system (BSS) usually involves two types of managerial decisions, namely, bicycle relocation and bicycle maintenance. Bicycle relocation is to ease the mismatch between the shared bike demand and the bike distribution by relocating bikes from stations with excessive bikes (compared with shared bike demand) to stations with insufficient bikes. To accomplish the relocation tasks, a possible way is to offer static or dynamic incentives to the shared-bike users to motivate them in utilizing the system resources effectively and voluntarily participating in system regulation. Static incentives are usually stationary to affect the users' decisions and the associated decisions have a longer implementation period, nearly independent of time, and do not consider look-ahead issues [1]. Examples of static incentives include the parking space reservation policy [2] and location-based pricing [3]. The former allows the operator to determine whether a cycling trip with a reserved bike rack can be accepted before the trip starts. The latter is to set the different rental fares for each OD pair such that a higher charge can be imposed on stations with low bike demand while a lower charge can be imposed on stations that have high bike demand. Dynamic incentives are opposite to the static ones, which involve short-term, time-dependent, and demand-responsive decisions that take the current and forecasted states of the system into consideration. The most common dynamic incentive is dynamic pricing, in which the price can be set at the return location at a particular time interval based on the information of the current state of the BSS and the estimated destination of the trip

[4][5]. Although offering these incentives can potentially benefit the system at a very low cost, user-based relocation has a limited effect on bike relocation operation because it highly relies on the willingness of the users [6].

Vehicle-based relocation is regarded as a more reliable and effective method in redistributing a large number of bikes. Trucks are deployed to relocate the bikes from bike surplus stations to bike deficit stations. This problem is called the bike repositioning problem (BRP), in which the operator needs to determine simultaneously the routes of the vehicles and the loading and unloading quantities at each visited station. Usually, the BRP can be classified into two groups, namely static and dynamic, according to the time of the relocation operation [7]. Dynamic BRP focuses on daytime relocation in which the customers' demand is taken into consideration. Both the routing and the loading decisions of the trucks need to be updated regularly subject to the changes among the stations and the estimated demand in the upcoming time intervals. Nevertheless, in practice, it is revealed that dynamic relocation cannot completely replace static relocation [8]. Static BRP focuses on nighttime relocation in which the cycling demand is negligible. The negligible variations of the bike inventory levels enable the operator to execute large-scale bike relocation because the number of bikes at each station is expected to be unchanged throughout the operation period. Moreover, the travel time at nighttime is assumed to be constant due to a very low level of traffic. So, in the literature, static BRP has received much more attention than dynamic BRP because it is more widely adopted in practice.

The static BRPs can be further classified according to their design objectives and design constraints. Regarding design objectives, demand dissatisfaction and operating cost are generally included in most studies. Demand dissatisfaction is associated with the level of service of the system as it represents the bike demand which cannot be satisfied after the repositioning operation. It can be represented as sole bike deficit (the negative deviation from the target inventory level) [9][10], the sum of bike surplus and bike deficit (the positive and negative deviations from the target inventory level) [11] [12], or a convex penalty function [7] [13], whereas these representations are all aiming at minimizing the total deviations from the target inventory levels of the stations in the system. The operating cost can be broken down into vehicle-related and station-related costs. Vehicle-related costs are always included in the studies which are usually represented by the travel cost of the vehicle [14-16] and very often to be the greenhouse gas emission cost of the vehicles [17] [18]. Different from the classical vehicle routing problem, the fixed cost per dispatched vehicle is seldom considered in bike repositioning literature. The reason is three-folded: first, the operator has usually the highest priority in minimizing the total deviations of all stations; second, the service time is usually the scarcer resources than operating cost (e.g., transport cost and vehicle deployment cost) in the relocation operation; third, the operator always uses its self-owned fleet for relocation and thus no additional cost due to outsourcing is required. These three reasons drive the operator to deploy more vehicles to accomplish the relocation service within a shorter service period. Station-related cost is usually related to the loading and unloading times at all stations, which can be calculated by imposing a constant per visited station [10] [11] or per loaded and unloaded bike [19]. The operating cost is usually the sum of the vehicle-based and station-based costs. To conclude, compared with other types of objectives, such as minimum loading activities [10] [20], which are considered in limited studies, it is sufficient for a BRP to formulate the objective function that jointly considers the total deviations from the target inventory level and the operating cost as it can cover both the system performance and the expenses of the system operator. The design constraints can be categorized into service level constraints and operational constraints. Service level constraints are associated with the final station inventory level, which includes perfect balance [21] and meeting a pre-defined inventory interval [20]. Operational constraints are related to the limitation on the relocation service and the loading and unloading strategies. The relocation service consists of decisions with relocation fleet size, truck capacity, and operation time. The latter includes the number of visits per station (e.g., multiple visits, or at most one visit), temporary storage, monotonicity, depot supply/ demand, multiple bike types, and split delivery. These constraints can cover most of the situations in an operating system, but they implicitly assume that all bikes in the system are in good condition and do not require maintenance.

In practice, bicycle maintenance is essential to maintain the level of service of the BSS by removing all faulty or broken bikes in the system. These broken bikes can be reported by the users via mobile apps, discovered by the crews during the repositioning operation, or estimated based on historical data [22]. Removing broken bikes is crucial because they reduce the station capacity which can be used by the normal bikes and deter the service quality [23]. The removed bikes are usually brought back by the relocation truck to the depot for maintenance. From the operator's perspective, to reduce the number of trucks deployed, the broken bike collection is usually accomplished along with the normal bike relocation to utilize the residual capacity of the truck. As a trade-off, the loaded broken bikes reduce the truck capacity for normal bike relocation as they cannot solve the normal bike demand and can only be unloaded at the depot.

Handling broken bikes during repositioning operation in the bike-sharing system has been a hot topic arisen in recent years. Table1 lists the papers that handle broken bikes with normal bike relocation and classifies them with respect to the design objective, the way to handle broken bikes, and the inclusion of normal bike replenishment in bike relocation operation. Regarding the design objectives, they can be divided into operational and service-related objectives. Operational objectives include minimal total service time/ cost (which includes travel time and loading and unloading times) [19] [24] [25], minimal vehicle deployment cost [26], minimal recycling and manual handling cost [27], the time span of the fleet [28], greenhouse emission [18], revenue maximization [29], and the total routing costs of the vehicles [30]. Service-related objectives are usually related to the inventory levels of normal bikes and broken bikes at all stations, which can be expressed in terms of minimal violation of service level requirement [25] and minimal total deviation from the target inventory level [30]. Whereas the service-related objectives are beneficial to the users, the operator has a larger interest in the operational objectives which can improve their profit. To capture the service-related objectives, some studies formulate them as the design constraints, such as the compulsory resolution of all bike demand [18] and loading all broken bikes in the system [19] [30]. Another approach is to combine both types of objectives in the objective function using a weighted sum method to optimize simultaneously the benefits of the operator and the users [19] [30]. The latter approach outperforms the former approach because it provides flexibility for the operator to impose weights on the two types of objectives to obtain an optimal solution.

In terms of the ways of handling broken bikes, they can be classified into two ways: recycling and collection. Recycling is to dissemble the broken bikes at the depot to preserve the usable parts (i.e., steel) of those bikes [24] whereas collection is to check and repair the abnormalities of the broken bikes at the depot. In other words, both ways require vehicles to relocate the broken bikes back to the depot in the first step despite the difference in the subsequent treatment of the broken bikes. In practice, the choices between recycling and collection are usually related to the costs of these two ways: when purchasing a bike costs less than repairing a broken bike, or the bike is broken severely, recycling can be a better option than collection for repair. Moreover, recycling is more often considered in the station-less BSSs [26] [27], whereas collection can be adopted in both station-less and station-based BSSs. Coupled with broken bike collection, some studies consider the replenishment of normal bikes to the BSS to maintain its service level [24] [29]. A sufficient number of normal bikes are implicitly assumed to be readily available at the beginning of the repositioning operation such that the vehicles leaving the depot can have a normal bike load on them. For some systems with a limited number of bikes, the replenishment can only be accomplished by firstly collecting the broken bikes back to the depot for repairing such that these repaired bikes can be relocated later on to satisfy the unmet demand [25]. This consideration is practical under a multi-day operation scenario. To conclude, the usual way to handle the broken bikes in current studies is to collect the broken bikes at the stations and send them back to the depot for repair while initially available normal bikes are used to fulfill the bike demand. However, in practice, as most of the broken bikes are associated with minor defects (e.g., tire replacement and chain lubrication) [31], on-site repairing can be a better alternative than sending all broken bikes back to the depot. Repairing broken bikes on-site can save both the vehicle capacity occupied by the broken bikes and the transporting time for redistributing the repaired bike back to the system. As a trade-off, the service times of the routes increase because on-site repairing must require more time than collection. So, to strike a balance between the service time and the vehicle capacity, a combined on-site repair and collection repositioning strategy has the potential to reduce operating costs and improve the service quality compared with mere collection or recycling. Nevertheless, no existing studies have considered this potentially efficient combined strategy in handling broken bikes.

Table 1　Summary of the characteristics of bike repositioning problems with broken bikes in the literature

Reference	Objective	Broken bike handling approach	Replenishment of normal bikes
Alvarez-Valdes et al. (2016)	Minimize the weighted sum of total service time and the coefficient of variations of the duration of the routes	Collection	Depot replacement
Wang & Szeto (2018)	Minimize total greenhouse gas emission	Collection	Depot replacement
Chang et al. (2018)	Minimize total recycling and manual handling cost	Recycling	Nil
Zhang et al. (2018)	Minimize total traveling cost	Collection	Depot replacement
Usama et al. (2019)	Minimize the weighted sum of total deviation from the target inventory level, repositioning vehicle routing cost, and service vehicle routing cost	Collection and pre-relocation shift	Depot replacement
Du et al. (2020)	Minimize the timespan of the repositioning operation of the fleet	Collection	Nil
Jin et al. (2020)	Minimize the weighted sum of the total travel time, loading time and unloading time, the penalty of violating the service level requirement, and reward of saved inventory	Collection and repair at the depot	Depends on the collected quantity
Zhang et al. (2020)	Minimize total travel cost and vehicle deployment cost	Recycling	Nil
Teng et al. (2020)	Maximize the total expected revenue of all hotspots in a period	Collection	Depot Replenishment
This study	Minimize the weighted sum of the deviations from the target inventory level and the total service time	Collection, on-site repair	Nil

This paper, therefore, proposes a novel static bike repositioning and maintenance problem (SBRMP) that includes five decisions, namely the routing decision of the repositioning trucks, loading and unloading decisions of normal bikes, and the loading and repairing decisions of the broken bikes. This SBRMP is formulated as a mixed-integer linear programming problem that aims to minimize the weighted sum of the total deviation and the total service time (which includes the travel time and the handling time of normal and broken bikes). The problem properties of this SBRMP is studied about the effect of the combined repair and collection strategy on the performance of the repositioning operation in a system with broken bikes. Moreover, a revised formulation that includes the time constraint is proposed. The optimal repositioning strategies under different service times are compared.

There are three contributions to this study:

1. It introduces a new practical problem in bike-sharing system – a static bike repositioning and maintenance problem (SBRMP) with the on-site repair;
2. It introduces a new formulation to the SBRMP and a revised version that includes the service time constraint;
3. It introduces a case study in Taiwan to show the applicability of the model and demonstrates the effect of the on-site repair and collection strategy and the service time towards the optimal repositioning plan and offers sensitivity analyses on key parameters of the operation.

The outline of the paper is stated as follows. Section 2 provides the problem statement and formulates the problem as a mixed-integer linear programming problem. Section 3 provides a case study of Taipei City to apply the model in a bike-sharing system with broken bikes. Section 4 provides a revised model with the service time constraint and demonstrates the effect of service time constraints on the optimal repositioning strategy. Section 5 gives a conclusion.

II. Problem formulation

2.1 Problem setting

This problem considers a bike-sharing system with a fleet of homogeneous vehicles, N stations, and a single depot (denoted as 0) operating at nighttime (in which cyclists' demand can be assumed as negligible) which the numbers of normal and broken bikes at each station are known in advance. Though arguably, a bike is recognized as broken until checking, there are four objective criterion to determine the number of broken bikes at a station [26] [29]: (1) user feedback, (2) lost location due to the damaged GPS equipment on the bike, (3) inactive for a pre-defined period (e.g., 24 hours), and (4) end of service life. By adopting these criteria, the number of broken bikes can be determined in advance. The operator needs to determine the number of deployed trucks and their corresponding routes. All vehicles must start and end their travels at the depot where no spare normal bike is available there. At every visited station, the operator needs to determine not only the number of loaded and unloaded normal bikes but also the number of broken bikes loaded and repaired on-site. Six assumptions related to the problem setting are stated as below.

A1 No broken bikes should be left in the system. All broken bikes must be either collected or repaired on-site.

A2 The repositioning crew can repair a broken bike at the station the bike stays. In other words, no location restrictions on the broken bike repairing.

A3 The repairing times of all broken bikes are identical and represented as R, so the severity of the broken bikes are assumed to be identical.

A4 A broken bike loaded on the vehicle can only be unloaded at the depot.

A5 Bike stations without bike deviations and broken bikes must not be visited. These stations can be understood as “complete balance stations”.

A6 The relocation truck in the fleet is homogeneous with a maximum of $|V|$ vehicles.

The first four assumptions (A1-A4) focus on the operational strategy in handling broken bikes. These can be adjusted subject to the preferences of the operator and the actual scenario. The fifth assumption (A5) is intuitive because visiting complete balance stations is not efficient in improving the level of service of the system. The sixth assumption (A6) can also be relaxed while the homogeneous fleet is often found in practice (e.g., Youbike). According to these assumptions, the next subsection formulates the problem as a mixed-integer linear programming problem.

2.2 Mathematical model

The above problem can be formulated based on the below notations.

Sets

N	Set of stations
N_0	Set of nodes (i.e., all stations and the depot)
N_A	Set of nodes that do not have the deviation from targeted inventory level and broken bikes, i.e., $N_A \subset N$

Parameters

t_{ij}	Travel time between nodes i and j
K_i	Number of broken bikes at station i

I_i^0 Initial inventory level at station *i*

O_i Target inventory level at station *i*

Q Capacity of a relocation truck

L Loading time of a bike

U Unloading time of a bike

R Repairing time of a bike

M A very large positive constant

ε A very small positive constant

α, β Non-negative weight

Decision variables

x_{ij} 1 if a vehicle travels directly between node *i* and node *j*; 0 otherwise

p_i Number of *normal* bikes picked up at station *i*

b_i Number of *broken* bikes picked up at station *i*

d_i Number of bikes dropped off at station *i*

r_i Number of *broken* bikes repaired at station *i*

u_i Auxiliary variable for node *i* used for sub-tour elimination constraints

φ_i 1 if station *i* does not have bike deficit after bike repair; 0 otherwise

Auxiliary variables

q_{ij} Total number of *normal* bikes on the vehicle traveling between node *i* and node *j*

θ_{ij} Total number of *broken* bikes on the vehicle traveling between node *i* and node *j*

f_i^+ Bike surplus at the end of the operation

f_i^- Bike deficit at the end of the operation

I_i Inventory level at the end of the operation

The model can then be expressed as

$$\min Z = \sum_{i \in N} \left(\alpha f_i^+ + \beta f_i^- \right) + \sum_{i \in N_0} \sum_{j \in N_0} x_{ij} t_{ij} + \sum_{i \in N} \left[L\left(p_i + b_i \right) + U d_i + R r_i \right], \tag{1}$$

subject to

$$I_i = I_i^0 + \left(d_i + r_i \right) - p_i, \forall i \in N, \tag{2}$$

$$f_i^+ - f_i^- = I_i - O_i , \forall i \in N , \tag{3}$$

$$\sum_{j \in N_0 \setminus N_A} x_{ij} = 1 , \forall i \in N \setminus N_A , \tag{4}$$

$$\sum_{j \in N_0 \setminus N_A} x_{ji} = 1 , \forall i \in N \setminus N_A , \tag{5}$$

$$\sum_{j \in N \setminus N_A} x_{j0} = \sum_{j \in N \setminus N_A} x_{0j} , \tag{6}$$

$$1 \le \sum_{j \in N \setminus N_A} x_{0j} \le |V| , \tag{7}$$

$$\varepsilon \left(I_i^0 - O_i + r_i \right) + \varepsilon^2 \le \phi_i \le \varepsilon \left(I_i^0 - O_i + r_i \right) + 1 , \forall i \in N \setminus N_A , \tag{8}$$

$$p_i \le M \phi_i , \forall i \in N \setminus N_A , \tag{9}$$

$$d_i \le M \left(1 - \phi_i \right) , \forall i \in N \setminus N_A , \tag{10}$$

$$p_i + d_i \le \left(2\phi_i - 1 \right) \left(I_i^0 - O_i + r_i \right) , \forall i \in N \setminus N_A , \tag{11}$$

$$b_i = r_i = 0 , \forall i \in N_A , \tag{12}$$

$$b_i + r_i = K_i , \forall i \in N , \tag{13}$$

$$\sum_{i \in N} p_i = \sum_{i \in N} d_i , \tag{14}$$

$$\sum_{i \in N_0 \setminus N_A} q_{ji} - \sum_{i \in N_0 \setminus N_A} q_{ij} = p_j - d_j , \forall j \in N \setminus N_A , \tag{15}$$

$$\sum_{i \in N_0 \setminus N_A} \theta_{ji} - \sum_{i \in N_0 \setminus N_A} \theta_{ij} = b_j , \forall j \in N \setminus N_A , \tag{16}$$

$$q_{ij} + \theta_{ij} \le Q x_{ij} , \forall i, j \in N_0 , \tag{17}$$

$$u_j \ge u_i + 1 - M \left(1 - x_{ij} \right) , \forall i \in N_0 , j \in N \setminus \{ i \} , \tag{18}$$

$$x_{ii} = 0 , \forall i \in N_0 , \tag{19}$$

$$x_{ji} = x_{ij} = 0 , \forall i \in N_A , j \in N_0 , \tag{20}$$

$$q_{0i} = q_{i0} = \theta_{0i} = 0 , \forall i \in N , \tag{21}$$

$$x_{ij} \in \{0,1\}, \forall i, j \in N_0, \tag{22}$$

$$p_i, d_i, r_i, b_i, f_i^+, f_i^-, I_i \geq 0, \forall i \in N, \tag{23}$$

$$u_i \geq 0, \forall i \in N_0, \tag{24}$$

$$q_{ij}, \theta_{ij} \geq 0, \forall i, j \in N_0, \tag{25}$$

The design objective (1) is to minimize the weighted sum of total deviations from the target inventory level at the system and the total service time of the vehicles. Equation (2) computes the inventory level of each station and equation (3) determines the deviations from the optimal inventory level at the end of the operation. Constraints (4) and (5) guarantee all stations (except the balanced stations without broken bikes) must be visited exactly once. Constraint (6) ensures that all vehicles leaving the depot must return to the depot. Constraint (7) defines the range of the number of vehicles that should be deployed for relocation. Constraints (8) to (14) define the loading, unloading, and repairing operations at all stations. Constraint (8) determines the status of the bike station after bike repairs. For bike deficit stations that have broken bikes, bike repair can increase its number of normal bikes. When the number of repaired bikes is greater than or equal to the bike deficit in that station, the station is no longer in bike deficit. So, constraints (8) to (10) define that a station which is still in bike deficit after bike repair has drop-off quantities whereas a station which does not have deficit after bike repair has pickup quantity. Constraint (11) defines that either the pickup or drop-off quantity does not exceed the deviation from the target inventory level. Constraints (12) and (13) ensures that the sum of the collected and repaired bikes is equal to the number of broken bikes at the station. Constraint (14) ensures that the total pickup and drop-off quantities are equal. Constraints (15) and (16) define the number of normal and broken bikes on the truck when leaving a station respectively. Constraint (17) ensures that there are normal bike flows and broken bike flows on travel arcs only. Constraint (18) is the sub-tour elimination constraint, which can be referred to Miller et al. [32]. Constraint (19) ensures no self-visit. Constraint (20) ensures no visit to balanced stations without broken bikes. Constraint (21) ensures that the truck leaves the depot without any bikes and returns to the depot only with broken bikes. Constraints (22) to (25) are the definitional constraints.

By expanding constraint (11), in which $p_i + d_i \leq (2\phi_i - 1)\left(I_i^0 - O_i\right) + 2\phi_i r_i - r_i$, it can be seen that the above equation is bilinear due to the term $r_i\varphi_i$, so a further linearization is required to make the model linear. Denote $\overline{r_i} = r_i\phi_i$, and thus the linearized form becomes

$$\begin{aligned}
&\bar{r}_i \le M\phi_i, \\
&\bar{r}_i \le r_i, \\
&\bar{r}_i \ge r_i - \left(1-\phi_i\right)M, \\
&\bar{r}_i \ge 0.
\end{aligned} \quad \forall i \in N \setminus N_A., \tag{26}$$

In the equation set (26), the value of $\bar{r}_i$ is either 0 or r_i according to the value of φ_i. So, constraint (11) is rewritten as

$$p_i + d_i \le \left(2\phi_i - 1\right)\left(I_i^0 - O_i\right) + 2\bar{r}_i - r_i, \forall i \in N \setminus N_A. \tag{27}$$

And thus the final model is written to be having an objective function (1) subject to constraints (2)-(10) and (12)-(27).

III. Case study

3.1 Case setting

This study adopts the Youbike BSS in Taipei to demonstrate the applicability of the problem. To be specific, this paper adopts the distance matrix provided by Lin & Yang [33] with 11 bike stations and picks maintenance center D as the only depot in this BSS. Figure 1 displays the distributions of the stations on the map which is obtained from the Open Street Map[2]. To convert the distance matrix (in meters) into a time matrix, we assume that the vehicle moves at a speed of 27 km/h (450 m/min) because the network is located in the urban area with traffic lights and this time includes the parking time and setup time at each station. Four scenarios with different levels of bike demand for normal bikes and broken bikes are proposed and investigated in this study. The data of the four scenarios can refer to Table I.1 in Appendix I for details. To reduce the effect of spatial distribution, the broken bikes are only allocated evenly to the odd-indexed stations while the changes in the normal bike level are only allocated to the even-indexed stations.

The values of other parameters are provided as follows unless specified in the section: Q = 25 vehicles; α = 10/ vehicle; β = 20/ vehicle; $L = U$ = 1 minutes; R = 3 minutes; M = 10,000; ε = 0.0001; and $|V|$ = 5 vehicles. The problem is solved by IBM ILOG CPLEX 12.9.0 coded with Visual C++ language and run in a Desktop computer equipped with Windows 10, an Intel (R) Core (TM) i9-9900K@ 3.60GHz, and a 32.00GB of RAM.

2. https://www.openstreetmap.org/

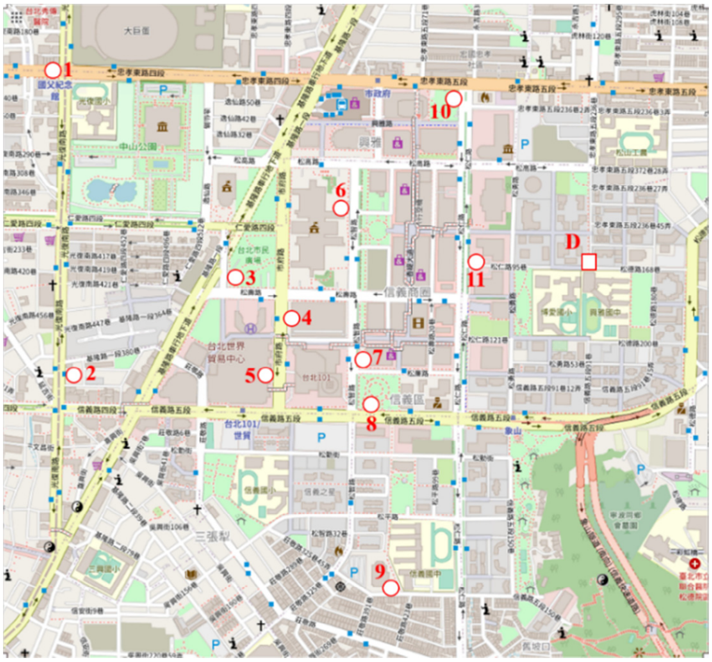


Figure 1　Distribution of the stations and depot

3.2 Comparisons between on-site repairing and broken bike collection relocation strategies

With the above problem setting, this section demonstrates the significance of the combined on-site repairing and broken bike collection strategy towards the repositioning strategy. The repair and collection (R&C) strategy is compared with the collection only (CO) and the repair only (RO) strategies. We adopt scenario 1 as the base case (in which the data can be obtained from Appendix I). In this scenario, there is a total bike net deficit of 15 bikes and 30 broken bikes in the system. Table 2 presents the results of the three strategies and Figures 2-4 demonstrate the optimal solutions for R&C, RO, and CO strategies respectively. In Table 2, “Obj.”, “Dev.”, “TT”, “HT”, “UV”, “P/D” and “C/R” denote the objective value, total deviations (where ‘+’ denotes surplus and ‘–’ denotes deficit), travel time, bike handling time, used vehicles, the number of normal bikes picked up and dropped off, and the number of collected and repaired bikes. To implement RO and CO strategies, we can impose the constraints $b_i = 0$ and $r_i = 0$ respectively in these two strategies separately.

R&C strategy outperforms the other two strategies in achieving the lowest objective value, followed by RO strategy and then CO strategy. The lowest objective value for the R&C strategy is mainly due to the zero deviation from the targeted inventory level. Regarding the travel time, RO strategy is the lowest and only one vehicle is deployed to serve all the 11 nodes, as shown in

Figure 3. The CO strategy deploys 3 vehicles to collect all broken bikes without hindering the normal bike relocation while the R&C strategy deploys one vehicle less than the CO strategy. For the total time for handling bikes, the CO strategy is the minimum because no time-consuming on-site repairing is required. In contrast, RO must have the largest handling time because all bikes are required to be repaired. The handling time of the R&C strategy lies between RO and CO while it is larger than the average of RO and CO (i.e., 218) because some repaired bikes can be transported to other stations to reduce the deviation. From the P/D column, the R&C strategy has more pickup and drop-off normal bikes than RO and CO strategies and therefore reduces the total deviations.

Table 2　Comparisons of the three relocation strategies

Strategy	Obj.	Dev.	TT	LT	UV	P/D	C/R
R&C	250.747	0	24.747	226	2	83/83	15/15
RO	416.404	+15	18.404	248	1	79/79	0/30
CO	517.835	-15	29.835	188	3	79/79	30/0

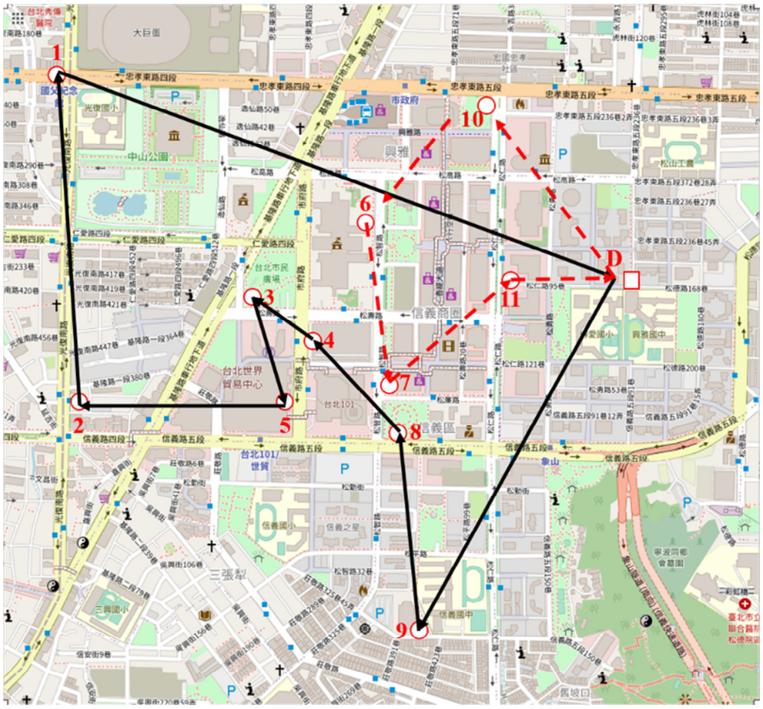


Figure 2　Optimal routes of R&C strategy

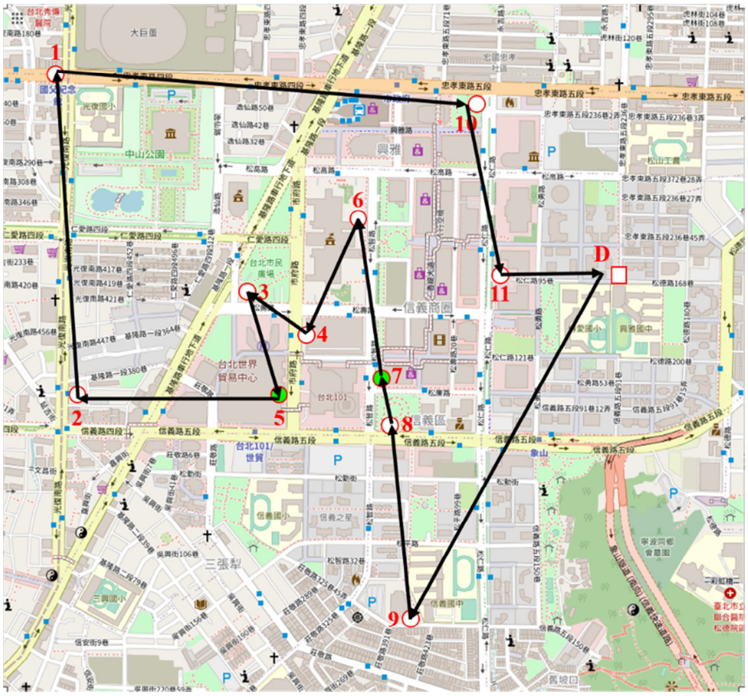


Figure 3　Optimal routes of RO strategy

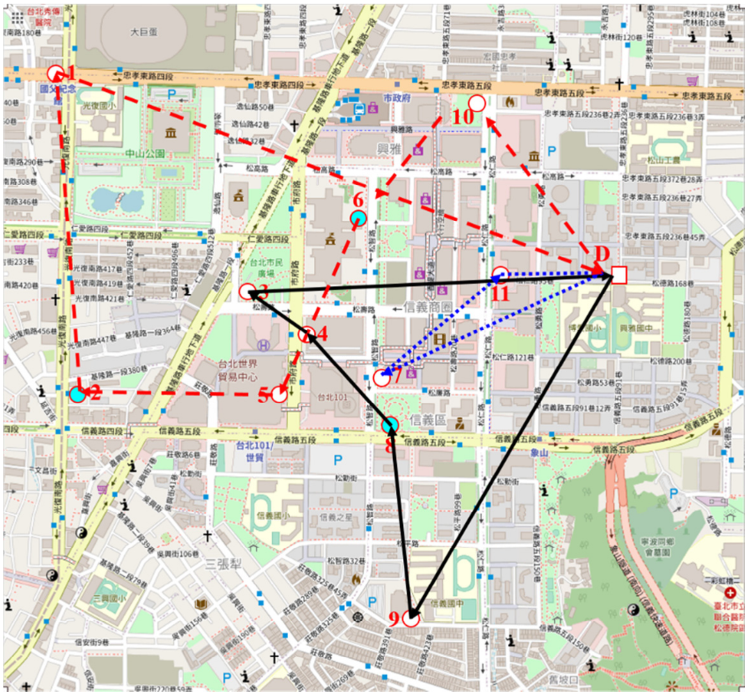


Figure 4　Optimal routes of CO strategy

Figures 2-4 illustrate the vehicle routes of the three strategies. In Figure 2, the two routes under the R&C strategy can solve all positive and negative deviations. In Figure 3, there is only one route for the RO strategy. The broken bikes are all repaired so no broken bikes are loaded on the vehicle, so the vehicle capacity can be maximized to load and unload the broken bikes. The two stations in green (stations 5 and 7) are the two stations that have positive deviations from the target inventory level. In Figure 4, three vehicles are deployed to collect all the broken bikes and relocate normal bikes. Three stations in blue (stations 2, 6, and 8) have negative deviations from the target inventory level because there are insufficient bikes to fulfill all bike demand in the system.

Three other types of instances are also considered. The first one is that the total number of normal bikes can fulfill the total bike demand while broken bikes are distributed among stations. The second one is that the total number of normal and broken bikes is not more than the total bike demand. The third one is that no broken bikes are found in the system while the total number of normal bikes is equal to the total bike demand. The data of these three instances are provided in Appendix I. Table 3 presents the results of the three instances. For the first instance, if the normal bikes are sufficient to handle all bike deficit, the solution is similar to the CO solution because the repair is not required. Three vehicles are deployed to ensure sufficient vehicle capacity for normal bike relocation in line with broken bike collection and thus result in higher travel time. For the second instance, the objective value is the highest due to the largest deviation. The bike handling time is also the highest because most of the broken bikes are repaired to meet the bike deficit. Meanwhile, two broken bikes are collected but not repaired because these repaired bikes cannot further reduce the total bike deficit. In other words, instance 2 displays a similar repositioning strategy with the RO case in Table 2. Finally, the third instance has the lowest handling time and the largest normal bikes pickup and drop-off. The objective value is the lowest among all three instances because it does not need to handle any broken bikes.

Table 3　Optimal relocation strategy under different scenarios

Scenario	Obj.	Dev.	TT	HT	UV	P/D	C/R
2	278.153	+5	34.153	194	3	82/82	30/0
3	414.933	-7	22.933	252	1	84/84	2/28
4	189.751	0	19.751	170	1	85/85	0/0

These results can firstly direct to the following conclusions about the choice of strategies: (1) R&C strategy is the best strategy when the total number of normal bikes is smaller than the total targeted number of normal bikes in the system while the sum of normal and broken bikes exceeds the targeted number; (2) RO is the best strategy when the sum of normal and broken bikes does not exceed the targeted number; (3) CO is the best strategy when the total number of normal bikes is greater than the targeted number. Regarding the implementation of these strategies, (1) CO strategy deploys the largest number of vehicles than RO and R&C and thus has the largest total travel time; (2) RO usually has the largest handling time due to the on-site repairing but the smallest travel time because it does not have broken bike delivery; (3) R&C can always achieve the minimal deviation in all scenarios while its travel time is not smaller than RO and its handling time is not smaller than CO. Furthermore, the above results demonstrates that the proposed formulation for R&C strategy can obtain the results for RO and CO strategy in certain demand scenario patterns and simultaneously determine the number of deployed vehicles.

3.3 Sensitivity analysis

This section analyzes the effects of several parameters towards the optimal repositioning strategies. To be specific, regarding the objective function, the ratio between the weights on the service time and the deviations from the target inventory levels are also investigated. For the operational parameters, the ratio between repairing time and the loading and unloading time, and the vehicle capacity are investigated. Unless specified, this section adopts scenario 1 for illustrative purposes and the parametric values except for the parameter in interest follow the definition in section 3.1.

To compare the ratio between the deviation from the target inventory level and the service time, here defines that the positive and negative deviations from the target inventory level (i.e., α and β) share the same weight. Table 4 shows the trade-off with respect to the weight of the deviation ranged from 0.01 to 100. It can be observed that when the weight of deviation increases, (1) the objective value increases; (2) the total deviation is non-increasing; (3) the pickup, drop-off, and repaired quantities are non-decreasing; (4) the total service time (i.e., the sum of travel time and handling time) is non-decreasing; and (5) the collected broken bike is non-increasing. For a smaller weight for deviation (e.g., 0.1), the total deviation is maximized because no pickup and drop-off operation is implemented and the travel distance is minimized under the restriction that all broken bikes are required to be collected. When the weight is between 1 and 1.5, there are pick-up and drop-off operations to reduce the total deviation. In addition to the increase in the handling time, the travel time increases such that the route can satisfy more unmet demand. When the weight is greater than 1.5, the number of repaired bikes increases to reduce the total deviation. Three vehicles are deployed when the weight is between 1.5 and 2.5 to achieve the optimal solution. When the weight is greater than 3, the objective value remains unchanged as the system is completely balanced. For the optimal solution when the weight is greater than 3, the numbers of pickup, drop-off, and repaired quantities (and thus the handling time) are at the maximum whereas the objective value is also the maximum.

Table 4　Optimal relocation strategy under different values of α

Value of α	Obj.	Dev.	TT	HT	UV	P/D	C/R
0.1	66.358	173	19.058	30	2	0/0	30/0
1	222.058	173	19.058	30	2	0/0	30/0
1.02	223.958	95	19.058	108	2	39/39	30/0
1.05	226.797	85	19.547	118	2	44/44	30/0
1.08	228.338	45	21.738	158	2	64/64	30/0
1.1	229.238	45	21.738	158	2	64/64	30/0
1.25	234.414	23	25.664	180	2	75/75	30/0
1.5	240.164	23	25.664	180	2	75/75	30/0
1.6	243.406	11	28.156	196	3	79/79	26/4
2	246.156	10	28.156	198	3	79/79	25/5
2.4	249.756	9	28.156	200	3	79/79	24/6
2.5	250.196	4	30.196	210	3	79/79	19/11
2.8	250.691	1	23.891	224	2	83/83	16/14
3	250.747	0	24.747	226	2	83/83	15/15
5	250.747	0	24.747	226	2	83/83	15/15
10	250.747	0	24.747	226	2	83/83	15/15

To investigate the effect of handling time towards the operational strategies, this analysis focuses on the ratio of repairing time with the loading and unloading time instead of adjusting both repairing time and loading and unloading time because the loading and unloading time is usually constant compared with the repairing time. Here assumes that the loading time and unloading time are equal and set to be 1 minute, and the repairing time intuitively must be equal to or greater than the loading and unloading time. Moreover, the value of β is set to be 10. Table 5 shows that this ratio does not have a significant impact on the solution. Meanwhile, the ratio between R and (α, β) has a significant impact on the optimal strategy. When $R \leq 10$, the optimal strategy remains unchanged despite the change in the value of R. Then, when $10 < R \leq 11$, the optimal strategy changes by sending more vehicles, having a larger total deviation, loading and unloading fewer normal bikes, and repairing fewer broken bikes (and collecting more broken bikes). When $R \geq 11.5$, the optimal strategy and the objective value remain unchanged because no broken bike repair is required in the strategy, so the increment in repairing time does not increase the objective value. In other words, the results here demonstrate that (1) the ratio between the repairing time and loading time has little effect on the optimal strategy; and (2) the ratio between the repairing time and the weight for deviation has a significant effect on the optimal strategy. So, the operator should focus on the ratio between R and (α, β) in determining the best relocation strategy.

Table 5 Optimal relocation strategy under different ratios between repairing time and loading and unloading times

Value of R	Obj.	Dev.	TT	HT	UV	P/D	C/R
1	220.747	0	24.747	226	2	83/83	15/15
3	250.747	0	24.747	226	2	83/83	15/15
10	355.747	0	24.747	226	2	83/83	15/15
10.5	362.696	4	30.196	210	3	79/79	19/11
11	366.156	10	28.156	198	3	79/79	25/5
11.5	367.784	15	29.784	188	3	79/79	30/0
12	367.784	15	29.784	188	3	79/79	30/0
15	367.784	15	29.784	188	3	79/79	30/0

The last analysis is to demonstrate the effect of vehicle capacity on the optimal strategy. In Table 6, the results can be separated into four ranges: (1) single vehicle with high deviation ($Q \leq 20$), (2) two vehicles with decreasing deviation ($20 < Q \leq 24$), (3) no deviations with decreasing travel distance ($24 < Q \leq 40$), and (4) no deviations and shortest path ($Q \geq 40$). For the first range, the increase in vehicle capacity results in the reduction of objective value and total deviation and the increment in loading and unloading quantity and total service time. The expansion of the vehicle capacity enables more vehicles to be relocated from bike surplus stations to bike deficit stations. As a trade-off, total service time increases due to the change in visit sequence. For the second range, two vehicles are deployed and some broken bikes are collected instead of repaired. As the collected broken bikes occupy spaces of the vehicle that hinder the bike relocation, two vehicles are deployed to share the broken bike collection. In the third range which does not have deviation, the increase in vehicle capacity does not increase the number of collected broken bikes but has a reduction on the deployed vehicle (when $Q \geq 30$) and the loading and unloading quantities (when $Q \geq 35$). The number of the collected bike remains unchanged because the number of the repaired bike is sufficient to satisfy the bike deficit of the whole system. The reduction in the deployed vehicle is that one vehicle has sufficient capacity to accommodate all the loading and unloading operations and at the meanwhile shorten the total travel distance. A further increase in the vehicle capacity (compare Q = 30 and Q = 40) can shorten the route because the vehicle can travel to closer stations for consecutive pickups or drop-offs. Finally, the fourth stage shows an intuitive result that a larger vehicle cannot shorten the travel distance and reduce the number of pickup and drop-offs.

Table 6　Optimal relocation strategy under different vehicle capacities

Q	Obj.	Dev.	TT	HT	UV	P/D	C/R
5	1927.509	123	17.509	140	1	25/25	0/30
10	1367.424	83	17.424	180	1	45/45	0/30
15	807.971	43	17.971	220	1	65/65	0/30
20	469.904	18	19.904	230	1	75/75	5/25
22	373.891	10	23.891	230	2	79/79	9/21
24	273.891	2	23.891	228	2	83/83	13/17
25	250.747	0	24.747	226	2	83/83	15/15
28	250.747	0	24.747	226	2	83/83	15/15
30	246.069	0	20.069	226	1	83/83	15/15
35	237.22	0	19.22	218	1	79/79	15/15
40	236.404	0	18.404	218	1	79/79	15/15
50	236.404	0	18.404	218	1	79/79	15/15

IV. Model extension

The above model can determine the optimal route under a weighted sum objective of the total deviation and the total service time. However, the achieved minimal total service time in the objective function may not satisfy the service time constraint because it does not limit the travel time of each route. This section introduces a revised formulation to handle the service time constraint and performs numerical experiments to establish the effect of the time constraint on the optimal R&C strategy.

4.1 Revised formulation

To determine the service time of a route, adding a subscript for the vehicle set in the routing and loading decision variables is the most straightforward method for calculation because the total service time is equivalent to the sum of the product of all service time attributes with all the values of variables associated with that vehicle. However, there are two drawbacks to this method. First, the number of decision variables is increased by $|V|$ times ($|V|$ denotes the number of the relocation truck) while most of them are zero finally. Second, the number of vehicles must be deterministic (or at least, the maximum fleet size). This section, therefore, introduces another way to formulate the service time constraint which can bypass the addition of the subscript for the vehicles under this problem.

The rationale of the formulation can be described as follows. Denote T as the maximum service time of a truck. In this problem, each station is visited once by one of the vehicles, so there must be one and only one arrival time at each station. The vehicle arrival time at a station can then be determined by the sum of (1) the vehicle arrival time at the preceding station, (2) the service time at the preceding station, and (3) the travel time between the preceding station and the arrival station. Furthermore, the maximum service time implies that the vehicle must return to the depot at or before T. Three constraints are then introduced to capture the arrival time at each station and the service time constraint.

$$a_j \le a_i + L\left(p_i + b_i\right) + Ud_i + Rr_i + t_{ij} + T\left(1 - x_{ij}\right), \forall i \in N_0, j \in N \setminus \{i\} \tag{28}$$

$$a_j \ge a_i + L\left(p_i + b_i\right) + Ud_i + Rr_i + t_{ij} - T\left(1 - x_{ij}\right), \forall i \in N_0, j \in N \setminus \{i\} \tag{29}$$

$$a_i + L\left(p_i + b_i\right) + Ud_i + Rr_i + t_{i0} \le T, \forall i \in N, \tag{30}$$

$$0 \le a_i \le T, \forall i \in N, \tag{31}$$

$$a_0 = 0, \tag{32}$$

Constraints (28) and (29) define the arrival time a_i at every station i. They define the difference of the arrival time between two consecutive stations to be the handling time of bikes at the preceding station and the travel time. Constraint (30) is the service time constraint of all stations. For a vehicle arriving at a station, its handling time at that station and the time to return to the depot must not exceed the maximal service time. Constraints (31) and (32) are the definitional constraints for service time, in which the arrival time at the depot is zero and the arrival times to all other stations should be non-negative and smaller than T.

When service time becomes a design constraint, the service time term in the constraint can be kept optionally. If it is removed, the objective function becomes minimizing solely the deviations from the target inventory level, which is

$$\min Z = \sum_{i \in N} \left(\alpha f_i^+ + \beta f_i^- \right) \tag{33}$$

If the initial objective function (1) is kept, the optimal solution can simultaneously minimize the total service times of all routes and guarantee that all the routes do not exceed the service time limitation. The mathematical model using the initial objective and the service time constraints can be presented as

Objective function (1)

subject to constraints (2)–(10), (12)–(17), (19)–(32).

In this model, it is noted that the sub-tour elimination constraint is replaced by the new set of constraints because the arrival time can restrict the number of visits to be once and avoid sub-tours.

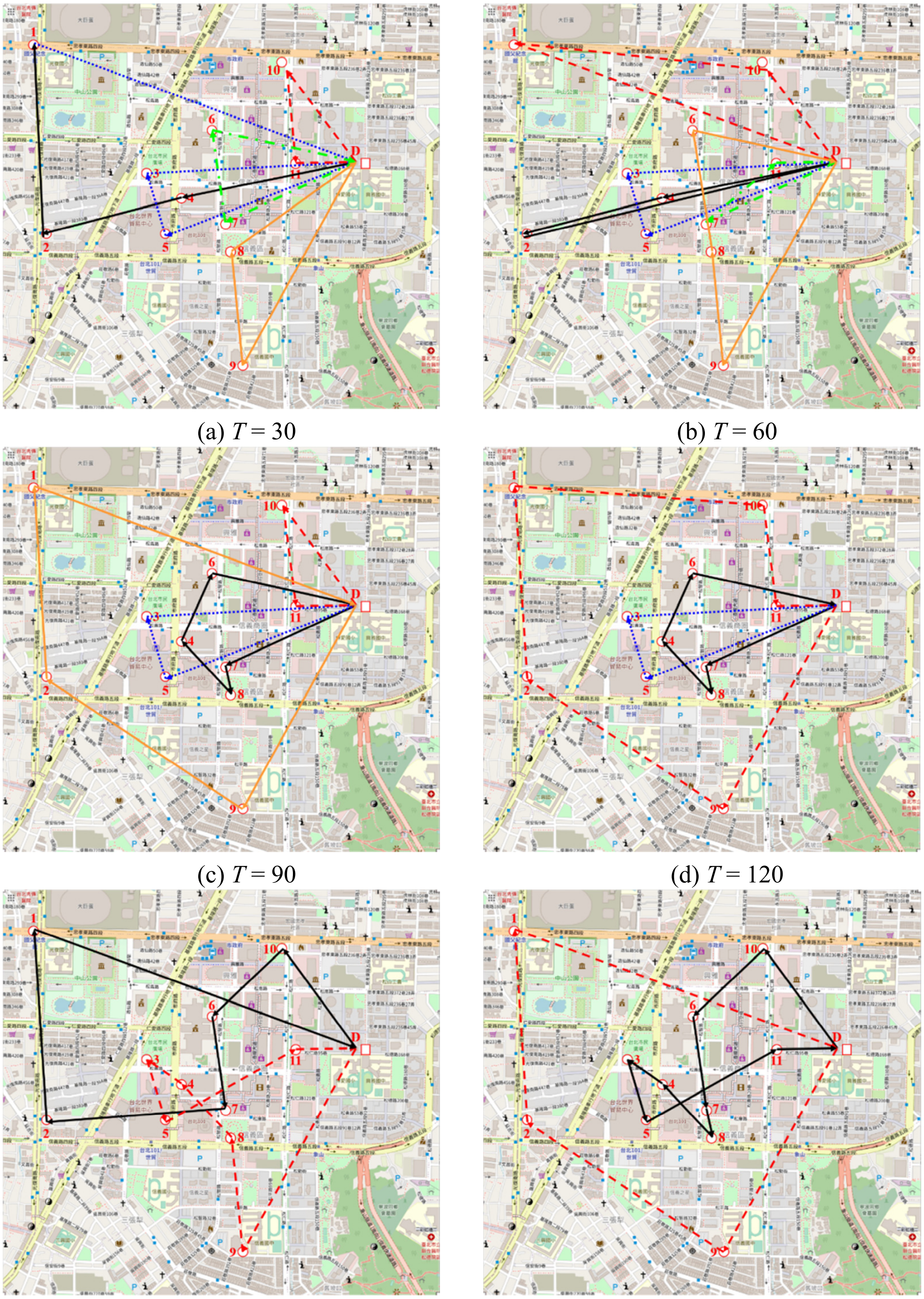
4.2 Comparison of the optimal tactics under maximal service time

This section compares the effect of the maximal service time towards the repositioning strategy. Table 7 demonstrates the results under different maximal service time *T*, where the column notations follow Table 2.

Table 7　Optimal relocation strategy under different maximal service times *T*

T	Obj.	Dev.	TT	HT	UV	P/D	C/R
30	1684.798	98	38.798	106	5	37/37	29/1
60	405.811	8	41.811	204	5	80/80	23/7
90	259.858	0	33.858	226	4	83/83	15/15
120	256.196	0	30.196	226	3	83/83	15/15
150	250.831	0	24.831	226	2	83/83	15/15
180	250.747	0	24.767	226	2	83/83	15/15
210	250.747	0	24.767	226	2	83/83	15/15

Figure5 (a)-(f) show the optimal routes under different service times. It can be seen that the objective value reduces when the service time increases from 30 to 180 while a further increase of *T* does not improve the objective value. When $T = 30$ minutes, very few bikes are repaired and relocated due to the service time limitation, and thus results in a large total deviation. When $T = 60$, the deviation is significantly reduced because more bikes are relocated and repaired. As a trade-off, the travel time is slightly longer for relocating more bikes. When *T* is greater than or equal to 90, the total deviations reach 0. When *T* increases from 90 to 180, the number of



(a) $T = 30$ (b) $T = 60$

(c) $T = 90$ (d) $T = 120$

(e) $T = 150$ (f) $T = 180$

Figure 5 Optimal relocation strategies under different maximal service times deployed vehicles decreases from 4 to 2 and the total travel distance is shorter. When T is greater than or equal to 150, the number of deployed vehicles is reduced to 2 while the total travel time decreases when T increases to 180. By examining the optimal routes shown in figures 5(e) (T = 150) and 5(f) (T = 180), a higher service time allows one of the routes to be longer such that the total traveling distance is minimized. These results show that a longer service time limitation can reduce firstly the total deviations, then the number of deployed vehicles, and finally the total travel distance in a multi-vehicle bike repositioning operation.

V. Conclusions

Handling broken bikes is an unavoidable practical operation in BSSs. Despite removing them back to the depot for repairing, this study considers on-site repair as another practical approach that can potentially save the time cost and loading and unloading time of the bike repositioning operation. This paper, therefore, proposes a novel static bike repositioning with maintenance operation which aims to minimize the weighted sum of the penalties for the positive and negative deviations from the targeted inventory level and the total service time by determining the vehicle routes, the loading and unloading quantities of normal bikes, and the number of collected and repaired broken bikes at each visited stations. A modified formulation that includes the service time constraint is also proposed. Numerical studies show that the combined collection and repair strategy can outperform the collection only and the repair only strategies in terms of deviation minimization, especially in the case that the existing normal bikes cannot meet the bike demand while the total number of normal and broken bikes is greater than the total bike demand. Sensitivity analyses demonstrate that (i) a larger weight for the deviation (and coupled with a larger ratio with the repairing time) can achieve an optimal strategy with minimal deviation; (ii) the effect of the ratio between repairing time and loading and unloading time is not significant; and (iii) an increasing vehicle size can reduce the deviation, the number of deployed vehicles, the travel distance, and the number of loaded bikes sequentially. When service time constraint is imposed, the proposed model establishes that, when the allowable service time increases, the total deviation is reduced firstly, followed by the number of deployed vehicles, and finally the total travel distance.

This study can be extended in several directions. First, broken bikes can be of different types of problems (for example, replacing broken/ stolen seats versus applying lubricating oil) that possess different repairing time or may require a compulsory repair at the depot. Sometimes the severity of the problems is uncertain such that the condition is revealed when the crew visits the station. So, a potential extension is to consider the optimal repositioning strategy in a system with broken bikes of different types or under uncertain conditions. Second, the repair operation can have further restrictions. These can be limitations on the repairing operation on a particular vehicle, type of crew, station, or depot (for multiple depot case). Third, more maintenance operations can be considered in addition to repair and collection. In practice, normal bikes may require regular checks subject to their usage conditions (e.g., number of rentals or total riding time). For some bikes which violate at least one of the usage conditions, they are not available to the users before having a check even if they are functional. These bikes may then be regarded as another type of 'broken bikes' which require on-site checking and further repairs if necessary. Forth, depot loading and unloading during the trip are possible such that the broken bikes can be delivered to the depot earlier and the service time may be potentially reduced. Fifth, as the service level can further deteriorate when the broken bikes are left idle in the system for a long time, a practical extension is to embed broken bike maintenance in daytime bike repositioning operation such that the idle time of the broken bikes can be reduced significantly. Finally, as this study only uses a system with a small number of stations, new solution methods can be developed to handle instances with larger network sizes, as commercial solvers are usually effective in instances with small networks. Heuristics or meta-heuristics are potentially more efficient and effective method in handling large-instance problem.

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Appendix I – Time matrix and demand intervals

Table I.1　Distance matrix (presented by Lin & Yang (2020))

i \ j	D	1	2	3	4	5	6	7	8	9	10	11
D	0	1901	2480	1333	1552	1681	1006	1394	1571	1949	849	363
1	1901	0	857	1302	1234	1333	1173	1495	1709	2276	1102	1598
2	2480	857	0	1807	1244	1426	1562	1533	1283	1824	1928	1830
3	1333	1302	1807	0	159	362	571	445	664	1226	1517	727
4	1552	1234	1244	159	0	186	444	327	543	1088	1014	602
5	1681	1333	1426	362	186	0	648	523	744	1301	1209	806
6	1006	1173	1562	571	444	648	0	338	557	1112	962	688
7	1394	1495	1533	445	327	523	338	0	223	776	1292	490
8	1571	1709	1283	664	543	744	557	223	0	552	1521	731
9	1949	2276	1824	1226	1088	1301	1112	776	552	0	1671	1144
10	849	1102	1928	1517	1014	1209	962	1292	1521	1671	0	507
11	363	1598	1830	727	602	806	668	490	731	1144	507	0

Table I.2　Normal and broken bike distributions and target inventory levels of all stations

Scene	Info	**D**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
1	II	0	10	20	15	20	30	10	50	10	30	80	20
	FI	0	30	25	35	15	11	25	30	24	10	65	40
	BB	0	5	0	5	0	5	0	5	0	5	0	5
2	II	0	10	24	15	24	30	14	50	14	30	84	20
	FI	0	30	25	35	15	11	25	30	24	10	65	40
	BB	0	5	0	5	0	5	0	5	0	5	0	5
3	II	0	10	16	15	16	30	6	50	6	30	76	20
	FI	0	30	25	35	15	11	25	30	24	10	65	40
	BB	0	5	0	5	0	5	0	5	0	5	0	5
4	II	0	10	23	15	23	30	13	50	13	30	83	20
	FI	0	30	25	35	15	11	25	30	24	10	65	40
	BB	0	0	0	0	0	0	0	0	0	0	0	0

II: Initial inventory level; FI: final inventory level; BB: broken bikes