運輸計劃季刊
第四十九卷 第二期
民國一〇九年六月
頁 147～頁 171

Transportation Planning Journal
Vo1. 49 No. 2
June 2020
PP. 147～171

不確定天氣條件下離島運輸服務的實質選擇權定價模型[1]

A REAL-OPTION PRICING MODEL FOR OFFSHORE ISLAND TRANSPORTATION SERVICE UNDER UNCERTAINTY WEATHER CONDITIONS

褚志鵬 Chih-Peng Chu[2]
蕭義龍 Yi-Long Hsiao[3]
饒玉玲 Yu-Ling Jao[4]

(107 年 7 月 26 日收稿，108 年 1 月 28 日第 1 次修改，108 年 9 月 23 日第 2 次修改，108 年 11 月 22 日第 3 次修改，109 年 2 月 16 日第 4 次修改，109 年 4 月 22 日定稿)

摘　要

惡劣天氣會擾亂人們的日常行程，也會給旅行者帶來更多的成本。對於離島運輸系統，天氣影響往往是旅運服務不確定的主因之一。例如，台灣在霧季期間，離島和主島之間常常有運輸風險。本研究通過實質選擇權處理不確定性的特性來評估天氣風險，以台灣為個案，提出解決其離島地區因天氣造成的交通不便的方式，因為低能見度可能導致飛機和船運輸服務的關閉。本研究根據 Black-Scholes 模型為旅行者設計了一個選擇權產品以對沖他們的旅行風險。本研究提出了計算框架並以收集到的實際資料計算了風險價格。

1. 本研究承科技部專題計畫 (NSC 101-2410-H-259-011-MY2 及 NSC 103-2410-H-259-028-MY2) 補助，特此致謝。
2. 國立東華大學企業管理學系暨運籌管理研究所教授(聯絡地址：97401 花蓮縣壽豐鄉志學村大學路二段 1 號 國立東華大學企業管理學系；電話：03-8903029；E-mail：chpchu@gms.ndhu.edu.tw)。
3. 國立東華大學財務金融學系副教授。
4. 國立東華大學企業管理學研究所研究生。

關鍵詞： 實質選擇權；離島運輸；天氣不確定性

ABSTRACT

Bad weather situations disturb people's daily schedules, but also cause more costs for travelers. For the offshore island transport system, the weather is one of the major uncertain factors affecting travel services. For example, in Taiwan during fog seasons, the transportation between offshore islands and the main island could be risky. This study values the risk of weather uncertainty by the advantage of real options. We, taking Taiwan's situation as an example, resolve the traffic inconvenience caused by weather in offshore island regions. The low visibility could lead to the shut-down of aircraft and vessel services. We design an option product for the travelers to hedge their travel risk based on Black-Scholes model. In this study, we proposal the computational framework and compute the price of risk.

Key Words: *Real option; Offshore island transportation; Weather uncertainty*

I. INTRODUCTION

Nowadays, weather plays a decisive role in around four-fifths of the world's economic activity. The United Nations Intergovernmental Panel on Climate Change (IPCC)[1] released a report in 2007 that there is evidence that climate change is causing heat waves, record high temperatures, heavy rainfall, hurricanes and other extreme weather events. The frequency of occurrence, intensity, duration of the events and the extent of such changes have appeared to expand in the past half century. Global climate change is now not only affecting national security, but also impacting our individual daily routines as well as commercial and leisure activities. Extreme weather also causes transport systems to be seriously stalled all over the world. For example, in the United States, snowstorms often cause air and ground traffic chaos, and passengers were stranded in the airport because of flights cancelled.

Due to such changes in the weather, IPCC have warmed that the earth has entered the era of rapid climate change and that the world needs to effectively manage the extremes of climate change and the risk of disaster. So, in order to respond to weather uncertainty, relatively preventive or compensatory measures have been developed. Disaster risk management and adaptation to climate changes mainly attempts to reduce exposure and vulnerability, and raise awareness of the potential adverse effects of extreme climate resilience.

For the offshore islands, the impacts of weather uncertainty could be even significant. One of the many reasons is that transportation is vital and indispensable for the offshore islands, and the condition of the weather is the most important factor for transportation. Take Taiwan as an example. The main meteorological factors that influence the offshore islands in Taiwan are typhoon and fog, with individual islands affected by different circumstances depending on their different locations. There is a need to minimize the uncertainty for travelers, such as whether the aircraft have the ability to be substituted for some other means of transport or by other compensation programs. Travel insurance programs are currently available products to reduce to cost of travel inconveniences, however, those programs only provide post hoc monetary compensation. Travelers still suffer the most of costs of weather risk in their trips.

In this paper we apply the concept of real options to minimize travelers' costs in case of flight suspensions, and give them the right to choose alternatives. The mechanism we proposed here not only develops a risk hedge for the passengers between offshore islands but could also benefits the option providers: airline companies, the cruise lines (ferry companies), and hotels. For these option providers, they can usually earn a premium in advance, and increase cruise passenger and hotel occupancy rates.

The mitigation of risks can be carried out with various tools or techniques. One of common tools is insurance. Table 1 Provides comparison of travel insurance and the real option proposed in this study. This table clearly points out the real options could provide an incentive for travelers, travel agency, as well as the hotels owners and ferries owners. Those stakeholders would reduce their risks due the weather and reserve the supply for travelers in the bad days.

Table 1　Differences between the insurance and option

	insurance	option
Risk Compensate Timing	After the event	In the event
Stakeholders	travelers and airline company	travelers, airline company, hotels industry, cruise industry
Advantages	Simple	A better incentive for the whole tourism supply chain to provide comfort service and share the risk
Disadvantages	Only monetary compensated. Do not solve the travel problem	Travelers have to be able to evaluate their risk
Compensated	Monetary	An alternative trip plan

II. Literature Review

2.1　Inter-island Transportation

For hundreds of years, the need to move people or goods from one place to another has increased dramatically. With the improvement of transportation technologies and the complication of geographical development, the travel of people or goods may involve one or more modes of transport in the process. Each mode of transport has its advantages or disadvantages in terms of cost, capacity, surface constraints, and so on. For the transportation systems among islands, air transport and sea transport are the most basic means.

Rutz and Coull [2] collected the inter-island shipping information of Indonesia. From their research, it shows that the island's route network is dependent on ship (ferries), and that shipping is the most basic and important means of Indonesia's transportation. Due to the amount of the regional population, geographical differences, and the degree of economic development, the frequency of ship service changes. Luis [3] using the case of air transport in the Canary Islands showed that if an offshore island is far from any other islands, the offshore island will mainly rely on aircraft. It also shows a positive relationship between how frequency of the flight/cruises and the island's economic development.

To close the service discrepancy, Seo et al. [4] suggested that the government should support the development of the offshore island, and reduce transportation costs to stimulate tourist arrivals using low cost airlines. The competitive prices enable the offshore island to remain competitive domestically.

According to the Word Bank's 2005 publication, Tsai and Chen [5] pointed out that "Natural Disaster Hotspots - A Global Risk Analysis," there is at least 73 % of the land area and population of Taiwan facing more than three types of natural disaster threats, which will increase the occurrence of risks to travelers and unpredictable losses for carriers.

2.2 Weather Uncertainty on Transportation and Tourism

More than 80 % of economic activities depend on the weather, and the losses caused by weather fluctuations can be up to 30 percent of the U.S. gross domestic product (Edrich [6]). Coombes and Jones [7] indicated that the impact of global climate change may increase the vulnerability of the environment. Weather changes have affected not only industrial development, but also the operation of the transportation. Research by Pisano et. al. [8] shows that accident between 1993- 2002 were mainly caused by weather in USA, and the phenomena of snow, fog and rain are the primary events that cause accidents. Koetse and Rietveld [9] pointed out that climate change may cause different results in terms of transport demand, and will change the tourism mode for travelers. Khattak and de Palma [10] had studied the influence of adverse weather to travelers' decision on their mode, routes and departure time changes. Rossello [11] investigated the link between the North Atlantic oscillation index and international air travel. The results show the climate condition is one of the important facts in determining tourist decision. In addition, Saneinjad et al. [12] provided a multinormal logit model to capture the impact of weather condition on home-based work travel behavior.

On the air transport side, the weather events may cause delays or aircraft accidents, caused by strong wind. However, the wind is just one of the reasons and the poor visibility will also influence the taking off and landing of aircraft. In San Francisco International airport, the runways are shorter than that in most others airports, so the Ministry of Transportation has ruled there must be a certain amount of visibility in order for planes to take off the land. Therefore, this means that a number of takeoffs and landings will be delayed or cancelled due to fog.

The option strategy is a practical tool in freight industry (Hummels and Schaur [13]; Feki et el. [14]). For airline industry, it is also used to hedge the risk of fuel cost (Morrell and Swan [15]; Berghöfer and Lucey [16]). In the late 1990s, the concept of weather hedging was formed gradually in the industry due to the losses during El Nino. Compared with other industries, the energy industry was the first one to use weather hedging and is the largest demander for the weather hedging market.

Richards et al. [17] present a method to use the Brownian motion process. It defines a pricing model for the temperature of low temperature days to avoid disasters. Wang et al. [18] introduce an extended financial market as an independent random process model including weather risk and validity. It also provides a price of weather derivatives arbitrage approach. Xu et al. [19] point out that it is not easy to make a price for weather derivative, and the study develop a model that uses the indifference pricing concept to calculate the willingness to pay for weather insurance. For example, Brandenburg in northeastern Germany is highly affected by weather risks because due to drought. Depending on the data of weather and temperature, it uses the indifference pricing model to set the price in order to avoid disasters. Example of this type of weather derivative are the most common.

Nilim et al [20] provided a dynamic path policy which uses a trajectory-based air traffic management system (TB-ATM) to reduce the expected delay time of aircraft when the aircraft flight path will be affected by bad weather. Steiner et al.[21] discussed creating weather forecast data with high-resolution in order to combine the relevant information in using an ensemble model and aviation-related weather information.

To the authors' knowledge, there are few studies on applying option pricing in passenger air transportation, especially for hedging the weather risk. Their study establishes representative probability forecasts for the expected results of air traffic management.

2.3 The Real Option

The option pricing model has evolved into a sophisticated financial product, thus making enterprises involved in financial or operational risk arbitrage much easier. As option pricing models have become increasingly popular, the applications of real options have gradually expanded the scope of industries to capital budgeting decisions.

Jain and Cox [22] presented a study whereby visitors can obtain lower prices when buying a ticket. The study collected the prices on fourteen aircrafts from four U.S. airlines, and analyzed the risk caused by the volatility of fares for passengers. Air travelers will face the risk of ticket price rises in the future. If the passengers buy the ticket now, the ticket price may fall later. The buyer can cancel or execute their plan flexibly in order to avoid the uncertainty of fares in the future by using the concept of options. Treanor [23] described how to use real options in the aviation industry as a risk management tool to help the company in terms of capital budgeting and risk management. He also explained how to prevent fuel price rises under uncertainty by means of route diversification.

This study proposes a pricing mechanism to circumvent the cancellation of aircraft caused by the suspension of services for travelers. In this study, the airlines offer two kinds of ticket which have different types of price. One is a normal ticket, while there is another option for aircraft tickets which are much more expensive than the average price. And the contract will use the visibility as implement standard because it is the norm for the operating by airplane and ships[5]. When visitors purchase tickets with option rights, they can perform their rights when dense fog causes poor visibility which makes the aircraft unable to take off. The options will give visitors the right to take ships to proceed with their trip. However, there may occur two to three days of traffic interruptions when the fog is in the area. In this case, the right to choose to complete the journey before the due date will remain a week or so, and in the meantime, if the next day, the traffic is still interrupted, people can continue to stay in the original hotel.

This paper follows the modeling process of Black–Scholes equation which utilizes the partial differential equation to give an estimate of the price of options. Broadly speaking, the term of Black-Scholes may refer to a similar PDE that can be derived for a variety of options. Since that there is no incentive for early exercise, the price of an American and European call option without dividends should not diverge. Therefore, our evaluation model is a series of PDE with initial condition. To fit in financial market, the several assumptions are set for mathematical process. To analogy, in our model, the option can only be exercised at the beginning of the vocation; the market is the weather which the conditions cannot be fully predicted; the risk-free rate and volatility of the underlying are known and assumed to be constant.

In this study, we used the Bermudan option to formulate our pricing option model. The Bermudan option is one of the path dependent options. It is an option between the American and

5. The runway visual range (RVR) for airport and the restricted visibility for seaport are safe standards for maintaining operations. Different airport and seaport have various standards.

European options because the contract to perform uses different ways. The contract performance period is more than just one point in time which means the buyer has the right to choose to perform, and the strike price of the set point may vary according to performance. So, the concept of the Bermudan option is helpful to support our model.

We formulate the substitution or compensation scheme options pricing model when the aircraft is suspended, and we will create a way to find out the value of the contract, and to measure the various changes to the compliance period. For these purposes, we use the Black-Scholes model on the application of the options as a basis to extend our model. So, according to the assumptions of the Black-Scholes model, we will modify and develop the assumptions of our model as below:

(1) No transaction costs or tax means that it can be converted to others of the same trade.
(2) The option is the Bermudan option (quasi-American option) and it has a different contract compliance period.
(3) The visibility of the airport or seaport follows the log normal distribution because it changes as a percentage when the visibility changes.
(4) Fixed risk free interest rate means the risk free interest rate will not change for the duration of the contract period.
(5) The trade is continuous because the contract will be implemented a continuous trading opportunities.
(6) The volatility of the contract is fixed and known during the contract period.

III. Framework and Model Formulation

In this section, we introduce the framework of how the weather option works by using a case study to interpret the process. Then, we give more details on the model formulation to explain the computation formulation and parameters.

To avoid misunderstand, we use Black-Scholes model instead of standard Black-Scholes model. In our study, there is no dividend for the weather risk option. Therefore, without dividend in the system, there is no incentive to early exercise, so the early exercise feature of an American call has no value. Hence, the price of an American and European call option without dividends should not diverge. Besides, Bermudan option can be seen as an option between an American-style option, which can be exercised at any point before expiration, and a European-style option, which can only be exercised on its expiration date; just as Bermuda itself is located between the US and Europe. That is in our case, the Block-Scholes model would fit to our research structure.

3.1 The Framework of Weather Option

We assume that airline companies offered two programs when the traveler purchases the airline ticket from Taiwan to Matsu, an offshore island of Taiwan. We assume that a traveler decided to purchase the three-day period option one month before his itinerary in order to reduce the risk of being unable to return to Taiwan. Figure 1 shows the framework of two programs for the travelers. The first program is only a regular airline ticket; the second program is the air ticket with the option. For the second program, of course, the traveler has to pay an extra fee, the premium.

For example, on March 1st, if the traveler buys a return airline ticket with option and plan to return from Matsu to Taiwan on March 28, the contract would cover from March 28 to March 30 (the maturity day). During these three days, if the weather is too bad to fly back, the traveler could exercise his option right.

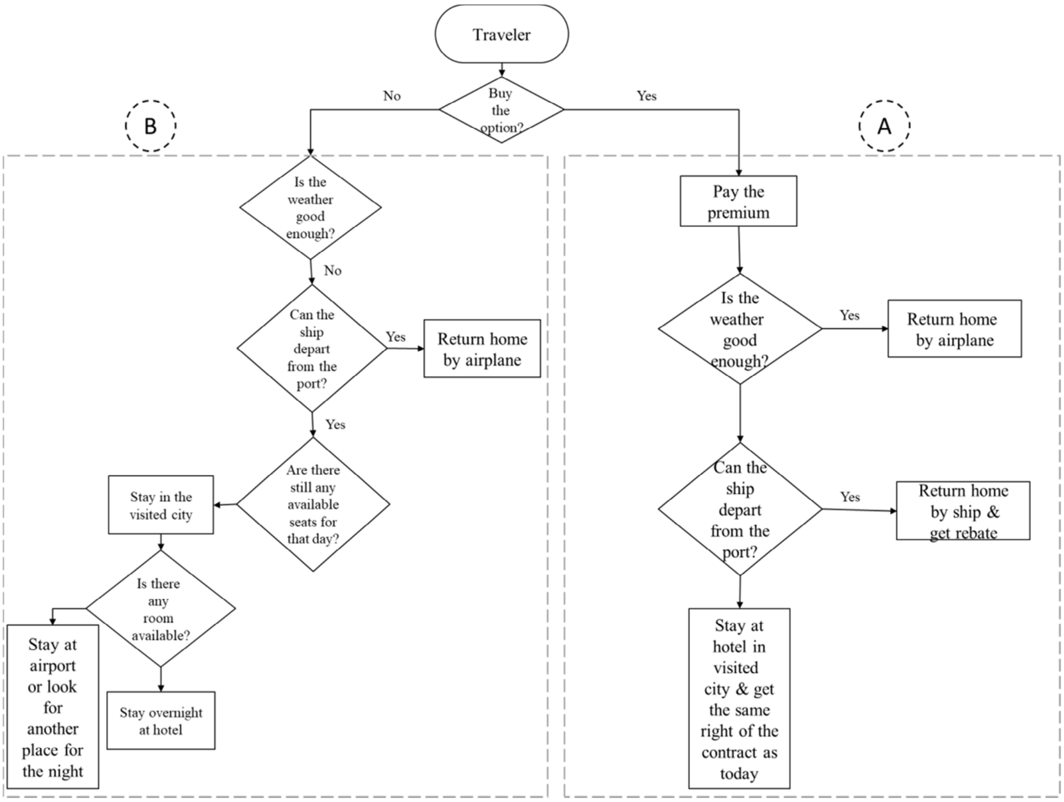


Figure 1　Illustration of the process of the option contract.

Now we assume that on March 28, if the visibility is low and the airport is closed but the harbor is not, the traveler has priority to buy the cruise ticket (but only pay partially) to return back to Taiwan and the traveler will also receive a compensation depended on the level of visibility[6]. If the visibility is too low such that even the harbor failed to operate. In that case, the travelers have to stay in Matsu for at least one more night. Those travelers with option will have the priority to stay at the pre-specified hotel. The traveler has to pay the price of the hotel rate (but only pay partially) to stay. Additionally, the travelers still have the same option tomorrow until the date of maturity. Note that the premium also depends on the percentage of the full price that the traveler wants to pay for the ferry and hotel room. The three-day decision process shows a figure in Appendix.

3.2 Model Formulation

From reports, we know that fog could cause airport to shut-down for a couple of days (Erdman [24], Machidon et al. [25]). Therefore, the contract period should be more than one day. In this research, we assume the contract lasts for three days. The travelers, if they want, can buy the option in advance, say on day t_0. The period of implementation starts from t_1 to T. The last day of contract, T, is the maturity day. Figure 2 shows the timeline of the purchase day, and the period of the contract from the first executable day to the maturity day.

As in Figure 1, we set $\tau = T - t$, is the time to maturity. The initial condition is when $\tau = 0$ because that the payoff is known. Then we can use American-style option to calculate the value of option at $\tau = T - t_1$, for $t \in (t_1, T)$. Then, the value of $\tau = T - t_1$ is the initial condition for us to calculate the option value at the purchase day $t = 0$, that is $\tau = T - 0$. For this time period, we use European-style option for value calculation. However, since there is no incentive for early exercise, the price of an American and European call option without dividends should not diverge. Additionally, Bermudan option can be seen as an option between an American-style option, which can be exercised at any point before expiration, and a European-style option, which can only be exercised on its expiration date; just as Bermuda itself is located between the US and Europe. That is in our case, the Block-Scholes model would fit to our research structure.

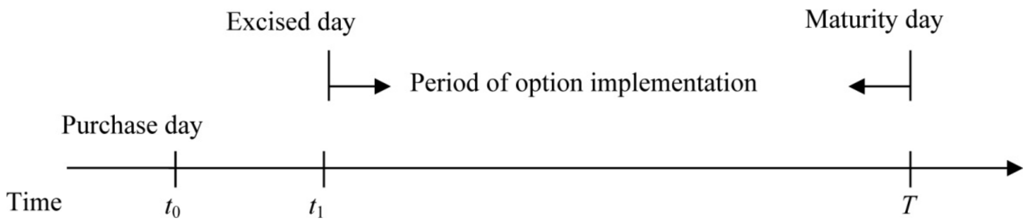


Figure 2 Timeline of the options.

6. The complete scenarios will be introduced in Section 3.2.

There are many factors to decide whether the airport can operate normally in the event of fog. In order to focus on the fog's impact, and to make it simpler, we assume that the visibility is the only factor involved.

(1) If the visibility (*V*) is higher than the visibility limit of airport (*B*), the airport can function well and let aircrafts take off; in this case, the option buyers cannot exercise the option.

(2) If the visibility (*V*) lies between the limit of the airport and the limit of harbor (*K*), the traveler can take a ferry to leave but not an airplane (because the airport will be closed). In this case, the option buyers obtain a ferry ticket (M_S) and compensation, because it takes more time to travel by sea than by air.

(3) Finally, if the visibility (*V*) is lower than the visibility limit of the harbor, the contract will provide the traveler with a hotel room (M_H) and an airline ticket for the next day.

For the setting of the contract, we subsidized part of the fare, α_s, when the visibility is in the workable range for ferries. If the visibility (*V*) is too low and causes a shut-down of transportation, some part of the accommodation expenses, α_H, will be subsidized. The reason for subsiding only part of the fare is because if we subside the full price for the ticket or rate, it will lead to higher premiums for travelers, and decrease travelers' willingness to buy (as in Figure 3).

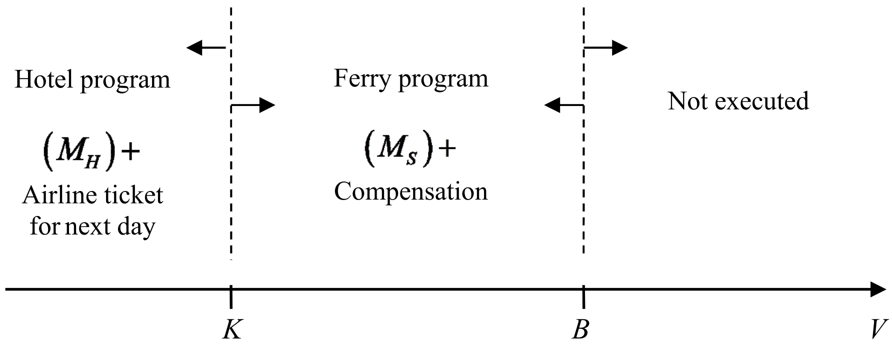


Figure 3　Implementation of a contract at different ranges of visibility.

Table 2 shows the rewards of partners involved in this contract at different visibility. If all transportation shut down when the visibility is too low, the hotel operators can earn the room rates; if it is only able to return to the destination by ferry, then the ferry operators will be able to earn the fare. But if the aircraft can take off smoothly, and then the hotel industry and ferry industry can get the basic right of premium.

To solve the price model of the option, the parameters needed are shown in Table 3. We use backward processes to construct the payoff for travelers with the consideration of the level of visibility. In Equation (1), the payoff on the maturity day is shown.

Table 2 Rights of industry and the traveler.

	$V \geq B$	$B > V \geq K$	$V < K$
Traveler	No	Right for take ship & compensation	Right for stay in hotel & airline ticket for tomorrow
Airline	Air fare & Premium	Get overcharge	Lose overcharge
Ship	Premium	Fare	Premium
Hotel	Premium	Premium	Room rate

Table 3 Definition of the parameters.

Parameters	**Definition**
$\boldsymbol{C}$	Price of the option
$\boldsymbol{B}$	Visibility limit of the airport
$\boldsymbol{K}$	Visibility limit of the ferry
α_H	Compensation ratio for the hotel room
α_S	Compensation ratio for ferry ship ticket
$\boldsymbol{M_H}$	Hotel room price
$\boldsymbol{M_S}$	Ferry seat price
$\boldsymbol{M_F}$	Airline ticket price
$\boldsymbol{R_H}$	Return of the hotel industry
$\boldsymbol{R_S}$	Return of the ferry industry
$\boldsymbol{R_P}$	Return of personal savings in bank
μ	Average increment of the visibility
σ	Average volatility of the visibility

$$\text{Payoff}_T^*(V)=\begin{cases}\alpha_{\mathrm{H}} M_H & ,\text{if } V<K \\ \alpha_{\mathrm{S}} M_S+\dfrac{V-K}{B-K}\times\left(M_F-M_S\right) & ,\text{if } K\leq V<B \\ 0 & ,\text{if } V\geq B\end{cases} \tag{1}$$

Because the size of water vapor in the air forms a dense fog and affects the visibility, the visibility of the concentration of fog is geometrically changed with the concentration of water. The change in visibility is in accordance with a log-normal model (or geometric Brownian motion) (Mohan and Payra [26]). To authors knowledge, there is no direct study about the distribution of visibility. Mohan and Payra [26] is one of the papers prove the aerosol number concentration (ANC) is log-normal distributed and also made a connection between ANC and visibility. In addition, we have tested the visibility data from CWB Observation Data Inquire System, it shows that their log values present as a normal distribution.

We assume that the level of visibility follows a log-normal distribution (Bullough and Rea [27]; Thies et al. [28]) [7]. That is, the change of visibility is not proportionally inversed to the absolute value of change of fog concentration; when the visibility is 300 meters to 200 meters, the change of fog concentration is many times more than that when the visibility is reduced from 3300 meters to 3200 meters. Gultepe et al. [29] shown that the relationship of visibility and fog concentration would be simplified as a logarithm function. Therefore, we transform the visibility to the variable *x* as shown in Equation (2).

$$x = \ln V + \mu \cdot \tau \tag{2}$$

where the transformation of visibility *x* considers the drift μ, and $\tau = T - t$ is the time to maturity [8].

When the transformed visibility *x is* large enough ($x \geq \ln B$), the airport functions well and the traveler can go back by air. The payoff to the traveler in this case is zero. If the transformed visibility *x* is low enough to shut-down the airport but still high enough for ferries to operate ($\ln K \leq x < \ln B$), the traveler will have discount on the ferry with a compensation (to cover the difference in speed between air and sea transportation). If the transformed visibility *x* is too low to shut down both the airport and the harbor, the traveler will have a discount on a reserved hotel room. Since it is at the maturity date, the option will end. Thus the transformed payoff at the maturity is rewritten in Equation (3), which describes the three scenarios for different visibility levels

$$\text{Payoff}_T(x) = \begin{cases} \alpha_{\mathrm{H}} M_H & ,\text{if } x < \ln K \\ \alpha_S M_S + \dfrac{e^x - K}{B - K} \times \left(M_F - M_S \right) & ,\text{if } \ln K \leq x < \ln B \\ 0 & ,\text{if } x \geq \ln B \end{cases} \tag{3}$$

7 The heat conduction equation, $\frac{\sigma^2}{2} u_{xx}(x,\tau) = u_\tau(x,\tau)$ is derived by the transformation of variables and the introduction of Green's function. Eq. (4) is then used to describe the expected value for the option at maturity.

8 The relationship of visibility and fog concentration is simplified as a logarithm function. (Gultepe et al. [29])

The recursive process of computing the value of the option is shown in Figure 4. The value of the option on the day before the maturity is

$$\begin{aligned} \mathrm{C}_{n-1}(x_{n-1},\tau_{n-1}) &= e^{-R_H\cdot(\tau_{n-1}-0)}\int_{-\infty}^{\ln K}\alpha_H M_H\cdot \mathrm{g}(x,0;x_{n-1},\tau_{n-1})\mathrm{d}x \\ &+e^{-R_S\cdot(\tau_{n-1}-0)}\int_{\ln K}^{\ln B}\alpha_S M_S\cdot \mathrm{g}(x,0;x_{n-1},\tau_{n-1})\mathrm{d}x \\ &+e^{-R_P\cdot(\tau_{n-1}-0)}\int_{\ln K}^{\ln B}\frac{e^x-K}{B-K}\times(M_F-M_S)\cdot \mathrm{g}(x,0;x_{n-1},\tau_{n-1})\mathrm{d}x \end{aligned} \tag{4}$$

where the function $g(.)$ is the log-normal probability density function of visibility as follows.

$$\mathrm{g}(x,\tau;\bar{x},\bar{\tau})=\frac{1}{\sqrt{2\pi\sigma^2(\bar{\tau}-\tau)}}\exp\left(-\frac{(x-\bar{x})^2}{2\sigma^2(\bar{\tau}-\tau)}\right)\cdot \mathrm{H}(\bar{\tau}-\tau)\,, \tag{5}$$

where the Heaviside step function $H(.)$ is

$$\mathrm{H}(\bar{\tau}-\tau)=\begin{cases}1 & ,\text{if } \bar{\tau}-\tau>0 \\ 0 & ,\text{if } \bar{\tau}-\tau\le 0\end{cases}. \tag{6}$$

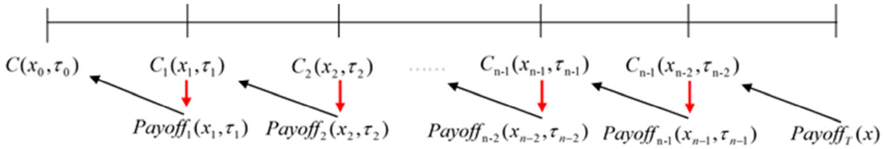


Figure 4 Recursive process of computing the value of the option.

The expected value of the transformed payoff on the maturity day is the integrals in Equation (4). There are three kinds of discount factors of concern: for hotels, for ferries, and for travelers. Since the premium for reserving and using a room can be reinvested in the hotel, we use the return rate of the hotel industry (R_H) for the discount rate. Similarly, for the ferry industry, we set the return rate of the ferry service industry (R_S) as the discount factor. The discount factor (R_P) for travelers is computed as the present value of the future income (the compensation due to the bad weather situation), therefore, the bank saving rate is applied. The option price C(.) is the sum of the discounted value with respect to every discount factor (R_H, R_S and R_P) at the time of maturity τ_{n-1}. In Equation (4), the first item is the share of premium for the hotel industry with the return rate of this industry. The second item is the share of premium for the ferry industry with the return rate of this industry. The third item is the share of premium for the traveler with the bank savings return rate.

Since the time of maturity τ_{n-1} is not the last day, the next day's option will be considered a part of the transformed payoff. In detail, Equation (7) shows the transformed payoff on the day before the maturity day.

$$\text{Payoff}_{n-1}(x_{n-1},\tau_{n-1})=\begin{cases}\alpha_H M_H+\mathrm{C}_{n-1}\left(x_{n-1},\tau_{n-1}\right) & ,\text{if } x_{n\text{-}1}<k_{n-1}\\ \alpha_S M_S+\dfrac{e^{(x_{n\text{-}1}\text{-}\mu\cdot\tau_{n\text{-}1})}-K}{B-K}\times\left(M_F-M_S\right) & ,\text{if } k_{n-1}\le x_{n\text{-}1}<b_{n-1}\,,\\ 0 & ,\text{if } x_{n\text{-}1}\ge b_{n-1}\end{cases} \tag{7}$$

where the transformed visibility limits of the ferry and airport at time of maturity τ_{n-1} are $k_{n\text{-}1}$ and $b_{n\text{-}1}$, respectively, where $k_{n-1}=\ln K+\mu\tau_{n-1}$ and $b_{n-1}=\ln B+\mu\tau_{n-1}$.

Similarly, we can obtain the option premium value two days before the maturity day in Equation (8) with respect to each discount factor (R_H, R_S and R_P).

$$\begin{aligned}&\mathrm{C}_{n\text{-}2}(x_{n-2},\tau_{n-2})=e^{-R_H\left(\tau_{n-2}-\tau_{n-1}\right)}\int_{-\infty}^{k_{n-1}}\alpha_H M_H\cdot \mathrm{g}(x_{n-1},\tau_{n-1};x_{n-2},\tau_{n-2})\mathrm{d}x_{n-1}\\&+e^{-R_S\left(\tau_{n-2}-\tau_{n-1}\right)}\int_{k_{n-1}}^{b_{n-1}}\alpha_S M_S\cdot \mathrm{g}(x_{n-1},\tau_{n-1};x_{n-2},\tau_{n-2})\mathrm{d}x_{n-1}\\&+e^{-R_P\left(\tau_{n-2}-\tau_{n-1}\right)}\int_{k_{n-1}}^{b_{n-1}}\frac{e^{\left(x_{n-1}-\mu\tau_{n-1}\right)}-K}{B-K}\times(M_F-M_S)\cdot \mathrm{g}(x_{n-1},\tau_{n-1};x_{n-2},\tau_{n-2})\mathrm{d}x_{n-1}\\&+e^{-R_H\tau_{n-2}}\int_{-\infty}^{k_{n-1}}\int_{-\infty}^{\ln K}\alpha_H M_H \mathrm{g}(x,0;x_{n-1},\tau_{n-1})\cdot \mathrm{g}(x_{n-1},\tau_{n-1};x_{n-2},\tau_{n-2})\mathrm{d}x\mathrm{d}x_{n-1}\\&+e^{-R_S\tau_{n-2}}\int_{-\infty}^{k_{n-1}}\left(\int_{\ln K}^{\ln B}\alpha_S M_S \mathrm{g}(x,0;x_{n-1},\tau_{n-1})\mathrm{d}x\right)\cdot \mathrm{g}(x_{n-1},\tau_{n-1};x_{n-2},\tau_{n-2})\mathrm{d}x\mathrm{d}x_{n-1}\\&+e^{-R_P\tau_{n-2}}\int_{-\infty}^{k_{n-1}}\int_{\ln K}^{\ln B}\frac{e^{x}-K}{B-K}\times\left(M_F-M_S\right)\cdot \mathrm{g}(x,0;x_{n-1},\tau_{n-1})\cdot \mathrm{g}(x_{n-1},\tau_{n-1};x_{n-2},\tau_{n-2})\mathrm{d}x\mathrm{d}x_{n-1}\end{aligned} \tag{8}$$

The transformed payoff for two days before the maturity is shown in Equation (9).

$$\text{Payoff}_{n-2}(x_{n-2},\tau_{n-2})=\begin{cases}\alpha_H M_H+\mathrm{C}_{n-2}(x_{n-2},\tau_{n-2}) & ,\text{if } x_{n-2}<k_{n-2}\\ \alpha_S M_S+\dfrac{e^{\left(x_{n\text{-}2}\text{-}\mu\cdot\tau_{n\text{-}2}\right)}-K}{B\text{ - }K}\times\left(M_F-M_S\right) & ,\text{if } k_{n-2}\le x_{n-2}<b_{n-2}\\ 0 & ,\text{if } x_{n-2}\ge b_{n-2}\end{cases} \tag{9}$$

where the transformed visibility limits of the ferry and airport at time to maturity τ_{n-2} are $k_{n\text{-}2}$ and $b_{n\text{-}2}$, respectively, where $k_{n-2}=\ln K+\mu\tau_{n-2}$ and $b_{n-2}=\ln B+\mu\tau_{n-2}$.

In addition, the method of calculation will repeat day after day in the period of the contract. That means the process will continue until the first day of the contract. The transformed payoff on the first day of the contract is

$$\text{Payoff}_1\left(x_1,\tau_1\right)=\begin{cases}\alpha_H M_H+\mathrm{C}_1\left(x_1,\tau_1\right) & ,\text{if } x_1<k_1\\ \alpha_S M_S+\dfrac{e^{\left(x_1-\mu\cdot\tau_1\right)}-K}{B-K}\times\left(M_F-M_S\right) & ,\text{if } k_1\le x_1<b_1\\ 0 & ,\text{if } x_1\ge b_1\end{cases} \tag{10}$$

where the transformed visibility limits of the ship and airport at time to maturity τ_1 are k_1 and b_1, respectively, where $k_1=\ln K+\mu\tau_1$ and $b_1=\ln B+\mu\tau_1$.

After continuing above processes, we obtain the expected value at the first day of the contract, $\mathrm{E}_1(x_1,\tau_1)$, which is the cumulated transform payoffs with the probability of events happened. The mathematic format is shown in Equation (11):

$$\mathrm{E}_1(x_1,\tau_1)=\int_{-\infty}^{\infty}\text{Payoff}_1(x_1,\tau_1)\cdot \mathrm{g}(x_2,\tau_2;x_1,\tau_1)\mathrm{d}x_1 \tag{11}$$

Then the premium of the option price at the purchase day, $\mathrm{C}(x_0,\tau_0)$, is the discount value of the expected value (11) of cumulated transformed payoffs with the probability of events happening from τ_1 to τ_0 and considering discount factors (R_H, R_S and R_P) with respect to each event.

IV. Case Study

4.1 Data Collection and Cost Analysis

Due to the seasonal fog, the external traffic of areas around Matsu is often affected by low visibility. We use the transportation between Matsu and Taiwan in our case study. To calculate the discount factor (R_H) for the hotel, we used the rate of return to the hotel as a variable because for most travelers to Matsu, the hotels are the major choice for overnight lodging. We calculated R_H by using the data of the Tourism Bureau in 2013.

The rate of return for the ferry R_S is from a suggestion from a study released by the Ministry of Transportation and Communications. Finally, the calculation the return rate for the traveler R_P is based on the deposit rate of the bank. For the setting of the contract, the parameters are set as shown in Table 4.

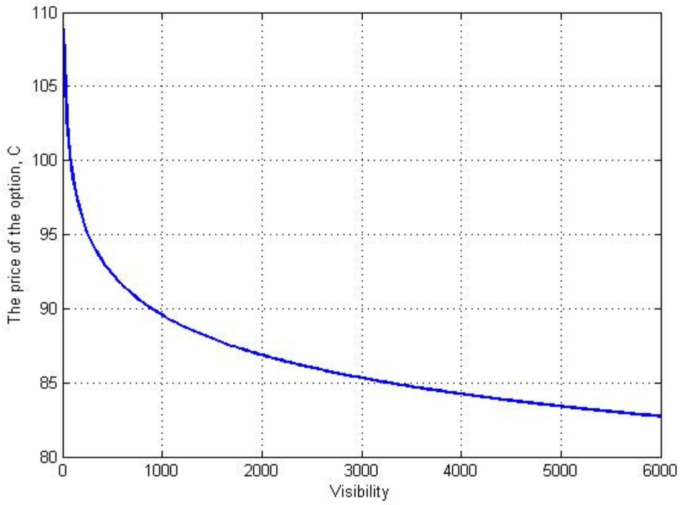
Table 4　Parameter settings.

Parameter	Definition	Number	Source
α_H	Compensation ratio for hotel room	20 %	Our assumption
α_S	Compensation ratio for Taiwan-Matsu Ferry ticket price	30 %	Our assumption
M_H	Average hotel room price in Matsu	1358(NT)	Tourism Bureau, Rep. of China
M_S	Taiwan-Matsu Ferry ticket price	1000(NT)	Tourism Bureau, Rep. of China
M_F	Taiwan-Matsu air ticket price	1896(NT)	Airline Website
B	Lowest limit to takeoff in Nangan airport	4800(meters)	Aeronautical Meteorological Service Page
K	Lowest limit to let ferry sail	300(meters)	Fu-ao port in Matsu
R_H	Return of hotel industry	0.1568 %	Tourism Bureau, Rep. of China
R_S	Return of ferry industry	1.425 %	Ministry of Transportation and Communications
R_P	Return on personal savings in bank	0.88 %	Bank of Taiwan
τ	Purchase time to maturity day	30(days)	Our assumption
μ	Average increment of visibility	1.671	
σ	Average volatility of visibility	7.333	Air Navigation and Weather Services Civil Aeronautics Administration R.O.C

4.2 Basic Case

In this section, we apply scenario analysis to realize the impacts of certain parameter on the option price.

Figure 5 shows the option price for the visibility on the purchase day one month ago. This curve is like a sharply bending curve. When the visibility increases, the option price will decline. This figure shows that when the visibility is below about 1000 meters, the option price decreases faster than the visibility changes. When the visibility is close to zero, the option price will reach almost NTD109. On the other hand, when the visibility reaches 6000 meters, the option price will decline to NTD83. Because of the uncertainty about weather, the option price declines quickly when the visibility is below about 1000 meters. The probability of traffic shut-down will increase in the future. Hence, the traveler will need to pay a high option price. Some results are also shown in Table 5 for reference.



Note: B=4800, K=3000, α_H =20 %, α_S =30 %, R_H=1.568 %, R_S=1.425 %, R_P=0.88 %, σ =7.333, μ =1.671, τ_0=30, τ_1=2, τ_2=1

Figure 5 Relationship between the option price and visibility.

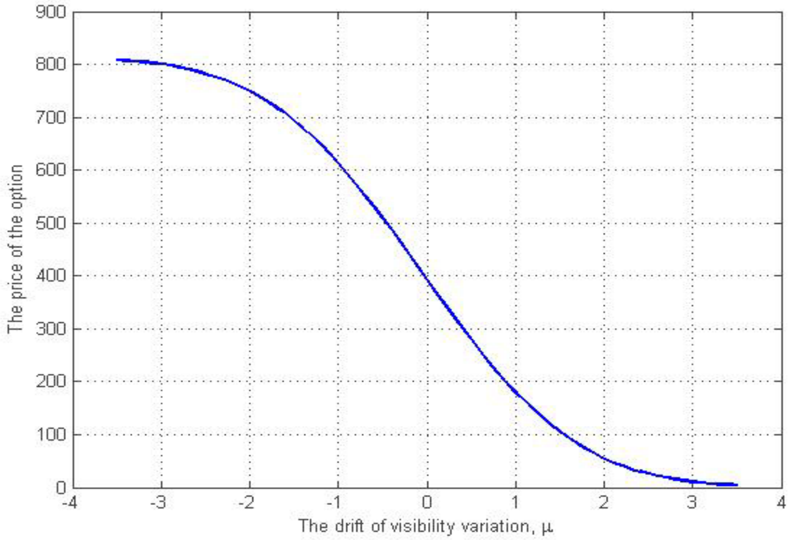
Table 5 Relationship between option price and visibility.

	Deep in the money	In the money	Out of money
Visibility level	150	3000	6000
Option price	97.3329	85.4297	82.8323

Note: B=4800, K=3000, α_H =20 %, α_S =30 %, R_H=1.568 %, R_S=1.425 %, R_P=0.88 %, σ =7.333, μ =1.671, τ_0=30, τ_1=2, τ_2=1

4.3 The Impacts of Drift of Visibility

The drift of visibility means the tendency of future visibility. When the drift of visibility is positive, the visibility may possibly increase in near future. That is, the probability of increment of visibility is positive with its drift. In Figure 5 and Table 6, we discuss the relationship between option price and the drift of visibility variation. We set the current visibility as 3000 meters, and find that different amount of drift of visibility variation will cause changes in the option price. We see that the option price declines when the drift of visibility variation increases. In Figure 6, when the drift of visibility variation reaches about 2, the decline in the option price gradually slows down. The growth of the option price slows down when the drift of visibility variation fells to -2. We also find that the impact of changes in the option price with different levels of drift of visibility variation is quite significant.



Note: B=4800, K=300, α_H =20 %, α_S =30 %, R_H=1.568 %, R_S=1.425 %, R_P=0.88 %, σ =7.333, τ_0=30, τ_1=2, τ_2=1, V_0=3000

Figure 6　Relationship between drift of visibility variation and option.

Table 6　Relationship between drift of visibility variation and option price.

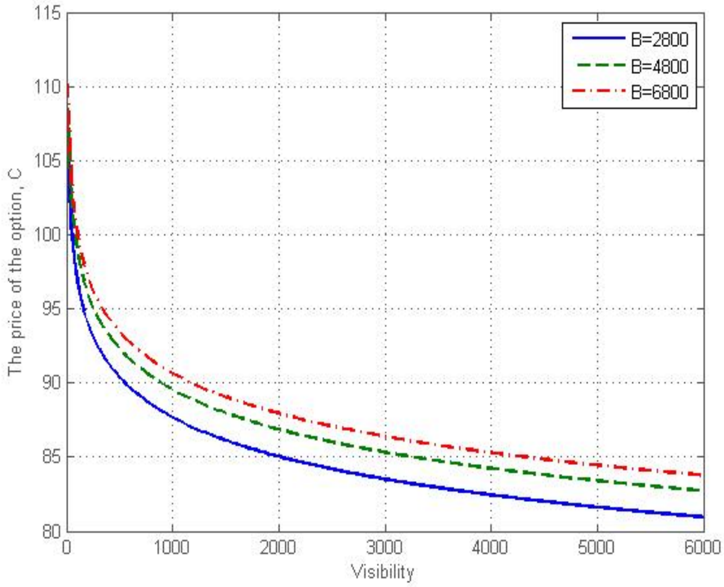
Drift of visibility variation	1.671	0.671	-0.329
Option price	85.4297	242.9039	470.2044

Note: B=4800, K=300, α_H =20 %, α_S =30 %, R_H=1.568 %, R_S=1.425 % R_P=0.88 %, σ =7.333, τ_0 =30, τ_1=2, τ_2=1, V_0=3000

4.4 Impact of Visibility Limit of the Airport (*B*).

The visibility limit of the airport is affects whether travelers can come back to Taiwan or not. The probability of executing the option right also considers the limit visibility of the airport.

Figure 7 shows the price curve under different degrees of visibility limit at the airport. In Table 7, we test the option value for different visibility limit of the airport, 2800, 4800 and 6800 separately. When the boundary of the visibility falls to 2800 from 4800, the curve of the option price is much lower than the original one with the visibility limit of 4800. In the figure, we see that the option price increases with the visibility limit.



Note: K=300, α_H =20 %, α_S =30 %, R_H=1.568 %, R_S=1.425 % R_P=0.88 %, μ =1.671, τ_0=30

Figure 7 Price curves for variant *B*.

Table 7 Option price for different visibility limits of airport (*B*).

Visibility limit of airport	Scenario (A)	Scenario (B)	Scenario (C)		
	150	3000	6000	(A) — (B)	(B) — (C)
2800	95.2102	83.5026	80.9494	11.7076	2.5532
4800	97.2008	85.3146	82.7209	11.8862	2.5937
6800	98.3699	86.3804	83.7632	11.9895	2.6172

Note: K=300, α_H =20 %, α_S =30 %, R_H=1.568 %, R_S=1.425 % R_P=0.88 %, μ =1.671, τ_0=30

V. Conclusion

5.1 Major Findings and Implications

With the increasing threat of uncertainty of global climates, numerous severe disasters occur in various regions. The climate factors have resulted in inconvenience to people's life, involving daily schedule and also the traffic situation caused by sudden change in the weather, which may cause traffic obstruction. Therefore, the study tries to find out solutions to alleviate the inconveniences caused by weather. Although the modern technology is advanced, it is still hard to forecast the weather accurately. Even there is modern protection insurance can only compensation be done after the event. There are not contingency measures and solutions for the potential loss in sight.

To reduce the impacts of weather on people's lives, the study attempts to evade the risks of weather uncertainty in a novel way. It applies the concept of real option theory to resolve the traffic inconvenience at offshore islands districts in Taiwan. Among these options, the traffics are shut down in western offshore islands during fog season. With Matsu region as the main range for implementation and visibility as main driving standard, aircraft or vessels are usually suspended due to poor visibility. Based on the concept of Bermuda options, we take the advantages of Black-Scholes model to set up a model with three stages: the first, high visibility and successful operation as scheduled; second, poor visibility, under which only vessels can work, so the rights of boarding for passengers are reserved, and considering that it takes more time by ship, different compensations will be provided according to the visibility at that time. Finally, extremely low visibility, so all traffics are shut down, the contract will provide rights for accommodation, and purchaser can re-perform the above rights next day.

From the decision tree, it finds that under the condition of unknown purchase options, more costs of uncertainty in the future will be saved. In the models with three different stages, it can calculate the price of reasonable premium to be paid by the buyer; at the same time, different variables in the model are analyzed to learn the impacts of different visibilities on variables.

The results of our study show that in addition to visibility, rate of change is also one major factor affecting the change of premium price. Finally, it explores changes in contract period and knows that longer contract period results in great impact on the premium price. Under our contract, it is not only can make an extra income for the hotel or ship industry, but more stable source of income can be. For travelers, it can reduce the risk of their trip, and provide an alternative immediately that improves passenger quality. It is helpful for the airline be more reliable for the traveler.

5.2 Contributions

Numerous severe disasters have occurred in various regions around the world, with an increasing threat of global climate uncertainty. Climatic factors have resulted in an inconvenience to people's life, involving daily schedules and also traffic situations caused by sudden changes in the weather, which may cause traffic obstruction. Therefore, this study proposes a solution to alleviate some of the inconveniences caused by weather.

The concept of real option theory is applied to resolve traffic inconvenience at offshore islands in Taiwan. Such a contract can help travelers reduce the weather risk for their trip. The framework proposed here provides a computational mechanism for as a market for stakeholders to improve their welfare.

5.3 Research limitations and Suggestion for Further Studies.

5.3.1 Limitations of current study

The study pursues rigorousness on all aspects; however, there are also limitations as below:

1. Limitation on research location

The study intends to relieve traffic problems due to uncertain factors of weather in offshore islands regions in Taiwan. However, it conducts analysis on the premise that both vessels and aircraft operate. However, in Kinmen and Liouciou (offshore islands in Taiwan) there is only single traffic mode, so it's not applicable to all offshore island regions in Taiwan. But similar concept can be used for these two islands to further set up their own models of traffic modes.

2. Limitation on data obtaining

In terms of visibility served as the research subjects, it mainly uses historical data from Matsu Nangan Airport, and data collected contains only four years. If more historical data can be obtained, it will be more close to actual situation, which can improve the analysis accuracy and detail the research results.

3. Limitation on study scope

The study takes the level of visibility to determine the operation state of transport vehicles. Consequently, for judgment on whether vessels can be operated or not, no marine meteorology is included in the scope of discussion. And the wind meteorology is not included, too. So, we set the influence factor for this contract only because of the visibility.

5.3.2 Suggestion for Future Studies

Based on the conclusions above, the study proposes the following three suggestions as reference for similar researches in the future.

1. Weather types change

The study mainly focuses on conditions of heavy fog weather, so future research is recommended to conduct further discussion on different weather types based on the proposed concept, and develop contingency measures for weather uncertainties under the impacts of different factors.

2. Wide application in other modes of transportation

The proposed study hypothesis applies air and marine transportation vehicles to response to poor weather conditions. In the future, land transportation vehicles can be included in the scope of study, so that greater transportation network can be formed, making implementation of contingency measures more flexible and extensive.

3. Applications in different countries

Although the study takes offshore island regions in Taiwan as principal subjects, the proposed research scheme can be applied to other countries such as Malaysia, to make concept of options be more widely used. As we illustrated in response for question 2, the drift rate μ and the volatility δ are two parameters which have significant impact on the price of underlying asset in the Black-Scholes model and are depended on the geographical and seasonal environments of the location. Take Machu, our study case, for example, the foggy season is from January to June. From the data collected in the CWB Observation Data Inquire System, different locations have different drift rates and volatilities of visibility. That is the reason, we present the process of estimate these two parameters.

REFERENCES

1. Intergovernmental Panel on Climate Change, *Climate Change 2014–Impacts, Adaptation and Vulnerability: Regional Aspects, Cambridge University Press*, UK, 2014.
2. Rutz, W. O. A. and Coull, J. R., “Inter-Island Passenger Shipping in Indonesia: Development of the System”, *Journal of Transport Geography*, Vol. 4, No. 4, 1996, pp. 275-286.
3. Luis, J. A. H., “The Role of Inter-Island Air Transport in the Canary Islands”, *Journal of Transport Geography*, Vol. 12, 2004, pp. 235-244.
4. Seo, J. H., Park, S. Y., and Yu, L., “The Analysis of the Relationships of Korean Outbound Tourism Demand: Jeju Island and Three International Destinations”, *Tourism Management,* Vol. 30, No. 4, 2009, pp. 530-543.
5. Tsai, C. H. and Chen, C. W., “The Establishment of a Rapid Natural Disaster Risk Assessment Model for the Tourism Industry”, *Tourism Management*, Vol. 32, No. 1, 2011, pp. 158-171.
6. Edrich, C., “Weather Risk Management”, *Journal of Financial Regulation and Compliance*, Vol. 11, No. 2, 2003, pp. 164-168.
7. Coombes, E. G. and Jones, A. P.,. “Assessing the Impact of Climate Change on Visitor Behaviour and Habitat Use at the Coast: A UK Case Study”, *Global Environmental Change-Human and Policy Dimensions*, Vol. 20, No. 2, 2010, pp. 303-313.
8. Pisano, P. A., Goodwin, L. C., and Rossetti, M. A., “US Highway Crashes in Adverse Road Weather Conditions”, 24th Conference on International Interactive Information and Processing Systems for Meteorology, Oceanography and Hydrology, New Orleans, LA, 2008.
9. Koetse, M. J. and Rietveld, P., “The Impact of Climate Change and Weather on Transport: An Overview Of Empirical Findings”, *Transportation Research Part D*, Vol. 14, 2009, pp. 205-221.

10. Khattak, A. J. and De Palma, A., "The Impact of Adverse Weather Conditions on the Propensity to Change Travel Decisions: A Survey of Brussels Commuters", *Transportation Research Part A: Policy and Practice*, Vol. 31, No. 3, 1997, pp. 181-203.
11. Rossello, J., "North Atlantic Oscillation Influences on European Airline Traffic", *Transportation Research Part D: Transport and Environment*, Vol. 16, No. 2, 2011, pp. 183-187.
12. Saneinejad, S., Roorda, M. J., and Kennedy, C., "Modelling the Impact of Weather Conditions on Active Transportation Travel Behaviour", *Transportation Research Part D: Transport and Environment*, Vo. 17, No. 2, 2012, pp. 129-137.
13. Hummels, D. L. and Schaur, G., "Hedging Price Volatility Using Fast Transport", *Journal of International Economics*, Vol. 82, No. 1, 2010, pp. 15-25.
14. Feki, Y., Hajji, A., and Rekik, M., "A Hedging Policy for Carriers' Selection under Availability and Demand Uncertainty", *Transportation Research Part E: Logistics and Transportation Review*, Vol. 85, 2016, pp. 149-165.
15. Morrell, P. and Swan, W., "Airline Jet Fuel Hedging: Theory and Practice", *Transport Reviews*, Vol. 26, No. 6, 2006, pp. 713-730.
16. Berghöfer, B. and Lucey, B., "Fuel Hedging, Operational Hedging and Risk Exposure−Evidence from the Global Airline Industry", *International Review of Financial Analysis*, Vol. 34, 2014, pp. 124-139.
17. Richards, T. J., Manfredo, M. R., and Sanders, D. R., "Pricing Weather Derivatives", *American Journal of Agricultural Economics*, Vol. 86, No. 4, 2004, pp. 1005-1017.
18. Wang, M., Wen, M. M., and Yang, C. C., "Weather Derivatives, Price Forwards, and Corporate Risk Management", *The Journal of Risk Finance,* 2010, Vol. 11, No. 4, pp. 358-376.
19. Xu, W., Odening, M., and Musshoff, O., "Indifference Pricing of Weather Derivatives", *American Journal of Agricultural Economics*, 2008, Vol. 90, No.4, pp. 979-993.
20. Nilim, A., El Ghaoui, L., Duong, V., and Hansen, M. "Trajectory-Based Air Traffic Management (Tb-Atm) under Weather Uncertainty", Proceedings of the 4th USA/Europe Air Traffic Management R&D Seminar, Federal Aviation Administration and EUROCONTROL, 2001.
21. Steiner, M., Bateman, R., Megenhardt, D., Liu, Y., Xu, M., Pocernich, M., and Krozel, J., "Translation of Ensemble Weather Forecasts into Probabilistic Air Traffic Capacity Impact", *Air Traffic Control Quarterly*, Vol. 18, No. 3, 2009, pp. 229-254.
22. Jain, A. K., and Cox, R. A. K., "Airfare Price Insurance: A Real Option Model", *The Journal of Risk Finance*, Vol. 12, No. 1, 2011, pp. 5-14.
23. Treanor, S. D., "The Flexibility and Benefits of Operating a Diverse Fleet: An Analysis Using Real Options", *Management Rsearch Review*, Vol. 35, No. 6, 2012, pp. 462-472.
24. Erdman, J., "The Most Weather-Delayed Major Airports in the U.S.", weather.com, https://weather.com/travel/news/2018-11-19-most-weather-delayed-major-us-airports, 2018.
25. Machidon, D., Apostol, L., and Buruiană, D., "Considerations of the Fog Regime in the Airports Area from Moldavia Territory (Romania)", *Aerul şi Apa: Componente ale Mediului*, Vol. 2014, No. 1, 2014, pp. 142-149.

26. Mohan, M. and Payra, S., "Aerosol Number Concentrations And Visibility During Dense Fog Over A Subtropical Urban Site", *Journal of Nanomaterials*, Vol. 2014, Article ID: 495457, 2014.
27. Bullough, J. D. and Rea, M. S., "Impacts of Fog Characteristics, forward Illumination, and Warning Beacon Intensity Distribution on Roadway Hazard Visibility", *The Scientific World Journal*, Vol.8, Article ID: 4687816, 2016.
28. Thies, B., Egli, S., and Bendix, J., "The Influence of Drop Size Distributions on the Relationship between Liquid Water Content and Radar Reflectivity in Radiation Fogs", *Atmosphere*, Vol. 8, No. 8, 2017, pp. 1-23.
29. Gultepe, I., Milbrandt, J. A., and Zhou, B., "Marine Fog: A Review on Microphysics and Visibility Prediction", *Marine Fog: Challenges and Advancements in Observations, Modeling, and Forecasting*, Springer International Publishing, 2017, pp. 345-394.

Appendix: The flow chart of the three-day contract