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碩 士 論 文

單一供應商多採購商供應鏈中之寄售與供應商管理庫存新模型

A New Model for Consignment and Vendor Managed Inventory in Single-Vendor Multiple Buyers Supply Chains

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ABSTRACT

This study is interested in finding the optimal replenishment schedule using a "vendor managed inventory and consignment" (VMI & CS) policy in a supply chain with a single vendor and multiple buyers. Different from other studies in the literature, we propose to plan the vendor's production schedule under Power-of-Two (PoT) policy in which the vendor's replenishment cycle time for each buyer must be a PoT-integer multiple of a basic period. This study investigates two kinds of vendor-buyers partnership: (1) the vendor has a VMI & CS partnership with buyers, and (2) the vendor and the buyers belong to the same vertically integrated firm. We formulate the corresponding mixed binary integer programming models for both scenarios to determine the vendor's replenishment cycle times for all the buyers and its own production schedule so as to minimize the average total costs (including ordering, inventory holding, and setup costs), while the vendor meets all the buyers' demand. We evaluated the effectiveness of our proposed models by comparing their optimal solutions with those from the models in the literature. The results of our numerical experiments show that the proposed models may bring solutions with more than 8% cost-saving for some parameter settings as comparing with the models in the literature.

Keywords : Consignment, vendor managed inventory, supply chain, power-of-two policy

摘要

本研究乃探討在單一供應商及多採購商供應鏈中運用「供應商管理庫存與寄售」政策下的最佳補貨排程。有異於文獻其他研究，我們運用「二冪政策」規劃供應商的生產排程，其中二冪政策要求供應商對於每個採購商的補貨週期必須為一個基本週期之二冪整數的倍數。本研究探討兩種供應商與採購商的伙伴關係：(1)供應商與採購商運用供應商管理庫存與寄售的伙伴關係，及(2)供應商與採購商屬於相同的垂直整合廠商。對於以上兩種情境，在滿足所有採購商需求的前提下，我們建構對應的混合二元整數規劃模式，求取供應商對於所有採購商的補貨週期及其自身的生產排程，以達到最小化平均總成本（包括訂購成本，存貨持有成本及設置成本）。 為評估本研究所提出模式的效能，將所求得的最佳解與文獻中之模式最佳解進行比較。我們的數據實驗結果顯示，在某些參數設定下，本研究提出之模式相較文獻中之模式，可以獲得超過 8%之成本節省。

關鍵字：寄售、供應商管理庫存、供應鏈管理、二冪政策

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CHAPTER I Introduction

I.1 Background

Studies in literature indicate that inventory cost is one of the most consuming costs in the overall production costs. Approximately, 30% of the annual cost are dedicated to keep in stock of materials or finished products (Ghiani, Laporte, & Musmanno, 2004). As an opportunity point of view, the inventory's investment can be considered as non-profitable investment since it is not able to gain any interest at all. Although it is unprofitable, keeping inventory is crucial to the company as it may improve service level, reduce overall logistics costs, and overcome the uncertainties, etc. Therefore, how to manage the inventory is very essential because it needs to carefully balance the tradeoff between cost efficiency and service level effectiveness.

Researchers have been investing their efforts in inventory control problems for many years. The core problem of inventory control is to match supply with demand by efficiently coordinating the production and the distribution of goods to meet customers' demand. However, it is usually very difficult to predict customers' demand in the real world due to its characteristics of fluctuation, variation, and also dynamic all the time. It may lead to the so-called "bullwhip effect" in a supply chain that indicates inherent fluctuation in demand keeps increasing as customers' demand backtrack to the distributor, the manufacturer, and the suppliers in the up-stream of the supply chain. Coordination among the firms in a supply chain is one of the most effective ways to surmount the bullwhip effect. The coordination between vendor and buyer may reduce or even eliminate the bullwhip effect (Chen, et al. 2002), and it could cut costs (i.e., inventory, transportation) and increase customer responsiveness (Chopra & Meindl, 2004). Therefore, it serves as an essential capability for companies to survive in a global market.

In order to increase the efficiency and effectiveness, several types of supply chain coordination have been developed. Vendor managed inventory (VMI) and consignment (CS) may help to improve performance in supply chain management (SCM) by decreasing the inventory levels and increasing fill rates (Emigh, 1999). Theses coordination mechanisms aims to coordinate the operations of individual members in the supply chain and improve their profits.

When applying VMI, vendor manage buyer's inventory (namely, decide the time and the quantity of replenishment) based on the information of the buyer's inventory. Usually, vendor will have opportunities of pursuing economy of scale in their operations since vendor does not need to worry about the fulfillment of the buyers' orders, but only need to avoid stock-out happens at the buyers'. VMI has become common practices of supply chain coordination between Wal-Mart and its vendors, like Procter & Gamble and Kimberly Clark (Harsono, 2017).

Consignment is another mechanism of supply chain coordination in which vendor places goods at buyer's location without receiving payment until after the goods are used or sold (Blackstone & Cox, 2004). Note that buyer is the decision maker under consignment coordination, and the ownership of the goods transfer from vendor to buyer only when the goods are consumed. Obviously, the buyer enjoys the advantage of no need to invest in the inventory, so that it incurs no inventory carrying costs. By making use of consignment, buyer can reduce its annual inventory carrying cots from 36% to 18% (cost of capital and taxes) (Williams, 2017). Therefore, consignment is usually more beneficial for the buyer.

Both VMI and CS are coordination mechanism helping to reduce risks and earn more profit for the firms in the supply chain. However, there exist several differences between VMI and CS, such as the decision maker and the ownership of goods. Zanoni, et al. (2012) proposed another strategy, which is a combination of VMI and CS (abbreviated as VMI & CS), as another coordination mechanism in supply chain. In the scenario of VMI & CS, the vendor is the decision maker who determines the ordering time and quality, but still owns the goods until they are used or sold at buyer's location. Khan, et al. (2016) indicate that using VMI & CS mechanism benefits the buyer by saving the investment in purchasing inventories, ensuring the vendor of an almost captive buyer, and also, making sure that the supply is available for the buyer. Interestingly, VMI & CS partnership could be more economical for both vendor (Zanoni, et al. 2012).

Of course, coordination management of the inventory could become more complicated as the number of buyers increase. The vendor has to guarantee satisfying the demand of all the buyers. On the other hand, the vendor is constrained by the available resources, such as the production capacity, etc., to accommodate the production for all the buyers. Also, the vendor would deal with the replenishment so as to keep the ordering and inventory holding cost as low as possible. Both parties would to like to minimize their costs incurred in the supply chain at the same time.

This study would like to propose a new mathematical model for the VMI & CS scenario in a supply chain with a single vendor and multiple buyers. We are concerned with the optimal replenishment strategy of the vendor under two scenarios, (1) the vendor has VMI & CS partnership with buyers, and (2) the vendor and the buyers belong to a centralized supply chain.

I.2 Purpose of Study

This study was inspired by Ben-Daya, et al. (2013) in which they proposed an inventory model under VMI & CS partnership in a supply chain with a single-vendor multiple buyers. In their model, all of the buyers share an Equal Number of Shipments from the vendor (abbreviated as ENS policy), and the vendor determines the optimal replenishment policy. They formulated a mathematical model and studied three types of coordination mechanism between the vendor and the buyers: (1) the vendor and the buyers act independently, (2) the vendor and the buyers are use VMI & CS partnership, (3) the vendor and the buyers belong to a centralized supply chain.

We consider it that there should be some replenishment policy that could be more flexible than the ENS policy proposed in Ben-Daya, et al. (2013) as implementing VMI & CS partnership in a supply chain. Intuitively, when using a more flexible replenishment policy, a supply chain could be more effective and the firms could benefit more from VMI & CS partnership. Therefore, we were motivated to propose a more flexible replenishment policy, formulate a mathematical model corresponding to the new replenishment policy, and suggest a solution approach for solving the mathematical model. We hope that the proposed model would be able to obtain solutions better than those Ben-Daya, et al. (2013), at least, for some parameter settings.

I.3 Research Framework

We started this study from the definition of the problem. Then, we have through review on the models in the paper of Ben-Daya, et al. (2013). Then, we would propose a decision-making scenario that is more flexible than that used in Ben-Daya, et al. (2013) for VMI & CS partnership and centralized partnership in a supply chain with a single vendor and multiple buyers.

The rest of this thesis is organized as shown in Figure I-1. CHAPTER I relative studies in the literature. CHAPTER III describes the decision making scenario, the assumptions, and the details of the proposed mathematical model; CHAPTER IV presents our numerical experiment based on the case studies in Ben-Daya, et al. (2013). Finally, CHAPTER V shows the conclusions of this study and some suggestions for future extensions.

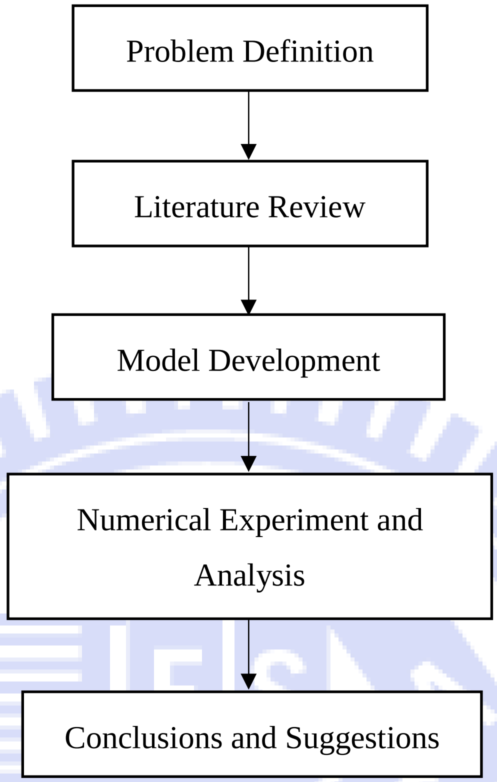


Figure I-1 Research framework

CHAPTER II Literature Review

Building collaboration between vendor and buyer could be one of the key success factors in a supply chain system since it helps in reducing uncertainty in demand and getting better control in replenishment by sharing information. Several types of collaborative initiatives between vendor and buyer, e.g., Consignment (CS) and Vendor Managed Inventory (VMI), had been proposed and practiced recently.

We divide this chapter into four parts. Section II.1 and II.2 present full discussions on the collaboration using CS and VMI policies. Then, we introduce an integrated inventory model in single-vendor and multiple-buyers in Section II.3. Finally, Section II.4 summarizes our literature review.

II.1 Consignment Policy

When using consignment policy, the vendor not only owns, but also manage the inventory of goods until the buyer removes them from its warehouse for use. Therefore, the buyer usually will hold a storage space or shelves in its warehouse for keeping consignment goods. The vendor has an access of the information on the daily consumptions of products to manage the refill of the stock in the buyer's warehouse.

Sharing information is an essential aspect in consignment coordination. With the shared information, the vendor and the buyers shall have opportunity of saving ordering and inventory holding cost. The consignment policy pushes vendor and buyers to exchange of information and share the risks. As a result of sharing information, the vendor and the buyer will be able to share the ordering cost and holding cost to minimize their costs.

Many studies presented successful application of consignment coordination industries. For example, Siemens Automation and Drives, Controls and Distribution (A&D CD) in Germany supplies products, systems, and solutions, starting from switching devices, through to complete cabinet systems (Gümüş, 2017). The purchasing department of Siemens A&D CD enjoyed the benefit from consignment partnership by saving their inventory costs, and consignment is usually applied to items with high purchasing volume.

Braglia and Zavanella (2003) presented a case study in automotive industry. They investigated a model in a single-vendor and single-buyer supply chain, would see the potentiality of consignment policy in practice. They identified that CS policy might be a strategic and profitable approach to stock management in uncertain environments, i.e. where delivery lead times or market demand vary over time. Valentini and Zavanella (2003) took another case study in automotive industry, and analyzed the minimum and maximum inventory levels in the supply chain. The models developed in these two studies demonstrated that the inventory system using consignment policy may outperform other traditional ones. Both studies also showed that the vendor and the buyer have two contrasting objectives. The buyer would like to keep the minimum level of inventory as high as possible to guarantee a required service level and the maximum level of inventory close to the minimum level. On the contrary, the vendor attempts to maintain the minimum level of inventory as low as possible to reduce inventory holding costs and the maximum level of inventory as high as possible to increase production flexibility and reduce shipping costs.

Persona et al. (2005) proposed a model to manage obsolescence of product in an integrated single-vendor single-buyer supply chain. Their model is an extension of Braglia and Zavanella (2003). Obsolescence is due to a finite lifetime of products, in terms of some contextual situations, such as the employment of new technologies, consistent market changes, and strong competition. It occurs when the product is no longer required or some item performs similar function. Their model determines the optimal shipment quantity and the maximum inventory level at the buyer's warehouse. They presented comparison analysis between the optimal solution from the proposed model with a non-obsolescence optimal solution and also with that considering stochastic behavior or the product lifetime estimation.

Chen and Liu (2208) developed a model for determining an optimal consignment policy for a manufacturer that comprises a fixed fee and a per-unit commission to offer its retailers a mutually beneficial consignment scheme. Their analysis demonstrates that a consignment policy not only generates a higher manufacturer's profit than the traditional system, but also coordinates the retailer to achieve a large supply-chain profit. Also, the consignment policy becomes more efficient, and the manufacturer is able to make more profit when the demand is sensitive to the markdown.

Yu, et al. (2012) extended Chen and Liu (2008) to a supply chain with single vendor and multiple buyers under stochastic market demand. Their model is generic to allow any type of demand distribution, but they tested two scenarios in which the demand distributions of the retailers conforms uniform and exponential distributions. Their analysis shows consignment policy not only helps the manufacturer to generate higher profit, but also coordinates retailers so that each retailers earns at least as much as they do in traditional uncoordinated case.

Battini, et al. (2010) developed an inventory model for the optimal replenishment of a single product in a single-vendor and multiple-buyers supply chain in which many clients can establish a CS policy with the same vendor. Their proposed model determines the maximum and minimum inventory stock levels to store at the buyers' plant warehouse and the optimal quantity delivered from the vendor to each buyer in order to minimize total supply chain costs.

II.2 Vendor Managed Inventory Policy

Vendor managed inventory (VMI) has been known as one of the most successful practices that develops supply chain integration. The potential advantages from VMI coordination can be summarized as reducing inventory costs for vendor and buyer and improving customer service levels, such as shorter order cycle times and higher fulfillment rates.

Parmalat, which offers milk and dairy products, fruit juices, table spreads, and cookies, is one of largest food companies in Canada. Parmalat manages the inventory of its products sold to buyers who have agreed to a VMI partnership and controls the replenishment time and order quantity shipped to the buyers (Gümüş, 2017). Through the VMI partnership, the vendor and the buyers set targets for service-levels as well as inventory turns. The buyers measure their service levels and prevent to stock-out by carefully monitoring their inventory level. The firm may save costs from pursuing for economic of scale through more effective truck utilization by full truck loads and stable production from coordination of replenishment lots of buyers.

Some researchers investigated the factors affecting the advantages of VMI policy. Yao, et al. (2007) used an analytical model to determine the influence of ordering cost and inventory carrying. They had the following observations: First, the supply chain enjoys benefit more if the vendor's ordering cost is relatively small comparing with the buyers', and the carrying charge of the supplier is larger than the buyers'. Second, the distribution of the benefits between the supplier and the buyers is disproportional. Dong and Xu (2002) evaluated the short-term and long-term impacts of

VMI on supply chain profitability. They investigate VMI issues with a focus on inventory systems, purchase prices, and purchase quantities. The authors formulated a mathematical model for a vendor-buyer channel structure, and examined the effects of a VMI policy on the various cost components of both parties. They concluded that VMI always leads to a higher buyer’s profit, but supplier’s profit varies. In the short-term, VMI is found to reduce total costs of the channel system, but under certain cost conditions between buyer and supplier, it could decrease the purchasing price and supplier’s profit. In the long-run, it could more likely increase supplier’s profit than in the short-run.

Zhang, et al. (2007) presented an integrated VMI model for a supply chain with a single vendor and multiple buyers with constant production and demand rates under the assumption that the buyers’ ordering cycles may be different and that each buyer can replenish more than once in one production cycle. In their scenario, the vendor also considered investment decision of ordering cost reduction. The authors proposed a solution approach solving the optimal investment amount and replenishment decision for all the buyers and the vendor. They discussed three numerical examples with an exponential ordering cost function to get managerial insights.

Contract designs is another important issue when implementing VMI policy. Nagarajan and Rajagolan (2008) indicated that the contracting terms determine not only ownership of the inventory, but also the responsibility of inventory replenishment decisions.

Nachiappan and Jawahar (2007) formulated a mathematical model for a two-echelon supply chain with a single vendor and multiple buyers, determining the channel profit of the supply chain, and contract price when practicing VMI policy. They derived optimal sales price and acceptable contract price at different revenue share with the optimal sales quantity. They proposed a genetic algorithm for solving this problem.

Nagarajan and Rajagolan (2008) considered a supply chain in which a manufacturer supplies a single product to a retailer facing random demand, and assumed that both players are rational and act noncooperatively. They proposed holding cost subsidy (HCS)-type contracts on inventories offered by the retailer in the VMI systems, and evaluated three inventory control policies, viz., deterministic economic order quantity, continuous review (Q, r) policies, and periodic review (base stock) policies. They concluded that such (HCS)-type contracts may improve channel performance.

II.3 VMI & CS Coordination in Supply Chain with a Single-Vendor and Multiple-Buyers

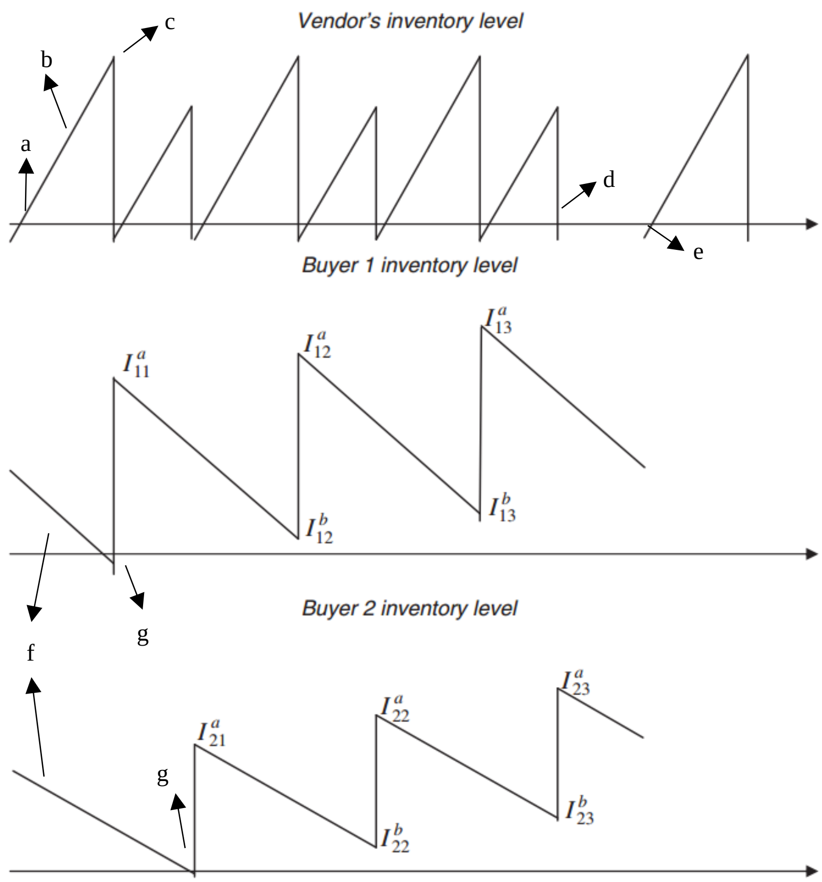
Under VMI & CS coordination, the vendor is the decision maker, who should be careful about the shipment size of replenishment to buyers with high physical storage costs. Usually, a good agreement of VMI & CS coordination relies on information sharing between vendor and buyers.

Several researchers have been working on integrated inventory models for single-vendor and multiple-buyers in recent years. Zavenella and Zanoni (2009), which was corrected in Zavanella and Zanoni (2010), considered a model for a single-vendor multiple-buyers supply chain system, and solved the optimal replenishment policy under two conditions: (1) At least one shipment has to be sent to each buyer within one cycle and if there are two or more shipments, they need to be sent consecutively, and (2) the sequence of the shipments deliveries to the buyers is predetermined.

Ben-Daya, et al. (2013) formulated a model for a single vendor and multiple buyers supply chain applying VMI and CS policy. They studied three scenarios of vendor-buyers partnerships: (i) the vendor and the buyers act independently, (ii) the vendor has a VMI & CS partnership with the buyers and (iii) the vendor and the buyer belong to a vertically integrated or called as centralized partnership. In their scenarios, the vendor adopts a cyclic delivery policy in which each buyer receives an equal-size shipment from the vendor every time, and the vendor ships the goods to all the buyers in a cycle and repeat this cycle until all shipments are delivered. (The sequence of the buyers does not matter in these scenarios).

Figure II-1 shows the inventory levels of the vendor and the buyers for a supply chain with the vendor and two buyers in the scenarios of Ben-Daya, et al. (2013). The first two triangles indicating the vendor's inventory level describes the production lots for buyer 1 and buyer 2, respectively, and these triangles repeats following the cyclic pattern. The vendor will send the goods to the buyer right after its production lot is finished. The shipment size from the vendor can cover the demand of buyer during the replenishment time, even the buyer may has excess inventory at the end of the replenishment cycles, for example, I_{12}^{b} and I_{13}^{b} for buyer 1 and I_{22}^{b} and I_{23}^{b} for buyer 2. The vendor may has idle time before starting the production in the next cycle, and obviously, there is no shipment to the buyers during idle time. The buyers will use their excessive inventory to cover their demand during the idle time.

From their numerical experiments, Ben-Daya, et al. (2013) conclude that VMI & CS policy is more beneficial when the vendor has a flexible capacity, and it is also more attractive to buyers when they have significant order costs and the vendor's setup cost is not large. Also, they find that the vendor will tend to make more frequent shipments with smaller lots under VMI & CS policy.



a = production begins; b = vendor produces; c = shipment dispatched; d = production stops; e = new production run begins; f = stocks of buyer decrease at demand rate; g = order placed; I_{ij}^b = inventory level for *i*th buyer before receipt of *j*th shipment; I_{ij}^a = inventory level for *i*th buyer after receipt of *j*th shipment

Figure II-1 Inventory profiles for vendor and buyers

Table II-1 summarizes the differences between our study and Ben-Daya, et al. (2013) in terms of the decision environment and the context. (We will present the details of our decision-making scenario of this thesis in Chapter III later.). One may find that Ben-Daya, et al. (2013) implements VMI & CS policy in a more restricted fashion since the replenishment cycles for all the buyers are of the same. Also, the scenario of Ben-Daya, et al. (2013) is not practical by assuming that the setup time is not considered in the vendor's replenishment cycle, and the vendor's capacity is unlimited. Therefore, we are motivated to present a novel approach of implementing VMI & CS coordination in this study. In our proposed scenario, we would allow the replenishment cycles of buyers could be different, and will take into accounts the vendor's available capacity and the setup time.

Table II-1 The differences between Ben Daya, et al. (2013) and this study (2017)

No	Ben Daya, et al. (2013)	This Thesis (2017)
1.	Do not matter in which sequence the buyers are replenished.	The vendor needs to consider the sequence of the buyers' replenishment in its production schedule.
2.	The replenishment cycles for all the buyers are of the same.	The replenishment cycle could be different for each buyer. It depends on the multipliers of basic period.
3.	The setup time is not considered in the vendor's replenishment cycle.	The setup time is counted in the beginning of each basic period.
4.	The vendor's capacity is unlimited.	The vendor's capacity is limited.
5.	The buyers may have excess inventory more than demand in the vendor's replenishment cycle.	The shipment size from the vendor is equal to the buyer's demand in the buyer's replenishment cycle.
6.	There is an idle time in the vendor's replenishment cycle.	There may exist idle time in each basic period.

II.4 Summary

Based on the review in section II.1-II.3, many studies have been developed by consignment, VMI, and integrated inventory model in single-vendor and multiple-buyers. Table II-2 summarizes the reviewed studies on a supply chain with a single vendor and multiple buyers in the literature.

Table II-2 Reviewed studies on a supply chain with a single vendor and multiple buyers

Papers	Policy	Objective function	Decision variables
Battini, et al. (2010)	CS	Minimize average total costs	Delivery quantity of vendor
Yu, et al. (2012)	CS	Maximize profit	Order quantity, delivery quantity, commission, and incentive fee
Zhang, et al. (2007)	VMI	Minimize average total costs	Number of batches and cycle time
Nachiappan and Jawahar (2007)	VMI	Maximize profit	Sales quantity and sales price
Zavanella and Zanoni (2009)	VMI & CS	Minimize average total costs	Cycle time and number of transport operations per production cycle time

Table II-3 Reviewed studies on a supply chain with a single vendor and multiple buyers (cont.)

Ben-Daya, et al. (2013)	VMI & CS	Minimize average total costs	Number of shipments and replenishment cycle length
This study (2017)	VMI & CS	Minimize average total costs	Multipliers of basic period and basic period

Following our review, VMI & CS coordination possess advantages over CS or VMI coordination. Ben-Daya, et al. (2013) presented a mathematical model for a supply chain with a single vendor and multiple buyers VMI & CS coordination. But, as shown in Figure II-1, the replenishment policy could be restrictive as all the buyers uses the same number of shipments, and the decision making scenario could be more practical by considering setup time and available capacity when the vendor makes replenishment. In CHAPTER III, we would propose a new approach of implementing VMI & CS coordination and present the corresponding mathematical model.

CHAPTER III Decision-Making Scenario and Mathematical Model

This chapter investigates the inventory control problem in a single-vendor multiple-buyer supply chain. We consider two scenarios, namely, VMI & CS and centralized policies, and compare the cost components for both scenarios, respectively. Also we formulate the mathematical models for both scenarios with a basic period, b and a vector of multipliers $(k_i, \dots, k_n)$, being the decision variables under Power-of-Two (PoT) policy, i.e., $k_i = 2^p, p \in \aleph, \forall i.$ A common objective for both models is to determine the optimal replenishment strategy, denoted b^* and $(k_1^*, \dots, k_n^*)$, to minimize the average total costs for the vendor and buyers in the supply chain.

III.1 Problem Definition

Both the vendor and the buyers attempt to minimize their costs in supply chain system. Gümüş, et al. (2006) analyzed that VMI is one of most popular strategies for the coordination of supply chain to pursue cost-savings for each party. Under VMI coordination, the vendors may place orders and make replenishment decisions according their preferences to minimize their costs effectively, especially, when the vendor is a dominant player in the supply chain

Under consignment (CS) coordination, the buyers may act as a dominant player and manage the replenishment strategy (i.e., the replenishment cycle time and/or quantity). Goods are owned by the vendor until the goods are sold, and the buyers are only responsible for the storage costs, but not including the inventory holding costs. Therefore, CS strategies are usually more beneficial to the buyers rather than the vendor.

Ben-Daya, et al. (2013) proposed a hybrid strategy that builds partnership between the vendor and buyers and balances their costs and benefits between VMI and CS in this study. It is so-called "VMI & CS coordination", in which the vendor not only manages the replenishment strategy, but also shares part of the inventory holding costs for those goods stored at the buyers. Such arrangements shall be beneficial to the buyer since they do not have to bear all the inventory holding costs. In their proposed model, the vendors replenishes the buyers for a common number of times (say, one time or several times, with equal replenishment intervals) in a shared cycle time. We would like to propose a new operation strategy that plans the replenishment for each buyer i with its replenishment cycle (T_i) being a Power-of-Two integer times a basic period, b, namely, $T_i = k_i b = 2^p b$, where $p \in \mathbb{N}, \forall i$.

We assume that the vendor has enough production capacity to fulfill all buyers' demands in this study. Since shortage is not allowed, the vendor has to determine its production cycle to satisfy the accumulated demands of the buyers. Importantly, the vendor should not only decide the replenishment cycles for all the buyers, but also manage the scheduling of the replenishment of each buyer carefully so that the production capacity for meeting the required replenishment will not exceed the available capacity of the vendor in each basic period. Therefore, the decision variables include the basic variable, the vector of the multipliers, and the scheduling of the replenishment for each buyer in the decision-making scenario.

III.2 Characteristics of Decision Making Scenario

We will present the details of our proposed VMI & CS (Vendor Managed Inventory and Consignment) coordination strategy, and compare it with the Centralized strategy.

Under our proposed VMI & CS coordination strategy, the vendor is not only the decision maker, and but also has strong links with the buyers since the vendor shares part of the ordering and inventory holding costs for the buyers. On the other hand, no cost-sharing exists between the vendor and buyers under the Centralized strategy. Table III-1 shows the comparison of the decision maker and the cost ownership in different types of supply chain coordination.

Table III-1 Cost sharing and decision makers in different types of supply chain coordination

Supply Chain Coordination	Decision Maker	Cost Ownership	
		Ordering	**Holding**
VMI	Vendor	Shared	Buyer
CS	Buyer	Buyer	Shared
VMI & CS	Vendor	Shared	Shared
Centralized	Vendor	Vendor	Vendor

The most important and interesting part of the VMI & CS coordination strategy is the cost-sharing between the vendor and buyers. The vendor and buyers share the ordering and inventory holding cost. We denote as A_i^{br} and A_i^{bp} the shared ordering cost for buyer *i* and the vendor's ordering cost (shared with buyer *i*) in the VMI & CS scenario, respectively. In the Centralized scenario, the sum of these two cost components is denoted by A_i^b. The inventory holding costs are denoted as H_i^{bo} and H_i^{bs} in the VMI & CS scenario for the vendor and buyer i, respectively. In the Centralized scenario, the sum of these two cost components is denoted by H_i^b. Also, note that the vendor is responsible for all of the ordering and inventory holding costs in the Centralized scenario. Table III-2 summarizes the ownership of the ordering and holding costs in the independent, the VMI & CS scenario and the Centralized scenario. Note that the symbols of the parameters in our study are as same as in Ben-Daya, et al. (2013), but we will employ a quite different decision-making scenario from theirs.

Table III-2 The ownership of the ordering and inventory holding costs in supply chain

Supply Chain Partner	Coordination Scenario in Supply Chain					
	Independent		VMI & CS		Centralized	
	Ordering	Holding	Ordering	Holding	Ordering	Holding
Vendor	A^{vs}	H^v	A_i^{bp}	H_i^{bo}	A_i^b	H_i^b
	A_i^{vr}	-	A^{vs}	H^v	A^{vs}	H^v
			A_i^{vr}	-	A_i^{vr}	-
Buyer	A_i^b		A_i^{br}	H_i^{bs}		

Notes: $A_i^b = A_i^{bp} + A_i^{br}$ and $H_i^b = H_i^{bo} + H_i^{bs}$

III.3 Assumptions and Notations

Before discussing our mathematical model, we would present the following assumptions made for the model formulation.

- **Vendor**

1. The production rate is constant and known.
2. The vendor could make production to satisfy the demand for one or several buyers in one basic period b.
3. Whenever any production lot is scheduled in a basic period, a setup time will lead the production durations for all the production lots scheduled in that basic period.

- **Buyer**

1. The demand rate of each buyer is constant and known.
2. Shortage is not allowed.
3. The warehouse capacity of each buyer is unlimited.

4. The replenishment cycle of the i^{th} buyer, i.e., T_i, is an integer multiplier (denoted by k_i) of a basic period b, i.e., $T_i = k_i b$.

Here are the notations used for the model formulation (adopted from Ben-Daya, et al. 2013).

Parameters

D_i	The demand rate of buyer i (units/unit time)
P	The production rate of the vendor (unit/unit time)
V_i	The maximum possible power-value of the multiplier for buyer i
S	The setup time of the vendor
H^v	The vendor's holding cost per unit per unit time ($/unit/unit time)
H_i^{bo}	The buyer i's opportunity holding cost per unit per unit time ($/unit/unit time)
H_i^{bs}	The buyer i's physical storage cost per unit per unit time ($/unit/unit time)
H_i^b	The buyer i's total holding cost per unit per unit time ($/unit/unit time)
A_i^{bp}	The cost of placing an order by the i^{th} buyer ($/order)
A_i^{br}	The cost of receiving a shipment by the i^{th} buyer ($/order)
A_i^b	The total ordering cost composed of the cost placing an order and the cost of receiving a shipment for the i^{th} buyer ($/order)
A^{vs}	The vendor's setup cost ($/order)
A_i^{vr}	The vendor's shipment release cost to the i^{th} buyer ($/order)

Variables

q_i	The shipment size for the buyer i $q_i = D_i k_i b$
T_i	The replenishment cycle for the buyer i $T_i = k_i b$
y_i	The vendor's production duration for buyer i $y_i = \frac{q_i}{P} = \frac{D_i k_i b}{P}$

TC_z^s The average total cost for a firm z in the supply chain, where $z = v$ (vendor) and $z = bi$ (buyer i) in scenario s where $s = 1$ and $s = 2$ indicate the VMI & CS and Centralized policy, respectively.

TC^s The average total cost of the whole supply chain in scenario s where $s = 1$ and $s = 2$ indicate the VMI & CS and Centralized policy, respectively.

$$TC^s = TC_v^s + \sum_{i=1}^{N} TC_{bi}^s$$

Decision Variables

b Basic period

k_i An integer multiplier of basic period for the buyer i

w_{ilj} The (binary) variable representing the scheduling of a production lot for the buyer i using a multiplier l in basic period j.

III.4 General-Integer Policy

This section presents the mathematical models for both the VMI & CS and the centralized scenarios. The proposed models are formulated under General-Integer (GI) policy. General-Integer policy mandates the multiplier of each buyer must take a (positive and general) integer value. One may refer to Appendix 1 for the details of the mathematical models under General-Integer policy. We note that there are a total of $\text{lcm}(k_1, \dots, k_n)$ constraints for the capacity restrictions in the planning horizon of $\text{lcm}(k_1, \dots, k_n)b$ where $\text{lcm}(k_1, \dots, k_n)$ is the least common multiplier (*lcm*) of all the buyer's multipliers. However, since all the $k_i's$ are actually the decision variables, there is no way to know the value of $\text{lcm}(k_1, \dots, k_n)$ in advance. In such a case, one is not able to solve the models using any commercial software since the number of constraints is unknown to the decision maker. One may concern about another issue, namely, the value of $\text{lcm}(k_1, \dots, k_n)$ could become unpractically large when many of $k_i's$ are prime numbers. Therefore, we propose to use another policy, called as Power-of-Two (PoT) policy, in this study so that we are able to employ commercial software for solving the mathematical model under PoT policy.

III.5 Power-of-Two Policy

We presents the mathematical models for the VMI & CS and Centralized scenarios under Power-of-Two (PoT) policy in this section. The PoT policy requires all the multipliers (i.e., $k_i's$) must take a positive, power-of-two integer value, or $k_i = 2^l, l \in \mathbb{N}, \forall i$. The PoT policy is very useful since they facilitate the construction of feasible production schedules, but with only insignificant cost increase from the restriction of taking only PoT values.

In this section, we formulate the mixed binary integer programming models for the VMI & CS and Centralized scenarios. Instead of directly using $k_i's$ as the decision variables, we employ the binary variables w_{ilj}'s to determine both the value of the multiplier (i.e., k_i) and the scheduling of the (cyclic) production lots for the buyer i. A binary variable w_{ilj} represents the production lot for buyer i with its multiplier being 2^l being scheduled at the basic period j. Table III-3 shows the relationship between the multipliers k_i and the binary variables w_{ilj}'s for the buyer i for an example with the maximum possible value k_i being $k_i = 2^3$ (or, equivalently, $V_i = 3$, following the definition of V_i).

Table III-3 The relationship between $k_i's$ and the binary variables $w_{ilj}'s$

k_i		Basic Period j								k_i
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
2^l	2^0	w_{i01}								$k_i = 1$
	2^1	w_{i11}	w_{i12}							$k_i = 2$
	2^2	w_{i21}	w_{i22}	w_{i23}	w_{i24}					$k_i = 4$
	2^3	w_{i31}	w_{i32}	w_{i33}	w_{i34}	w_{i35}	w_{i36}	w_{i37}	w_{i8}	$k_i = 8$

With $k_i = 2^l, l \in \mathbb{N}, \forall i$ the equivalent values of k_i upon deciding the values of w_{ilj}'s show in the right-most column Table III-3. The columns (indexed by *j*) correspond to those basic periods in a planning horizon of 2^{V_i} basic periods. Recall that $w_{ilj} = 1$ indicates that the multiplier of buyer *i* is $k_i = 2^1$, and its production lot is scheduled at the basic period *j*. If the chosen multipliers for buyer i is $2^1 = 2$, there are two possible schedules for the replenishment of the buyer i, namely, the vendor may start its production for the buyer i at either basic period 1 or 2. For the example in

Table III-3, there are a total of 15 options (corresponding to 15 $w_{ilj}{}'$s, where $2^{V_i+1}-1=15$) to determine the value of multipliers (i.e., k_i) and the first replenishment basic period for the buyer i.

Table III-4 shows the cyclic pattern of the production schedule determined by the binary variables $w_{ilj}{}'$s. One may observe that w_{112} is the only binary variable being 1 (among those 15 options) which indicates the multiplier for the buyer i is $k_i=2^1=2$, and the vendor replenishes the buyer *i* starting at basic period 2, and resumes the replenishment at basic periods 4, 6, and 8.

Table III-4, we color the cells corresponding to all the 15 options ($w_{ilj}{}'$s) in grey so that one may easily observe their cyclic patterns.

Table III-4 The cyclic pattern of the production schedule corresponding to $w_{ilj}{}'s$

Buyer *i*	2^l	**Basic Period *j***								k_i
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
Buyer *i*	2^0	w_{101}	w_{101}	w_{101}	w_{101}	w_{101}	w_{101}	w_{101}	w_{101}	**1**
		0	0	0	0	0	0	0	0	
	2^1	w_{111}	w_{112}	w_{111}	w_{112}	w_{111}	w_{112}	w_{111}	w_{112}	**2**
		0	**1**	0	**1**	0	**1**	0	**1**	
	2^2	w_{121}	w_{122}	w_{123}	w_{124}	w_{121}	w_{122}	w_{123}	w_{124}	**4**
		0	0	0	0	0	0	0	0	
	2^3	w_{131}	w_{132}	w_{133}	w_{134}	w_{135}	w_{136}	w_{137}	w_{138}	**8**
		0	0	0	0	0	0	0	0	

III.5.1 Objective Function

1. VMI & CS Scenario

In the proposed VMI & CS scenario, the two objective functions are to minimize the average total costs for the vendor and the buyers, respectively.

The equation (3.1) expresses the average total costs for the vendor.

Minimize

$$\sum_{i=1}^{N}\sum_{l=0}^{V_i}\sum_{j=1}^{2^{V_i}}\frac{D_i b 2^l w_{ilj}}{2}\left(\frac{D_i}{P}H^v + H_i^{bo}\right) + \sum_{i=1}^{N}\sum_{l=0}^{V_i}\sum_{j=1}^{2^{V_i}}\frac{2^{-l} w_{ilj}}{b}\left(A_i^{vr} + A_i^{bp}\right) + \sum_{j=1}^{2^{maxV_i}} m_j \frac{A^{vs}}{2^{maxV_i} b} \quad (3.1)$$

The first term in (3.1) shows the inventory holding cost and the shared inventory holding cost. The second term represents the shipment cost and the order cost sharing cost with the buyers. The last term expresses the setup cost.

The equation (3.2) gives the average total costs for all the buyers.

$$\text{Minimize } \sum_{i=1}^{N}\sum_{l=0}^{V_i}\sum_{j=1}^{2^{V_i}}\frac{H_i^{bs}}{2} D_i 2^l w_{ilj} b + \sum_{i=1}^{N}\sum_{l=0}^{V_i}\sum_{j=1}^{2^{V_i}}\frac{A_i^{br}}{b} 2^{-l} w_{ilj} \quad (3.2)$$

The two cost terms in eq. (3.2) are the inventory holding cost and the order cost for all the buyers, respectively.

2. Centralized Scenario

In the Centralized scenario, the vendor is responsible to the average total costs of both the vendor and the all buyers. Therefore, there is only one objective function, *viz.*, (3.3) which is to minimize the average total costs, in such a scenario.

$$\text{Minimize } TC^2 = TC_v^2 + \sum_{i=1}^{N} TC_{bi}^2$$

$$= \frac{H^v}{2P}\sum_{i=1}^{N}\sum_{l=0}^{V_i}\sum_{j=1}^{2^{V_i}} D_i^2 2^l w_{ilj} b + \sum_{i=1}^{N}\sum_{l=0}^{V_i}\sum_{j=1}^{2^{V_i}}\frac{A_i^{vr}}{b} 2^{-l} w_{ilj} + \sum_{j=1}^{2^{maxV_i}} m_j \frac{A^{vs}}{2^{maxV_i} b}$$

$$+ \sum_{i=1}^{N}\sum_{l=0}^{V_i}\sum_{j=1}^{2^{V_i}}\frac{H_i^b}{2} D_i 2^l w_{ilj} b + \sum_{i=1}^{N}\sum_{l=0}^{V_i}\sum_{j=1}^{2^{V_i}}\frac{A_i^b}{b} 2^{-l} w_{ilj} \quad (3.3)$$

The first three terms in (3.3) are the average total costs for the vendor, and the others are the average total costs for the buyers. The first term in the objective function describes the inventory holding costs for the vendor. The second and the third terms represent the shipment costs and the setup costs for the vendor, respectively. The last two terms are the inventory costs and the order costs for the buyers, respectively.

III.5.2 Constraints

Since both the VMI & CS and Centralized scenarios use the same mechanism for production operations, two scenarios share the same set of constraints. We present the details on the constraints as follows.

The Constraints for the Scheduling of the Vendor's Setup

The vendor is charged a setup cost when the vendor needs to make production for at least one buyer in a basic period. The setup cost was incurred only one time although the vendor may make production for more than one buyer in a basic period (since all the buyers received the same product from the vendor). We present the constraint for the scheduling of the vendor's production (and its setups) in each basic period of the planning horizon of $2^{max\, V_i}$ basic periods in (3.4).

$$\sum_{i=1}^{N}\sum_{l=0}^{V_i} w_{ilj} \le m_j M \text{ , } for\ j = 1, \dots, 2^{maxV_i}. \tag{3.4}$$

The constraints (3.4) check the vendor's setup at every basic period using another binary variable (m_j). We use m_j to indicate if any production is scheduled in basic period j, namely, m_j is equal to 1 only if the vendor makes production for at least one buyer in basic period j; otherwise, m_j is equal to 0. Note that M in (3.4) is an extremely large number. Whenever the value of the sum at the left-side is positive (with some buyers i's with $w_{ilj} = 1$), the value of m_j must be equal to 1 to make the inequality hold. The vendor must set up the production system to makes production for at least one buyer in basic period j, in such a case.

The Capacity Constraints

The constraints in (3.5) guarantee that the total production duration plus the setup time for the production scheduled in one basic period does not exceed the length of the basic period for each basic period in the planning horizon (of $2^{max\, V_i}$ basic periods).

$$m_j S + \sum_{i=1}^{N}\sum_{l=0}^{V_i} w_{il\varphi(2^l,j)} \frac{D_i 2^l b}{P} \le b, for\ j = 1, \dots, 2^{maxV_i} \tag{3.5}$$

The first term of constraint (3.5) represents the setup time scheduled in basic period j. The second term expresses the sum of the production duration of all the buyer $i's$, i.e., $\frac{D_i 2^l b}{P}$, basic period j. Note that the vendor's production scheduled for each buyer i follows a cyclic pattern. We use the function $\varphi\left(2^l, j\right)$ to control the cyclic pattern of the production schedule where the function $\varphi\left(2^l, j\right)$ is defined in (3.6).

$$\varphi\left(2^l, j\right) = \begin{pmatrix} j \bmod 2^l, & \text{if } j \ne \gamma 2^l, \gamma \epsilon \mathrm{N} \\ 2^l, & \text{if } j = \gamma 2^l, \gamma \epsilon \mathrm{N} \end{pmatrix} \tag{3.6}$$

Figure III-1 and Figure III-2 depicts examples of the vendor's production schedules for buyer 1 and buyer 2, respectively. Both figures show that the production duration of each buyer is less than the length of basic period. As one may observe, the vendor needs to make production for both buyers 1 and 2 at basic period 1, 5, and 9. We must check if the vendor has sufficient capacity by assuring the sum of the production durations for buyer 1 and/or 2 plus the setup time in that basic period does not exceed the length of the basic period as defined in (3.5).

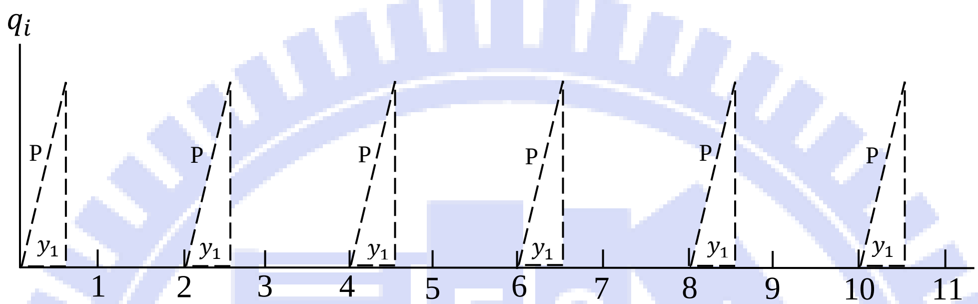


Figure III-1 The vendor's production schedule for buyer 1

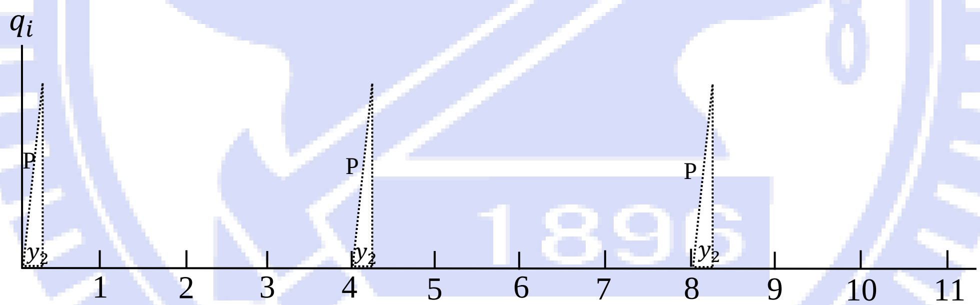


Figure III-2 The vendor's production schedule for buyer 2

The Constraints for Production Scheduling

The equations in (3.7) assure that each buyer will pick exactly one of the $(2^{V_i+1}-1)$ options, corresponding to $(2^{V_i+1}-1)$ w_{ilj}'s, to determine the value of multipliers (i.e., k_i) and the first replenishment basic period.

$$\sum_{l=0}^{V_i}\sum_{j=1}^{2^{V_i}} w_{ilj} = 1, for\ i = 1, \dots, N \qquad (3.7)$$

CHAPTER IV MODEL VERIFICATION AND NUMERICAL EXPERIMENTS

In this chapter, we first verify the formulation of our models proposed in CHAPTER III and, then present our numerical experiments. We will compare our experimental results with those in Ben-Daya, et al. (2013), and, particularly, investigate the ordering cost and holding cost shared by the vendor and the buyers, as well as the overall costs in the supply chain.

We solved the instances by Gurobi Optimizer® and a Notebook PC with an Intel® Core™ i7-4702MQ @ 2.20GHz CPU and 8GB RAM in our numerical experiments.

IV.1 Model Verification

IV.1.1 Constraint Verification

We would verify our results from Gurobi Optimizer® in this section. We will use a simple example check the consistency of its results with the model formulation.

The Constraints for the Scheduling of the Vendor's Setup

We expressed the constraints for the scheduling of the vendor's production (and its setup) in eq. (3.4), and we present it as (4.1) for the purpose of verification in this chapter again.

$$\sum_{i=1}^{N}\sum_{l=0}^{V_i} w_{ilj} \le m_j M, for\ j = 1, \dots, 2^{maxV_i} \quad (4.1)$$

We use a binary variable $m_j = 1$ to indicate when any production is scheduled in basic period j. Such a variable m_j assists to check if the vendor make production for any buyer at a particular basic period *j*. Table IV-1 shows the production schedule in our example.

In Table IV-1, the vendor makes production for buyer 1 and buyer 2 using the same cyclic pattern with $w_{ilj} = 1$ for *i* = 1, 2, and *j* = 1, 3, 5, 7, namely, at basic period 1, 3, 5, and 7 for buyer 1 and 2. Table IV-1 shows that one buyer has only one option of w_{ilj}*'s*, and the value of m_j will be 1 if there is at least one production lot scheduled at that particular time.

Table IV-1 The production schedule of an example

Buyer i	**2^l**	**j**								**k_i**
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
Buyer 1	**2^0**	w_{101}	w_{101}	w_{101}	w_{101}	w_{101}	w_{101}	w_{101}	w_{101}	**1**
		0	0	0	0	0	0	0	0	
	2^1	w_{111}	w_{112}	w_{111}	w_{112}	w_{111}	w_{112}	w_{111}	w_{112}	**2**
		$\underline{1}$	0	$\underline{1}$	0	$\underline{1}$	0	$\underline{1}$	0	
	2^2	w_{121}	w_{122}	w_{123}	w_{124}	w_{121}	w_{122}	w_{123}	w_{124}	**4**
		0	0	0	0	0	0	0	0	
	2^3	w_{131}	w_{132}	w_{133}	w_{134}	w_{135}	w_{136}	w_{137}	w_{138}	**8**
		0	0	0	0	0	0	0	0	
Buyer 2	**2^0**	w_{201}	w_{201}	w_{201}	w_{201}	w_{201}	w_{201}	w_{201}	w_{201}	**1**
		0	0	0	0	0	0	0	0	
	2^1	w_{211}	w_{212}	w_{211}	w_{212}	w_{211}	w_{212}	w_{211}	w_{212}	**2**
		$\underline{1}$	0	$\underline{1}$	0	$\underline{1}$	0	$\underline{1}$	0	
	2^2	w_{221}	w_{222}	w_{223}	w_{224}	w_{221}	w_{222}	w_{223}	w_{224}	**4**
		0	0	0	0	0	0	0	0	
	2^3	w_{231}	w_{232}	w_{233}	w_{234}	w_{235}	w_{236}	w_{237}	w_{238}	**8**
		0	0	0	0	0	0	0	0	
Buyer 3	**2^0**	w_{301}	w_{301}	w_{301}	w_{301}	w_{301}	w_{301}	w_{301}	w_{301}	**1**
		0	0	0	0	0	0	0	0	
	2^1	w_{311}	w_{312}	w_{311}	w_{312}	w_{311}	w_{312}	w_{311}	w_{312}	**2**
		0	$\underline{1}$	0	$\underline{1}$	0	$\underline{1}$	0	$\underline{1}$	
	2^2	w_{321}	w_{322}	w_{323}	w_{324}	w_{321}	w_{322}	w_{323}	w_{324}	**4**
		0	0	0	0	0	0	0	0	
	2^3	w_{331}	w_{332}	w_{333}	w_{334}	w_{335}	w_{336}	w_{337}	w_{338}	**8**
		0	0	0	0	0	0	0	0	

```
Constraint 1
Period  1
<gurobi.LinExpr: 2.0> <= 100000.0
Period  2
<gurobi.LinExpr: 1.0> <= 100000.0
Period  3
<gurobi.LinExpr: 2.0> <= 100000.0
Period  4
<gurobi.LinExpr: 1.0> <= 100000.0
Period  5
<gurobi.LinExpr: 2.0> <= 100000.0
Period  6
<gurobi.LinExpr: 1.0> <= 100000.0
Period  7
<gurobi.LinExpr: 2.0> <= 100000.0
Period  8
<gurobi.LinExpr: 1.0> <= 100000.0
```

Figure IV-1 The results of the constraints for scheduling of vendor's setup from Gurobi Optimizer®

Figure IV-1 shows the results of the constraint for the scheduling of vendor's setup from Gurobi Optimizer®. The left-side values indicate the sum of variable w_{ilj} for the corresponding basic periods. The values of m_j will be exactly one if the left-sides values greater than 0. Following Table IV-1, the vendor makes production in each of the 8 basic periods, and we have $m_j = 1$ for j=1, …, 8. The right-side values become 100000 for each of the 8 basic periods since we set M=100000. Therefore, our results match with the formulation proposed mathematical model in this study.

The Capacity Constraints

We present (3.5) as the capacity constraint in the proposed model, and show them in (4.2) for model verification as follows.

$$m_j S + \sum_{i=1}^{N} \sum_{l=0}^{V_i} w_{il\varphi(2^l,j)} \frac{D_i 2^l b}{P} \leq b, for\ j = 1, \dots, 2^{maxV_i} \tag{4.2}$$

Our key concern is to check the total production duration plus the setup time for the production scheduled in one basic period does not exceed the length of the basic period for each basic period in the planning horizon (of 2^{maxV_i} basic periods).

Table IV-2 represents the production duration for buyer i at specific basic period j and the required setup time at specific basic period j. The total required capacity (including setup time 2.0 time units and production duration 30 time units) by the vendor are 32 time units for all basic periods. Obviously, it may vary with the configuration of production schedule of each buyer (corresponding with their first replenishment basic period).

```
Constraint 2
Period  1
2.0 + <gurobi.LinExpr: 30.0>
Period  2
2.0 + <gurobi.LinExpr: 30.0>
Period  3
2.0 + <gurobi.LinExpr: 30.0>
Period  4
2.0 + <gurobi.LinExpr: 30.0>
Period  5
2.0 + <gurobi.LinExpr: 30.0>
Period  6
2.0 + <gurobi.LinExpr: 30.0>
Period  7
2.0 + <gurobi.LinExpr: 30.0>
Period  8
2.0 + <gurobi.LinExpr: 30.0>
```

Figure IV-2 The results for the capacity constraints from Gurobi Optimizer®

We present the results for capacity constraints from Gurobi Optimizer® in Figure IV-2. On the left-side, the first term describes the setup time and the second term is the total duration for the scheduled production. Therefore, we confirm the results of from Gurobi Optimizer® match with the model formulation, and we successfully verify the capacity constraints.

Table IV-2 The total required capacity of the vendor

Buyer i	$\mathbf{2^l}$	**j**								k_i
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
Buyer 1	2^0	w_{101}	w_{101}	w_{101}	w_{101}	w_{101}	w_{101}	w_{101}	w_{101}	**1**
		0	0	0	0	0	0	0	0	
	2^1	w_{111}	w_{112}	w_{111}	w_{112}	w_{111}	w_{112}	w_{111}	w_{112}	**2**
		$\underline{10}$	0	$\underline{10}$	0	$\underline{10}$	0	$\underline{10}$	0	
	2^2	w_{121}	w_{122}	w_{123}	w_{124}	w_{121}	w_{122}	w_{123}	w_{124}	**4**
		0	0	0	0	0	0	0	0	
	2^3	w_{131}	w_{132}	w_{133}	w_{134}	w_{135}	w_{136}	w_{137}	w_{138}	**8**
		0	0	0	0	0	0	0	0	
Buyer 2	2^0	w_{201}	w_{201}	w_{201}	w_{201}	w_{201}	w_{201}	w_{201}	w_{201}	**1**
		0	0	0	0	0	0	0	0	
	2^1	w_{211}	w_{212}	w_{211}	w_{212}	w_{211}	w_{212}	w_{211}	w_{212}	**2**
		$\underline{20}$	0	$\underline{20}$	0	$\underline{20}$	0	$\underline{20}$	0	
	2^2	w_{221}	w_{222}	w_{223}	w_{224}	w_{221}	w_{222}	w_{223}	w_{224}	**4**
		0	0	0	0	0	0	0	0	
	2^3	w_{231}	w_{232}	w_{233}	w_{234}	w_{235}	w_{236}	w_{237}	w_{238}	**8**
		0	0	0	0	0	0	0	0	
Buyer 3	2^0	w_{301}	w_{301}	w_{301}	w_{301}	w_{301}	w_{301}	w_{301}	w_{301}	**1**
		0	0	0	0	0	0	0	0	
	2^1	w_{311}	w_{312}	w_{311}	w_{312}	w_{311}	w_{312}	w_{311}	w_{312}	**2**
		0	$\underline{30}$	0	$\underline{30}$	0	$\underline{30}$	0	$\underline{30}$	
	2^2	w_{321}	w_{322}	w_{323}	w_{324}	w_{321}	w_{322}	w_{323}	w_{324}	**4**
		0	0	0	0	0	0	0	0	
	2^3	w_{331}	w_{332}	w_{333}	w_{334}	w_{335}	w_{336}	w_{337}	w_{338}	**8**
		0	0	0	0	0	0	0	0	

Table IV-2 The total required capacity of the vendor (cont.)

	j							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Total production duration at basic period *j*	30	30	30	30	30	30	30	30
Setup time at basic period *j*	2	2	2	2	2	2	2	2
Total production capacity required	32	32	32	32	32	32	32	32

The Constraints for Production Scheduling

We present (3.7) as the constraints for production scheduling in proposed model, and express as (4.3) again for model verification as follows.

$$\sum_{l=0}^{V_i}\sum_{j=1}^{2^{V_i}} w_{ilj} = 1, for\ j = 1, \ldots, 2^{maxV_i} \tag{4.3}$$

The constraint for production scheduling mandate that each buyer has to set only one binary variable to 1 to pick the value of k_i and its first replenishment basic period in the production schedule. The rest of the production schedule follows a cyclic pattern according to the value of k_i.

We display the results of the constraint for production scheduling from Gurobi Optimizer® in Figure IV-3. Figure IV-3 shows that the sum of the binary variables among all the periods j and multipliers l is equal to 1 across. Thus, the results from Gurobi Optimizer® match exactly with the proposed model.

```
Constraint 3
Buyer  1
<gurobi.TempConstr: <gurobi.LinExpr: 1.0> == 1>
Buyer  2
<gurobi.TempConstr: <gurobi.LinExpr: 1.0> == 1>
Buyer  3
<gurobi.TempConstr: <gurobi.LinExpr: 1.0> == 1>
```

Figure IV-3 The results of the constraint for production scheduling from Gurobi Optimizer®

IV.2 Numerical Experiment

IV.2.1 Parameter Setting

In this section, we present the parameter settings of our numerical experiments, by referring to the three data sets in Ben-Daya, et al. (2013). Table IV-3 shows all of the parameters in the three data sets in which we use bold-fonts to indicate those key different values in the settings. Then, we solve the proposed model for the VMI & CS and Centralized scenarios under Power-of-Two (PoT) policy using Gurobi Optimizer®.

Table IV-3 The three data sets from Ben-Daya, et al. (2013)

	Data Set 1		Data Set 2		Data Set 3	
Buyer	**1**	**2**	**1**	**2**	**1**	**2**
$\boldsymbol{D_i}$	500	1000	500	1000	500	1000
$\boldsymbol{A_i^b}$	25	75	25	75	25	75
$\boldsymbol{A_i^{bp}}$	**15**	**50**	**20**	**65**	**15**	**40**
$\boldsymbol{A_i^{br}}$	**10**	**25**	**5**	**10**	**10**	**35**
$\boldsymbol{H_i^b}$	5	5	5	5	5	5
$\boldsymbol{H_i^{bo}}$	**2.5**	**2**	**4.5**	**4.5**	**2.5**	**2**
$\boldsymbol{H_i^{bs}}$	**2.5**	**3**	**0.5**	**0.5**	**2.5**	**3**
$\boldsymbol{A_i^{vr}}$	0	0	0	0	0	0
$\boldsymbol{A^{vs}}$	400		400		400	
$\boldsymbol{H^v}$	4		4		4	
P	3200		3200		3200	

For each data set, we solve the proposed model for 200 values of b in the range between 0.01 and 1.005 with an increment value of 0.005 for these three data sets using Gurobi Optimizer® and compile the results of w_{ilj}*'s* to obtain the corresponding multipliers k_i*'s*.

IV.2.2 Results of the proposed model

The top part of Table IV-4 summarizes the results of the proposed model under PoT policy for the VMI & CS and Centralized scenario. The optimal solutions for all the three data sets in VMI & CS scenario are different. The optimal objection function value are \$3,130.11, \$3,011.21, and \$3,120.52 in the three data sets, respectively. One may observe that the optimal solutions for all the three data sets in Centralized scenario are the same with the optimal basic period b*=0.165, the vector of optimal multipliers $(k_1^*, k_2^*) = (2,2)$ and the optimal objection function value being \$3,010.46. Such interesting outcomes results from the settings that the values of total inventory holding ($H_i^b = H_i^{bo}$+H_i^{bs}) and ordering costs ($A_i^b = A_i^{bp}$+A_i^{br}) are same for the three data sets as shown in Table IV-3. We also present the optimal solution from Ben-Daya, et al. (2013) in the lower parts of Table IV-4. The models of Ben-Daya, et al. (2013) obtain better solutions than the proposed model by 6.81%, 6.80%, and 4.49% in the three data sets, respectively, as comparing the VMI & CS scenario. Also, the solutions of Ben-Daya, et al. (2013) are better than those the proposed model by 7.06% for all the three data sets in Centralized scenario. To get more insights, we present the details of all the cost components of the vendor and buyers in Table IV-5.

Table IV-4 Comparison of the optimal solutions of the three data sets

	The Proposed Model					
Data Set	Decision Variables			VMI & CS		
	b	k_1	k_2	ATC	ATBC	ATVC
Data Set 1	0.22	2	2	3130.11	1014.515	2115.57
Data Set 2	0.17	2	2	3011.21	171.6179	2839.6
Data Set 3	0.435	1	1	3120.52	1027.82	2092.70
Data Set	Decision Variables			Centralized		
	b	k_1	k_2	ATC	ATBC	ATVC
Data Set 1	0.165	2	2	3010.46	807.31	1469.93
Data Set 2	0.165	2	2	3010.46	169.20	1469.93
Data Set 3	0.165	2	2	3010.46	837.61	1469.93
	Ben-Daya, et al. (2013)					
Data Set	Decision Variables			VMI & CS		
	T	n		ATC (Advantage %)	ATBC	ATVC
Data Set 1	0.6572	3		2930.61 (6.81%)	1119.89	1810.72
Data Set 2	0.504	3		2819.50 (6.80%)	219.21	2600.29
Data Set 3	0.705	4		2986.29 (4.49%)	1226.3	1759.99
Data Set	Decision Variables			Centralized		
	T	n		ATC (Advantage %)	ATBC	ATVC
Data Set 1	0.4289	2		2797.88 (7.60%)	1697.72	1100.16
Data Set 2	0.4289	2		2797.88 (7.60%)	1697.72	1100.16
Data Set 3	0.4289	2		2797.88 (7.60%)	1697.72	1100.16

Notes: ATVC, Average Total Vendor Costs; ATBC, Average Total Buyer Costs; ATC, Average Total Costs.

Table IV-5 Comparison of details of cost components of the three data sets

VMI & CS	Cost	Data Set 1		Data Set 2		Data Set 3	
		Proposed model	Ben-Daya	Proposed model	Ben-Daya	Proposed model	Ben-Daya
Vendor Costs	H^v	343.74	171.15	265.625	131.20	339.84	137.61
	H_i^{bo}	715	734.21	1147.5	1168.95	706.88	742.39
	A_i^{vr}	0	0	0	0	0	0
	A_i^{bp}	147.727	296.71	250	506.16	126.44	312.26
	A^{vs}	909.09	608.64	1176.47	793.98	919.54	567.74
Buyer Costs	H_i^{bs}	935	960.13	127.5	129.88	924.38	970.82
	A_i^{br}	79.55	159.77	44.12	89.32	103.45	255.48
	Total	3130.11	2930.61	3011.21	2819.5	3120.52	2986.29

Table IV-5 Comparison of details of cost components of the three data sets (cont.)

Central-ized	Cost	Data Set 1		Data Set 2		Data Set 3	
		Proposed model	Ben-Daya	Proposed model	Ben-Daya	Proposed model	Ben-Daya
Vendor Costs	H^v	257.81	167.54	257.81	167.54	257.81	167.54
	A_i^{vr}	0	0	0	0	0	0
	A^{vs}	1212.12	932.63	1212.12	932.63	1212.12	932.63
Buyer Costs	H_i^{bs}	1237.5	1231.40	1237.5	1231.40	1237.5	1231.40
	A_i^{br}	303.03	466.31	303.03	466.31	303.03	466.31
	Total	3010.46	2797.88	3010.46	2797.88	3010.46	2797.88

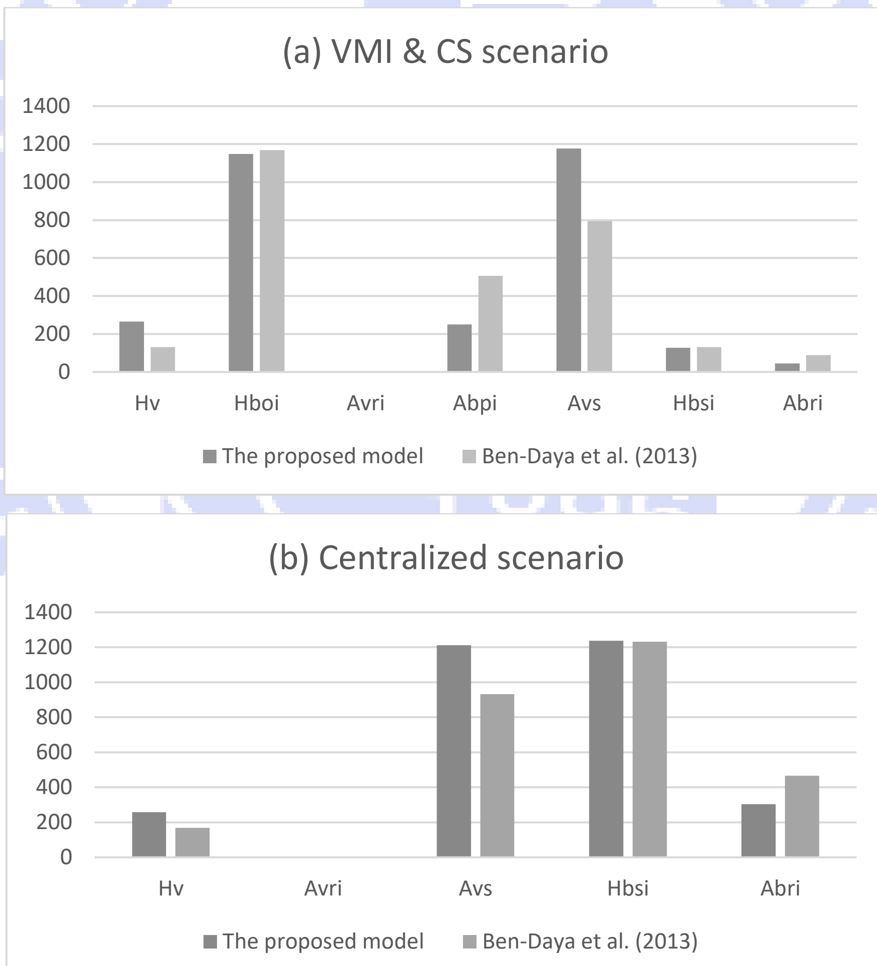


Figure IV-4 Comparison of the cost components using Data set 2 in (a) VMI & CS scenario and (b) Centralized scenario

Recall that Ben-Daya, et al. (2013) obtains the most significant advantage comparing with the proposed model in Data set 2. We would show the comparison of details cost components for both the VMI & CS and Centralized scenario between the proposed model and Ben-Daya, et al. (2013) in Figure IV-4. We may observe that the buyers are responsible for more costs in the model of Ben-Daya, et al. (2013) comparing to the proposed model in the VMI & CS scenario. The vendor will incur more significant cost for both setup and inventory holding cost in the proposed model. For the Centralized scenario, the inventory holding cost for the buyers is the same for both models. Again, the vendor will have more setup cost and inventory holding cost in the proposed model.

We are curious if Ben-Daya, et al. (2013) also obtains better solution than the proposed model. Therefore, we will conduct sensitivity analysis for further investigation on the impact of the parameter settings using the three data sets in VMI & CS and Centralized scenarios.

IV.3 Sensitivity Analysis

This section presents sensitivity analysis that compares the results from our proposed model with Ben-Daya, et al.'s (2013) for the VMI & CS and Centralized scenarios by changing the parameter settings in the three data sets in Section IV.2. We investigate the comparison between our proposed model and Ben-Daya, et al. (2013) with different levels of utilization ratio (*D/P*), order costs ratio (A^{vs}/A_i^b), setup time (*S*), and number of buyers (*N*). We conduct our numerical experiments using the three data sets presented in IV.2.1. Using the solutions of Ben-Daya, et al. (2013) as benchmarks, we will compute the percentage of cost saving as comparing with the solutions from our proposed models under VMI & CS and Centralized scenarios.

IV.3.1 Capacity Ratio *D/P*

We test 4 levels of utilization ratio, i.e., *D/P*=0.1, 0.2, 0.3, and 0.4 in our experiments for the three data sets. For each instance, we solve it by the proposed models and collect the cost saving by comparing with the results from the models in Ben-Daya et al.'s (2013).

We summarize our experimental results for Data set 1 in Table IV-6, also illustrate them in Figure IV-5. One may have two observations: (1) the average total costs for both scenarios are increase as *D/P* ratio increases, and (2) the cost saving decreases as *D/P* ratio increases for both scenarios. In the VMI & CS scenario, the proposed model obtains a better solution than Ben-Daya et al.'s (2013) only when *D/P*=0.2, but the other three are worse, and the buyers obtain more cost saving than the vendor. On the other hand, they are not able to obtain any cost saving in the Centralized scenario, either.

Table IV-6 The sensitivity analysis of *D/P* using Data set 1

D/P	Proposed Model							
	VMI & CS				Centralized			
	ATVC	ATBC	ATC	%Sav	ATVC	ATBC	ATC	%Sav
0.1	1825.51	1152.38	2977.89	-0.04%	1185.93	1612.94	2798.87	0.00%
0.2	1908.55	1102.79	3011.34	1.97%	1259.52	1598.21	2857.73	0.00%
0.3	1988.11	1073.22	3061.33	-1.64%	1331.92	1583.61	2915.53	-1.51%
0.4	2064.58	1034.03	3098.61	-3.04%	1417.36	1554.76	2972.12	-5.08%

Notes: ATVC, Average Total Vendor Costs; ATBC, Average Total Buyer Costs; ATC, Average Total Costs; %Sav, % saving.

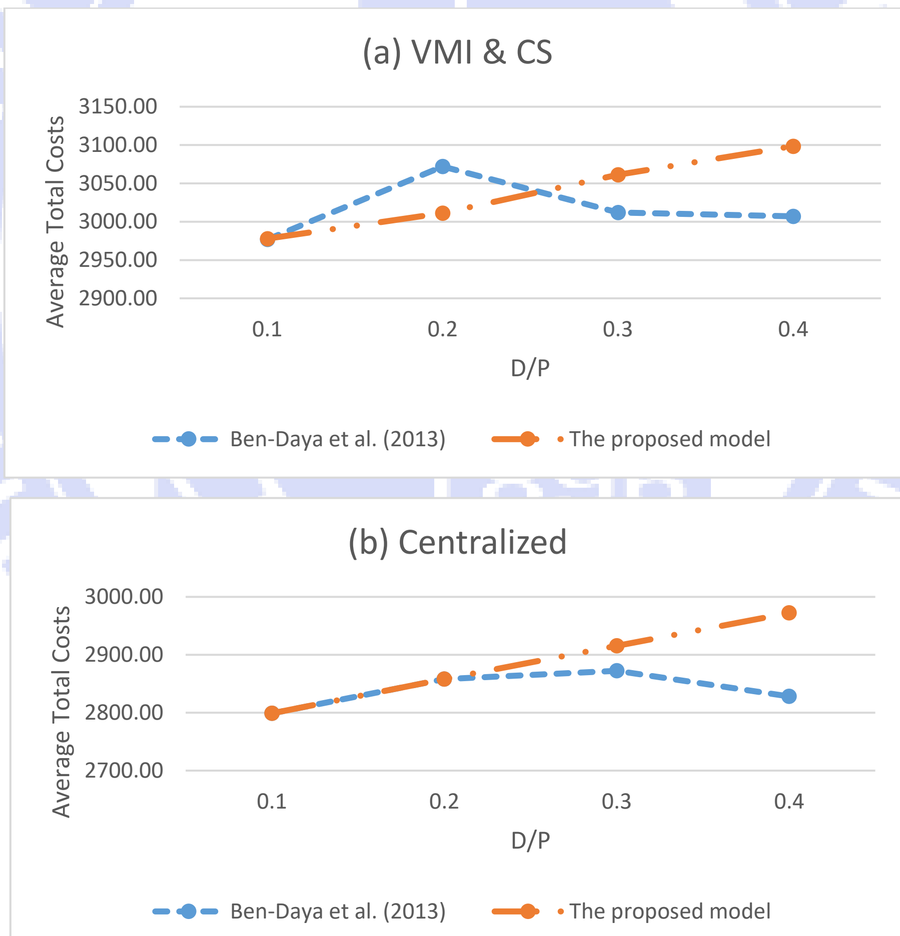


Figure IV-5 The sensitivity analysis of *D/P* using Data set 1: (a) VMI & CS (b) Centralized

Table IV-7 summarizes the results of the proposed model for Data set 2, and we also illustrate the results in Figure IV-6. One may observe that the proposed model obtain no results better than those Ben-Daya, et al. (2013) for both the VMI & CS and Centralized scenario.

Table IV-7 The sensitivity analysis of *D/P* using Data set 2

D/P	Proposed Model							
	VMI & CS				Centralized			
	ATVC	ATBC	ATC	%Sav	ATVC	ATBC	ATC	%Sav
0.1	2621.23	179.2905	2800.52	0.00%	1185.93	1612.94	2798.87	0.00%
0.2	2682.22	176.6669	2858.89	0.01%	1259.52	1598.21	2857.74	0.00%
0.3	2741.82	175.3785	2917.2	-1.54%	1331.92	1583.61	2915.53	-1.51%
0.4	2800.17	172.8538	2973.03	-5.09%	1417.36	1554.76	2972.12	-5.08%

Notes: ATVC, Average Total Vendor Costs; ATBC, Average Total Buyer Costs; ATC, Average Total Costs; %Sav, % saving.

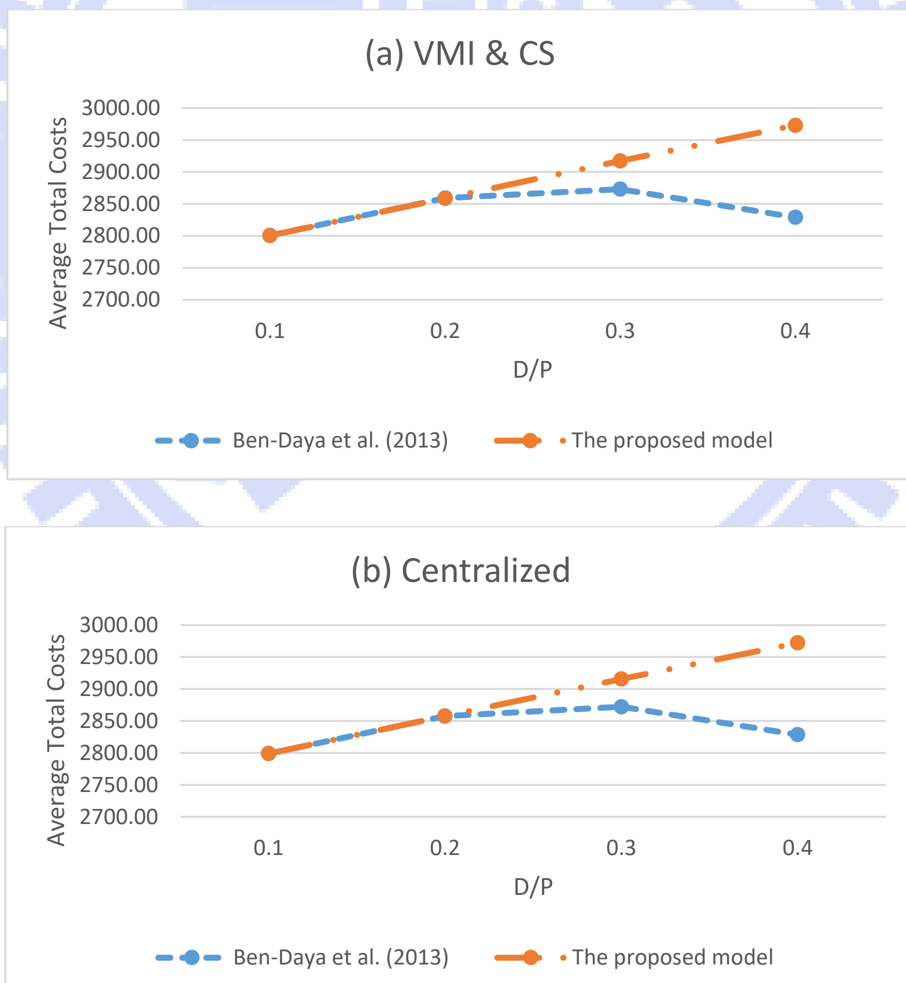


Figure IV-6 The sensitivity analysis of *D/P* using Data set 2: (a) VMI & CS (b) Centralized

The results of proposed model for Data set 3 are summarized in Table IV-8 and illustrated in Figure IV-7. In Table IV-8, we have similar observations for Data set 3 as we did in Table IV-6 for Data set 1. And, the cost savings for the VMI & CS range from 1.3% to 1.7%, and the proposed model obtain no results better than Ben-Daya, et al. (2013) for Centralized scenario.

Table IV-8 Effect of changing *D/P* on data set 3

D/P	Proposed Model							
	VMI & CS				Centralized			
	ATVC	ATBC	ATC	%Sav	ATVC	ATBC	ATC	%Sav
0.1	1805.78	1162.234	2968.02	-0.07%	1185.93	1612.94	2798.87	0.00%
0.2	1887.92	1113.75	3001.67	1.71%	1259.52	1598.21	2857.74	0.00%
0.3	1966.62	1084.899	3051.52	1.35%	1331.92	1583.61	2915.53	-1.51%
0.4	2042.26	1046.749	3089.01	-3.44%	1417.36	1554.76	2972.12	-5.08%

Notes: ATVC, Average Total Vendor Costs; ATBC, Average Total Buyer Costs; ATC, Average Total Costs; %Sav, % saving.

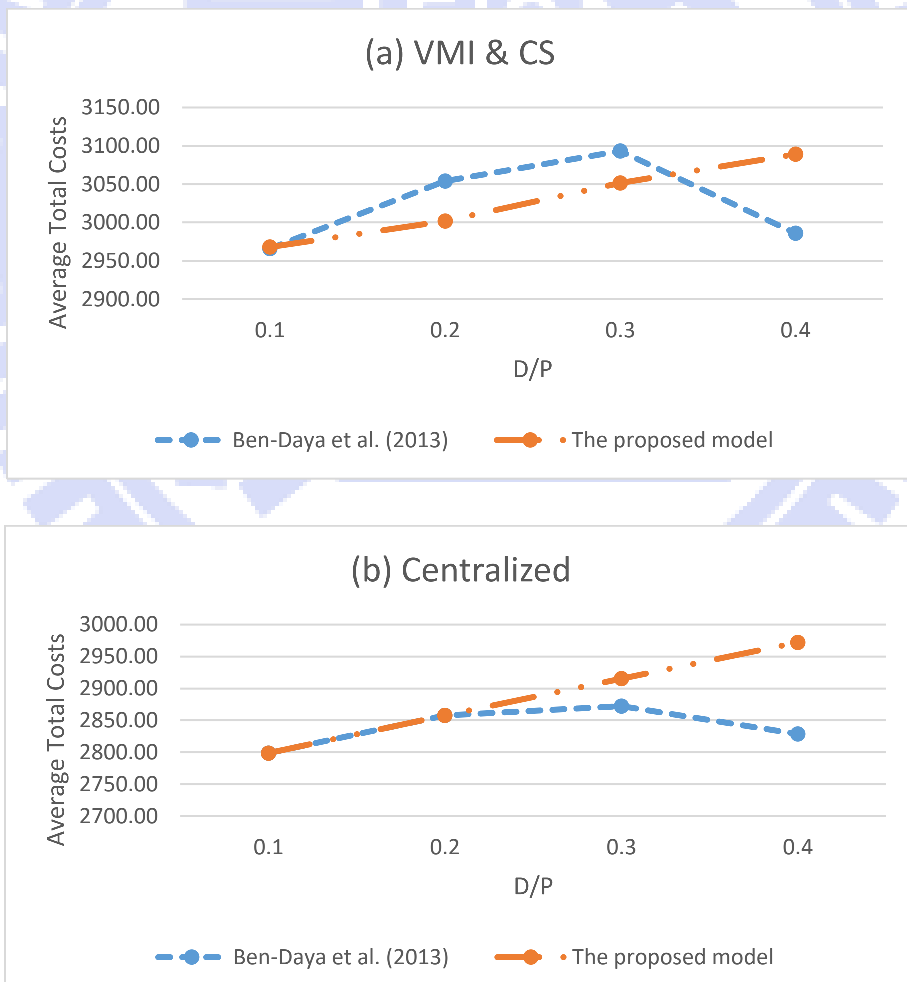


Figure IV-7 The sensitivity analysis of *D/P* using Data set 3: (a) VMI & CS (b) Centralized

IV.3.2 Order Costs Ratio A^{vs}/A_i^b

Referring to the three data sets, A^{vs} is one of the most significant cost components in the proposed model. Our conjecture is that when the vendor's setup cost is not large (comparing to the buyers'), the proposed model might have more cost-saving. To verify this conjecture, we run experiments with different levels of A^{vs}/A_i^b ratio for three data sets, and compare the objective function values between our proposed model and Ben-Daya, et al. (2013). We have a common observation among all the three data sets, namely, the average total costs for VMI & CS and Centralized scenario increase as A^{vs}/A_i^b ratio increases.

Table IV-9 summarizes our experimental results for Data set 1, and we also illustrate results in Figure IV-8. We observe that the cost savings for the VMI & CS scenario are in the range from 0% to 6% for VMI & CS scenario, and the cost savings decreases as A^{vs}/A_i^b ratio increases. However, the cost savings no results better than Ben-Daya, et al. (2013) for Centralized scenario.

Table IV-9 The sensitivity analysis of A^{vs}/A_i^b using Data set 1

A^{vs}/A_i^b	Proposed Model							
	VMI & CS				Centralized			
	ATVC	ATBC	ATC	%Sav	ATVC	ATBC	ATC	%Sav
0.75	1160.83	655.834	1816.67	5.95%	1187.17	593.862	1781.03	-0.003%
1.00	1260.24	687.116	1947.36	3.75%	1291.03	612.92	1903.94	0.000%
1.25	1352.32	720	2072.32	1.84%	1385.85	633.68	2019.53	-0.005%
1.50	1438.54	754.166	2192.71	0.13%	1480.36	648.311	2128.67	0.000%

Notes: ATVC, Average Total Vendor Costs; ATBC, Average Total Buyer Costs; ATC, Average Total Costs; %Sav, % saving.

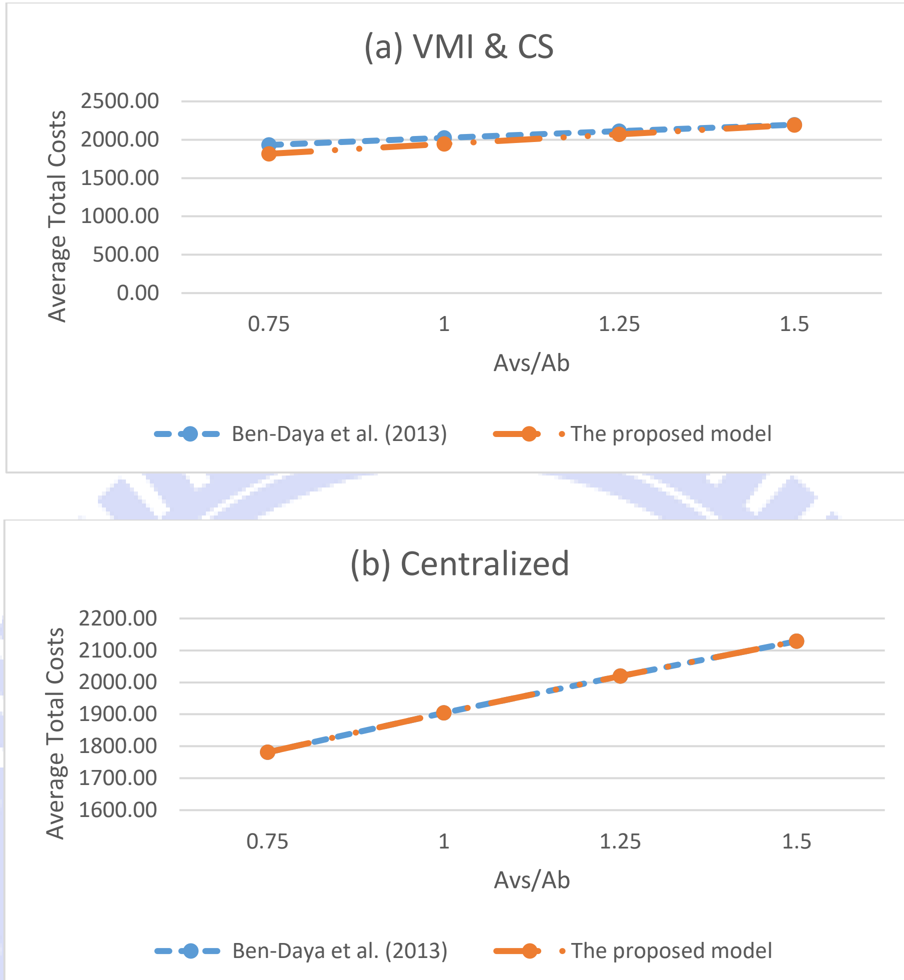


Figure IV-8 The sensitivity analysis of A^{vs}/A_i^b using Data set 1: (a) VMI & CS (b) Centralized

We summarize the sensitivity analysis of A^{vs}/A_i^b using Data set 2 in Table IV-10 and illustrated in Figure IV-9. The proposed model obtain no better solutions than Ben-Daya, et al. (2013) for Centralized scenario and most of the cases of VMI & CS scenario.

Table IV-10 The sensitivity analysis of A^{vs}/A_i^b using Data set 2

A^{vs}/A_i^b	Proposed Model							
	VMI & CS				Centralized			
	ATVC	**ATBC**	**ATC**	**%Sav**	**ATVC**	**ATBC**	**ATC**	**%Sav**
0.75	1630.98	150.048	1781.03	0.0%	1630.98	150.048	1781.03	0.0%
1.00	1753.76	150.1785	1903.94	0.0%	1753.76	150.1785	1903.94	0.0%
1.25	1868.49	151.0412	2019.53	0.0%	1868.49	151.0412	2019.53	0.0%
1.50	1976.67	152.5003	2129.17	0.4%	1976.72	151.9546	2128.67	0.0%

Notes: ATVC, Average Total Vendor Costs; ATBC, Average Total Buyer Costs; ATC, Average Total Costs; %Sav, % saving.

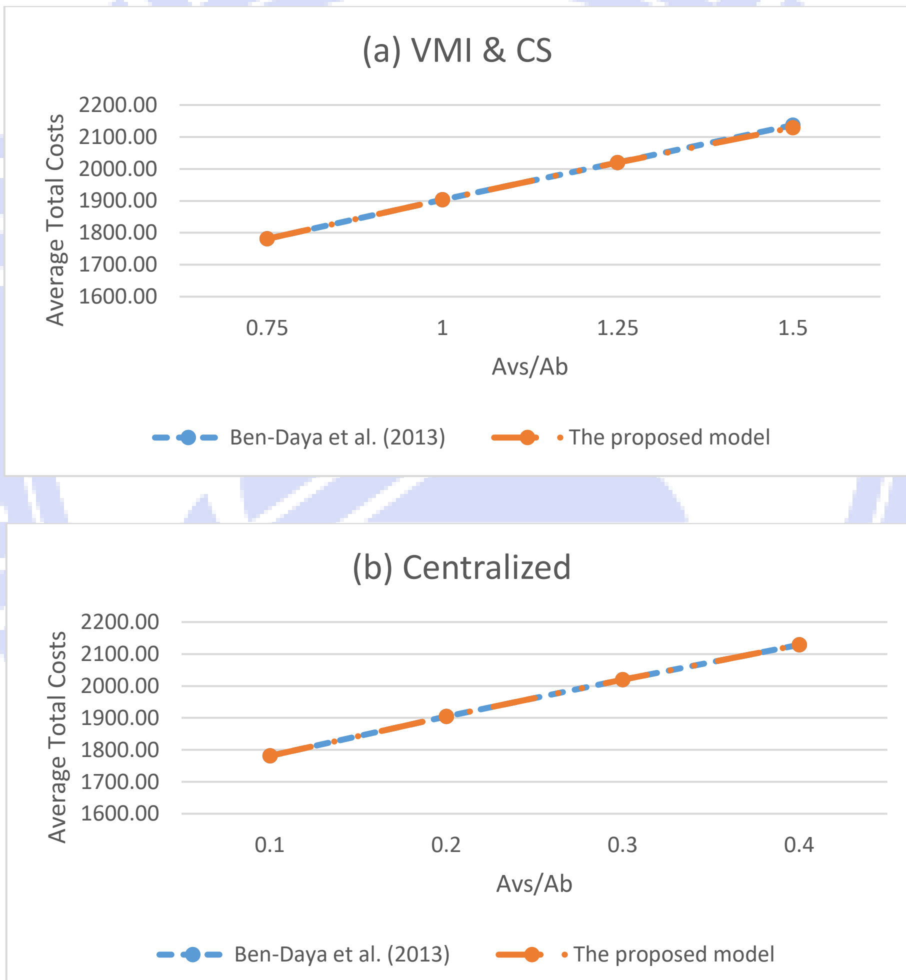


Figure IV-9 The sensitivity analysis of A^{vs}/A_i^b using Data set 2: (a) VMI & CS (b) Centralized

We summarize the results of the proposed model for Data set 3 in Table IV-11 and illustrated in Figure IV-10. The cost savings for the VMI & CS range from 0% to 5.8%, and the cost savings decreases as the A^{vs}/A_i^b ratio increases. We obtain no better cost savings than Ben-Daya, et al. (2013) for Centralized scenario.

Table IV-11 Effect of changing A^{vs}/A_i^b on Data set 3

A^{vs}/A_i^b	**Proposed Model**							
	VMI & CS				**Centralized**			
	ATVC	**ATBC**	**ATC**	**%Sav**	**ATVC**	**ATBC**	**ATC**	**%Sav**
0.75	1118.65	684.402	1803.06	5.83%	1135.89	645.144	1781.03	0.00%
1	1221.44	718.346	1939.78	3.30%	640.253	1263.69	1903.943	0.00%
1.25	1319.48	718.346	2037.82	2.68%	1341.41	678.125	2019.53	0.00%
1.5	1404.71	771.423	2176.13	0.10%	1437.81	690.864	2128.67	0.00%

Notes: ATVC, Average Total Vendor Costs; ATBC, Average Total Buyer Costs; ATC, Average Total Costs; %Sav, % saving.

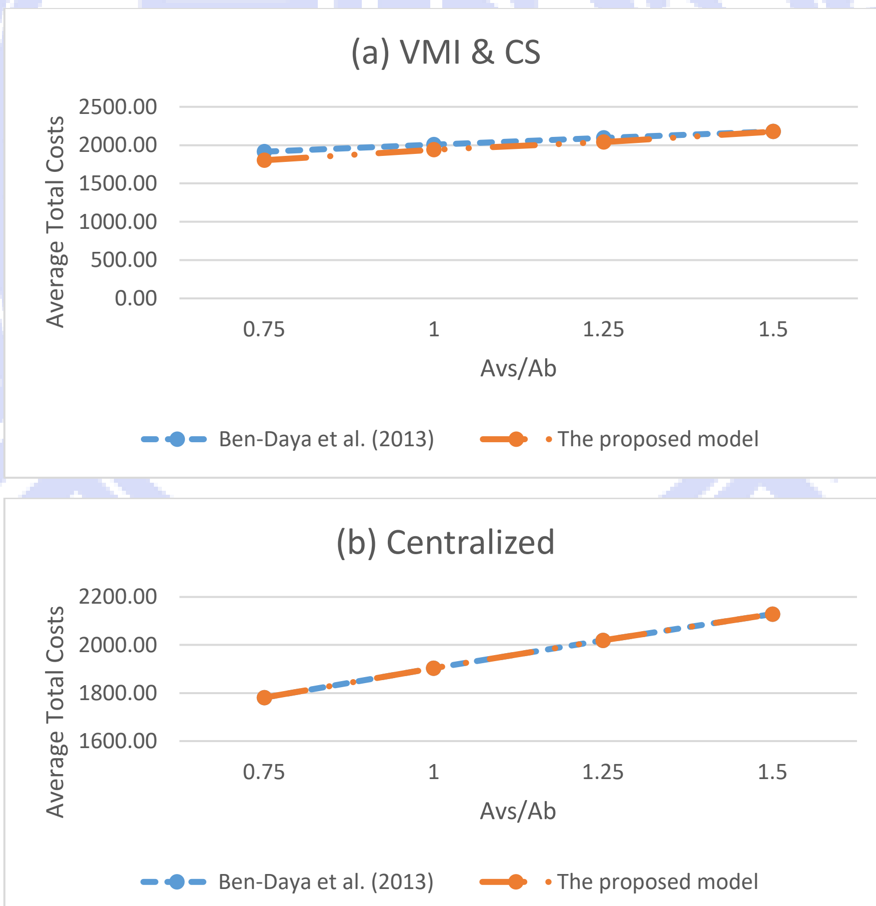


Figure IV-10 The sensitivity analysis of A^{vs}/A_i^b using Data set 3: (a) VMI & CS (b) Centralized

IV.3.3 Setup Time *S*

Recall that we introduce setup time in our proposed model, but Ben-Daya, et al. (2013) did not include it in their model. A setup time will lead the production duration for all the scheduled production lots in a scheduled basic period in our proposed model. On the other hand, a setup time lead all the production lots included in the vendor's replenishment cycle *(T)* in the models of Ben-Daya, et al. (2013).

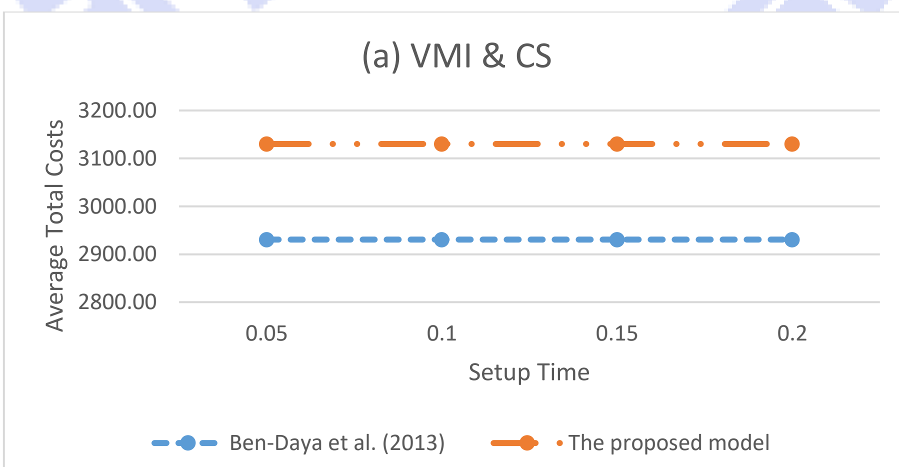
The average total costs may increase only when setup time becomes larger, and they are not even affected by the settings of setup time in both scenarios.

Table IV-12 summarizes the results of the proposed model for Data set 1, and we also illustrate the results in Figure IV-11. The proposed model obtain no results better than Ben-Daya, et al. (2013) for both VMI & CS and Centralized scenarios.

Table IV-12 The sensitivity analysis of setup time using Data set 1

S	Proposed Model							
	VMI & CS				Centralized			
	ATVC	ATBC	ATC	%Sav	ATVC	ATBC	ATC	%Sav
0.05	2115.57	1014.545	3130.11	-6.81%	1469.93	1540.53	3010.46	-7.60%
0.10	2115.57	1014.545	3130.11	-6.81%	1469.93	1540.53	3010.46	-7.60%
0.15	2115.57	1014.545	3130.11	-6.81%	1469.93	1540.53	3010.46	-7.60%
0.20	2115.57	1014.545	3130.11	-6.81%	1349.51	1688.16	3037.67	-8.57%

Notes: ATVC, Average Total Vendor Costs; ATBC, Average Total Buyer Costs; ATC, Average Total Costs; %Sav, % saving.



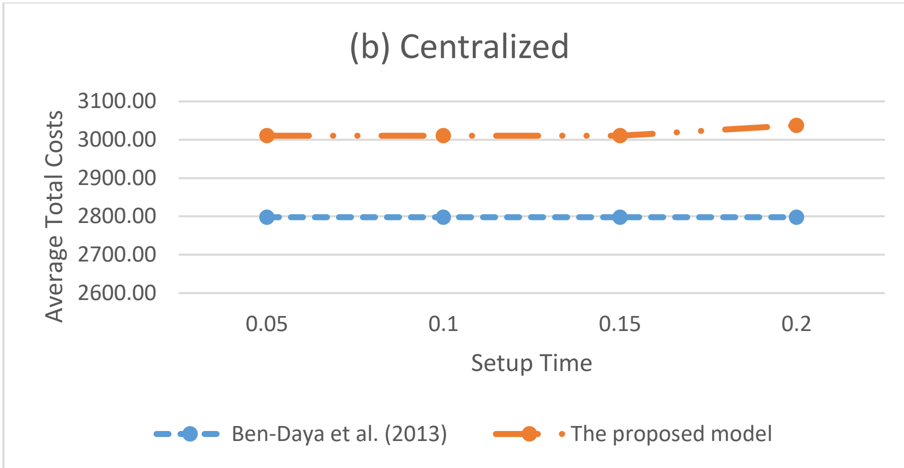


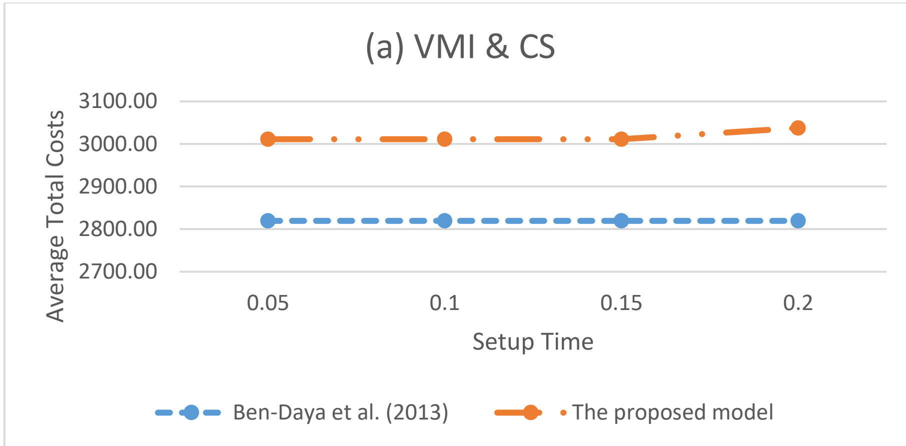
Figure IV-11 The sensitivity analysis of setup time using Data set 1: (a) VMI & CS (b) Centralized

We summarize the results of proposed model for Data set 2 in Table IV-13 and illustrated in Figure IV-12. As we did in Table IV-12 for Data set 1, the results from Ben-Daya, et al. (2013) are better than the proposed model's.

Table IV-13 The sensitivity analysis of setup time using Data set 2

S	Proposed Model							
	VMI & CS				Centralized			
	ATVC	ATBC	ATC	%Sav	ATVC	ATBC	ATC	%Sav
0.05	2839.6	171.6179	3011.21	-6.80%	1469.934	1540.53	3010.464	-7.60%
0.10	2839.6	171.6179	3011.21	-6.80%	1469.934	1540.53	3010.464	-7.60%
0.15	2839.6	171.6179	3011.21	-6.80%	1469.934	1540.53	3010.464	-7.60%
0.20	2855.69	181.9739	3037.66	-7.74%	1349.507	1688.158	3037.664	-8.57%

Notes: ATVC, Average Total Vendor Costs; ATBC, Average Total Buyer Costs; ATC, Average Total Costs; %Sav, % saving.



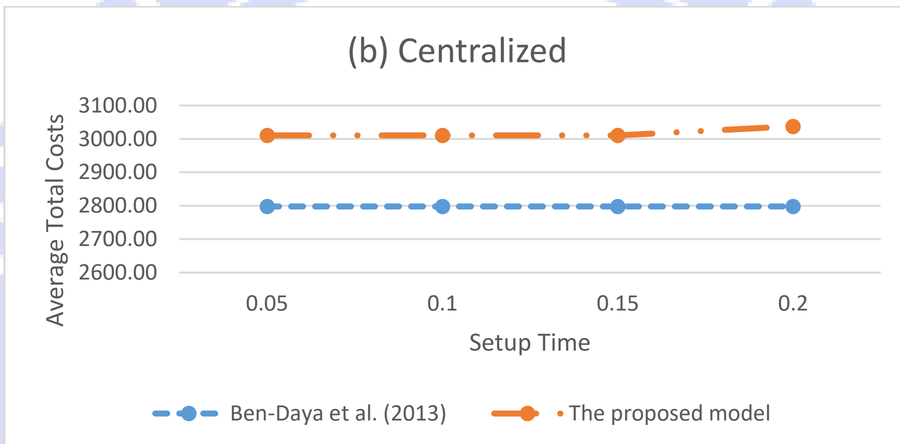


Figure IV-12 The sensitivity analysis of setup time using Data set 2: (a) VMI & CS (b) Centralized

The results of proposed model for data set 3 are summarized in Table IV-14, and illustrated in Figure IV-13. Our proposed models are not able to get the better than those from Ben-Daya, et al. (2013) which obtain cost-saving by 4.49% and 7.6% in VMI & CS and Centralized scenarios, respectively.

Table IV-14 The sensitivity analysis of setup time using Data set 3

S	Proposed Model							
	VMI & CS				Centralized			
	ATVC	ATBC	ATC	%Sav	ATVC	ATBC	ATC	%Sav
0.05	2092.7	1027.824	3120.52	-4.49%	1469.93	1540.53	3010.46	-7.60%
0.10	2092.7	1027.824	3120.52	-4.49%	1469.93	1540.53	3010.46	-7.60%
0.15	2092.7	1027.824	3120.52	-4.49%	1469.93	1540.53	3010.46	-7.60%
0.20	2092.7	1027.824	3120.52	-4.49%	1349.51	1688.16	3037.66	-8.57%

Notes: ATVC, Average Total Vendor Costs; ATBC, Average Total Buyer Costs; ATC, Average Total Costs; %Sav, % saving.

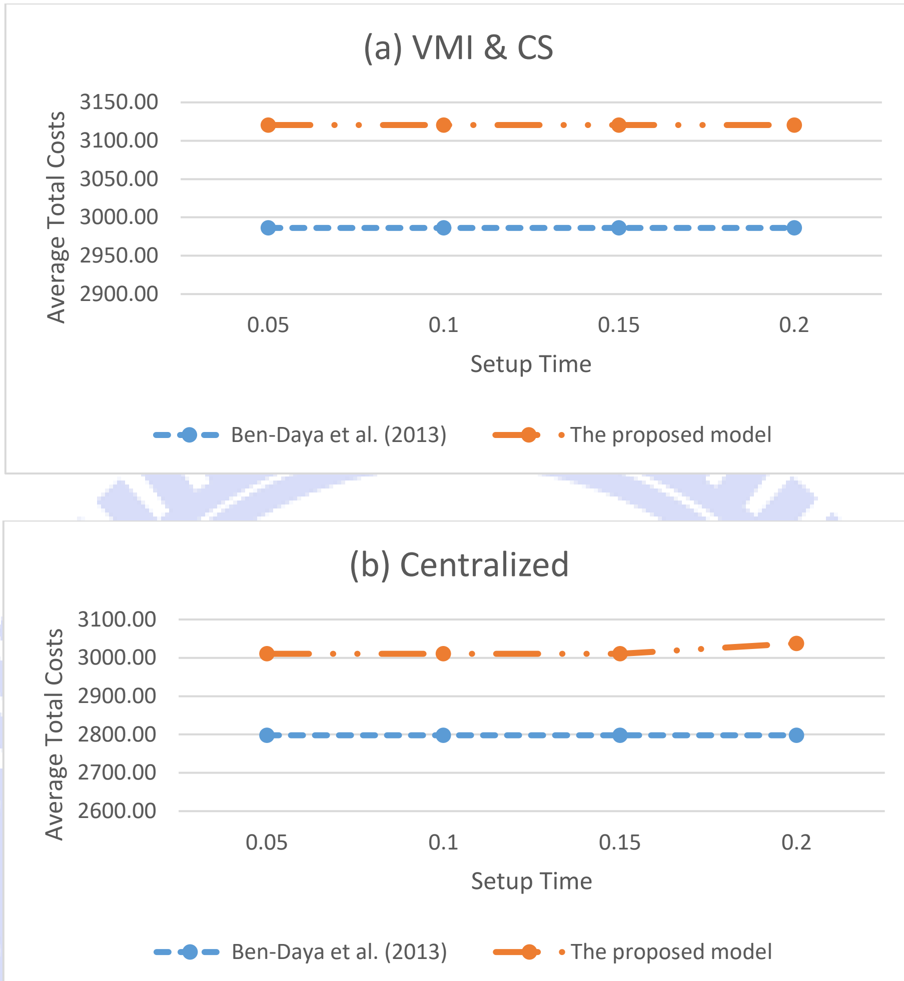


Figure IV-13 Effect of setup time on Data set 3. (a) VMI & CS (b) Centralized

IV.3.4 Number of Buyers

There are only two buyers in the three data sets of Ben-Daya, et al. (2013). However, a vendor may have to face larger number of buyers in the real world. We would like to compare the results of our proposed model with those from Ben-Daya, et al. (2013) when the number of buyers increases from 2 to 5. For new added buyers, we simply duplicate the values of demand rate, ordering cost, and holding costs as those presented in IV.2.1. We set both the vendor's shipment release costs to i^{th} buyer and its setup time as zero.

We solve Data set 1 and summarize the results of the proposed model in Table IV-15 (and also illustrated in Figure IV-14) after adding different number of buyers. Table IV-15 shows that the percentage of cost-savings (of our proposed model) is in the range from 0% to 8.6% for the VMI & CS scenario and flat on 0% for the Centralized scenario.

Table IV-15 The sensitivity analysis of number of buyers using Data set 1

N	Proposed Model							
	VMI & CS				Centralized			
	ATVC	ATBC	ATC	%Sav	ATVC	ATBC	ATC	%Sav
2	1867.5	1132.5	3000	3.29%	1215.51	1612.94	2828.45	0.00%
3	2212.7	1299.699	3512.4	8.61%	1364.34	1971.83	3336.17	0.00%
4	2819.58	1780.418	4600	0.08%	1592.05	2789.77	4381.82	0.00%
5	3105.89	1934.821	5040.71	3.94%	1681.46	3140.38	4821.84	0.00%

Notes: ATVC, Average Total Vendor Costs; ATBC, Average Total Buyer Costs; ATC, Average Total Costs; %Sav, % saving.

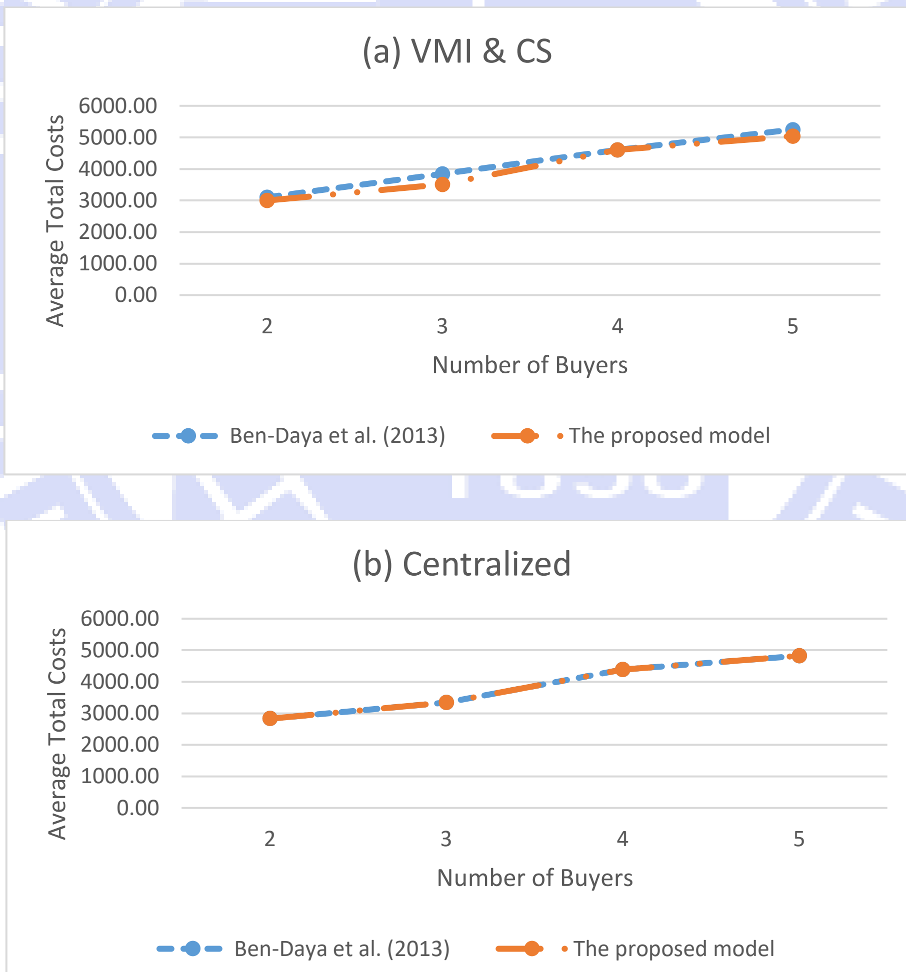


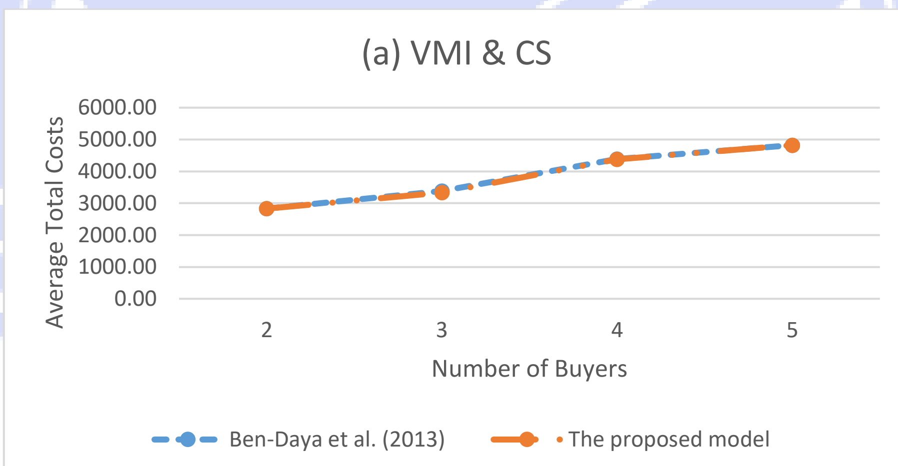
Figure IV-14 The sensitivity analysis of number of buyers using Data set 1: (a) VMI & CS (b) Centralized

The results of proposed model for different number of buyers using Data set 2 are summarized in Table IV-16 and illustrated in Figure IV-15. We may observe that the percentage of cost-saving are less sensitive for both VMI & CS and Centralized scenarios in Table IV-16. The proposed model obtain no results better than Ben-Daya, et al. (2013) for both VMI & CS and Centralized scenarios using Data set 2.

Table IV-16 The sensitivity analysis of number of buyers using Data set 2

N	Proposed Model							
	VMI & CS				Centralized			
	ATVC	ATBC	ATC	%Sav	ATVC	ATBC	ATC	%Sav
2	2651.89	177.9706	2829.86	0.01%	1215.51	1612.94	2828.45	0.00%
3	3113.85	224.0382	3337.88	-0.01%	1364.34	1971.83	3336.17	0.00%
4	4065.71	317.1422	4382.86	0.00%	1592.05	2789.77	4381.82	0.00%
5	4459.04	363.9507	4822.99	0.00%	1681.46	3140.38	4821.85	0.00%

Notes: ATVC, Average Total Vendor Costs; ATBC, Average Total Buyer Costs; ATC, Average Total Costs; %Sav, % saving.



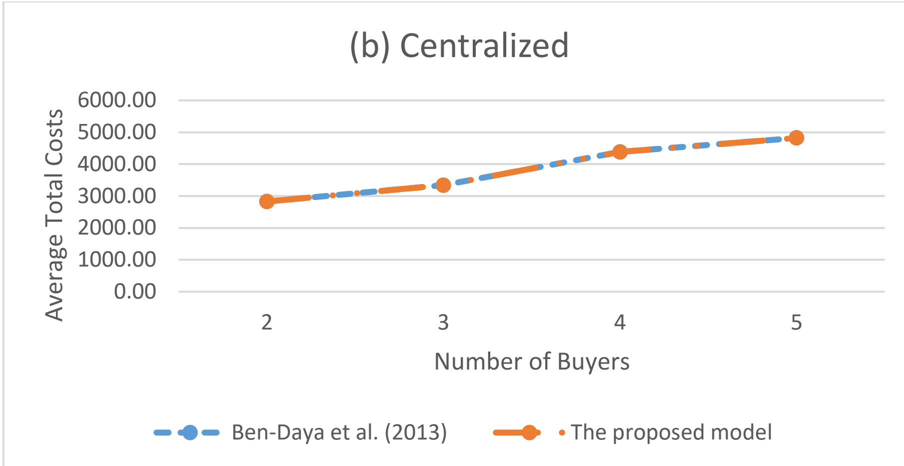


Figure IV-15 The sensitivity analysis of number of buyers using Data set 2: (a) VMI & CS (b) Centralized

We summarize the results of proposed models for different number of buyers using Data set 3 in Table IV-17 and illustrated in Figure IV-16. Similar with the results in Table IV-15, we observe the cost-saving for the VMI & CS scenario is around 3%, but no cost-saving for the Centralized scenario.

Table IV-17 The sensitivity analysis of number of buyers using Data set 3

N	Proposed Model							
	VMI & CS				Centralized			
	ATVC	ATBC	ATC	%Sav	ATVC	ATBC	ATC	%Sav
2	1847.32	1133.087	2980.41	3.34%	1215.51	1612.94	2828.45	0.00%
3	2189.52	1310.407	3499.93	3.36%	1364.34	1971.83	3336.17	0.00%
4	2765.88	1815.744	4581.62	3.93%	1592.05	2789.77	4381.82	0.00%
5	3048.36	1971.731	5020.09	3.59%	1681.46	3140.38	4821.85	0.00%

Notes: ATVC, Average Total Vendor Costs; ATBC, Average Total Buyer Costs; ATC, Average Total Costs; %Sav, % saving.

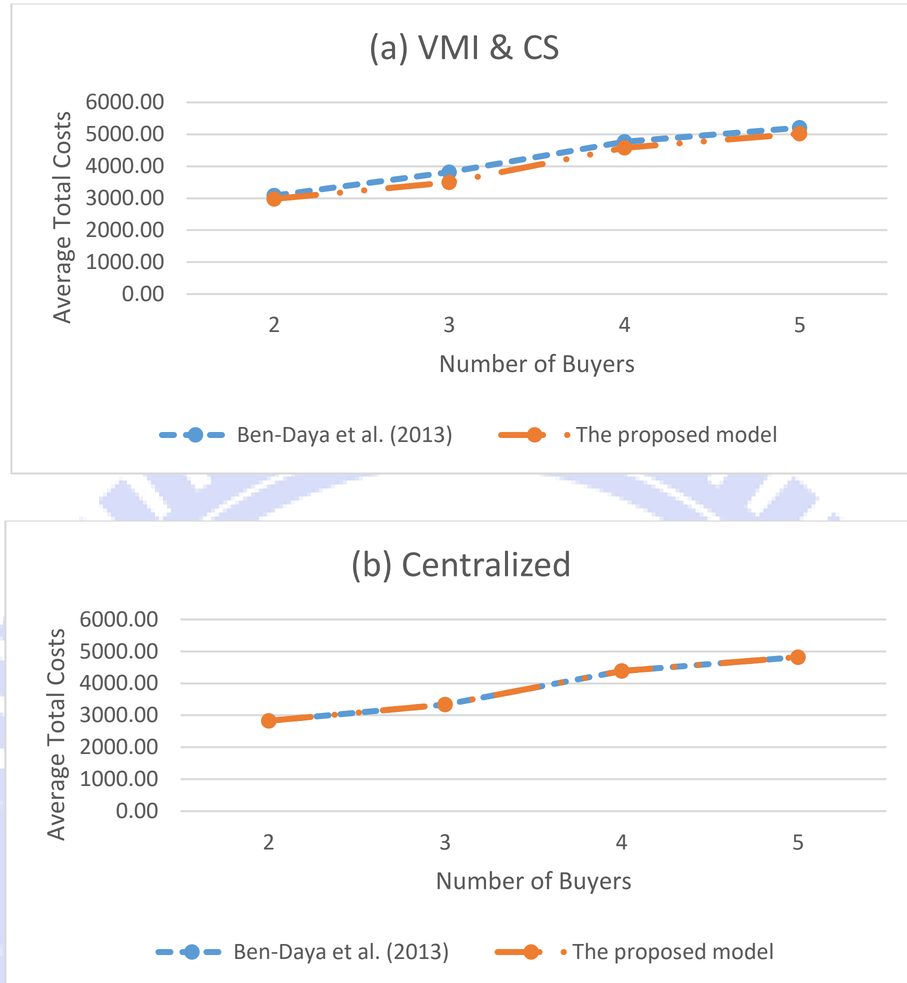


Figure IV-16 The sensitivity analysis of number of buyers using Data set 3: (a) VMI & CS (b) Centralized

CHAPTER V Conclusions and Future Works

V.1 Conclusions

This study proposes new mathematical models for VMI & CS and Centralized scenarios in single-vendor multiple-buyers supply chain by referring to Ben-Daya, et al. (2013). We formulate mixed binary integer programming models under Power of Two Policy (PoT) policy for both scenarios, and the decision variables are the basic period, the replenishment (integer) multiplier and the first replenishment basic period in the production schedule of each buyer. Note that the models in Ben-Daya, et al. (2013) did not take into account setup time leading the production lots for all the buyers in each replenishment cycle. Interestingly, following our numerical experiments, we observe that the cost savings could become significant (up to more than 40%) when the setup time is not significant (say, the setting of setup time is 0.05 and 0.10) for Centralized scenario.

V.2 Future Works

The interested researcher may extend this study by changing the features in the model formulation and/or propose more efficient solution approaches for solving the proposed models. The possible extension could pursue the following issues:

1. We may propose new solution approaches by investigating the theoretical properties of the objective functions for both VMI & CS and Centralized scenarios.
2. We may relax the restrictions of Power-of-Two policy by allowing the replenishment (integer) multipliers to be general (positive) integers.

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Appendix 1

a. Objective Function

1. VMI & CS Scenario

Minimize Average Total Vendor Cost

$$\sum_{i=1}^{N}\sum_{l=0}^{V_i}\sum_{j=1}^{l}\frac{D_i b l w_{ilj}}{2}\left(\frac{D_i}{P}H^v + H_i^{bo}\right) + \sum_{i=1}^{N}\sum_{l=0}^{V_i}\sum_{j=1}^{l}\frac{l^{-1}w_{ilj}}{b}\left(A_i^{vr} + A_i^{bp}\right) + \sum_{j=1}^{l} m_j\frac{A^{vs}}{lb}$$

Minimize Average Total Buyer Cost

$$\sum_{i=1}^{N}\sum_{l=0}^{V_i}\sum_{j=1}^{l}\frac{H_i^{bs}}{2}D_i l w_{ilj} b + \sum_{i=1}^{N}\sum_{l=0}^{V_i}\sum_{j=1}^{l}\frac{A_i^{br}}{b}l^{-1}w_{ilj}$$

2. Centralized Scenario

Minimize Average Total Cost

$$TC^2 = TC_v^2 + \sum_{i=1}^{N} TC_{bi}^2$$

$$TC^2 = \frac{H^v}{2P}\sum_{i=1}^{N}\sum_{l=0}^{V_i}\sum_{j=1}^{l} D_i^2 l w_{ilj} b + \sum_{i=1}^{N}\sum_{l=0}^{V_i}\sum_{j=1}^{l}\frac{A_i^{vr}}{b}l^{-1}w_{ilj} + \sum_{j=1}^{l} m_j\frac{A^{vs}}{lb}$$
$$+ \sum_{i=1}^{N}\sum_{l=0}^{V_i}\sum_{j=1}^{l}\frac{H_i^{b}}{2}D_i l w_{ilj} b + \sum_{i=1}^{N}\sum_{l=0}^{V_i}\sum_{j=1}^{l}\frac{A_i^{b}}{b}l^{-1}w_{ilj}$$

b. Constraints

The Constraints for the Scheduling of the Vendor's Setup

$$\sum_{i=1}^{N}\sum_{l=0}^{V_i} w_{ilj} \le m_j M, for\ j = 1,\ \dots,\ lcm(k_i)$$

The Capacity Constraints

$$m_j S + \sum_{i=1}^{N} \sum_{l=1}^{V_i} w_{il\varphi(l,j)} \frac{D_i lb}{P} \leq b, for\ j = 1,\ \ldots,\ lcm(k_i)$$

$$\varphi(\mathrm{l}, \mathrm{j}) = \begin{pmatrix} \mathrm{j\ mod\ l} & \mathrm{if\ j} \neq \gamma\mathrm{l},\ \gamma\epsilon\mathrm{N} \\ \mathrm{l} & \mathrm{if\ j} = \gamma\mathrm{l},\ \gamma\epsilon\mathrm{N} \end{pmatrix}$$

The Constraints for Production Scheduling

$$\sum_{l=0}^{V_i} \sum_{j=1}^{l} w_{ilj} = 1, for\ i = 1,\ \ldots,\ N$$